PrObEx and Internal Model
Calibrating dependencies among risks in Non-Life

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## PrObEx and Internal Model

1. Introduction
2. SCR and risk aggregation
3. PrObEx – a Bayesian model
4. Implementation in the Internal Model
5. Conclusion
SCOR

- SCOR is the 5th largest reinsurer in the world (Premium income of EUR 9.514 billion in 2012).

- SCOR operates worldwide via its six Hubs located in Paris, Zurich, Cologne, New York and Singapore.

- Ratings:
  - A+ S&P positive outlook
  - A A.M.Best stable outlook
  - A1 Moody’s stable outlook
  - A+ Fitch stable outlook

- Priority of SCOR is the delivery of the Internal Model and its approval by the ACPR (Autorité de Contrôle Prudentiel et de Résolution) for purpose of use under Solvency II.

- We illustrate a key innovation in SCOR’s Internal Model: PrObEx
### PrObEx and Internal Model

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SCR and risk aggregation

According to Solvency II, we need to determine the Solvency Capital Requirement (SCR).

The SCR is given by:

\[ \text{SCR} = -\text{VaR}_{0.5\%}(G) \]

where \( G \) is the change in the economic value over the measurement period (one year), i.e.

\[ G = \nu W_1 - W_0 \]

where \( \nu \) is the discount factor (risk-free) from the horizon date to the valuation date, and the economic value is given by:

\[ W_t = \sum_{p=1}^{P} A_p(t) - \sum_{q=1}^{Q} L_q(t) \]

\( W_0 \) and \( \nu \) are considered known at the valuation date, while \( W_1 \) is modeled as a random variable.
SCR and risk aggregation

- Monte-Carlo simulation methods are used to determine the stochastic value $W_1$.

- The valuation of $W_1$ requires to calculate the distribution of the Liabilities at time 1:

$$\sum_{q=1}^{Q} L_q(1)$$

- The latest financial crisis has dramatically shown that dependence among risks can not be ignored.

- We use copula models in order to prudently account for dependence (especially in the tail!).

- Copula estimation procedures usually contain a large parameter uncertainty if data is scarce.

- We developed a Bayesian model to calibrate copula parameters $\rightarrow$ PrObEx
PrObEx and Internal Model

1 Introduction
2 SCR and risk aggregation
3 PrObEx – a Bayesian model
4 Implementation in the Internal Model
5 Conclusion
Copula and dependence measure

- Let \((X, Y)\) be a bivariate random vector and assume the marginal distributions \(F(x)\) and \(G(y)\) are known.

- The joint cumulative distribution can be represented as
  \[
  H(x, y) = C(F(x), G(y))
  \]
  where \(C\) is the unique copula function that joins the two marginal distributions.

- There exist many copula families and some are relevant for modeling insurance risks. We focus on the most popular families characterized by one parameter.

- We assume the copula family is already known. Our aim is to estimate the copula parameter \(\gamma_0\).

- We chose a dependence measure \(\rho\) which is familiar to insurance business experts and which can be linked to the copula parameter.
  - Calculating an estimate \(\hat{\theta}_0\) of the value \(\theta_0\) of the dependence measure leads to an estimate of \(\gamma_0\).
PrObEx – Combining three sources of information

- (Up to) three sources of information can be combined:
  - **Prior**: A prior density $\pi(\theta)$, e.g. from previous years or from regulators.
  - **Observation**: $N$ independent observations of joint realizations from $(X, Y)$. The set of observation is denoted by $\mathcal{O}$.
  - **Experts**: $K$ experts, each providing one point estimate $\varphi_k$ of $\rho$. The set of expert assessments is denoted by $\mathcal{E}$.

- We replace the prior density $\pi(\theta)$ by a posterior density $\pi(\theta|\mathcal{O}, \mathcal{E})$ of $\theta$ given $\mathcal{O}$ and $\mathcal{E}$.

- Bayes’ Theorem leads to the relation
  $$\pi(\theta|\mathcal{O}, \mathcal{E})h(\mathcal{O}, \mathcal{E}) = h(\mathcal{O}, \mathcal{E}|\theta)\pi(\theta)$$
Our model

- We make the following assumptions:
  - The expert assessments and the observations are independent
  - The observations are independent
  - The experts form their opinion independently of each other

- Under these assumptions, the posterior distribution of the value of the dependence measure reads as:

\[
\pi(\theta|\mathcal{O}, \mathcal{E}) \propto \pi(\theta) \prod_{n=1}^{N} c(U_n, V_n|g(\theta)) \prod_{k=1}^{K} e_k(\varphi_k|\theta)
\]

- Through this posterior distribution we can:
  - Estimate \( \hat{\theta}_0 \), e.g. via \( \hat{\theta}_0 = \mathbb{E}[\theta|\mathcal{O}, \mathcal{E}] \).
  - Assess the uncertainty of our estimate, e.g. via \( \text{var}(\theta|\mathcal{O}, \mathcal{E}) \).
Prior information

- Suppose we can infer a point estimate $\hat{\theta}_p$ of $\theta_0$ from the prior source of information.
- We then model $\pi(\theta)$ with a shifted Beta distribution with mean $E[\theta] = \hat{\theta}_p$.

- If the source of information leading to $\hat{\theta}_p$ does not specify a measure of uncertainty, we determine $\text{var}(\theta)$ through a qualitative approach:
  
  $$\text{var}(\theta) = \begin{cases} 
  0.005(b - a)^2 & \text{for high confidence,} \\
  0.02(b - a)^2 & \text{for intermediate confidence,} \\
  0.05(b - a)^2 & \text{for low confidence.}
\end{cases}$$

- If no prior belief is available then $\pi(\theta)$ can be set uninformative.
- The four mentioned qualitative approaches:

![Graph showing different prior distributions for different confidence levels]
The elicitation of expert opinions

- An expert elicitation procedure needs to satisfy five principles in order to reach rational consensus, namely:
  - Reproducibility
  - Accountability
  - Empirical control
  - Neutrality
  - Fairness

- Psychological effects are involved and have to be considered carefully.

- The literature distinguishes between behavioral vs. mathematical approaches.
The modeling of expert opinions

- The conditional density of the k-th expert is modeled via a shifted Beta distribution.

- We model the expert estimates to be conditionally unbiased, i.e. $E[\varphi_k|\theta] = \theta$.

- To reflect the expert uncertainty we assign each expert a variance $\sigma_k^2$, which is assumed to be independent of $\theta$, i.e. $\text{var}(\varphi_k|\theta) = \sigma_k^2$.

- Three possible approaches to calculate estimates $\hat{\sigma}_k^2$ of $\sigma_k^2$ are considered:
  - Subjective variances
  - Homogeneous experts
  - Seed variables
An illustrative example (1 of 2)

- Let \( C \) be a T-copula* and the dependence measure \( \rho \) be Kendall's Tau.
- Then, the dependence measure is linked to the copula parameter by the function: \( g(\theta) = \sin(\theta \pi / 2) \)

- Suppose we have no prior information available.
- Let N=24 observations be given:

![Scatter plot]

- Experts opinions: \( \varphi_1 = 0.35 \), \( \varphi_2 = 0.5 \), \( \varphi_3 = 0.7 \). Moreover:

<table>
<thead>
<tr>
<th>True value</th>
<th>Seed variables</th>
<th>Estimated variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3</td>
<td>0.0625</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.0281</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

\* For the purpose of this example, we consider a T-copula with 3 degrees of freedom.
The best estimate using all information is then:

$$\hat{\gamma}_0 = g(\mathbb{E}[\theta|\sigma, \varepsilon]) = g(0.399) = 0.587$$
PrObEx: Two experts equally certain and no prior information...

Combining different sources of information
PrObEx: … what if we can use an informative prior? …

Combining different sources of information

[Graphs showing the combination of informative priors and expert likelihood functions]
PrObEx: ... confident experts increase further the precision
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Why “Optimal Dynamics”? 

The Group Internal Model (GIM) determines the profitability and solvency path of Optimal Dynamics

External forces  
Economic and financial environment  
Regulatory changes  
Reinsurance industry dynamics

Group Internal Model

- Strong Franchise
- High Diversification
- Controlled Risk Appetite
- Robust Capital Shield

The new strategic plan, Optimal Dynamics, aims to:
- Optimize capital utilization
- Maximize diversification benefits
- Reduce volatility
- Self-finance growth while remunerating shareholders

...with dynamic mechanisms in place to remain on the target path to profitability & solvency

Maximize Profitability:
1 000 bps above risk-free rate over the cycle

Respect solvency target:
185%-220% solvency ratio

---

1) "Risk-free rate" is based on 3-month risk-free rate
2) As per the Group Internal Model, it is the ratio of Available Capital over SCR (Solvency Capital Requirements); see page 21 for further details
The relevance of the project

- As part of SCOR internal model, PrObEx contributes to the determination of the SCR → it has an impact on key areas, such as capital allocation, underwriting and investment strategies.

- In line with SCOR's strategic plan “Optimal Dynamics”, PrObEx offers support for high diversification and controlled risk appetite.

- To ensure robustness of final results, the process of gathering the expert's opinion has been industrialized and fully documented.

- **33 workshops** were organized and more than 100 experts, scattered in 7 different locations around the World, were involved in the project.

- Overall, more than 1’300 dependence assessments were elicited, covering 16 different Lines of Business.
The calibration process

Overview → Training → Brainstorming → Questionnaire

Prior Information + Observations + Experts opinions

PrObEx

Dependence parameters

Risk aggregation

Solvency Capital Requirement (SCR)
The risk aggregation tree for **Specialty** Non-Life LoBs

**Group Level**

**Line of Business (LoB)**
(e.g. Aviation, Credit & Surety)

**Business Maturity**

**Reinsurance/Cover Type**

**Legal entity**

**Treaty for a certain LoB**
Dependence measure – what we asked to SCOR experts

- The experts were asked to answer a question like:
  
  “Suppose Y exceeds the 1-in-100 year threshold. What is the probability that also X exceeds its 1-in-100 year threshold?”

- This is equivalent to quantify the so called Quantile Exceedance Probability:

\[
P[X > VaR_{0.99}(X) \mid Y > VaR_{0.99}(Y)]
\]
Workshop agenda

Stage 0 – WORKSHOP AGENDA
presented by Davide Canestraro and Nigel Riley
Paris, September 29, 2011

Stage 1 - OVERVIEW
presented by Davide Canestraro and Nigel Riley
Paris, September 29, 2011

Stage 2 – TRAINING SESSION
presented by Davide Canestraro and Nigel Riley
Paris, September 29, 2011

Stage 3 – BRAINSTORMING
Facilitators: Davide Canestraro and Nigel Riley
Paris, September 29, 2011

Stage 4 – QUESTIONNAIRE
Facilitators: Davide Canestraro and Nigel Riley
Paris, September 29, 2011
Expert judgment and heuristics (1)

- Representativeness (1)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

Is it more likely that:

(A) Linda is a bank teller?

(B) Linda is a bank teller and active in the feminist movement?
Expert judgment and heuristics (2)

- Representativeness (2)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

There are 100 people who fit the description above. How many of them are:
(A) bank tellers?
(B) bank tellers and active in the feminist movement?

Answer:
Expert judgment and heuristics (3)

- Availability

Are there more words in the English language that begin with R or have R as their third letter?

Which hazard claims more lives in the United States: lightning or tornadoes?
Expert judgment and heuristics (3)

- Anchoring

  Is the population of Chicago more or less than 200,000? Estimate the population.

  Is the population of Chicago more or less than 5 million? Estimate the population.
Given that an extremely bad outcome is observed in the legal entity Switzerland, what is your estimate of the probability that the legal entity France will experience an extremely bad outcome?

Which are the risk drivers which can cause such a bad outcome in the legal entity Switzerland?

Assume they are:
- Eurowind
- European Earthquake
- North American Tropical Cyclone
Section A.1
Given that an extremely bad outcome is observed in the Legal Entity Switzerland, list some of the risk drivers for which ALSO Legal Entity France will experience an extremely bad outcome.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Risk driver</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>Eurowind</td>
<td>0.3</td>
</tr>
<tr>
<td>50%</td>
<td>European Earthquake</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Section A.2
Given that an extremely bad outcome is observed in the Legal Entity Switzerland, list some of the risk drivers for which Legal Entity France will NOT experience an extremely bad outcome.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Risk driver</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>North American Tropical Cyclone</td>
<td>0.4</td>
</tr>
</tbody>
</table>
The aggregation tree for Non-Life
Dependence parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Modelling Unit</th>
<th>Legal Entity</th>
<th>Cover Type</th>
<th>Reinsurance Type</th>
<th>Business Maturity</th>
<th>Region</th>
<th>Copula Type</th>
<th>Theta Parameter</th>
<th>Exceedance Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1262</td>
<td>Auto Scor Switzerland AG</td>
<td>Auto</td>
<td>Scor Switzerland AG</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>None</td>
<td>Clayton</td>
<td>0.83</td>
</tr>
<tr>
<td>1263</td>
<td>Auto Scor Switzerland AG</td>
<td>Auto</td>
<td>Scor Switzerland AG</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Reserves</td>
<td>None</td>
<td>0.88</td>
</tr>
<tr>
<td>1261</td>
<td>Auto NewBusiness</td>
<td>Auto</td>
<td>NewBusiness</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>NewBusiness</td>
<td>None</td>
<td>0.55</td>
</tr>
<tr>
<td>1260</td>
<td>Auto Treaty Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Proportional</td>
<td>Treaty</td>
<td>NewBusiness</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1259</td>
<td>Auto Treaty Non-Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Non-Proportional</td>
<td>Treaty</td>
<td>NewBusiness</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1258</td>
<td>Auto Reinsurance</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
</tr>
<tr>
<td>1257</td>
<td>Auto Treaty Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Proportional</td>
<td>Treaty</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1256</td>
<td>Auto Treaty Non-Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Non-Proportional</td>
<td>Treaty</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1255</td>
<td>Auto Reinsurance</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
</tr>
<tr>
<td>1254</td>
<td>Auto Treaty Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Proportional</td>
<td>Treaty</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1253</td>
<td>Auto Treaty Non-Proportional</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>Non-Proportional</td>
<td>Treaty</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td>1252</td>
<td>Auto Reinsurance</td>
<td>Auto</td>
<td>SCOR ITA</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Reinsurance</td>
<td>None</td>
<td>0.55</td>
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SCOR
# PrObEx and Internal Model

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</table>
Conclusion

- PrObEx provides a sound mathematical framework for estimating copula parameters.

- PrObEx allows to reduce the parameter uncertainty when estimating copula parameters.

- A statistical analysis conducted from Professor Sebastien Van Bellegem (Toulouse School of Economics) has demonstrated the robustness and the absence of bias in the results.

- PrObEx can be used to calibrate dependencies also in other contexts (e.g. Life, Economy, etc.).

- A scientific paper on PrObEx has been published in the ASTIN Bulletin.


Thank you for your attention!

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Thank you for your attention!

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Copula

Let \((X, Y) \in \mathbb{R}^2\) be a bivariate random vector. Assume the margins \(F(x) = \mathbb{P}[X \leq x]\) and \(G(y) = \mathbb{P}[Y \leq y]\) of \((X, Y)\) are continuous.

We can represent the joint cdf \(H(x, y) = \mathbb{P}[X \leq x, Y \leq y]\) as:

\[
H(x, y) = C(F(x), G(y)), \quad \text{for all } x, y \in \mathbb{R},
\]

where \(C : [0,1]^2 \to [0,1]\) is the unique copula function.

The copula \(C\) is also the cdf of the random vector \((U, V) = (F(X), G(Y)) \in [0,1]^2\), denoted with \((U, V) \sim C\).
Four popular copula families

Let $\mathcal{C}_0 = \{C_\gamma : \gamma \in \Gamma\}$ denote a family of bivariate copulas, with parameter set $\Gamma$ and density $c(\cdot | \gamma)$. We assume that

$$C = C_{\gamma_0} \in \mathcal{C}_0$$

where $\gamma_0$ is an unknown but fixed parameter. Our aim is to estimate $\gamma_0$.

<table>
<thead>
<tr>
<th>Copula family</th>
<th>Definition</th>
<th>Parameter range $\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>$C_{\gamma}^{Ga}(u,v) = \Phi_\gamma(\phi^{-1}(u), \phi^{-1}(v))$</td>
<td>$\gamma \in (-1,1)$</td>
</tr>
<tr>
<td>$t$</td>
<td>$C_{\nu,\gamma}^{t}(u,v) = t_{\nu,\gamma}(t_\nu^{-1}(u), t_\nu^{-1}(v))$</td>
<td>$\gamma \in (-1,1)$</td>
</tr>
<tr>
<td>Clayton</td>
<td>$C_{\gamma}^{Cl}(u,v) = (u^{-\gamma} + v^{-\gamma} - 1)^{-1/\gamma}$</td>
<td>$\gamma \in (0,\infty)$</td>
</tr>
<tr>
<td>Gumbel</td>
<td>$C_{\gamma}^{Gu}(u,v) = \exp\left(-((-\ln(u))^{\gamma} + (-\ln(v))^{1/\gamma})\right)$</td>
<td>$\gamma \in [1,\infty)$</td>
</tr>
</tbody>
</table>
Four popular copula families – rank scatter plots
Dependence measure

Let $\rho(\cdot, \cdot)$ denote a fixed dependence measure. Define the set of attainable values of $\rho$ for copulas in $\mathcal{C}_0$ by

$$\Theta = \{\rho(U^*, V^*): (U^*, V^*) \sim C^* \in \mathcal{C}_0\}.$$  

We assume that $\Theta$ is an interval, i.e. $\Theta = [a, b] \subset \mathbb{R}$

We focus on $\rho(\cdot, \cdot)$ which satisfy $\rho(U, V) = \rho(X, Y)$

We assume there exists $g : [a, b] \rightarrow \Gamma$ a bijective link function s.t.

$$g(\rho(U^*, V^*)) = \gamma^* \quad \text{for all} \quad (U^*, V^*) \sim C_{\gamma^*}.$$

Calculating an estimate $\widehat{\theta}_0$ of $\theta_0$ leads to an estimate $\widehat{\gamma}_0 = g(\widehat{\theta}_0)$ of $\gamma_0$. 

\text{SCOR}
PrObEx – Combining three sources of information

(Up to) three sources of information can be combined:

**Prior** A prior density \( \pi(\theta) : [a, b] \rightarrow [0, \infty) \) e.g. from previous years or from regulators.

**Observations** \( N \) independent observations \((U_n, V_n), n = 1, \ldots, N, \) of \((U, V) \sim C_{\gamma_0}\). The set of observations is denoted by \( \mathcal{O} = \{(U_n, V_n) : n = 1, \ldots, N\} \).

**Experts** \( K \) experts, each providing one point estimate \( \varphi_k, k = 1, \ldots, K, \) of \( \rho(U, V) \). The set of expert assessments is denoted by \( \mathcal{E} = \{\varphi_k : k = 1, \ldots, K\} \).

We replace the prior density \( \pi(\theta) \) by a posterior density \( \pi(\theta|\mathcal{O}, \mathcal{E}) \) of \( \theta \) given \( \mathcal{O} \) and \( \mathcal{E} \).

Bayes’ Theorem leads to the relation

\[
\pi(\theta|\mathcal{O}, \mathcal{E}) h(\mathcal{O}, \mathcal{E}) = h(\mathcal{O}, \mathcal{E}|\theta) \pi(\theta),
\]
Bayesian inference

We assume that the expert assessments and the observations are independent, thus:

\[ h(\bar{\theta}, \mathcal{E}|\theta) = h_O(\bar{\theta}|\theta) h_E(\mathcal{E}|\theta), \]

where \( h_O \) and \( h_E \) are the conditional densities, given \( \theta \), of \( \bar{\theta} \) and \( \mathcal{E} \), respectively.

As the observations \((U_n, V_n), n = 1, \ldots, N\), are independent,

\[ h_O(\bar{\theta}|\theta) = \prod_{n=1}^{N} c(U_n, V_n|g(\theta)), \]

where \( c(u, v|\gamma) = \frac{\partial}{\partial u} \frac{\partial}{\partial v} C_{\gamma}(u, v) \).

Assuming the experts form their opinions independently of each other, we have:

\[ h_E(\mathcal{E}|\theta) = \prod_{k=1}^{K} e_k(\varphi_k|\theta), \]

where \( e_k(\cdot|\theta) \) is the conditional density, given \( \theta \), of the \( k \)-th expert assessment.
Our model

Since

\[ \pi(\theta | \Theta, \mathcal{E}) \propto \pi(\theta) h(\Theta, \mathcal{E} | \theta) \]

we get:

\[ \pi(\theta | \Theta, \mathcal{E}) \propto \pi(\theta) \prod_{n=1}^{N} c(U_n, V_n | g(\theta)) \prod_{k=1}^{K} e_k(\varphi_k | \theta) \]

For sensitivity analysis or in case no expert opinions or observations are available, we can also compute:

\[ \pi(\theta | \Theta) \propto \pi(\theta) \prod_{n=1}^{N} c(U_n, V_n | g(\theta)), \quad \pi(\theta | \mathcal{E}) \propto \pi(\theta) \prod_{k=1}^{K} e_k(\varphi_k | \theta). \]

Through this posterior distribution we can:

- estimate \( \hat{\theta}_0 \), e.g. via \( \hat{\theta}_0 = \mathbb{E}[\theta | \Theta, \mathcal{E}] \).
- assess the uncertainty of our estimate, e.g. via \( \text{var}(\theta | \Theta, \mathcal{E}) \).
The modeling of expert opinions (1 of 2)

The conditional density $e_k(\cdot | \theta)$ is modeled via a shifted Beta distribution. We model the expert estimates to be conditionally unbiased, i.e.

$$E[\varphi_k|\theta] = \theta \quad \text{for all} \quad \theta \in [a, b],$$

To reflect the expert uncertainty we assign each expert a variance $\sigma^2_k$, $k = 1, \ldots, K$, which is assumed to be independent of $\theta$:

$$\text{var}(\varphi_k | \theta) = \sigma^2_k \quad \text{for all} \quad \theta \in [a, b].$$

Three possible approaches to calculate estimates $\widehat{\sigma^2_k}$ of $\sigma^2_k$:

- Subjective variances
- Homogeneous experts

$$\widehat{\sigma^2} = \frac{1}{K-1} \sum_{k=1}^{K} (\varphi_k - \overline{\varphi})^2,$$

where $\overline{\varphi} = \frac{1}{K} \sum_{k=1}^{K} \varphi_k$. 

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The modeling of expert opinions (2 of 2)

- Seed variables

<table>
<thead>
<tr>
<th>True value</th>
<th>Seed variables</th>
<th>Experts’ estimates</th>
<th>Estimated variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_0^{(1)}$</td>
<td>$\psi_0^{(H)}$</td>
<td></td>
</tr>
<tr>
<td>Expert 1</td>
<td>$\psi_1^{(1)}$</td>
<td>$\psi_1^{(H)}$</td>
<td>$\sigma_1^2 = \frac{1}{H} \sum_{h=1}^{H} (\psi_1^{(h)} - \psi_0^{(h)})^2$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>Expert M</td>
<td>$\psi_M^{(1)}$</td>
<td>$\psi_M^{(H)}$</td>
<td>$\sigma_M^2 = \frac{1}{H} \sum_{h=1}^{H} (\psi_M^{(h)} - \psi_0^{(h)})^2$</td>
</tr>
</tbody>
</table>
Investor’s day 2011

Strong Momentum V1.1 is consistent with the Group’s four strategic cornerstones

<table>
<thead>
<tr>
<th>SCOR consistent execution of its strategic cornerstones</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strong franchise</strong></td>
</tr>
<tr>
<td>❑ Is reaching a new perimeter</td>
</tr>
<tr>
<td>❑ Is deepening its global franchise</td>
</tr>
<tr>
<td>❑ Is pursuing the announced “Strong Momentum” growth initiatives</td>
</tr>
<tr>
<td><strong>Controlled risk appetite</strong></td>
</tr>
<tr>
<td>❑ Sticks to the “Strong Momentum” mid level risk appetite</td>
</tr>
<tr>
<td><strong>High diversification</strong></td>
</tr>
<tr>
<td>❑ Adds additional diversification benefits</td>
</tr>
<tr>
<td><strong>Robust capital shield</strong></td>
</tr>
<tr>
<td>❑ Proves the relevance of its capital shield policy</td>
</tr>
<tr>
<td>❑ Pursues active capital management</td>
</tr>
</tbody>
</table>
The risk aggregation tree for **Standard** Non-Life LoBs

- **Group Level**
  - Standard lines are inverted so that aggregation first occurs within a legal entity

- **Line of Business (LoB)**
  - (e.g. Auto)

- **Legal entity**

- **Business Maturity**

- **Reinsurance/Cover Type**

- **Treaty for a certain LoB**

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