The Taxpayer Burden of Bank Restructuring

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ABSTRACT

We formalize the taxpayer burden implied by various bank restructuring plans. Even assuming minimal frictions, in spirit of Modigliani and Miller (1958), when debt contracts cannot be changed, transfers from the taxpayer (in a Net Present Value sense) are necessary. Debt holders benefit from a lower default probability and a higher recovery given default. Absent government transfers, their gains imply a decrease in equity value. Shareholders will therefore oppose the restructuring unless they receive transfers from taxpayers. The taxpayer burden consists of the NPV of inflows and outflows of cash needed to persuade shareholders (or bank managers) to accept a change in capital structure. The government’s intervention aims at preventing systemic effects from a default of an important bank, and thus targets a default rate. Due to different implied recovery rates given default, the required transfer amounts vary across restructuring plans that achieve the same target default rate. In this regard, asset sales requires more transfers than recapitalization or asset guarantees, because asset sales support a higher recovery rate.
I. INTRODUCTION

What is the best policy option for rescuing a troubled systemically important bank? Various plans have been implemented around the world in the wake of 2008 financial turmoil. Examples include recapitalizations, asset purchases, and asset guarantees.

We rank these policy options from the viewpoint of taxpayers’ cost, primarily based on Modigliani and Miller (1958) framework in corporate finance and a compensation principle typically discussed in public economics (e.g., Laffont, 1988). Although in principle restructuring can be done without taxpayer contributions through debt-for-equity swap, in reality such restructuring is difficult and rare. When debt contracts cannot be renegotiated, any restructuring that lowers the default probability of a bank will increase value of debt. This increase, given the same asset values, implies the reduction of value of equity (Modigliani-Miller theorem). To obtain shareholder approval for the restructuring, the government needs to transfer some cash to the bank that shareholders become at least indifferent about the restructuring (compensation principle of a public program). We measure this type of public transfers to restore a systemic bank’s solvency to a given target for each policy option, and compare the size of the transfers. For example, unless assets are priced well below their fundamental values, it is easy to show that an asset sales program is likely to be more costly for taxpayers than other forms of restructuring (e.g., recapitalization).

Our results may be more applicable after market prices stop free-falling and after independent audits or stress tests reveal more of the true asset quality of banks, as we explore our theory in an economy with minimal frictions. However, to the best of our knowledge, the basic reasoning that we find in this paper is novel. In various recommendations given by many authors, most focus on the fiscal outlay from the government: how much money is needed to buy toxic assets v.s. to recapitalize a bank to lower the default risk (to presumably the same level). In this typical argument, missing is the value of assets or equity that government purchases. If the transaction is NPV equivalent, all the fiscal outlay is backed by the same NPV financial assets so that there is no net fiscal expenditure. If there is net fiscal expenditure, it must be the case that there is explicit or hidden subsidies, for example, in the form of inflated price to buy toxic assets from a bank or equities of a bank. What we argue is that this (hidden) subsidy is the true taxpayers’ cost, and that, in the best-possible public intervention, the size of subsidy should be set to make losing stakeholders of a bank (typically shareholders) to be indifferent by a policy intervention.

We believe this reasoning carries over any environment with more frictions. Indeed, the various restructuring options under other frictions (e.g., moral hazard and adverse selection) are evaluated and compared with each other in a review paper by Landier and Ueda (2009). They find that there is no magic bullet—a combination of tools is likely to be optimal with the exact mix depending on the balance of different frictions—but also find that our reasoning in minimal friction environment carries over in an economy with frictions. We thus focus on the environment with minimal frictions in this paper.
As we discuss the various recommendations given by many columns and Op-Eds, serious academic work like this paper is rare. Exceptions include Kocherlakota (2008), Philippon and Schnable (2009), and Uhlig (2009). Kocherlakota (2009) argues that, given de facto guarantees by the U.S. government at the end of 2008, additional scheme should lower the debt-overhang problem by buying toxic assets. Philippon and Schnabl (2009) show that recapitalization seems the best given the private information problem in asset quality (lemon’s problem) combined with debt-overhang problem. Uhlig (2009) argues that there seems mispricing in the market stemming from uncertainty, rather than lemon’s problem, and thus the scheme that reduce uncertainty is the best.

II. Setup and Objective

The goal of restructuring is assumed to be a lower probability of a bank’s default with a minimal taxpayer burden. We analyze the restructuring of a bank in a simple framework in the spirit of Modigliani and Miller (1958).

A. Setup

A bank manages an asset $A$ currently (time 0), which will have a final value $A_1$ next period (time 1). The final value $A_1$ is stochastic. It is drawn from a cumulative distribution function (CDF), $F$. The capital structure at time 0 is debt with face value $D$, which needs to be repaid at time 1. Equity has book value $E$ (see Figure 1a). Absent restructuring, the probability of default of the bank at time 1, $p$, is the probability that the next-period value $A_1$ will be less than the debt obligation $D$, that is, $p = F(D)$ (see Figure 1b).

The assumptions of Modigliani-Miller are complete and efficient markets, without any information frictions. Under these assumptions, the sum of the market values of debt and equity is independent of the bank’s capital structure and equals the market value of the asset: $V(A) = V(E) + V(D)$ (see Figure 1c). We also assume $D < V(A)$, implying that the bank is not currently insolvent, but we do assume a positive default probability. The market value of debt $V(D)$ is thus smaller than the book value $D$.

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2 A more practical definition of insolvency is regulatory insolvency. In this case, certain positive equity is required in order to be solvent, that is, a bank is solvent if the book value of assets is large enough ($A > D + \text{required capital}$). However, the thrust of the analysis would not change, and thus a simple condition of solvency, $V(A) > D$, is used throughout this note.
B. Objective

Assuming a large social cost associated with default of a systemically important bank, the government’s objective is to lower the default probability or, in practice, achieving a target default probability \( p^* = F(A^*) \). A bank restructuring problem amounts then to finding a way to achieve \( p = p^* \) starting from a higher default probability, \( p > p^* \).

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\(^3\) \( A^* = F^{-1}(p^*) \) is the marginal threshold of the realization of \( A_1 \) to achieve the target default probability. Put differently, if the debt is restructured to have face value \( A^* \), then the default probability will be \( p^* \). Note that the social costs associated with default are assumed not to be sensitive to the recovery rate of debt in the event of bankruptcy.
Each restructuring plan provides different recovery given default. However, recovery given default is a pure transfer to debt holders and the government should minimize that. Thus, we essentially rank restructuring plans by looking at the recovery given default.

For each restructuring plan, we identify the least costly plan. Formally, let the recovery rate, $R(y)$, an increasing function of the next period asset level $y$. Then, the objective of the government is to choose the recovery rate function $R(y)$ to minimize the taxpayer burden while achieving the target level of default, that is,

$$\min \int_0^\infty R(y)F(dy), \text{ subject to } R(A^*) = D. \quad (1)$$

We compare various recovery rate functions depending on the restructuring plans with the original, before-restructuring recovery function

$$R_o(y) = y, \text{ for } y \in [0, D],$$
$$= D, \text{ for } y \in (D, \infty). \quad (2)$$

Based on this, the before-restructuring value of debt can be expressed as

$$V_o(D) = \int_0^\infty R_o(y)F(dy). \quad (3)$$

We can think of the future cash flows of a bank as a *pie* shared between shareholders and debt holders. Indeed, the value of equity is just the value of residual claim after subtracting the value of debt (*conservation of value*). Formally, let the value of asset be

$$V_o(A) = \int_0^\infty yF(dy), \quad (4)$$

then, the value of equity is

$$V_o(E) = V_o(A) - V_o(D). \quad (5)$$

We assume here that the asset value is constant before and after the restructuring. This is because we would like to focus on the basic mechanism that we propose. In more general setting, although we do not explore the implication in this paper, restructuring may increase the size of the *pie*. The review paper by Landier and Ueda (2009) include detailed discussions, summarized as follows. The restructuring must bring social benefits; otherwise, there is no justification for the government involvement. Both private surplus and social surplus can be endogenous and function of design of restructuring plan. Among the various schemes that achieve the target default probability $p = p^*$, those that maximize the restructuring surplus are the most efficient—they will minimize the transfer required from taxpayers—and thus should be pursued.
Specifically, the government should pay attention to the various stakeholders’ payoffs and incentives. For example, decreasing the probability of default might attract customers who were previously worried about the bank’s high probability of failure. This potential private surplus from lowering the cost of financial distress can facilitate restructuring and reduce or may even eliminate the need for transfers from taxpayers.

III. FIRST BEST—VOLUNTARY DEBT RESTRUCTURING

The government’s objective is to decrease the probability of default $p$ to the target level while making no one financially worse off. We show that the bank can decrease its probability of default to any target level by converting some debt into equity. A restructuring can be carried out in such a way that both equity holders and debt holders are not financially worse off.

This is feasible by a change in the structure of claims, namely, the partial transformation of debt into equity. More specifically, a restructuring that leaves both debt and equity holders indifferent is the conversion of debt $D$ into a combination of lower-face-value debt ($D' = A^*$). This is a (partial) debt-for-equity swap.

Proposition 1

Debt-for-equity swap is the first best solution that can achieve any target level of default probability without government involvement.

Proof

The additional piece of equity has a value of $V_0(D) - V_0(D')$. The new financial stake of the initial debt holders is thus worth $V_0(D') + (V_0(D) - V_0(D'))$, which is by design unchanged from the original market value of debt $V_0(D)$. The firm’s future cash flows are unchanged, and only the sharing rule for these cash flows has changed, so that the total value of the firm is unchanged (following the Modigliani-Miller theorem). Because the value of the claims that belong to the initial debt holders is unchanged, the value of the equity of the initial shareholders remains the same as well.

QED

Because a debt-for-equity swap is the first best, we take the value of debt under this scheme to be the benchmark. Let $R_{opt}(y)$ denote the recovery function under the debt-for-equity swap, the value of debt is formally expressed as:

$$V_{opt}(D) = \int_0^\infty R_{opt}(y)F(dy),$$

where

(2)
Figure 2 illustrates the change in the liability structure induced by this partial debt-for-equity swap that makes the probability of default equal to $p^*$. The total payment promised to debt holders decreases from $D$ to $A^*$. This is illustrated by the downward shift of the horizontal line for debt payoff in Figure 2. After the restructuring, a fraction of the equity is held by the initial debt holders to compensate them for the decrease in the value of debt. Thus, when the bank does not default, equity accounts for a larger fraction of the asset’s payoffs. Graphically, the equity line shifts up. The full conversion of debt into equity against a fraction of equity would also be a solution to the restructuring problem. Either scheme can be implemented by means of a debt-for-equity swap.4

III. RESTRUCTURING WITH NO DEBT RENEGOTIATION

Although the proposed debt-for-equity swap is the first-best solution, it is often a difficult solution to implement in practice. A major reason is the speed of events, which leaves no time for negotiation. The possibility of a deposit run calls for speedy resolution, while dispersion of bank debt holders requires a lengthy negotiation process. An orderly bankruptcy might be the

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4 This scheme is possible only when debt holders and equity holders negotiate freely and reach agreement easily. In practice, this is difficult outside a bankruptcy regime. Zingales (2009) advocates this solution by changing the bankruptcy law for banks. Note that in this truly frictionless framework, it is sufficient to prevent default with an ex post debt-for-equity swap that triggers when the realized asset value is less than the debt obligation, $A_1 < D$. In other words, no ex-ante restructuring is needed.
most efficient way to structure the renegotiation process, but might negatively impact other systemically important institutions. In what follows, we assume that the government wants to avoid such a bankruptcy procedure because of the potential systemic costs.

We examine in this section various possible restructuring options that do not involve renegotiation of the debt contracts. We also assume that transactions with external parties other than the government are carried out at a fair price (i.e., reflecting expected discounted cash flows) and that markets are efficient. This means that, for these external parties, financial transactions must be zero net present value (NPV) projects.

With no renegotiation of debt contracts and no help from the government, a restructuring that reduces the probability of default increases the value of the debt and thus decreases the value of the equity. Therefore, it will be opposed by shareholders. A restructuring thus will not happen unless the government provides subsidies in some form or makes participation compulsory.

Many schemes, though not all, are equivalent in the required government transfers. The reason is that some imply a higher recovery rate for given default than others. Asset sales, for example, are more expensive than subsidizing the issuance of common equity. The second-best scheme is a form of partial insurance on the assets’ payoff. Changing the liability side by subsidized debt buyback is an option close to the second-best scheme.

A. Difficulty of Voluntary Restructuring

Without debt renegotiation and in the absence of transfers from the government, all restructuring that lowers the default probability \( p \) would be opposed by equity holders. This is because such restructuring increases the value of debt at the expense of equity (the debt overhang problem; see Myers, 1977). Indeed, debt holders are better off in every possible scenario—the default probability of a bank becomes lower and the recovery rate given default becomes higher. The value of debt thus increases from \( V_0(D) \) to \( V_R(D) \) and, without third-party involvement, the increase in debt value is precisely compensated by a decrease in equity value, \( V_R(E) - V_0(E) = - ( V_R(D) - V_0(D) ) < 0 \). The worse the bank’s default probability is initially, the larger \( V_0(D) - V_R(D) \) and the larger the loss imposed on shareholders. Shareholders of more distressed banks thus tend to be more reluctant to restructure.

Shareholders need to be either forced or induced through subsidies in some way by the government to approve such restructuring. Their approval is needed, because they have control rights as long as the bank does not default. The transfer needed from the government is equal to the increase in the value of debt, \( T = V_R(D) - V_0(D) \). This transfer equals the expected discounted value of immediate and future subsidies from the government. Under this transfer, the value of equity remains unchanged. We now examine in detail how this transfer varies across different restructuring schemes.
B. Preview of Comparison among Typical Plans

All restructuring schemes that achieve a target default probability $p^*$ must involve transfers from the government. The size of the transfers determines the degree of the debt’s safety. From this perspective, among the schemes we will examine, asset sales appear to be the most costly for taxpayers. This is because whatever the final realization of $A$, asset sales imply the largest increase in debt recovery and therefore the largest transfer to debt holders. Figure 3 gives a preview of our results, illustrating the recovery schedule of debt for various realizations of $A$ and various types of restructuring. Restructuring shifts the default threshold to the left (from $D$ to $A^*$) and changes the payoff to the debt holders in case of default $D<A^*$. This new recovery schedule can vary depending on the restructuring plan (three different slopes in Figure 3). Restructuring that creates higher recovery schedules is more costly to taxpayers, since it (indirectly) transfers more value to debt holders. The transfers are equivalent between recapitalization with security issues and asset guarantees with a cap cost. While debt buybacks turn out to cost less, asset sales cost more than recapitalization.

C. Second Best (Constrained Optimal): State-Contingent Insurance

We first describe the restructuring scheme that minimizes the transfer from taxpayers. The size of the transfer can be expressed graphically as a function of the asset’s realization $A_1$ (Figure 4a). Figure 4b shows the corresponding debt recovery. Because the objective is to decrease the probability of default, there is no need to improve the recovery of debt in case of default.

Proposition 2
Second best (constrained optimal) scheme takes the form of state-contingent insurance. Specifically, the recovery function is given by

\[ R_{\text{con}}(y) = y, \text{ for } y \in [0, A^*], \]
\[ = D, \text{ for } y \in [A^*, D]. \]

\[(3)\]

Proof

In the minimization problem (1), the government wants to set the recovery function equal to the realized asset value, \( y \), as in the first-best optimal recovery function \( R_{\text{opt}}(y) \) for every realization of the next period asset values. However, for the realization values between \( A^* \) and \( D \), the no-debt-renegotiation constraint \( R(y) = D \) binds.

\[ QED \]

Graphically, only difference of this recovery function from the first best \( R_{\text{opt}}(y) \) is that the full face value of debt \( D \) is honored when the bank does not default (see Figure 4a). Default occurs in the left part of the figure, \( y < A^* \). In this region, as in the debt-for-equity swap case, there is no change in debt value before the restructuring: the government makes no transfer (see Figure 4b). When the realized asset value \( y \) is between \( A^* \) and \( D \), the bank needs a transfer \( D - y \) from the government so that it is able to repay \( D \) to debt holders and avoid default. When the realized \( y \) is above \( D \), no subsidy is needed to avoid default (again, see Figure 4b). In other words, the optimal restructuring is a guarantee under which the government transfers money ex post only when the bank is in default but not far from solvency. This scheme would not provide any transfer to debt holders when default is inevitable \( (y < A^*) \) or when the bank can repay debt on its own \( (y > D) \).

The relative cost to taxpayers of various types of restructuring depends on how close they are to implementing this debt-recovery schedule. This scheme might be difficult to implement and calibrate in practice, but it provides three useful insights. First, to decrease the probability of default, the government does not have to subsidize the recovery rate for all the realized value of the assets. It should instead focus on avoiding default only when the bank is close to solvency. Second, it is not necessarily a bad deal that the taxpayers do not receive any upside or even any positive cash flow in exchange for their intervention. Some of the rescue schemes we will examine below occasionally provide payments to taxpayers. This optimal scheme never provides any payments to taxpayers, but its overall cost to taxpayers is the lowest. Third, more transfers could boost the share price, but a higher share price does not mean a good rescue plan from the point of view of taxpayers.

\[ \{ \text{ TC "Figure 4a. Transfer of the Optimal Subsidy} \quad \text{Figure 4b. Recovery Rate"} \} \]

\[ \{ \text{ TC "Figure 4a. Transfer of the Optimal Subsidy} \quad \text{Figure 4b. Recovery} \}

\[ ^5 \] Here, we assume that the social benefits from saving a systemically important bank are limited, and thus the government will not transfer funds beyond the upper limit \( D - A^* \). Apparently, if there is a need to transfer money to counterparties in case of default, a subsidy that gives higher debt recovery given default \( A < A^* \) may be optimal.
D. Recapitalization with Common Equity

One straightforward way of decreasing the default probability is to issue new equity and keep the proceeds as cash. This makes the debt less risky.

**Proposition 3**

The recovery function under recapitalization that achieves the target default probability $p^*$ is expressed as

$$R_{rec}(y) = \begin{cases} y + D - A^*, & \text{for } y \in [0, A^*] \\ D, & \text{for } y \in [A^*, D]. \end{cases} \quad (4)$$

Accordingly, the value of debt is

$$V_{rec}(D) = \int_0^\infty R_{rec}(y) F(dy).$$

**Proof**

Bankruptcy occurs with $prob(A + Cash < D)$, equivalently, $prob(A < D - Cash)$ or $F(D - Cash)$. The minimum amount of cash that has to be raised is such that $p^* = F(D - Cash)$, that is, $Cash = D - A^*$. Even if the next period value $y$ turns out to be zero, $Cash$ can be recovered. Thus, it is the intercept of the debt recovery schedule in Figure 5. For one more dollar realization of asset value $y=A_1$ in default region ($y < A^*$), the debt holders can recover exactly one more dollar in addition to $Cash$. Thus, the slope coefficient on the next-period value $y=A_1$ is one, the same as in the before-restructuring case.

$QED$
Because default occurs less often and the recovery rate is higher, the value of debt increases from $V_0(D)$ to $V_{\text{rec}}(D)$. The new equity holders do not make or lose money by investing in the newly issued equity, as the transaction is assumed to preserve the same NPV value. Assuming no government subsidy, the gain of debt holders $V_{\text{rec}}(D) - V_0(D)$ is obtained at the expense of the old equity holders, who will lose exactly that amount. This implies that they would oppose the restructuring. Issuance of equity is dilutive for preexisting shareholders not because an equally large pie is now divided among more shareholders—in fact, the pie is bigger because of the proceeds of the new equity issue—but because the debt holders receive more of the pie.

To make the restructuring acceptable to shareholders, the value of the equity should not decrease. To this end, a possible policy option is for the government to give the bank cash in the amount of $V_{\text{rec}}(D) - V_0(D)$ conditional on the bank’s issuance of equity of an amount $D - A^* - (V_{\text{rec}}(D) - V_0(D))$ at a fair price reflecting the expected discounted value of future payouts to shareholders. With the total new cash $D - A^*$, the probability of default becomes $p^*$. The market value of the debt jumps by $V_{\text{rec}}(D) - V_0(D)$ and the government loses exactly that amount, so that, as planned, shareholder value is unchanged.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Recapitalization}
\end{figure}

E. Recapitalization by Issuance of Preferred Stock or Convertible Debts

Instead of issuing equity, banks could issue hybrid securities such as convertible debt or preferred stock. This would not change the analysis done in the previous section.

**Proposition 4**

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\[ \text{If the newly issued equity is priced higher than the NPV value, no investors will buy them. If it is priced lower than the NPV value, there is an arbitrage opportunity by investors and the government loses more.} \]

\[ \text{Issuance of new (nonconvertible) debt would increase the default probability and is thus not a possible restructuring scheme.} \]
The impact of recapitalization with preferred stock or convertible debt on preexisting debt value, \( V_{rec}(D) - V_0(D) \), and thus the transfer of the taxpayer, is the same as in the recapitalization through the issuance of common equity.

Proof
To show that the recovery of preexisting debt is the same as in Figure 5, there are two cases to consider separately. In the first case, the new claims do not trigger default. This applies to preferred stock or convertible debt with a conversion option at the issuer’s discretion, since the dividends do not have to be paid out (preferred stock) or debt converted into equity (convertible debt) when the bank is unable to pay dividends or coupons. In this case, the amount of capital that needs to be raised to achieve the target default probability \( p = p^* \), and thus the recovery schedule of initial debt, remains the same as in the case of recapitalization with common equity.

The second case involves the issuance of convertible debt, with the conversion not determined by the issuer (i.e., the conversion is automatic or at the holder’s discretion) and seniority equal to that of preexisting debt. The recovery rate is in proportion to total debt (\( pari passu \))—so the slope of the recovery is the same as in the equity issue case (see Figure 6a). At the same time, the trigger point for defaults after restructuring is set to be \( A^* \) as in the equity issuance case. Thus, the recovery of preexisting debt is exactly the same as in the equity issue case.

To make equity holders willing to accept the restructuring, the government has to compensate them with a conditional transfer identical to the one needed in the case of an equity issuance. Indeed, total wealth before and after the restructuring remains the same (\( conservation of value \)). That is, the sum of the changes in wealth of initial equity holders, initial debt holders, new claim holders, and taxpayers is zero. Because new claims are issued at a fair price, the new claimholder’s wealth is unchanged. Provided the restructuring needs to leave initial equity holders’ wealth unchanged, the taxpayer transfer should be equal to the change in debt value. This is the same as in the case of an equity issue.

\( QED \)

Figure 6a. Same Seniority Convertible

\[ ^8 \text{Conversion options in hybrid securities are discussed further in section V A below.} \]

\[ ^9 \text{It is more costly for taxpayers to issue convertible subordinated debt (i.e., junior to preexisting debt) with conversion not determined by the issuer. In this case, although the trigger point is still the same as in Figure 6a, the preexisting debt holders will be given priority in case of default. This extra gain imposes an extra cost on taxpayers.} \]
An alternative way to decrease the default probability down to $p^*$ is for the government to offer full or partial insurance on the bank’s assets using simple asset guarantees. To limit the cost to the taxpayers, such insurance can have a cap (partial insurance).

**Proposition 5**

Asset guarantees with cap of transfer up to $D-A^*$ has exactly the same cost for the government as in the recapitalization program.

**Proof**

The implied debt recovery is identical to that in Figure 5. To reach a default probability $p^*$, the government can insure against the value of assets falling below $D$, with a maximum transfer of $D - A^*$. This guarantees that the bank is able to repay its debt fully as long as $y = A_1 \geq A^*$ (see Figure 8).

$QED$
Note that compared with the optimal partial insurance scheme in Section C, this plan is more costly, since it makes debt recovery higher in case of default. In contrast to the second-best scheme, however, this transfer will be paid even in the worst cases, \( y = A_1 < A^* \). A full insurance scheme (without the transfer cap) would cost taxpayers more, since it involves higher payments in the worst cases, \( y < A^* \).

It is always optimal for taxpayers to insure whole portfolio of assets as opposed to a specific subset of them. Future payoffs of a subset of assets do not perfectly predict the default of the bank as a whole. Thus, higher transfers (as a precautionary cushion) are necessary to achieve the same default probability.

**G. Asset Sales at Inflated Price**

Another popular alternative is to sell a fraction \( a \) of the assets to the government at an overvalued price with markup \( m \), that is, \( (1 + m) a V_0(A) \), to achieve the target default probability \( p = p^* \) without dilution for shareholders. The proceeds of the sale are again kept as cash on the balance sheet.

**Proposition 6**

The government transfer needed for the asset sales program, which buys “\( a \)” portion of bank asset, is larger than the recapitalization program.

**Proof**

The recovery rate function is

\[
R_a(y; a) = (1-a)y + D - (1-a) A^*, \quad \text{for} \quad y \in [0, A^*]
\]

\[
= D, \quad \text{for} \quad y \in [A^*, D].
\]

(4)

Accordingly, the value of debt is

\[
V_{\text{as}}(D; a) = \int_0^\infty R_a(y; a) F(dy).
\]

To see this, note that the new assets owned by the bank are cash and remaining old assets, \( (1 + m) a V_0(A) + (1 - a) A \), which have higher expected value and lower risk than the original assets \( A \) (see Figure 9a). The parameters \( a \) and \( m \) can be picked as the solutions of two equations. The first equation states that the probability of default is \( p^* \), \( (1 + m) a V_0(A) + (1 - a) A^* = D \). The second equation states that the negative NPV of the government’s injection covers the increase in the value of debt, \( a m V_0(A) = V_{\text{as}}(D; a) - V(D) \)—new value of debt, \( V_{\text{as}}(D; a) \), depends on the sales fraction \( a \).
Because default occurs less often than in the do-nothing case, the value of the debt increases by $V_{ap}(D;a) - V_0(D)$. This jump is larger than in the case of recapitalization with common-equity issuance with the same default probability $p^*$, since the recovery rate for every realization $A_1$ is larger. The probability of default is equal to $\text{prob}( (1 - a) A < D - \text{cash} )$, equivalently, $\text{prob}( A < (D - a V_0(A)) / (1 - a) )$. Hence, the required fraction of assets $a$ should solve $(1 - a) A^* = D - a V(A)$. For a given realization of asset value $A_1$ that makes the bank default ($y < A^*$), the debt holders recover cash $a V(A)$ and liquidation value $(1 - a) \ y$, that is, $D - (1 - a) (A^* - y)$, which is more than in the equity issue case, $D - (A^* - y)$.

This is illustrated by a simple graphical intuition showing that the slope of the recovery schedule in the default zone is now $(1 - a)$ instead of 1 (see Figure 9b). Note that it is irrelevant whether the government or private investors hold the assets, as long as the government subsidizes the price by a markup $m$ so that it provides the subsidy required to compensate equity holders.

QED
H. Subsidized Debt Buybacks

When equity or other securities are issued, banks do not have to keep the proceeds on their balance sheet and might as well use them to buy back some debt. Economic intuition suggests that buying back debt and converting it into equity is closer to the first-best solution (i.e., debt-for-equity swap agreed to by debt holders). Altering the liability structure decreases the size of the transfer required from the government (see Bulow and Klemperer, 2009). Indeed, this decreases the transfers from taxpayers required to implement \( p = p^* \). The government thus should utilize the buyback opportunity in designing the restructure plan.

**Proposition 7**

*Subsidized debt buybacks is more cost efficient than subsidized recapitalization program with cash injection by issuing common or preferred stocks or convertible debt.*

**Proof**

For the initial debt holders, the recovery rate function that achieves the target default probability \( p^* \) is

\[
R_{db}(y) = \frac{D}{A^*} y, \text{ for } y \in [0, A^*]
\]

\[
= D, \text{ for } y \in [A^*, D].
\]

(4)

Accordingly, the value of debt is

\[
V_{db}(D) = \int_0^* R_{db}(y)F(dy).
\]

To see this, note that the bondholders that sell to the bank are not assumed to be naïve—they know that the value of the debt will rise as a result of the restructuring and therefore agree to sell only at the fair price that reflects the post-restructuring value of their claim.\(^{10}\) The fraction \( \alpha \) of outstanding debt that needs to be bought is such that \((1 - \alpha) D = A^*\), and the remaining debt contracts are untouched, so the new aggregate face value of the debt is \((1 - \alpha) D = A^*\). After the announcement, the value of the initial debt should increase from \( V_{db}(D) \) to \( V_{db}(D) \), reflecting the lower default probability after the restructuring. Out of this initial debt, a fraction \( \alpha \) is bought by the firm at a value \( \alpha V_{db}(D) \), while a fraction \((1 - \alpha) \) remains outstanding, with market value \((1 - \alpha) V_{db}(D) \).

To leave the equity holders indifferent, the government needs to subsidize the buyback. In exchange for the transfer, the bank should be willing to issue equity to buy back a fraction \( \alpha \) of

\(^{10}\) Note that this is a conservative assumption in evaluating taxpayer transfers, since it implies that the bank is not able to buy back debt secretly and restructure afterward by surprise.
the debt. Equivalently, the government can directly buy debt at the post-restructuring market price and convert it into equity at a conversion rate that leaves equity holders indifferent.\textsuperscript{11} As in the other schemes, the optimal size of the government transfer is equal to the increase in debt holders’ wealth created by the restructuring, $V_{db}(D) - V_{0}(D)$. Whether they keep their bonds or sell them, all initial debt holders receive this gain on a \textit{pro rata} basis. The remaining debt is a fraction $(1 - \alpha)$ of the initial debt. The gains of the remaining debt holders are $(1 - \alpha)$ of the gains of all the initial debt holders. Thus, the transfer by the government can be calculated by rescaling the realized recovery of the remaining debt by a factor $1 / (1 - \alpha)$ (the upper line in Figure 7). This \textit{total implied recovery} reflects the restructuring effects on the \textit{full} initial debt.

The recovery schedule (upper line of Figure 7) of this scheme is lower than the recovery schedule of the recapitalization in Figure 5. Therefore, this scheme is less costly for taxpayers than a recapitalization in which cash from new issues is kept on the balance sheet. \hfill \textit{QED}

IV. DEEPER ASSUMPTIONS: WHICH MARKETS ARE OPEN?

There are cases in which the bank can act right after the restructuring. For example, if large amount of its debt is traded in the market at the fair value, a bank can buy back debt using the cash it obtained from recapitalization. Also, if it has easy access to debt market, albeit some premium, government’s recapitalization efforts can be nullified by the bank issuing debt to restore the high leverage ratio.

\textsuperscript{11} Note that this scheme is equivalent to finding some debt holders that agree to convert into equity at the postrestructuring price, which is higher than the current market price but below the face value.
In practice, the liability side operation can be easily monitored by shareholders and the government. So, it is unlikely pose a problem. Instead, if there is a chance to buyback debt in lower-than-face values, the government should make the bank to do so.

However, when asset-side market is open and highly liquid, then the monitoring the bank’s portfolio allocation is quite difficult. Indeed, in any second, the bank manager can rebalance their portfolio back to pre-restructuring composition. In this case, asset sales program targeting the specific assets (e.g., highly risky assets) can be nullified easily. Thus, only a non-targeted asset sales program is worth to consider.\(^\text{12}\)

A targeted asset sales program, however, may make sense in the case when the asset-side market is not well functioning; for example, traditional business loans without securitized market or securitized complex instruments which lost the market in 2008 financial turmoil. In this case, indeed, theoretical ambiguity arises about the relative costs of the asset sales program. A targeted asset sales program may be less costly than a recapitalization program. However, we show numerically that, under the plausible range of parameter values, a targeted asset sales program is more costly than a non-targeted program.

**A. Theoretical Prediction on Risky Asset Only Sales**

We now consider targeted asset sales in case there is no market open for assets. We cannot find a clear-cut theoretical prediction. The sales of high-risk assets ("toxic assets") only can be beneficial, as the government helps a bank to rebalance portfolio towards safer side. However, high-risk assets provide diversification benefits as long as they are not correlated perfectly with other assets. Also, getting rid of high-risk assets implies that the remaining asset becomes very safe compared to pre-restructuring assets or the simple asset purchase of the slice of portfolio. This implies that less fiscal outlays is needed to achieve the same probability of default. However, the question is how much debt holders will gain after this type of restructuring, not how much direct fiscal outlays are used. Because to make asset very safe implies large reduction in equity values, which the government needs to compensate.

In this case, assumptions on the distributions of next period assets become critical, in contrast to that the distribution \(F\) does not matter at all in the cases examined so far. Now assume that the pre-restructuring share of highly risky, “toxic” assets \(A_T\) is \(\beta\), so \((1 - \beta)\) portion is safer assets \(A_S\). The government purchases \(x\) portion of toxic assets for its net present value \(E(A_T)\). Given the cumulative distribution function \(F_T\) for \(A_T\) and \(F_S\) for \(A_S\), their convolution \(F(z|x)\) can be calculated as:

\[^{12}\text{With any restructuring plan, a bank manager maximizing the shareholder value is likely to choose safer composition of assets when he can rebalance the portfolio. When the default probability is higher, the shareholders will gain more with riskier investments due to the convex nature of payoff to shareholders. A restructuring gives less of this risk-shifting incentive.}\]
computed for the portfolio value, $z = (1 - \beta) A_t + \beta (1 - x) A_r + \beta x A_r$, for given purchase portion $x$.\(^{13}\)

The government needs to find $x$ to solve to achieve the target level of default probability $p^*$,

$$\text{prob}(z \leq D) = \int_0^D (1 - \beta) A_t + \beta x A_r + \beta (1 - x) E(A_r) dF(z | x) = p^*. \quad (5)$$

The value of debt can be computed as

$$V_{TA}(D) = \int_0^D R_{TA}(z) F(z | x), \quad (6)$$

given the recovery function $R_{TA}(z)$.

**Proposition 8**

* A high-risk asset only sales may be impossible to achieve the target default probability. However, if it achieves, it may be, though not always, less costly than a non-targeted asset sales scheme and even than a recapitalization scheme.

**Proof**

In general, there is a possibility that lower risk assets alone can generate high probability of default. In this case, even after removing all the high-risk assets, the bank cannot achieve the target level of default probability.

In case the high-risk asset only sales can achieve the target default probability, the costs are uncertain. Because removing the toxic assets from the balance sheet dramatically reduces the default probability, cash that is transferred to the bank should be less than those of non-targeted asset sales. With the distribution $F(z|x)$ changes, the target level asset (originally $A^*$) above which face value of debt is honored can also be closer to, or even as same as, $D$. Hence, the recovery function can be lower than both the non-targeted asset sales case and the recapitalization case.

However, the value of debt is the integration of the recovery function times the density function as described in equation (6). If there is little changes of the density function for the next-period assets, the ordering of reaction function can be preserved so that the targeted asset sales scheme is the least costly. However, with much less tail risk after the restructuring, the value of debt can be larger than even the non-targeted asset sales.

\[ QED \]

\(^{13}\) $E(A_r)$ is the value of toxic assets, and thus evaluated with its distribution function $F_r$. 

B. Numerical Analysis for Targeted Asset Sales

Since there is no clear-cut theoretical prediction (Proposition 8), we compute the required government transfers for a targeted asset sales program numerically. Then, we compare it with the case of non-targeted asset sales as well as a recapitalization program under the same parameter values.

Under the plausible range of parameter values, we find that the targeted asset sales are not so much different from non-targeted asset sales, and thereby more costly than recapitalization. This is because the costs from losing diversification effects and making debt too safe counteracts the benefits from rebalancing portfolio to make a bank more solvent with smaller amount of asset sales.

Specifically, a numerical approximation is used to evaluate the integration of convolution \( F(z|x) \) based on two cumulative distribution functions \( F_T \) and \( F_S \). This is done by Monte Carlo simulations of 10000 random draw for each of \( F_T \) and \( F_S \). Both \( F_T \) and \( F_S \) are assumed as log normal distributions. Toxic assets and safer assets are assumed to be positively correlated but not perfectly. Note that negative correlation would give larger benefits of having toxic assets in the portfolio, and thus would be against our findings.

Table 1 shows the benchmark parameter values. There are three types of assumptions: one for the balance sheet composition, other for asset returns, and the last for default probabilities. For
the balance sheet, we first assume that the pre-restructuring capital ratio is 4%, at the regulatory minimum in the U.S.. This is broadly representing the situation for U.S. big banks in 2008 financial turmoil. We set the target default probability to be 5%.

For the asset returns, to normalize, the means of absolute values of toxic and safer assets are assumed to be equal to one. That is, there is no change in the asset size on average. By technicality, this means that the log mean is equal to negative of half the associated variance for each type of assets.

Historical expected default probability provided by Moody’s (via commercial arrangement with the IMF in their proprietary method). It shows that the default probability went up from negligible values in fall of 2008 to near 25% in early 2009 before the Geithner plan was announced. It was then returned back to around 5% in May 2009. Thus, our target default probability is in line with a historical episode. Other parameter values are set so that together they produce pre-restructuring default probability around 25%. In particular, we pick $\alpha=60\%$ for the portfolio share of toxic assets, 0.04 for the standard deviation (square root of the variance) of the safer asset, $\kappa=4$ for the relative size of standard deviation (“toxity”) of the toxic assets, and $\rho=0.4$ for the correlation between the toxic and safer assets, which is likely to be positive in reality.

We vary $\alpha$ (the portfolio share of toxic assets), $\kappa$ (toxity of the toxic assets), and $\rho$ (the correlation between the toxic and safer assets), separately to see sensitivity of our analysis. Specifically, Figures 9-11 shows the computed transfers from the government to shareholders of targeted, risky-asset-only sales and non-targeted asset sales, relatively what is required in the case of recapitalization scheme.

Figure 9 shows that when the toxic assets are below 40 percent of the total assets, the bank cannot reach the target default probability of 5% by selling only risky assets. In this case, the sales of all types of assets is inevitable to achieve the goal. When the toxic assets consist more than 40 percent of the total assets, the risky asset only sales is slightly less costly than the non-targeted assets sales program.

Note that the economically those are significant. For example, as of end-2007, Citigroup had 1 trillion dollar asset and UBS had 2 trillion dollar asset in their balance sheet. As we normalized the unit to one, the difference of 0.0001 ($=1\times10^{-3}$) is about 100 million dollars.

Figure 10 shows that regardless of toxity of the toxic assets, there are only small differences in costs between the risky asset only sales and the non-targeted asset sales. The difference widens, risky asset only sales cost less, with higher toxity. However, when the toxity is about the same (one at the leftmost on x-axis), then the risky asset only sales is very slightly more costly compared with the non-targeted asset sales. This is because sales of one asset or combined assets are basically the same implication as they follow the same log normal distribution. But there is
small difference in that the correlation is not one, rather 0.2 in the benchmark case. That is, selling substantial amount of one asset will lose the diversification benefit to reduce the default probability, especially when the two assets have the same mean and variance, but not perfectly correlated.

Figure 11 shows that when the cost savings by toxic asset only sales are relatively robust over different correlations at the benchmark value.

Figure 12 to 14 show that the required cash amount to inject equity or to buy assets. This is typically the subject that many people use (e.g., IMF’s own GFSR, many commentators). However, this is NOT we think important, as we discussed so far. Whatever the cash amount is, if the exchanged equity or asset values are the same value as cash spent at NPV, then there is no taxpayers’ cost incurred.

Still, to make sure how the different recommendations would arise if we use this cash spending criteria, we examine Figure 12 to 14. First, in most cases, recapitalization is still cheapest in this criteria, too. Also, the risky only asset sales is second cheapest in most cases, though the difference among schemes appear much larger than our criteria.

Most importantly, when toxicity is high (rightmost in Figure 13), risky asset only sales look least expensive, even less costlier than recapitalization. Indeed, this type of claims were not uncommon in late 2008 to early 2009. However, again, the required cash to spend is not the right criteria to examine the taxpayers’ cost. And this example clearly shows how the choice of a wrong criteria could lead to a costlier policy.

<table>
<thead>
<tr>
<th>Assumptions on Balance Sheet</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>Pre-Restructuring Capital Ratio</td>
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<tr>
<td>Portfolio Share of Toxic Assets</td>
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<th>Assumptions on Asset Returns</th>
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<tr>
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<tr>
<td>Std. Dev. of Log Value of Safer Assets</td>
<td>$\sigma$</td>
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<tr>
<td>(Mean of Log Value of Safer Assets)</td>
<td>(-$\sigma^2$)</td>
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<tr>
<td>Relative Size of Std. Dev. (Toxicity) of Log Value of Toxic Assets</td>
<td>$\kappa$</td>
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<tr>
<td>(Std. Dev. of Log Value of Toxic Assets)</td>
<td>(5$\kappa$)</td>
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<tr>
<td>Correlation between Toxic and Safer Assets</td>
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<table>
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<th>Assumptions on Default Probability</th>
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<tr>
<td>Target Default Probability</td>
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V. Conclusion

When designing a restructuring plan for a systemically important bank, a key issue is to limit transfers from taxpayers. Given the reluctance of shareholders (and bank managers representing them or themselves as stock option holders), the public intervention is often voluntary. If so, as discussed in public economics literature, some compensation is needed for shareholders to agree with restructuring a bank because any restructuring lowers the value of equity by raising the value of debt through lower default probability.

We find that necessary transfers varies among restructuring options. In a Modigliani-Miller framework in which cash flows are independent of capital structure, restructuring is theoretically possible by converting some debt into equity. This is, however, difficult in practice. Without changing debt contracts, all restructuring involves transfers from the government. A plan subsidizing common equity issues and buying back debt is close to optimal. Subsidized asset sales are more costly to taxpayers, since debt holders benefit more. Numerical examples show that the difference is economically significant.

Our reasoning presented in this paper goes through any economic environment with frictions. For example, see discussions in the review paper (Landier and Ueda, 2009). There, they argue that the restructuring of a systemically important bank should combine several solutions to resolve multiple concerns and trade-offs on a case-by-case basis, but the main argument based on the compensation principles go throughout the analysis.
Figure 9. Value of Debt Under Various Portfolio Share of Risky Asset (Relative to Recapitalization)

Figure 10. Value of Debt Under Various Toxity of Risky Asset (Relative to Recapitalization)
Figure 11. Value of Debt Under Various Correlation between Risky and Safe Assets (Relative to Recapitalization)

Figure 12. Cash Required for Injecting Equity or to Buying Assets Under Various Portfolio Share of Risky Asset
Figure 13. Cash Required for Injecting Equity or to Buying Assets Under Various Toxicity of Risky Asset

Figure 14. Cash Required for Injecting Equity or to Buying Assets Under Various Correlation between Risky and Safe Assets
REFERENCES


