Abstract

In the current politico-economic context made unstable by the coming up of several different conflicts, it is interesting to deal with the use and the development of models. Indeed, it is necessary to pay special attention to projections obtained with current models because parameters are often estimated with data which no longer reflect the economic context they have to describe.

Moreover, a model is valid in a precise frame and for a given time horizon: universal models do not exist. It is necessary to keep in mind that a model, by definition, allows to produce simplified representations of unpredictable and very complex systems. That is why, the model presented in this paper is not meant to be adapted to all situations: for instance, the model is not adapted to a day to day dynamic management in a stable economic situation.

This study deals with the development of economic scenarios generators (ESG) and their use in asset management. We propose a tool or more globally a process allowing to take into account, during the optimization of an equity portfolio, the drastic increase of the major crises probability faced since the beginning of the 21st century.

At first, we present a brief analysis describing the world politico-economic context. Besides, we give some fundamental principles to be adopted to build or use an ESG. Then, we present the basic ESG structure retained and the method chosen to integrate external elements stemming from a fundamental market analysis. Finally, we present a simple illustration among a large number of possible applications: the optimization of an equity portfolio by following the created process.
1 Unstable economic context and combined approaches

1.1 A drastic increase of major crises risks

In the French actuarial magazine *L’actuariel nb.6*, Jean-Paul Delevoye said that *Future is not the projection of past anymore, but the calling into question of the latter*. This thinking illustrates the breaking noticed since the beginning of the 21st century with the apparition of several crises (financial, geopolitical...) at a high and unprecedented frequency. Below, we propose a brief review of the major crises having occurred since the beginning of the century. They will be classified according to their natures:

- Financial crises: they provoke a strong volatility on markets and participate in the collapse of stock exchanges. Firstly, let us remind the three speculative bubbles bursting: the technology stocks bubble in 2000 following the development of the Internet; and the real estate bubble and the credit one coming from the United States in 2007. Secondly, let us remind the crisis of the sovereign debts and the European crisis which have disturbed financial markets since 2010.

- Military conflicts, persistent or starting, national or international: these conflicts unsettle the world stability and badly affect the investor’s confidence. Some most known examples are: the Israeli-Palestinian conflict, wars in Iraq and Afghanistan, the Arab Spring riots in 2010, the revolts in Senegal as well as in Syria... The movements of revolt in Russia and the economic situation of China also participate in this instability. We would like to underline an element on this last point: an analysis of the budget of China shows that this country plans an important annual increase of its defence budget until 2015 to reach a 240 billion dollar total budget. This is four times more than the defence budget of Japan (which is the second one in Asia). This element tends to prove that the Asian situation is not peaceful. Is it a cold war with important economic consequences?

We can also add crisis scenarios which can burst out because of the unbalanced economic situation existing between BRIC and developing countries, for two main reasons:

- There is no reason so that the Chinese GDP per capita stays lower than the Japanese’s one. This implies a fourfold increase of the Chinese GDP in the next years.

- Because of BRIC, Europe is inevitably going to continue to lose market shares for the benefit of emerging countries and to pursue its process of deindustrialization.

Consequently, adaptation crises will be inevitable. That is why multiple and varied scenarios are to be taken into account. Olivier Blanchard, head economist of the International Monetary Fund and Christine Lagarde’s adviser even evoked the possibility of a *big depression*.

To conclude, we would like to mention an economic argument corroborating the fact that crises are to be planned. The demographic power of the Asian countries creates a modification of the world economy evolution and disturbs several economic theories such as the Kondratieff one: according to this theory, the economy follows a long cycle of about fifty six years including five seasons:
• Spring: inflationary boom and bullish cycle of quantities.
• Summer: cruising speed, the economy goes well.
• Stagflation: decreasing growth and hyperinflation.
• Indian summer: period of artificial prosperity ideal for speculative bubble creation ending by a crash.
• Depression: decrease of assets prices and solvency problems.

According to Francois-Xavier Chevalier’s study [4], we should be at the beginning of a long cycle but yet the growth decreases… This implies that several long-term scenarios are possible. This brief study of the politico-economic situation shows that it is essential to adapt the traditional asset management by giving priority to careful management focusing on the research of values which fall less than the others in crises period, the resilient values.

1.2 Asset management in crises period

One of the objectives of this paper is to describe a tool adapted to asset management in crises period. That is why it is necessary to define some fundamental principles. There were established from an a posteriori crises study allowing to understand some manager’s irrational behavior and to avoid some stumbling.

First of all, we would like to quote this sentence stemming from the book of Arnaud Clément-Grandcourt [3], economist specialist of the crises periods management: anticipating a crisis is really difficult and it is very dangerous to delude oneself about the subject. This idea perfectly introduces the two main qualities of a good crisis manager: cautiousness and modesty.

The study of crises leads us to define what is a careful asset manager: it is a manager who limits the number and the importance of his errors by taking measured risks and by taking into account all the important parameters to make decisions; moreover it is a person who does not overestimate his capacity of forecasting and understanding the events. To bring a real surplus value, the manager has to realize accurate market analysis and has to invest in resilient values, while keeping in mind that being careful sometimes requires being offensive: a good offense is sometimes prudent.

To reach an effective assets allocation, this paper proposes:

• The use of an economic scenarios generator allowing to take into account several crises scenarios,
• The consideration of leading economic indicators which help to give probability to each central scenario.

Before describing the economic scenario generator created, we would like to say some words about the use of models and to specify the favorite approaches to be adopted in crises periods, the combined approaches.
1.3 Combined approaches

At present, it seems coherent to believe that the evolution of past risks does not augur the evolution of the 21\textsuperscript{st} century risks anymore. That is why, it is necessary to use exogenous information in addition to historical data to get reliable projections. The main part of actual quantitative market models are only calibrated with past data and do not integrate the increase of the crises probability noticed since the beginning of the 21\textsuperscript{st} century. Consequently, these models do not seem adapted to the current economic environment. Besides, some approaches consist in selecting values by a fundamental study only. This sort of study do not allow an optimal assets allocation for several reasons:

- They are relatively subjective: the manager establishes his own vision of several firms which can be very different from an other manager's one.

- It is not possible to realize a specific analysis for many companies because it is time consuming. Consequently, it is possible to miss performing stocks.

The remarks formulated above explain the reasons why it is advisable to set up mixed approaches. This paper proposes a combined approach adapted to the management of a shares fund:

- Quantitative Part: realization of projections by the use of an economic scenarios generator and definition of utility function allowing the interpretation of the projections.

- Qualitative Part: definition of central scenarios integrated during the realization of the projections through a jumps function. These scenarios must be determined before hand.
2 The economic scenarios generator retained

Economic scenarios generators (ESG) have several advantages, particularly in period of crises:

- They facilitate the decision-making by giving additional indicators.
- They do not suffer from the lack of rationality.
- They are tools allowing to make decisions based on the past and the future.

In the following paragraphs, we justify the choice of the Wilkie’s generator structure and we describe the modifications we brought to adapt the model to the current economic context.

2.1 The Wilkie’s ESG, an efficient and simple cascading structure

The aim of this paper is not to draw a panorama of all the existing ESG. However, we would like to briefly remind the two possible ESG structures:

- Cascading structure: definition of a central value allowing to find the whole state of the market. For instance, as far as the Wilkie’s model is concerned, the inflation is the central value.
- Correlation structure: based on the idea of allowing the available data (historical data) to determine a simultaneous correlation structure between variables and then to model and calibrate them according to this structure.

The choice of the Wilkie’s ESG structure (1985) [1] results from several elements:

- In crises periods, it is better to choose cascading structures because they are more stable than the correlation ones. Besides, a major advantage of these structures is that they are more flexible: it is possible to adapt the ESG while keeping the same structure.
- Simple to use: the equations of the model are simple - no mathematical complexity.
- This ESG have fewer parameters than the Wilkie’s 1995 one [2]: a retrospective study was realized on the Wilkie’s model to test its robustness. It concluded that the Wilkie’s is more successful in its first version (1985) than in its second (1995).
- This ESG is a good balance between statistical complexity and speed of simulation. Let us remind that the complexity of the statistical extensions improves the adequacy to the past without improving inevitably the quality of the forecast.
- Allowing projections over 1 or 5 years.
Fair Valuation of risks

Under Solvency II, the Market Value Margin (MVM) is meant to bring technical provisions to a fair value, and is to be computed using the Cost of Capital approach. In the background lies the Market Consistent economic balance sheet which reflects what Solvency II seeks to achieve: a fair valuation of risks.

Limiting ourselves to the reserve risk only – as will be done in the rest of this note – the following graph shows that the Capital should be sufficient to restore the balance sheet to a fair value of liabilities after a 1 in 200 event:

The scheme below illustrates the ESG structure:

![ESG Structure Diagram]

**Figure 1: Wilkie’s model (1985)**

### 2.2 Critics of Wilkie’s model

The plentiful literature on this model allows avoiding some stumbling blocks and enables this model to evolve. The critics essentially concern the specification and the quality of the model adequacy. We shall limit us to remind the main problems, the reader can find precise details in [5].

Several English actuaries strongly criticized the model: in particular Kitts (1990), Huber (1995) and Geoghegan and al. (1992). The critics mainly concerned two elements:

- The model specification and the initial parameters dependency: Huber (1995) has underlined that the over parametrization of the model implies a strong dependency of the results on the initial parameters. The equations, containing a mean reverting process, and the choice of the mean condition the future results and the projections. Consequently, specifications choices must be made with caution.

- The biases linked to the estimation: Kitts (1990) has shown that the series of inflation, initially used by Wilkie, is not a stationary one. Therefore, he has criticized the modeling of the inflation by an auto-regressive one order process. Geoghegan and al. (1992) have demonstrated that the model residuals do not respect the normality, independence and constant variance hypotheses. The residuals distribution is asymmetric and leptokurtic and residuals have strong volatility periods alternating with low volatility ones. Finally, Huber (1995) has proved that the Wilkie’s model is biased by some data corresponding in the years of strong inflationary shocks in Great Britain (on 1920, 1940 and 1974). If these years are excluded from the regression, the correlation coefficient between dividend yields and inflation is not significant anymore.

There are also theoretical critics again Wilkie’s model. Indeed, two central economic theory assumptions are not respected: the financial markets efficiency and the absence of arbitrage opportunity. However, it turns out that these two critics are challenged in crisis period and that most of
the asset management models do not respect these two principles. The improvements brought to the initial ESG are described in the following paragraph. Some of them consist in answering the various critics presented above.

### 2.3 Adjustments on Wilkie’s model

To create an ESG adapted to the current context, several modifications have been brought to the basic Wilkie’s structure:

- Modification of data,
- Modification of the modeling of inflation,
- Modification of the residuals distribution,
- Integration of a jumps function.

The fourth element being an important point of this paper, we shall dedicate it the third paragraph.

#### 2.3.1 Modification of used data

To take into account Kitts’s critics and to obtain parameters reflecting the current economic context, monthly data over the period [2000; 2011] are considered. This period seems appropriate to take into account the politico-economic context changes that was discussed in the first paragraph. Besides, having defined this period, the consideration of annual or quarterly data would not be appropriate to obtain reliable modeling.

To illustrate our work, calculations were realized for three countries: France, Germany and the United States.

#### 2.3.2 Modification of the modeling of inflation

In a first approach, we have considered the inflation over a large period, including oil crises, the world wars... We quickly realized that modeling such a series by an auto-regressive order one process was not adapted. Furthermore, a brief study led to the choice to consider the core inflation because during the last ten years, the prices of foodstuffs and oil price were very volatile.

Then, a temporal series study for every country, by following the Box and Jenkins algorithm and by considering all the SARIMA processes, allows to get stationary series and to identify the optimal number of delays to be considered. The ACF and PACF let appear an annual seasonality, validated by the plot of Fourier’s spectrogram.
Fair Valuation of risks

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Limiting ourselves to the reserve risk only – as will be done in the rest of this note – the following graph shows that the Capital should be sufficient to restore the balance sheet to a fair value of liabilities after a 1 in 200 event:

For Solvency II, the Solvency Capital Requirement (SCR) is meant to cover one year of deterioration, meaning that only “shocks” applied to the following year are considered. The graph depicts, on the liability side of the economic balance sheet, how the capital funded at time $t=0$ is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year, where both the Best Estimate of Liabilities (BEL) and the MVM are subject to a distressed scenario.

Cost of Capital approach

The CoC approach takes the perspective that sufficient capital is needed to be able to run-off the business. Here, the risk margin is estimated by the present value of the expected SCR for non-hedgeable risks to support the complete run-off of all liabilities.

Schematically, the MVM calculation can be carried out in 4 steps:
- First, project the expected SCR until all liabilities run-off. This puts into the equations the fact that an undertaking taking over the portfolio has to put up future regulatory capital $\text{SCR}(1), \text{SCR}(2), \ldots, \text{SCR}(n-1)$ until the portfolio has run-off completely at time $t=n$;
- Second, multiply all current and future SCR by the Cost of Capital rate ($c$ or CoC). This captures the fact that the insurer selling the portfolio has to compensate the insurer taking over the portfolio for immobilizing future capital requirements;
- Third, discount everything to time $0$;
- The sum then gives the CoC risk margin.

Figure 2: ACF-PACF Inflation - France

From a macroeconomic point of view, it is clear that the inflation index is sensitive to the seasonality. Indeed, prices undergo different season variations: in December for example, Christmas has a cyclical effect on prices.

Finally, the modeling retained to model the inflation is the following seasonal auto-regressive integrated moving average process: SARIMA(1,1,1)(0,0,1)12.

To validate the use of this modeling, we have to make sure that the obtained residuals are white noises with the Breusch-Godfrey’s test: the latter concludes the rejection of the normality hypothesis. To resolve this problem, it is possible to model the residuals volatility by a GARCH process: Wilkie proposed this modeling in the 1995 version. Studies proved that the use of this method was less effective than the initial one.

In this paper, we propose to model the residuals by a Pareto Hybride distribution introduced in [6] as we developed in the following paragraphs.

2.3.3 Modification of the residuals distribution

To reach a statistically coherent model, it is interesting to look for distribution, easily implementable, and allowing to adapt itself better to the residuals of the equations. In our study, we have chosen to model residuals by an hybrid Pareto distribution.

This distribution is built in the following way: the center is modeled by a normal law and the distribution tails by a generalized Pareto distribution. By fixing continuity, differentiability and density conditions, a statistical law depending on four parameters (average, variance and parameters of tail) is obtained.

Schematically, below the density of an hybrid Pareto distribution:
Under Solvency II, the Market Value Margin (MVM) is meant to bring technical provisions to a fair value, and is to be computed using the Cost of Capital approach. In the background lies the Market Consistent economic balance sheet which reflects what Solvency II seeks to achieve: a fair valuation of risks.

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Cost of Capital approach

The CoC approach takes the perspective that sufficient capital is needed to be able to run-off the business. Here, the risk margin is estimated by the present value of the expected cost of current and future SCRs for non-hedgeable risks to support the complete run-off of all liabilities.

Schematically, the MVM calculation can be carried out in 4 steps:
- First, project the expected SCR until all liabilities run-off. This puts into the equations the fact that an undertaking taking over the portfolio has to put up future regulatory capital \( \text{SCR}(1), \text{SCR}(2), \ldots, \text{SCR}(n-1) \) until the portfolio has run-off completely at time \( t=n \);
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- Third, discount everything to time \( 0 \);
- The sum then gives the CoC risk margin.

Figure 3: Hybrid-Pareto distribution

It is possible to adjust an hybrid Pareto distribution to a set of data by applying classical methods such as the moments method or the maximum likelihood. Other more specific methods exist as the Zhang’s estimator.

The reader will find more precise information on various methods of adjustment by consulting [6]. The modeling by an hybrid Pareto distribution is effective mainly in period of crises. Indeed, for these periods, the normal law leads to an under-estimate of extreme values.
3 Integrate exogenous elements into the ESG structure

The aim of this paper is to be adapted to the current economic context. Naturally, the modified Wilkie’s ESG presented in the previous paragraph is not reasonable and it is advisable to add a jumps function allowing to integrate crises scenarios during the projections. At first, a study to compare the performance of various leading indicators and to select one is realized. It is based on [10] and [11]. These indicators turn out to be a precious tool on whom the assets managers can lean to realize the scenarios. Secondly, various methods allowing to integrate jumps are presented and the choice of one of them justified.

3.1 Composite leading indicators

The composite leading indicators are descriptive chronological series which provide early signals of turning points (peaks and troughs) between economic activity expansions and slowdowns. The underlying principle is simple: the delay registered to provide the main economic statistics given, we try to identify a set of variables (called leading indicators) available faster and which evolve before the reference series. We would like to underline that use of leading indicators faces the risks of false signals and missing signals. It is thus essential to test their reliability and their forecasting power.

The study carried out consist in considering successively two reference series (the gross domestic product and the consumer price index) to test the reliability of four leading indicators (the CLI, the composite leading indicator of OECD; the SP500 index; the ECRI; and the conference Board). The consideration of the one or other one of these series is equivalent to plan the crises or the recessions. The study shows that, over the period [2000;2011], the Composite Leading Indicator (CLI) is the most successful to forecast peaks and troughs, in particular for the United States. However, it is advisable to remain watchful because this indicator tends to plan too many peaks.

Besides, the study has allowed to define the average lead for the studied countries:

- France: 6 months
- Germany: 4 months
- The United States: 5 months.

Let us add that the CLI is not successful for BRICs and that it is only robust for the western countries.

Moreover, the consideration of the monthly calculated index of the Institute **fur Wirtschaftsforschung** (IFO), which allows to follow the confidence of 7000 German managers, is also interested. Indeed, due to the German’s economy importance in the Eurozone, the IFO value can influence largely the euro level on the foreign exchange market and thus the market evolution. The average lead of this
index is about six months. Moreover, the IFO possesses the particularity to have not missed any recession since 1993. Consequently, we shall be interested in its evolution to create our scenarios.

Before having studied economic leading indicators and having built his own opinion of the market, the fund manager is able to define several scenarios of the following type: 

*There is a probability of a\% that a crisis generating a decrease/increase of shares prices of b\% and a decrease/increase of the inflation of c\% on a temporal horizon of d month with a return to the balance of e month(s) occurred.*

It is to integrate this type of scenarios during the projections that we define a jumps function.

### 3.2 Non retained methods

The originality of this paper is the fact that we consider the current economic environment and we allow taking into account scenario of crises linked to exogenous events. Since the beginning of the 21st century, the crises risks have been grown considerably as well as their probability. Consequently, it is not conceivable to build models without being able to take into account them anymore.

The literature concerning the introduction of jumps in a temporal series is not very plentiful. Indeed, these techniques were not too much studied yet, in particular for financial series. That is why, at first, we have tested simple methods which we did not retain afterward.

#### 3.2.1 Modification of residuals parameters

This method consists in allowing jumps introduction into a temporal series by modifying the residuals parameters according to the realization of an uniform distribution.

This method was proposed in [9] and applied to an order one auto regressive process. In this case, residuals are only modeled by a normal random variable. We applied this method to our model but it seems not adapted for two reasons:

- The part *moving average* of the processes into which we wish to integrate crises is not as simple as the one of an auto-regressive order one process,
- This method, even in the case of a modeling by an order one auto regressive process, seems inadequate: indeed, a potential shock is translated by a modification of the mathematical expectation and the variance of the normal random variable. So, the obtained residuals can lose their character *white noises*. Besides, how to modify the variance in an effective way?

#### 3.2.2 The use of Poisson distribution

Naturally, the use of a Poisson process, rare events distribution, seems adapted to the crises modeling. For around thirty years, the scope of this distribution has considerably widened, which makes us think that it is possible to adapt it in our case.
This method was not retained for two reasons:

- The consideration of a jump translates the whole curve and does not plan a return to balance. Consequently, the inflation decrease, for example, is never really compensated. This does not correspond to the reality. Indeed, usually, crisis owes the time required for the correction of speculative excesses and a return to balance is observed after the disappearance of these excesses.

- Adding jumps inside the process implies that predicted values take into account possible jumps and this lead to a biased modeling.

In conclusion, the integration of a simple Poisson process can be envisaged. However, it is possible to define a method which allows, for instance, a return to the balance. This is the object of the next part.

### 3.3 The chosen method

The Box and Tiao’s algorithm allows to take into account disturbances arising on chronological series. The contribution of the application of this method in a SARIMA modeling is mainly due to the quantity of information available to model this series: the Box and Jenkins’s approach uses the quantitative information contained in the data whereas this one allows to add qualitative information via exogenous binary variables. Ferrara and Guegan describe it with RATP data in [7].

The intervention model proposed by Box and Tiao in [8] is written as:

\[
X_t = C + \frac{\omega(B) \times B^b}{\delta(B)} \xi_t + \text{SARIMA}_t
\]

With:

- \(X_t\): series to be modeled,
- \(\omega\) and \(\delta\): polynomials which degrees are to be determined,
- \(B\): delay operator,
- \(C\): constant to be determined.

The determinist function \(\delta^{-1}(B)\omega(B)B^b\xi_t\) represents the intervention effect which is added to the model SARIMA. It is called transfer function.

More generally, the chronological series can be disturbed by \(k\) interventions. With the previous notations, the intervention model is written as:
Fair Valuation of risks

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- Third, discount everything to time \( 0 \);
- The sum then gives the CoC risk margin.

Moreover, various transfer function can be considered.

### 3.3.1 Additive outlier model

First of all, we assume that we observe a trajectory which is the realization of a SARIMA process. Then, we assume that at a given time, \( t = t' \), the evolution of this trajectory is disturbed by an outside shock.

The intervention model is written as:

\[
X_t = C + \sum_{j=1}^{k} \frac{\omega_j(B) \times B_j^{(j)}}{\delta_j(B)} \xi_j^{(T_j)} + \text{SARIMA}_t
\]

In this paper, the perturbations dates, the scale modifications and the speed of return to balance are not known: it is by attribute probabilities to each parameters that the manager will be able to integrate his scenarios.

The parameter \( \omega_0 \) represents the impact of the outside shock on the series. It estimation allows to give a measure to the impact. In our case, the value is determined in the scenarios.

### 3.3.2 Temporary change model

We can also assume that the intervention comes to disrupt the series with an effect which progressively exponentially decreases. This intervention can be illustrated by the graph below:

![Figure 4: Temporary change model](image)

The intervention model is written as:

\[
X_t = C + \frac{\omega_0}{1 - \lambda B} \times P_t^{(t')} + \text{SARIMA}_t
\]

The parameter \( \lambda \) is within the range \([0;1]\): it measures the speed of the intervention effect erasing. If it is close to zero, it means that the impact is almost punctual (previous case) and if it is close to one, it means that the impact goes on in time.
3.3.3 Transient change model with level Shift

Finally, it is possible to assume that the perturbation will affect the evolution of the series with a temporary change and level shift.

The intervention model is written as:

\[ X_t = C + \left( \frac{\omega_0}{1 - \lambda B} + \frac{\omega_1}{1 - B} \right) \times P^{(r)}_t + \text{SARIMA}_t \]

The parameter \( \omega_1 \) represents the level of shift consecutive to the shock: it is supposed to be positive if the change observed after the shock is upper than the level observed before and negative otherwise. The sum of \( s0 \) and \( s1 \) represents the impact of the outside intervention on the series. As well as previously, \( \lambda \) measures the speed of the intervention effect erasing.

3.4 Advantages of the transient change function

We have experienced the various forms of transfer function to finally retain the transient change function. Indeed, this function perfectly represented what we wish to model i.e. shocks not immediately reduced. As we have seen above, it is possible to define a more precise transfer function by considering a temporary change function with level shift. We did not retain this model in order not to over parameter the process. Moreover, we consider that the estimation of the level shift according to experts introduces too much subjectivity.

It is through three jumps parameters (amplitude, probability and parameter of reduction), integrated during the projection by an interface, that the fund manager can integrate his macroeconomic analysis. The coefficients values are determined according to experts.
4 Results of the approach and application

Having described theoretically the model, we would like to illustrate the study by an example. After having defined the frame of the illustration (choice of scenarios etc.), we propose an interpretation of the projections by the establishment of an utility function replacing the traditional average-variance optimization.

4.1 Determination of scenarios and choice of countries

We chose to compare the projections of three countries index: CAC 40 (France), DAX 30 (Germany) and SP500 (The United States). We chose to define five central scenarios presented in the table below:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Inflation</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>-1%</td>
<td>-40%</td>
</tr>
<tr>
<td>2</td>
<td>15%</td>
<td>2%</td>
<td>-20%</td>
</tr>
<tr>
<td>3</td>
<td>45%</td>
<td>0%</td>
<td>-20%</td>
</tr>
<tr>
<td>4</td>
<td>20%</td>
<td>0%</td>
<td>-10%</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
<td>0%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

We would like to make it clear that these scenarios were defined from an economic analysis at one point. The inflation shocks stated in the table above correspond to a curve translation whereas the shock on the shares is defined in percentage of the share value.

4.2 Results of the projections

Graphs below correspond to the evolution of the inflation (1000 projections average) for the three selected countries considering five central scenarios.
Fair Valuation of risks

Under Solvency II, the Market Value Margin (MVM) is meant to bring technical provisions to a fair value, and is to be computed using the Cost of Capital approach. In the background lies the Market Consistent economic balance sheet which reflects what Solvency II seeks to achieve: a fair valuation of risks.

Limiting ourselves to the reserve risk only – as will be done in the rest of this note – the following graph shows that the Capital should be sufficient to restore the balance sheet to a fair value of liabilities after a 1 in 200 event:

For Solvency II, the Solvency Capital Requirement (SCR) is meant to cover one year of deterioration, meaning that only “shocks” applied to the following year are considered. The graph depicts, on the liability side of the economic balance sheet, how the capital funded at time \( t=0 \) is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year, where both the Best Estimate of Liabilities (BEL) and the MVM are subject to a distressed scenario.

Cost of Capital approach

The CoC approach takes the perspective that sufficient capital is needed to be able to run-off the business. Here, the risk margin is estimated by the present value of the expected SCR for non-hedgeable risks to support the complete run-off of all liabilities.

Schematically, the MVM calculation can be carried out in 4 steps:

1. First, project the expected SCR until all liabilities run-off. This puts into the equations the fact that an undertaking taking over the portfolio has to put up future regulatory capital \( \text{SCR}(1), \text{SCR}(2), \ldots, \text{SCR}(n-1) \) until the portfolio has run-off completely at time \( t=n \);
2. Second, multiply all current and future SCR by the Cost of Capital rate (c or CoC). This captures the fact that the insurer selling the portfolio has to compensate the insurer taking over the portfolio for immobilizing future capital requirements;
3. Third, discount everything to time \( 0 \);
4. The sum then gives the CoC risk margin.

4.3 Interpretation of projections

To interpret the projections, we could define an utility function allowing to classify the shares from the more resilient share to the less one. Let us precise that the function used in this paper is quite simple and that a more thorough research will be realized in a next article to define a more adequate utility function.

Traditionally, we distinguish three indicators of risk: the variance, the skewness and the value at risk. We chose not to retain the variance because we consider that this indicator is not relevant in crises periods: no distinction between the negative and positive variations, lot of noise which make the estimation really complicated. We thus preferred a combination of the VAR and of the skewness. This approach tends to favor countries with strong automatic stabilizers (as France).

By considering the following utility function (Expectancy + 0.5VaR + 0.5Skewness), and the central scenarios (Table 1), the calculation leads to favor an investment on France, then on the United States and finally on Germany.

At first sight, this result can be surprising: indeed, at present, Germany seems stronger to face the crises than France, as evidenced by the very low German government bond which even sometimes gives a negative real yield! However, it is possible to explain this result which tends to call into question the skewness use in the utility calculation. Indeed, France, contrary to both countries presented above, has strong stabilizing pillars: the French social security system is very protective and it can turn out useful in times of crisis, in particular to avoid social crises. Consequently, the optimization of a portfolio by a combination of means and skewness favors countries with strong automatic stabilizers and a more detailed study could be interesting.
5 Conclusion

This study gives a way to manage an equity portfolio in an unstable economic context by adopting a cautious approach. The main idea is the presentation of an economic scenarios generator integrated in a general process: we propose a combined approach with both qualitative and quantitative considerations.

On the one hand, we have demonstrated that for the stocks picking and the definition of central scenario, the composite leading indicators are a very useful tool. On the other hand, we have created an ESG adapted to integrate crises scenarios.

To follow up on this study, several works could be led: how can we define a more accurate utility function? How can we modify the ESG to take into account the succession of crises? What processes can be put in place in order to build suitable scenarios for the projections? All these questions will be addressed in a book published in the coming months with Arnaud Clément-Grandcourt. In particular, we will propose a detailed analysis on the management of crises when a change of paradigm occurred. We also provide a large range of possible scenarios for several countries.
Fair Valuation of risks

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Cost of Capital approach

The CoC approach takes the perspective that sufficient capital is needed to be able to run-off the business. Here, the risk margin is estimated by the present value of the expected Cost of Capital (CoC) for current and future SCRs for non-hedgeable risks to support the complete run-off of all liabilities.

Schematically, the MVM calculation can be carried out in 4 steps:

- First, project the expected SCR until all liabilities run-off. This puts into the equations the fact that an undertaking taking over the portfolio has to put up future regulatory capital $\text{SCR}(1), \text{SCR}(2), \ldots, \text{SCR}(n-1)$ until the portfolio has run-off completely at time $t=n$;
- Second, multiply all current and future SCR by the Cost of Capital rate ($c$ or CoC). This captures the fact that the insurer selling the portfolio has to compensate the insurer taking over the portfolio for immobilizing future capital requirements;
- Third, discount everything to time $t=0$;
- The sum then gives the CoC risk margin.

References

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