FROM PRINCIPLE-BASED RISK MANAGEMENT TO SOLVENCY REQUIREMENTS

Analytical Framework for the Swiss Solvency Test

SCOR Switzerland AG*


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Prefaces

In an international well diversified reinsurance company, the ability to model properly the different types of risks is central for the strategic management and the key business decision processes.

SCOR has put this modelling skill at the heart of its organisation and is actively developing a coherent and complete Enterprise Risk Management framework based on its internal modelling capabilities.

Diversification of risks and optimal capital allocation are key for our company and has been central during the elaboration of the strategic Dynamic Lift Plan. The diversification of risks has been one of the main triggers for the mergers with Revios in 2006 and Converium in 2007.

SCOR’s own way in the development of appropriate tools and sound methodology has been fertilized through the intense development of academic papers and practical developments on capital allocation and risk measurement in regard with the new European insurance regulation schemes, namely the Swiss Solvency Test in Switzerland and the Solvency II project in the European Union.

The merger with Converium gave to the SCOR Group the benefit of all the studies, actuarial, financial and information technology investments implemented in order to fulfill all the requirements of the Swiss Solvency Test regulation. It offers to the entire Group the opportunity of acquiring a concrete experience of the implementation of the Asset Liabilities Management process in line with the Solvency II project in 2012.

For the sake of transparency, it has been decided to disclose the entire documentation regarding the methodologies used for the needs of financial and risks modelling. It is the main reason for the publication of this book which is a testimony of the talent and the skill of the teams in charge and which will give to any expert the possibility to understand the way SCOR has chosen to take into account every risk and the dependencies between risks.
I shall not go into the different chapters of the book, the executive summary chapter helps to understand the general architecture and the key points. I will just put some emphasis on the Economic Scenario Generator (ESG) which has been created and developed by the modelling team itself using a resampling methodology through the bootstrap techniques. Usually, modellers make use of economic scenario generators developed by software firms or economic information providers. In our case this innovative approach is crucial as a masterpiece in the modelling scheme and is entirely embedded in the algorithms, which gives an additional efficiency to the ALM process.

We are particularly proud to have been able to extend this internal model through an integrated group organisation which allows now the entire group to benefit from the intellectual and technical skills which had been invested in the past.

After this first success, new projects are already launched in order to maintain state-of-the-art modelling approaches in our company. We are today happy and proud to share this information with different communities: academic, regulators, actuaries and clients of SCOR.

Denis Kessler
Chief Executive Officer
SCOR Group

July, 2008
The merger of the SCOR group with Converium brought about, as part of the “wedding arrangement”, a well-advanced corporate asset and liability management according to the requirements of the Swiss Solvency Test (SST).

Putting into place and making such a significant project work, requires the collaboration of a variety of highly skilled professionals, who bring along many years of work experience in the various fields of the project. The fact of sharing this broad knowledge is already an asset to the company processes.

It is important to acknowledge the competence and the attention to quality which have guided the management of this project. In reading this book, anyone can appreciate the quality of work done and the competence of the project’s participants who compiled this extensive SST documentation.

Beyond the accomplishment of the Zurich entity, which is remarkable in itself, the SCOR group finds it a very useful experience for its other European entities. This will enable the group to comply with the risk model required by Solvency II in the future, which is based on concepts very close to those of the SST. For satisfying the future requirements by the European Union’s regulations, the rapid implementation of the SST will allow us to bring valuable experience to the group for choosing the appropriate tools and methods going forward.

Thus, the important investment and the accumulation of knowledge made over time by the teams of SCOR Switzerland (former Converium) will enable us not only to satisfy the expectations of the Swiss regulator but undoubtedly represent an advantage for the SCOR group as a whole.

Beyond the regulatory requirements, the understanding of the concepts as well as results of risk modeling is essential in all sectors of the group. The whole set of rules, processes, and methods that accompany the implementation of those models are vital in developing a coherent Entreprise Risk Management process. This process allows the company to optimally allo-
cate the capital necessary for the business; a framework of a successful and coherent risk and reward strategy.

This decisive step for the future of the group is now operational. Here-with we would like to thank all the people who have contributed to the SST documentation, in particular Michel Dacorogna and his team whose talent and enthusiasm have brought this project to a successful end which was complex by nature and required careful and skilful handling.

Jean-Luc Besson
Chief Risk Officer
SCOR Group

April, 2008
Acknowledgements

This documentation is a testimony to the long-term commitment of Zurich Re, Converium and SCOR with regard to the development of strong quantitative tools and processes for managing the risks of our company. This five hundred page book describes the results of many years of discussions and work on the best way to model the great variety of risks in a reinsurance company. The Swiss Solvency Test (SST) required us to describe our models and processes and gave us the opportunity to look back critically at all the developments in those areas. When it comes to acknowledging the many contributions, it is difficult not to forget someone, since so many people were involved in this large project over the years.

First of all our thanks go out to all the contributors of this book who are also listed in a separate section. They have spent time and energy and shown their enthusiasm in describing the content of their daily work. Markus Gut was responsible for editing the modelling part. Although coming from a very different background, he gained the respect of the experts he worked with through his dedication and diligence he brought to this task. The quality of the set up of this documentation testifies to his important contribution. Martin Skalsky was the editor for the process part. Thanks to his quick grasp of the facts and his independent view, he managed to describe the SST-related processes in easily comprehensible and illustrative ways. He was supported by Roland Bürgi who added the technical element thanks to his thorough understanding of processes and systems. Roland was also the author of the system description which demonstrates his qualities as a system architect. The executive summary was written by Michael Moller. His technical know-how as well as his detailed knowledge of the internal model were indispensable in the description of the concept the model is built on. All the other experts cited in the list of contributors have produced equally high quality descriptions of their areas of responsibilities that make this text a living document explaining in great detail the various aspects of modelling our business.

We would also like here to acknowledge the work of all the members of the SST core team who have contributed to the progress of the project.
and gave their support to this long-term effort over the years since April 2006. The members of the steering committee have been a great support for this endeavour. We want to particularly thank Andy Zdrenyk and Paolo de Martin whose strong support reflected the importance of this documentation for the entire company. Last but not least, we would like to point out the two project managers who accompanied us along these months with their commitment and dedication through all the ups and downs of such a demanding project: Jindrich Wagner and Viviane Engelmayer.

You all made this project possible and gave us a sense of pride for the development of our internal model and its enhancements that will ultimately allow us to fulfil the regulators’ requirements.

Michel Dacorogna
Head of Group Financial Modeling
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April, 2008
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Executive Summary

This SST-model documentation will give the reader of the SST report the orientation about the origin, purpose and interpretation of the results stemming from SCOR’s internal model. As the results of the internal model are used by SCOR’s top management for their decision findings, a clear understanding of the model framework should be supported by this documentation. To mention some of the major decisions backed by the internal model results, there are the need for the overall capital and hybrid debt requirements (implying an assessment of the equity dimension of hybrid instruments), the strategic asset allocation (based on different risk-return outcomes of the company’s value depending on different asset allocations), the capital allocation to major risk sources (based on the contributions to the overall risk of a risk driver) and, therefore, the future direction of SCOR’s business and, last but not least, risk mitigation measures (such as the impact of retrocession programmes).

However, the focus of the SST-Report lies in the thorough investigation and analysis of the Solvency capital requirements (SCR), i.e. the difference between risk-bearing capital and target capital, based on the valuation of the economic balance sheet of the legal entities of the company at the beginning and the end of the related period.

The definitions of the terms related to the SCR are outlined in several documents published by the FOPI (see, for instance, “Technisches Dokument zum Swiss Solvency Test” Version 2 October 2006, p. 3/4 and also in Part I of this documentation). In general, risk-bearing capital is measured as the difference of the market value of assets and the best estimate of liabilities assuming a run-off perspective after one year, even if some assets or liabilities are not on the formal balance sheet after one year, e.g. embedded value of the life business, whereas the target capital requirement is computed as the difference between the expected economic risk-bearing capital at the beginning of the period and the expected shortfall at risk tolerance level 1% of the risk-bearing capital at the end of the period under consideration plus the cost of capital required to settle the associated run-off portfolio, i.e. the market value margin (MVM).

Any assessment of the future development of assets and liabilities depending on all relevant risk factors is based on a Monte Carlo simulation
approach. The major risk factor categories set by the FOPI comprise insurance risk, market risk and credit risk which are further split into several sub-risks. Many scenarios are stochastically generated for all these individual risk factors and applied to the affected economic balance sheet quantities by aggregating the individual items using a hierarchical copula-dependency tree approach for the new business risk, correlation for the dependencies between run-off lobs and regression-type dependencies between assets and liabilities, if necessary. It has to be noted that the different risks are consistently generated and their mutual dependence is fully taken into consideration. Other risks like operational risk or liquidity risk are also considered in the internal model but are partly not (yet) included in the detailed SCR calculations.

The general outline of this documentation is as follows: The first three parts of this SST-documentation investigate in detail the modeling approaches for the insurance, market and credit risk. Part IV deals with operational risk and emerging risks. Part V describes the valuation of the economic balance-sheet items from the output perspective. The resulting economic shareholders-equity position (or “ultimate” economic valuation of the legal entity) constitutes the basis for the ultimate SCR computations that are outlined in Part VI. The last part will give the reader a brief overview on qualitative risk factors that are analyzed in the context of ERM but not explicitly considered for the SST from a quantitative point of view. It finishes with an outlook to further enhancements of the model that are envisaged in the short to mid-term.

The internal modeling of the insurance risk affects several dimensions. The main categories are Life versus Non-life risks and New business versus Run-off risks, respectively. Non-Life insurance risks are generally modeled as aggregate loss distributions per major line of business (e.g. Motor, Liability, Property and Property Cat). The aggregate loss distribution of a line of business is itself the convolution of the ultimate loss distributions per contract level. The aggregation process of all the loss distributions follows a hierarchical dependency tree that assumes at each of its nodes a copula dependency.

For lines of business with major excess of loss retrocession, a frequency-severity distribution model approach is used. A very important example of this is currently (i.e. 2007) SCOR’s cat business where cat frequency scenarios are linked to the regional severity distribution which is itself a function of the underlying exposure data. Details related to the modeling of the nat cat risks can be found in Section 3.8. The Credit & Surety loss model is based on a compound Poisson loss model. For details on the generation of the probability to default and the loss given default distribution the reader may refer to Section 3.6.
The Non-Life Run-off risk is basically modeled using the Merz-Wüthrich approach (Section 3.9 of this documentation). Contrary to the original Mack-method estimating reserve risk volatility based on the ultimate reserve risk, the Merz-Wüthrich approach focuses on the reserve risk volatility of the one year change (“volatility of the triangle diagonal”) which is the appropriate view from the SST perspective.

Several sub-risk models have been built for the Life risk, see Chapter 4 for details. The main building blocks are risk models for guaranteed minimum death benefits (GMDB), Financial Reinsurance, Fluctuation risk, trend risk and life reserve risk. The GMDB-exposure is in general affected by capital market risks and therefore modeled using a replicating portfolio. The other inherent risks, namely mortality risk and policyholder behavior are either deemed immaterial or modeled together with other risk buckets. The current Financial Reinsurance contracts mainly exhibit credit risk as the cedent could eventually go bust. Other risks like higher mortality or higher lapse rates are negligible due to the specific contract features. Stochastic credit defaults stemming from the credit risk model are applied to the deterministic cash flow projection of the non-default case of the financial reinsurance. Trend risk mainly (e.g. parameter uncertainty) affects long-duration business and is modeled as the distribution of NPVs of future stochastic cash flows. Fluctuation risk resulting from morbidity, mortality or accident risks is modeled as an aggregate loss distribution and relates especially to short- and long duration business to reflect short-term loss variability.

Capital allocation is provided according to the contribution to expected shortfall principle, see Section 8.3.

The market risk, i.e. the risk of changes of the economic environment, relates primarily, but not exclusively, to the economic value change of the invested assets of the company. Of course, the change in interest rates affects economically all balance sheet positions that are or have to be discounted like non-life reserves or reinsurance assets and the valuation of bond investments or interest rate hedges simultaneously.

Most important changes of the market environment are related to a change in interest rates, a drop in equity investments, a crisis of the real estate market, the risk of hedge fund investments and, last but not least, the risk of changes in one or more currencies. In order to cope with these risks in a consistent manner, SCOR uses a bootstrapping Economic Scenario Generator (Chapter 10, “Bootstrapping the Economy”) whose basic idea consists of the simultaneous resampling of historical innovations for all economic variables for a certain point in time. This procedure reproduces the dependency structure between all the different economic variables given a certain point in time. Specific treatments have to be applied to guarantee the generation of positive interest rates over time (“weak mean reversion
force”), the volatility clustering of periods with extreme equity returns or to adjust currency movements via interest rate differential and purchase power parity over time. Special emphasis has to be given to the “fat tail correction” during the economic scenario generation. This ensures the occurrence of very bad economic outcomes even though they are not part of the historical sample on which the bootstrapping mechanism is based.

It is assumed that the invested assets of the legal entities are sufficiently diversified to be reasonably represented by bootstrapped economic variables like MSCI-equity indices or Hedge Fund indices for the different currency regions. The generation of Economic Scenarios is outlined in detail in Chapter 10 of this documentation.

The following three chapters “Interest Rate Hedges” (Chapter 11), “Bond portfolio management” (Chapter 12) and “Accounting of an equity portfolio” (Chapter 13) analyze in detail the impact of economic variables on the business logic of bonds and equity-type investments (including hedge funds and real estate investment) from all accounting perspectives.

Interest rate hedges consist here of swaptions and structured notes. As their value increases with an increase of the interest rates, the purpose of these instruments is a lowering of the nominal interest rate risk of the fixed income instruments (see the Section 6). The modeling of the bond portfolio analyzes the effects of using stylized bond portfolios (“model points”), rollover strategies and the effects of interest rate and currency changes on the accounting and economic valuation of the bonds.

Similarly, the model of the stylized equity portfolio contains the translation of economic equity return and currency changes to the behavior of the equity investments of the company including all relevant accounting figures like dividend payments, realized and unrealized gains, currency translation adjustments etc.

The Chapter “Foreign Exchange Risk” (Chapter 15) outlines to the reader the approximation formula currently used to assess the overall impact of changes in foreign exchange rates on the ultimate shareholders equity position of the balance sheet. The formula gives the estimate of the FX rate change effects under the condition that the liability side is currently just modeled in the accounting currency.

The modeling of credit risk, i.e. the risk that obligors from a very general perspective do not fulfill their obligations, uses a unified approach to the major different types of credit risk and is outlined in Part III of this documentation. The credit risk model basically relates a general market credit worthiness to the individual rating or default probability of the related individual asset under credit risk. The asset types considered contain the risk of default of (mainly corporate) bonds, reinsurance assets or other assets held by cedents.
Part V, “economic balance sheet,” focuses on the output perspective. It describes the dynamic aggregation process of the individual stochastic economic building blocks (like “new business premiums, expenses and losses,” “new reserves,” “development of invested assets”) in order to arrive at the valuation of the economic balance sheet items of the company. A summary of the valuation of the economic balance sheet items of the legal entities subject to the individual risk factors is given. Most of the invested assets are marked to market whereas the majority of the liability side of the balance sheet is marked to model. Some off-balance sheet items, especially for life like present value of future profits are additionally considered and can not be categorized as “asset” or “liability” item. Other existing balance sheet items like certain deferred acquisition costs in life will be replaced by the net present value of the related future cash flows.

Furthermore, basic considerations related to the fungibility of capital are explained in Part V as the ability or contractual duties to transfer capital or risk from one legal entity to the other immediately affects the risk-bearing capital capacity of these legal entities. In terms of economic balance sheet items the valuation of participations (based on the economic run-off book value of the subsidiary), internal retrocession and loans are considered as they appear on the non-consolidated balance sheet of a legal entity. But also the off-balance sheet effect of parental guarantees and the ability to sell a participation is taken into account with respect to the assessment of risk-bearing capital of a legal entity.

The valuation of the economic balance sheet results of the legal entities is used as input for the Part VI where the framework how the Solvency Capital Requirements based on risk-bearing capital, target capital and market value margin are calculated is established. The quantification of the capital requirements constitutes the ultimate results of the SST-report.

The SST and ALM processes are documented in Part VII. The main purpose is to explain the processes which lead to the SST results. The ALM process at SCOR Switzerland has been enhanced to be fully in compliance with the SST requirements, set up by the Federal Office of Private Insurance. The design of the ALM modules and the process steps in particular are documented. The high-level data flows and process steps are specified with an emphasis also on the roles, responsibilities and sign-off procedures.

An overview of the IT systems which are relevant for the calculation of SST results is given in Part VIII. This part of the documentation discusses the high level architecture alongside with the some new developments. The IT systems are set in relation with the process modules of Part VII.
Part IX gives an outlook on the current stage of the internal risk model and the potential for future improvements. Model implementation of management rules related to dividend policy, capital management (hybrid debt versus equity), strategic asset allocation, risk mitigation or future business mix during the reinsurance cycle are some of the possibilities for future enhancements.
I

Insurance Risk
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1 Preliminaries

Financial instruments are denoted by letters such as $X, Y, Z, L$ etc. Monetary amounts corresponding to cash flows from these financial instruments are in general random variables, and we denote them by $X, Y, Z, L$, etc. The random variables are defined on a suitable probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with probability measure $\mathbb{P}$.

Let time in years be denoted by $t$, starting at $t = 0$. Year $i = 0$ denotes the period from $t = 0$ to $t = 1$, year $i = 1$ from $t = 1$ to $t = 2$ etc.

We express the information available at time $t \in [0, T]$ by a filtration $\mathcal{F}_t$ on the probability space, where for any $t \geq 0$, $\mathcal{F}_t$ is a $\sigma$-algebra on $\Omega$ and, with $T$ denoting the terminal time,

$$ \mathcal{F}_0 = \{\emptyset, \Omega\}, \quad \mathcal{F}_T = \mathcal{A}, \quad \mathcal{F}_s \subseteq \mathcal{F}_t \quad \text{for} \quad s \leq t. $$

For a random variable $X$ we denote by $F_X$ its distribution function

$$ F_X : \mathbb{R} \to [0, 1], \quad x \mapsto \mathbb{P}[X \leq x]. $$

To express risk-free discounting of a cash flow $x_t$ occurring at time $t$ discounted to time $s \leq t$, we write

$$ \text{pv}_{(t \to s)}(x_t), $$

which is to be understood as the value at time $s$ of a risk-free zero-coupon bond with face value $x_t$ maturing at time $t$ in the appropriate currency.
2

Valuation of Insurance Liabilities

The SST is based on an economic balance sheet. This implies in particular that insurance liabilities have to be valued market-consistently. There is no obvious answer to date on how to calculate the value of insurance liabilities. For this reason, we attempt in the following to provide a very general discussion of valuation, which will then lead to the methodology proposed for the SST.

This section is structured as follows. In Section 2.1.1 we introduce the crucial concept of replication for the valuation of financial instruments. In Section 2.1.2 we consider how to treat instruments which cannot be (perfectly) replicated, such as most insurance liabilities. This introduces so-called basis risk. The following Section 2.1.3 specializes this discussion to one particular method of accounting for the basis risk, which in turn can be translated into a cost of capital margin approach.

Section 2.1.4 introduces one particular and common “replicating” portfolio, which is often used (sometimes implicitly) in the context of valuation of insurance liabilities.

In Section 2.2.1, the preceding discussion is applied to present the SST methodology and its underlying assumptions. Our proposed implementation of the SST methodology is outlined in Sections 2.2.2, 2.2.3, and 2.2.4.

Section 2.3.1 outlines the proposed treatment of external retrocession and fungibility in the implementation of the SST. Section 2.3.2 discusses limitations of the proposed approach, and, finally, Section 2.3.3 outlines similarities and differences between the valuation approaches used in the SST and in the pricing of contracts.

We refer the reader to Philipp Keller’s article on internal models for SST, Keller [2007].
2.1 Basics of Valuation

2.1.1 Valuation by Perfect Replication and Allocation

Given a stochastic model for the future cash flows of a liability, the valuation of the liability at time $t$ consists in assigning a monetary amount, the value, to the liability. According to SST principles, the valuation has to be market-consistent.

The valuation of (re)insurance liabilities is a special case of the more general valuation of contingent claims. Contingent claims are financial instruments for which the future claims amount is not fixed, but depends, in the meaning of probability theory, on the future state of the world. To highlight the overall context, we provide in the following some examples and remarks on the valuation of contingent claims, and its relation to the valuation of (re)insurance liabilities.

The market-consistent value of a liability, in case the liability is traded in a liquid market, is simply its market price, i.e. the observed transfer price. Typically, there is no liquid market for the liabilities. In this case, the market-consistent value of such a non-traded financial instrument is taken to be the market value of a replicating portfolio, that is, a portfolio of liquidly and deeply traded financial instruments whose cash flows perfectly replicate the stochastic cash flows of the given instrument. In other words, the replicating portfolio replicates the (future) cash flows of the instrument exactly for any future state of the world. We call this approach valuation by perfect replication.

Of course, market-consistent valuation in this sense is possible only if a replicating portfolio exists. From the point of view of probability theory, perfect replication means replication for any state of the world $\omega \in \Omega$. The actual probabilities assigned to each of these states of the world then become irrelevant. If perfect replication is not possible, a different form of replication could be considered, which we call law replication. Law replication means that we construct a portfolio which has the same probability distribution as the given financial instrument. Assume that valuation of instruments relies on a monetary utility function $U$, or alternatively on a convex risk measure $\rho = -U$, and that the functions $U$, $\rho$ are law-invariant, i.e. they depend only on the law of a random variable, that is, on the distribution of the random variable. Then, with respect to such a monetary utility function, the initial instrument and the law-replicating portfolio have the same value.

Clearly, perfect replication implies law replication but not vice versa. Regarding the latter statement, dependencies to other random variables are the same for the instrument and the portfolio in the case of perfect replication, but not for law replication. In particular, valuation of an instrument by a law-invariant monetary utility function or risk measure will disregard dependencies to other instruments.
We now return to the notion of perfect replication. We can distinguish two types of replicating portfolios: static portfolios, where the replicating portfolio is set up at the beginning and kept constant over time (although, of course, its value probably changes over time); and dynamic portfolios, where the initial portfolio is adjusted over time in a self-financing way (i.e. with no in- or out-flow of money, with the exception of the liability cash flows themselves) as information is revealed.

Uniqueness of the value derived by perfect replication follows from a no-arbitrage argument: If two portfolios of liquidly traded financial instruments perfectly replicate a given instrument, their values have to be equal, because otherwise an arbitrage opportunity would exist. To be more precise, because the market is liquid, the two portfolios can be bought and sold for their value (which is the market value), so the theoretical arbitrage opportunity can be realized.

We mention two well-known examples of valuation by perfect replication. As a first example, consider the value at time $t$ of a deterministic future payment at time $t + d$. This instrument is perfectly replicated by a default-free zero-coupon bond with maturity $d$. Hence, the value of the future payment is equal to the price of the bond at time $t$. The replicating portfolio is clearly static.

As a second example, consider, at time $t$, a stock option with strike date $t + d$. The cash flow at $t + d$ of the option is stochastic, and depends on the value of the stock at $t + d$. Nonetheless, option pricing theory shows that, under certain model assumptions, the option cash flow at $t + d$ can be perfectly replicated by holding at time $t$ a portfolio composed of certain quantities (long or short) of cash and the stock, and by dynamically adjusting this portfolio over time in a self-financing way. So this is a dynamic replicating portfolio. The value of the option at time $t$ is again equal to the price of the replicating portfolio at $t$.

In this example, the stochasticity of the cash flow becomes irrelevant, since perfect replication means that replication works for any state of the world. Assuming the option is part of a larger portfolio of financial instruments, its value is independent of the other instruments in the portfolio, on how their cash flows depend on each other, and on the volume of the portfolio.

Observe that particularly the second example is not realistic in that it does not take into account transaction costs, which are inevitably part of the replication process. Crucially, when taking into account transaction costs, the costs of the replication are likely no longer independent of the overall portfolio. It might be, for instance, that certain transactions needed to dynamically adjust the replicating portfolio for different instruments cancel each other out, so that overall transaction costs are reduced. Therefore, the value of one instrument depends on the overall portfolio of instruments and is no longer unique - even though the replication is perfect and stochasticity
has been removed.

In the option example, the transaction costs cannot be directly assigned to individual instruments due to dependencies between the transactions. I.e. the transaction costs are not a linear function of the financial instruments, in the sense that the transaction costs for two instruments together are not equal to the sum of the transaction costs for each instrument on its own. Thus, a potential benefit (positive or negative) in costs has to be allocated to each instrument. The value of an instrument is then a consequence of this allocation. A reasonable condition on the allocation could be, for instance, that any sub-portfolio of instruments partake in the benefit, i.e. would not be better off on its own. In the language of game theory, the allocation would have to be in the core of a corresponding cooperative game (Lemaire [1984], Denault [2001]).

Having presented two examples of valuation by perfect replication, we return to the general question of valuation. In analyzing the second example above of a stock option, we have seen that the introduction of transaction costs might imply that the value of two replicating portfolios is no longer equal to the sum of the values of the two replicating portfolios on their own.

Thus, a crucial issue with regards to valuation of individual instruments by perfect replication is whether the value function is linear in the replicating portfolio, in the sense that the value (i.e. the price) of a collection of replicating portfolios is equal to the sum of the values of the stand-alone replicating portfolios of that collection.

In the case of non-linearity, valuation by perfect replication makes sense only for the whole liability side of an economic balance sheet, and not for individual liabilities in that balance sheet\(^1\). In contrast, the value of an individual liability is derived from the value of the whole liabilities by allocation. In other words:

- The value of the portfolio of liabilities in an economic balance sheet is derived by perfect replication, if perfect replication is possible. The value is calculated by setting the value equal to the value of the asset side of the balance sheet, where the assets consist of a replicating portfolio.

- The value of an individual instrument in the balance sheet is derived from the value of the whole portfolio by allocation.

As a consequence, assuming perfect replication is possible, a unique value can be assigned to a portfolio of instruments, by considering the portfolio as the liability side of an economic balance sheet. On the other hand, the value of an individual instrument in the portfolio in general depends both

---

\(^1\)In fact, transaction costs have been used in economics (Ronald Coase etc.) to explain why firms exist and to explain their size. I.e. why not all transactions are just carried out in the market, and why the whole economy is not just one big firm.
on the portfolio itself and on the allocation method selected. It will thus, in
general, not be unique. In fact, it is reasonable to require of the allocation
the property that the sum of the individual values is equal to the overall
value. However, this will normally not uniquely determine the allocation
method, and additional requirements will have to be imposed.

The view that the value is defined only with regards to a balance sheet
makes sense when considering the values of produced physical goods: The
price of a good has to take into account all assets of the producing factory
(such as machines, buildings, . . . ) required to produce the good, as well as
all other goods produced by the factory.

There are, however, two aspects not yet considered: perfect replication
may not be possible; and the asset side of the actual balance sheet might
not be equal to the (perfect or imperfect) replicating portfolio. Both aspects
introduce a mismatch between asset and liability side, and are relevant for
the SST.

2.1.2 Imperfect Replication – Optimal Replicating Portfolio,
Basis Risk, Law Replication

We now focus on the valuation of insurance and reinsurance liabilities. In
view of the preceding Section 2.1.1, the value of the whole liability side of an
economic balance sheet is derived by (perfect) replication, and the value of
an individual liability in the balance sheet is derived by allocation from the
value of the whole liability side. In the context of the pricing of reinsurance
contracts, the corresponding allocation methodology we use is described in
Section 8.3 (pp. 168). A comparison of the valuation approaches in the SST
and in (non-life) pricing is given in Section 2.3.3.

Note that the value of the whole liability side of an economic balance
sheet is not independent of whether additional liabilities are added to the
balance sheet in the future. The assumption underlying the calculation of
the value of the liabilities thus has to be that no such additional liabilities
are added in the future.

Typically, insurance and reinsurance liabilities cannot be perfectly repli-
cated. By definition, this means that the cash flows cannot be replicated
for every state of the world by a portfolio of deeply and liquidly traded
instruments.

An optimal replicating portfolio (ORP) denotes a portfolio of deeply and
liquidly traded instruments “best replicating” the cash flows of the liabilities.
We call the mismatch between the actual liability cash flows and the ORP
cash flows the basis risk. We keep open for the moment what exactly is
meant by “best” replication; the idea is to minimize the basis risk. Note that
the condition that the ORP instruments be deeply and liquidly traded is, of
course, essential; otherwise, one could just select the liabilities themselves
for the ORP and have no basis risk left.
Different types of “imperfect replication” are possible: The cash flows might only be replicated for a limited number of possible outcomes; they might be replicated for all outcomes only within a certain error tolerance; or, as a combination of the preceding two, replication might work only for a limited number of outcomes within a certain error tolerance.

The probabilities assigned to those states of the world for which we do not have perfect replication obviously have an impact on the basis risk. Thus, imperfect replication introduces an inherent stochasticity into the task of valuation, expressed by the basis risk. By definition, the stochasticity of the basis risk cannot be replicated by traded instruments, and this means that a different approach is needed to value the basis risk.

Notice here that we do not assume that the expected cash flow mismatch is zero, i.e. we do not per se require the expected values of the ORP and the actual liability cash flows to be equal. This will hold for special types of ORPs, in particular for the “expected cash-flow-replicating portfolio” introduced in Section 2.1.4.

We call a pseudo-replicating portfolio a portfolio of financial instruments which replicates the cash flows for any state of the world, but which contains instruments which are not deeply and liquidly traded. In the following, we look for pseudo-replicating portfolios or, equivalently, try to express the basis risk in terms of financial instruments. So we need additional instruments to account for the difference between liability and ORP cash flows. The instrument replicating one particular cash flow is composed of a cash-in option, providing the difference if the liability cash flow exceeds the ORP cash flow (a call option), and a cash-out option, allowing to pay out the excess amount otherwise (a put option). Thus, the basis risk is replicated by the sequence of cash-in and cash-out options, with one pair of options for every time a cash flow occurs. Note that the optionality in the cash option is due to the fact that the subject amounts are stochastic.

In reality, firms of course do not guarantee the liability cash flows under every possible state of the world. Given certain outcomes, the firm has the right not to pay out the liability cash flows, so the firm owns a certain default option. Because of this option, the cash flows need to be replicated only on the complement of these outcomes. We denote the set of these states of the world where the default option is not exercised by \( \Omega^* \). From the point of view of the policyholder, whose claims the liability cash flows are supposed to pay, the default option (short) reduces the value of the contract for the policyholder. However, the default option is an integral part of the contract between policyholder and the firm, i.e. the (re)insurance company.

The pseudo-replicating portfolio for the liabilities thus consists of the following financial instruments:

\(^2\)This right, of course, comes from the limited liability of the owners of the firm.
• An optimal replicating portfolio (ORP)
• The cash-in options
• The cash-out options
• The default option

The portfolio of the four instruments above is a pseudo-replicating portfolio since it perfectly replicates the liability cash flows for any state of the world, but it is, of course, not a replicating portfolio as defined in the preceding section for perfect replication, since the cash options are, by definition, not traded\(^3\).

The market-consistent value of the liabilities is then equal to the value of these four financial instruments. For convenience, we call the instrument composed of the cash options and the default option the defaultable cash options.

Note that the value of one particular cash option (for time \(t_k\)) likely depends on time, because, normally, information is revealed as time \(t\) approaches \(t_k\). This introduces an inherent temporality into the valuation, since it matters when a cash option is purchased. It seems reasonable to suppose that the value (the price) of the cash option tends to the actual mismatch amount as \(t \to t_k\). (This makes sense in particular because there is some flexibility with regards to the times the payments have to be made.) As a consequence, a cash option is useless unless it is purchased in advance of the payment time.

Now, the main task is: How to value the defaultable cash options? Since the defaultable cash options account for the basis risk, whose stochasticity cannot be replicated by traded instruments, valuation needs to explicitly take into account the probabilities of different outcomes. A possible approach is by law replication as introduced in the preceding section. By definition, a law-replicating portfolio for the defaultable cash options is a portfolio of traded instruments which has the same law as the defaultable cash options, i.e. the same probability distribution\(^4\). Valuation by law-replication then means that the value of the basis risk (i.e. the value of the defaultable cash options) is defined to be the value of such a law-replicating portfolio.

We have a special case of valuation by law-replication if the value is calculated from a law-invariant risk measure \(\rho\), i.e. a risk measure which

---

\(^3\)The cash options are not traded by definition because they capture the basis risk which, due to the definition of the ORP, constitutes the part of the liabilities which cannot be replicated by traded instruments.

\(^4\)To be more precise, the law of a random variable \(X\) is the probability measure \(P \circ X^{-1}\) induced by \(X\).
only depends on the law, i.e. the distribution of the random variable. For instance (as in Föllmer and Schied [2004], Jouini et al. [2005b], Jouini et al. [2005a]) we might use a monetary utility function $U$, or alternatively a convex risk measure $\rho = -U$, where $U$, $\rho$ are law-invariant.

Valuation by law-replication raises additional issues. First of all, several law-replicating portfolios might exist, and there is a priori no guarantee that they all have the same value. This issue is exacerbated when law-invariant risk measures are used, since risk measures collapse a whole distribution to one number, so instruments with different laws might have the same risk.

Furthermore, because only the law of a random variable is considered in law replication, dependencies to other random variables are not taken into account. Consequently, valuation by law-replication again makes sense only for a whole portfolio of instruments, and the value of an instrument is no longer unique since it both depends on the other instruments in the portfolio and on the allocation method used to derive the value of the instrument from the value of the portfolio.

2.1.3 One-Year Max Covers, Cost of Capital

As introduced in the previous section, the cash options have to provide for the basis risk, i.e. for the cash flow mismatch between liability and ORP cash flows. In conjunction with the default option, the cash options have to provide for the mismatch only for those states of the world for which the default option does not apply. In particular, the total maximal amount to be provided by the defaultable cash options is finite.

Consequently, one possible way to cover the defaultable cash options during one year is by an instrument we call the one-year max cover:

- At the start of the year $i$, borrow the maximum possible positive mismatch amount $K_i$ (based on the available information) for this year.
- At the end of the year, pay back the borrowed amount $K_i$ minus the actual cash flow mismatch (positive or negative), plus a coupon $c_i$.

Valuation of the basis risk then becomes calculation of the required coupons $c_i$ for the one-year max covers.

We can make two important observations concerning the one-year max covers. First of all, notice that the amount $K_i$ is borrowed for a whole year even though this would in theory not be necessary – the amount is only needed at the time the cash flow mismatch occurs. In particular, the selection of a one year time horizon is per se arbitrary. It makes sense, however, as we will see below, in view of the fact that balance sheets are published once a year.

This remark is a first reason why covering the basis risk by the one-year max covers is not optimal. A second reason is a consequence of the following second observation.
The second observation is that the determination of the maximal mismatch amount $K_i$ introduces a temporal aspect with respect to available information: Depending on the point in time $t < i$ the amount $K_i$ is calculated, different amounts will result, because the available information will be different. The temporal stochasticity can be visualized by a graph in the form of a tree branching at each year. States of the world $\omega \in \Omega$ then correspond to paths in the tree. We come back to this observation again below.

To express the cover of the basis risk provided by the one-year max covers, define a super-replicating portfolio for a financial instrument to be a portfolio which provides at least the cash flows required for the instrument, but where either the cash flows are potentially higher than needed, or where money to ensure the cash flows is held longer than necessary. The (market-consistent) value of the liabilities is then bound above by the value of the super-replicating portfolio. The portfolios derived using the one-year max cover will be super-replicating for the reasons mentioned above.

To make the one-year max covers more precise, denote by $X_i$ the liability cash flow in year $i$. Let $\text{ORP}_i$ be the optimal replicating portfolio in year $i$, and denote its cash flow in year $j$ by $f_j(\text{ORP}_i)$. Let $t \leq i$ be the time the basis risk is evaluated, and let $\Omega_{t}^* \subseteq \Omega$ be those states of the world possible given the information at time $t$ where the default option is not exercised. The one-year max cover for year $i$ then has to cover on $\Omega_{t}^*$ the cash flow mismatch

$$M_i := X_i - f_i(\text{ORP}_i).$$

Thus, the borrowed amount $K_i$ is given by

$$K_i = \sup_{\Omega_{t}^*} M_i.$$

Disregarding the coupon payment, the pay-back amount $K_i - M_i$ of the one-year max cover determined at time $t$ satisfies

$$0 \leq K_i - M_i \leq \sup_{\Omega_{t}^*} M_i - \inf_{\Omega_{t}^*} M_i.$$

Note that the expected pay-back amount is not necessarily equal to $K_i$, because we have not imposed the requirement that the expected cash flow mismatch be zero. This implies that the expected value of the one-year max cover cash flow is not necessarily equal to the coupon payment $c_i$, so we cannot in general interpret the coupon payment as the “expected return.” In fact, in theory, the coupon might even be negative.
For this reason, we define an *unbiased optimal replicating portfolio* to be an ORP for which the expected value of any mismatch $M_i$ is zero. Note that this condition has to hold on the more general mismatch amount introduced in (2.8) below.

Nonetheless, the price for a one-year max cover is expressed by the coupon $c_i$ to be paid at the end of year $i$. Thus, at time $t < i$, a portfolio containing a one-year max cover for year $i$ is equivalent to a portfolio containing a *risk-free zero-coupon bond with face value $c_i$ in the appropriate currency and maturity $i-t$*, provided that a *liquidity assumption* is taken concerning the availability of the one-year max covers (since, by definition, there is no deep and liquid market for them). To express the liquidity assumption by a financial instrument, define the *purchase guarantee* to be the guarantee that the one-year max cover for year $i$ can be purchased at the beginning of year $i$. The one-year max cover is then equivalent to the respective zero-coupon bond together with the purchase guarantee.

There are two different strategies for the one-year max covers: In the *first strategy*, all one-year max covers are available at the beginning of year $i = 0$ for all years $i = 0, 1, 2, \ldots$. The *super-replicating portfolio for the first strategy* at $t = 0$ for the liabilities is then given by

- An optimal replicating portfolio
- The one-year max covers
- The default option

In this case, the calculation of the $K_i$ from (2.2) is based on the states of the world $\Omega^*_0$. Visualizing the states of the world by paths in a tree, each $K_i$ needs to be the maximal positive mismatch amount over all nodes in the tree belonging to the year $i$. The sum of these $K_i$s will in general be larger than the maximal amount that would be needed along every path, and thus, the required coupons for the one-year max covers will in general be larger than needed\(^5\).

If the one-year max covers are replaced by the zero-coupon bond and the purchase guarantee, we get a second *super-replicating portfolio* at time $t = 0$ given by

- An optimal replicating portfolio
- The risk-free zero-coupon bonds
- The purchase guarantees

\(^5\)As a simple example, consider a binary tree and binary cash flow mismatches with mismatch amount $K_i > 0$ with probability $p_i > 0$, and $K_i = 0$ otherwise. The required coupon $c_i$ is then a function of $(K_i, p_i)$, and it is reasonable to assume that the coupon is monotone in the sense that, for $K'_i > K_i$ and $p'_i > p_i$, we have $c'_i > c_i$. 

18
The default option

To this super-replicating portfolio corresponds an alternative strategy. In this second strategy, each one-year max cover for year \( i = 0, 1, 2, \ldots \) is purchased at the beginning of the respective year \( i \). The covers are guaranteed to be available due to the purchase guarantee, and the risk-free zero-coupon bonds allow to pay the required coupons \( c_i \) for the one-year max covers.

Denote by \( C_i \) the zero-coupon bond with face value equal to the monetary coupon amount \( c_i \) for the one-year max cover for year \( i \) in the currency of the liability cash flows of year \( i \) maturing at time \( t = i + 1 \). Then, under the second strategy, the value of the basis risk at time \( t \leq i \) is estimated by the value at time \( t \) of the zero-coupon bonds

\[
\sum_{i \geq t} V_t(C_i)
\]

plus the value of the purchase guarantees and the value of the default option.

Consider now for the second strategy the question of how to calculate the face values for the zero-coupon bonds. Per se, the amount has to correspond to the maximal possible mismatch for each year just like in the first strategy, so the nominal sum of the face values is equal to

\[
\sum_{i \geq 0} \sup_{\Omega_0} c_i.
\]

However, because the portfolio of zero-coupon bonds is more “fungible,” the face values can be reduced. This observation is obvious if we assume for a moment a risk-free interest rate of zero. Then, the zero-coupon bond portfolio at \( t = 0 \) can be replaced by a cash amount equal to

\[
\sup_{\Omega_0} \sum_{i \geq 0} c_i,
\]

and (2.5) will typically be smaller than (2.4). Expressed in terms of the tree graph, the sum of the maximum over all nodes of a year can be replaced by the maximum of the sum over all paths. In case the risk-free interest rate is non-zero, the optimal composition of the zero-coupon bond portfolio is more sophisticated, but the same conclusion still applies. For this reason, typically, the second strategy results in a lower value of the liabilities, with the value defined as the value of the super-replicating portfolio.

The one-year max covers ensure that the deviations of the actual liability cash flows to the ORP cash flows can be covered. However, for liabilities assumed by any actual (re)insurance company, an additional restriction applies. The one-year max covers described so far ensure “cash flow” solvency of the company: Any cash flow that needs to be paid can be paid. However, an actual firm needs also to be “balance-sheet” solvent, that is, the
value of the assets has to exceed the value of the liabilities (to demonstrate that its future cash flows can be paid). Formulated differently, every time a balance sheet is published (i.e. at the end of every year), the reserves are (re)calculated as a “best estimate” (according to the applicable accounting rules), given the available information, of the future liability cash flows. The value of the assets then needs to be larger than the reserves; otherwise the company is considered not able to meet its future obligations – even though it would be by purchasing the one-year max covers.

For this reason, the one-year max covers must not only cover deviations in the cash flows for the current year but also changes in the estimate of the future cash flows. To ease notation, we will call this extension of the one-year max cover again the one-year max cover. Note also that the structure of the covers remains essentially the same, and so the remarks from above remain valid.

In a truly economic balance sheet, this best estimate of the future cash flows at time \( t \) can reasonably be taken to correspond to the optimal replicating portfolio \( ORP_t \) set up at time \( t \), taking into account all information available up to time \( t \). Under this assumption, the additional restriction from above is translated into the condition that the existing \( ORP_i \) (set up at time \( t = i \)) be transformed at time \( t = i + 1 \) into the \( ORP_{i+1} \), which is optimal for time \( t = i + 1 \) and thus additionally takes into account the information gained in year \( i \). Whether this can be done in a self-financing way depends on the (market-consistent) value \( V_i \) at time \( t = i + 1 \) of the two portfolios. As a consequence, the one-year max covers for year \( i \) additionally have to cover the difference

\[
V_{i+1}(ORP_{i+1}) - V_{i+1}(ORP_i).
\tag{2.6}
\]

It is reasonable to construct the portfolio \( ORP_i \) so that, given the information at time \( t = i \), the expected portfolio reshuffling is self-financing; i.e. so that the expected value at \( t = i \) of the random variable (2.6) is zero.

For such an economic balance sheet, the amount \( K_i \) that has to be borrowed at the start of year \( i \) for the one-year max cover, determined at time \( t < i \), is given by

\[
K_i = \sup_{\Omega_t^i} M_i
\tag{2.7}
\]

where the mismatch amount \( M_i \) is given by

\[
M_i = \left( \text{pv}_{(i+1/2- i)}(X_i - f_i(ORP_i)) + \right.
\]

\[
\left. + \text{pv}_{(i+1-i)}(V_{i+1}(ORP_{i+1}) - V_{i+1}(ORP_i)) \right),
\tag{2.8}
\]

with “pv” denoting risk-free present value, and where we assume for simplicity that the actual cash flow \( X_i \) occurs at mid year \( i + 1/2 \). Recall that \( \Omega_t^i \) denotes the states of the world possible at time \( t \) where the default option
is not exercised, and that $f_i(\text{ORP}_i)$ denotes the pay out from the ORP in year $i$.

For balance sheets which are not economic in the sense above, it might not be possible to express the additional restriction in terms of values and ORPs. At any rate, the restriction introduces additional frictional costs.

There is an obvious instrument to provide the one-year max covers: the shareholders equity. The equity corresponds to capital put up by outside investors, which in an economic balance sheet corresponds to the difference between the market-consistent value of the assets less the market-consistent value of the liabilities.

In this situation, the composition of the asset side is per se arbitrary, and so the assets do not necessarily replicate the liabilities in the sense of the ORP. This results in an additional cash flow mismatch between assets and liabilities. However, for simplicity we assume in the following that the assets correspond to the required super-replicating portfolio.

Valuation of the basis risk, which we reformulated as valuation of the required coupon for the one-year max covers, is then further translated into the cost of capital margin CoCM approach. In this approach, one needs to determine

- The required shareholder capital $\tilde{K}_i$ for all years $i$
- The cost of capital, i.e. the required return on the capital $\eta_i$

The required return on capital $\eta_i$ is a proportionality factor applied to the required capital $\tilde{K}_i$. Per se, the returns can be different for different years. However, in practice, one usually assumes that the required return is prescribed externally and is constant over the years equal to some $\eta$. The required return on capital has to be understood as an expected value: The expected value of the actual return has to be equal to the required return.

The required capital corresponds to the amounts $K_i$ for the one-year max covers. Usually, to calculate the amounts $K_i$, the supremum over a set $\Omega^*_t$ of states of the world is expressed by choosing an appropriate, usually law-invariant risk measure. For instance, selecting the same law-invariant risk measure $\rho$ for any year $i$, the formula (2.7) for the amounts $K_i$ determined at time $t$ becomes

$$K_i = \rho(M_i)$$

(2.9)

where the distribution of the mismatch $M_i$ given by (2.8) is conditional on the information available at time $t \leq i$.

Since the set $\Omega^*_t$ reflects the states of the world where the default option is not exercised, the selection of the risk measure is directly connected to the (value of the) default option. For instance, selecting as risk measure the expected shortfall at a less conservative level $\alpha = 10\%$ instead of at $\alpha = 1\%$
reduces the amounts $K_i$ and thus the value of the corresponding one-year max cover, but increases the value of the default option\(^6\).

If one chooses the first of the two strategies for the one-year max covers presented above, the required capital $\tilde{K}_i$ for any year $i$ is equal to the discounted sum of future required amount $K_j$ for $j = i, i+1, i+2 \ldots$. On the other hand, if the second strategy is selected, the required capital $\tilde{K}_i$ for any year $i$ is just the amount $K_i$.

In the context of shareholder capital, the two strategies correspond to the question of whether buyers can be found in the future who are going to put up the required capital. In the first strategy, the full amount of required capital for all years $j \geq i$ is put up at time $i$, and no further capital providers have to be found in the future. In the second strategy, on the contrary, the purchase guarantees are used for every year $j \geq i$ to find a provider for the capital required for that year $j$ according to (2.7) with $t = j$.

Thus, using shareholder capital, the super-replicating portfolio for the first strategy at the start of year $i$ is given by

- An optimal replicating portfolio $ORP_i$
- The default option
- Raised shareholder capital $\tilde{K}_i$ equal to

$$\tilde{K}_i = \sum_{j \geq i} pv_{(j-i)}(K_j).$$

- The risk-free zero-coupon bonds $C_j$ for $j \geq i$ to pay out the future required returns on capital

The super-replicating portfolio for the second strategy at the start of year $i$ is given by

- An optimal replicating portfolio $ORP_i$
- The default option
- Raised shareholder capital $\tilde{K}_i = K_i$ to cover deviations in year $i$
- The risk-free zero-coupon bonds $C_j$ for $j \geq i$ to pay out the future required returns on capital
- The purchase guarantees

\(^6\)Thus, a somewhat unsatisfactory aspect of the cost of capital margin approach is that it separates the required capital from the required (percentage) return on capital. The two are related because the amount of capital needed is determined by the default option (or, as a special case, by the selection of the risk measure). But depending on this choice, the resulting distribution of the return on capital will be different and this will impact the requirement on the expected value of the return, i.e. the required return on capital.
Note that the capital is not part of the value of the liabilities, but it is obviously required as part of the super-replicating portfolio.

Further note that the face values $c_i$ of the zero-coupon bonds $C_i$ are not necessarily equal to the required return on capital $\eta \tilde{K}_i$ since the latter has to be the expected value of the actual return. However, the statement holds if the ORP is *unbiased* as defined earlier.

Since the equity has a market value, its value can be compared to the concept of the one-year max covers. The question is then: What instrument is an investor into equity buying, and how does this compare to one-year max covers? For instance, for a firm following the second strategy above, the expected return on the capital amount raised from investors corresponds to a single one-year max cover.

### 2.1.4 Expected Cash Flow Replicating Portfolio

A simple static, unbiased optimal replicating portfolio at time $t$ consists of risk-free zero-coupon bonds in the currency corresponding to the liability cash flows, with maturities matching the expected future cash flows of the liabilities, where the expected cash flows are based on the expected value given the information available up to time $t$. For simplicity, we assume that the cash flow $X_i$ in year $i$ occurs at time $i + 1/2$. We call this ORP the *expected cash-flow-replicating portfolio* $ERP_t$ at time $t$. It consists at time $s \geq t$ of the financial instruments

$$ERP_t = \{ D^{(t)}_i \mid i \geq s \}$$

where $D^{(t)}_i$ is a risk-free zero-coupon bond with maturity $i + 1/2 - s$ and face value $d^{(t)}_i$ given by the expected cash flow in year $i$,

$$d^{(t)}_i = f_i(ERP_t) = \mathbb{E}[X_i \mid \mathcal{F}_t],$$

where $\mathcal{F}_t$ is the information available at time $t$. The value of the zero coupon bond $D^{(t)}_i$ with $t \leq i$ at time $s \leq i$ is thus given by

$$V_s(D^{(t)}_i) = \text{pv}_{(i+1/2 \rightarrow s)}(\mathbb{E}[X_i \mid \mathcal{F}_t]).$$

For the $ERP_t$, the expected values (given information up to time $t$) of the future cash flows are matched with certainty, and basis risk comes from the mismatch between actual cash flows and expected cash flows. The value of the $ERP_t$ is equal to the present value of the expected future cash flows, discounted using the risk-free yield curves of the corresponding currency and the respective maturity of the cash flows. This in turn is equal to the *discounted reserves* in an economic balance sheet, in the sense of discounted best estimate of the liabilities.
Hence, basis risk arises when the actual liability cash flows exceed the expected cash flows, i.e. when the discounted reserves are too small.

The expression (2.6) demonstrating balance sheet solvency for year $i$ in an economic balance sheet becomes

$$V_{i+1}(ERP_{i+1}) - V_{i+1}(ERP_i) = \mathbb{E}[R_{i+1} \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_{i+1} \mid \mathcal{F}_i],$$

where $R_i$ denotes the discounted outstanding reserves at time $i$,

$$R_i := \sum_{j \geq i} \text{pv}_{(j+1/2 \rightarrow i)}(X_j). \quad (2.10)$$

Assuming that we use a law-invariant risk measure $\rho$ to capture the default option, the formula (2.9) for the amounts $K_i$ available at the start of year $i$, determined at time $t \leq i$, can then be written as

$$K_i = \rho \left( \text{pv}_{(i+1/2 \rightarrow i)}(X_i - \mathbb{E}[X_i \mid \mathcal{F}_i]) + \text{pv}_{(i+1 \rightarrow i)}(\mathbb{E}[R_{i+1} \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_{i+1} \mid \mathcal{F}_i]) \right), \quad (2.11)$$

where the distributions of the involved random variables are conditional on the information available at time $t \leq i$. Because $X_i$ is known at time $i + 1$, we have $X_i = \mathbb{E}[X_i \mid \mathcal{F}_{i+1}]$, which implies that (2.11) for the amounts $K_i$ can be written

$$K_i = \rho(M_i),$$

where $M_i$ denotes the random variable of the mismatch

$$M_i = \mathbb{E}[R_i \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_i \mid \mathcal{F}_i],$$

conditional on the information available at time $t \leq i$. The mismatches $M_i$ can be written as the change of the estimated ultimate amounts over a one-year period

$$M_i = \mathbb{E}[R_0 \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_0 \mid \mathcal{F}_i], \quad (2.12)$$

because the cash flows $X_j$ for $j = 0 \ldots i - 1$ are known at times $t = i, i + 1$.

Considering the cash flow of the corresponding one-year max cover, the expected value of the mismatch $M_i$ conditional on the information at time $t \leq i$ is equal to zero. That is, the ERP is an unbiased optimal replicating portfolio. Consequently, the expected value of one-year max cover cash flow is equal to the coupon payment. In case we use a cost of capital margin approach, the expected return on the capital $\tilde{K}_i$ for year $i$ is then equal to the required return if the required return is provided by the coupon payment. Denoting the required return by $\eta$, the coupon face values $c_i$ are equal to

$$c_i = \eta \tilde{K}_i.$$
2.2 SST Valuation

2.2.1 Market Value Margin and Solvency Capital Requirement

In SST terminology, the value of the basis risk is called the *market value margin* (*MVM*), and is equal to the difference between the market-consistent value of the liabilities and the value of the ORP. Denoting the portfolio of liabilities by $L$, the *market-consistent value of the liabilities* $V_t(L)$ at time $t$ is given by

$$V_t(L) = V_t^0(L) + MVM_t(L),$$

where

$$V_t^0(L) = V_t(ORP_t) = \text{value of the optimal replicating portfolio},$$

$$MVM_t(L) = \text{market value margin} = \text{value of the basis risk}.$$

The SST Solvency Condition

The basic idea of the SST to determine required regulatory capital is the following: A firm is considered *solvent* at time $t = 0$ if, at time $t = 1$, with sufficiently high probability, the market(-consistent) value of the assets exceeds the market-consistent value of the liabilities, i.e.

$$V_1(A_1) \geq V_1(L_1). \quad (2.13)$$

Let us quickly comment on this solvency condition. Since we have expressed the value of the liabilities by means of a (super-)replicating portfolio, condition (2.13) guarantees that such a (super-)replicating portfolio can be purchased at time $t = 1$ by converting the assets to the (super-)replicating portfolio. This then ensures a regular run-off of the liabilities, which implies that the obligations towards the policyholders can be fulfilled\(^7\). It is assumed that the conversion of the assets can be achieved *instantly*.

The SST approach is based on the *cost of capital margin*, and so the corresponding super-replicating portfolios have to be considered. These require capital, whose size depends on whether the first or the second strategy for the one-year max covers is selected (see Section 2.1.3)\(^8\). The SST is based on the *second strategy*, and thus the *super-replicating portfolio for the liabilities* at time $t = 1$ is given by

- an unbiased optimal replicating portfolio $ORP_1$

\(^7\)I.e. the policyholder’s claims can be paid, except when the default option is exercised. However, the default option is an integral part of the contract between policyholder and (re)insurance company.

\(^8\)Such capital would typically be provided by another (re)insurance company which takes over the run-off of the liabilities.
the default option

- shareholder capital \( \tilde{K}_1 = K_1 \) (see (2.2)) to cover deviations in year \( i = 1 \)

- the risk-free zero-coupon bonds \( C_j \) with face values \( c_j \) for \( j \geq 1 \) to pay out the future required returns on capital

- the purchase guarantees

where the purchase guarantees ensure that the capital providers (or run-off buyers) potentially needed in the future can be found.

Note that we require the ORP to be unbiased above in order to be able to interpret the coupon payments as the expected return on capital. Further recall that, in particular, the expected cash flow replicating portfolio \( ERP \) is unbiased.

In view of (2.13), there will be situations where the difference

\[
V_1(A_1) - V_1(L_1) \tag{2.14}
\]

is not zero but strictly positive. Thus, when the assets are converted at \( t = 1 \) to the optimal replicating portfolio, there might be shareholder capital left, given by the above difference. This capital can be either paid out to the shareholder; used to finance new business; or used to reduce the amount of capital \( \tilde{K}_1 \) which has to be available at \( t = 1 \) to cover the run-off of the portfolio \( L_1 \). Regardless of which of these options is selected, if the capital is not paid back to the shareholder it has to earn the required capital costs either from new business or from the run-off business.

Recall in this context from the discussion in earlier sections that the market-consistent value of a book of liabilities contained in a whole portfolio of liabilities in the balance sheet is not unique in the sense that it depends on the whole portfolio of liabilities. In particular, the value \( V_1(L_1) \) of the “run-off” liabilities in the balance sheet at the end of year \( i = 0 \) depends on whether new business is written in the years \( i = 1, 2 \ldots \) or not. Presumably, such new business would increase the diversification of the whole portfolio, thus reduce the capital required for the “run-off” part \( L_1 \), and hence decrease the value \( V_1(L_1) \).

However, acquisition of future new business is not guaranteed, and from the perspective of the regulator, it makes most sense to require that the existing obligations can be fulfilled (even) if no future new business is written. For this reason, the stipulation in the SST is that

- The value \( V_1(L_1) \) has to be calculated under the assumption that no new business is written.
Returning to the question of how to use potential capital from (2.14), in view of the preceding remarks, it can either be paid back to the shareholder or used to reduce the amount of capital $\tilde{K}_1$ which has to be available at $t = 1$ to cover the run-off. In the latter case, however, required capital costs still have to be earned on the capital. Consequently, it is irrelevant to the value $V_1(\mathcal{L}_1)$ of the liabilities which of these two options is selected, and we assume that potential capital is fully paid back.

In this sense, the SST methodology is based on a run-off situation but not necessarily one under duress\(^9\).

**Enlarged Capital, Solvency Capital Requirement**

We now reformulate condition (2.13) in order to obtain a condition on the balance sheet at time $t = 0$. To this end, define the *enlarged capital* $AC_0(t)$ at time $t$ by

$$AC_0(t) := V_t(A_t) - V^0_t(\mathcal{L}_t),$$

where $V^0_t(\mathcal{L}_t)$ is the value of the optimal replicating portfolio at time $t$ and *not* the market-consistent value of the liabilities. The solvency condition (2.13) can then be rewritten in terms of the available capital and the market value margin as the requirement that, with “sufficiently high probability.”

$$AC_0(1) \geq MVM(\mathcal{L}_1).$$

The “sufficiently high probability” condition is reflected by selecting as risk measure the *expected shortfall* $ES_\alpha$ at safety level $\alpha = 1\%$. This selection specifies the states of the world $\Omega^*_1$ (see (2.1)) possible at time $t = 1$ for which the default option is not exercised.

Writing $AC_0(1) = AC_0(0) + (AC_0(1) - AC_0(0))$, the solvency condition (2.13) can then be further reformulated as the condition on the balance sheet at time $t = 0$ that

$$AC_0(0) - SCR(0) \geq pv_{(1+0)}(MVM(1)),$$

where the *solvency capital requirement* $SCR(t)$ at time $t$ is defined by

$$SCR(t) := ES_\alpha[pv_{(t+1-t)}(AC_0(t+1) - AC_0(t))],$$

and where we define the market value margin, by slight abuse of notation, by

$$MVM(1) := \sup_{\Omega^*_1} MVM(\mathcal{L}_1).$$

\(^9\)However, note that the super-replicating portfolio also contains the purchase guarantees ensuring that the future required capital will be provided. It could be argued that the value of these guarantees is not independent of whether the company is under duress or not.
Note that the set $\Omega^*_1$ exclude those states of the world where default occurs in year $i = 0$. For this reason, we do not have to consider the larger set $\Omega^*_0$. However, for convenience, it might be simpler to consider the whole set $\Omega^*_0$, which is a more conservative approach.

The solvency condition is then expressed by the requirement that the enlarged capital $AC_0(0)$ at time $t = 0$ satisfies

$$AC_0(0) \geq SCR(0) + pv_{(1-\eta)}(MVM(1)).$$

(2.15)

To calculate $SCR(0)$ and $AC_0(0)$, the actual asset portfolio of the firm is considered. Note that the two quantities $SCR(0)$ and $AC_0(0)$ do not require the calculation of the market-consistent value of the liabilities.

**Calculation of the Market Value Margin**

It remains to calculate $MVM(1)$, which corresponds to calculating the market-consistent value of the liabilities at time $t = 1$. As mentioned above, the SST is based on a cost of capital margin approach following the second strategy for the one-year max covers, with a super-replicating portfolio as shown above. In particular, the asset portfolio is assumed to be equal to the super-replicating portfolio.

In view of (2.3), the market value margin is given by the value at $t = 1$ of the risk-free zero-coupon bonds $C_i$ for $i \geq 1$,

$$MVM(1) = \sum_{i \geq 1} V_1(C_i),$$

(2.16)

with the face values $c_i$ of the bonds $C_i$ given by the required expected return $\eta = 6\%$ over risk-free,

$$c_i = \eta \bar{K}_i = \eta K_i.$$

Recall from (2.7) and (2.9) that, selecting the risk measure $\rho = ES_\alpha$,

$$K_i = \sup_{\Omega^*_i} M_i = ES_\alpha[M_i],$$

where $\Omega^*_i$ (see [2.1]) denotes the states of the world possible at time $t = 1$ for which the default option is not exercised, and $M_i$ for $i \geq 1$ (see [2.8]) denotes the random variable of the mismatch

$$M_i = pv_{(i+1/2-i)}(X_i - f_i(ORP_i)) + pv_{(i+1-i)}(V_{i+1}(ORP_{i+1}) - V_{i+1}(ORP_i))$$

conditional on the information given at time $t = 1$. In the case the ORP is selected to be the expected cash flow replicating portfolio ERP, defined in Section 2.1.4, $M_i$ is given by (2.12),

$$M_i = E[R_0 | F_{i+1}] - E[R_0 | F_i],$$

28
conditional on the information available at time \( t \leq i \), where \( R_i \) denotes the discounted outstanding reserves at time \( i \) defined in (2.10).

Since the asset side is assumed to consist of the appropriate super-replicating portfolio, we have \( ES_\alpha[M_i] = SCR(i) \) for \( i \geq 1 \), and hence the market value margin \( MVM(1) \) from (2.16) is given by

\[
MVM(1) = \eta \sum_{i \geq 1} pv_{i+1} (SCR(i)) = \eta \sum_{i \geq 1} pv_{i+1} (ES_\alpha[M_i]). 
\] (2.17)

Recall Equations (2.4) and (2.5) from Section 2.1.3, expressing the difference between the sum of the maximum of \( K_i \) over all nodes of year \( i \) in the tree, and the maximum over the sum of the \( K_i \) over all possible paths in the tree. In the SST, the potentially more conservative first option corresponding to (2.4) is selected by calculating the amounts \( K_i \) as above (2.17) by the solvency capital requirement \( SCR(i) \) given the information at \( t = 1 \).

This observation can be expressed in a different way in terms of “temporal dependencies” between different \( M_i \), where \( i \) denotes the calendar year under consideration: No diversification benefit is considered between the different mismatches \( M_i \). The market value margin formula (2.17) corresponds to the assumption that the mismatches \( M_i \) are co-monotone.

### 2.2.2 Calculation of SCR and MVM

In this section we describe our proposal to calculate the necessary components for the SST.

In view of the preceding Section 2.2.1, we have to consider the following **liability portfolios**:

- \( L_0 \) = the liability business in the balance sheet of the end of year \( i = -1 \)
- \( N_0 \) = the new liability business in year \( i = 0 \).
- \( L_1 \) = the liability business in the balance sheet of the end of year \( i = 0 \)
- \( L_i \) = for \( i \geq 2 \), the “run-off” of the portfolio \( L_1 \) in year \( i \)

and the following **asset portfolios**:

- \( A_0 \) = the actual asset portfolio in the balance sheet of the end of year \( i = -1 \)
- \( A_1 \) = the actual asset portfolio in the balance sheet of the end of year \( i = 0 \).

We decompose the liability portfolio into disjoint modeling units which we call **baskets**, \( L_i = L^1_i \cup L^2_i \cup \ldots \) (2.18)
In view of the solvency condition (2.15) and the remarks on the calculation of the market value margin $MVM(1)$, we have to determine the following quantities:

1. The enlarged capital $AC_0(0)$ at time $t = 0$ given by

$$AC_0(0) = V_0(A_0) - V_0^0(L_0),$$

where $V_0^0(L_0)$ denotes the value of the optimal replicating portfolio selected for the liabilities at time $t = 0$.

2. The enlarged capital $AC_0(1)$ at time $t = 1$ given by

$$AC_0(1) = V_1(A_1) - V_1^0(L_1),$$

where $V_1^0(L_1)$ denotes the value of the optimal replicating portfolio selected for the liabilities at time $t = 1$.

3. The random variable $M_i$ of the mismatch amounts for $i \geq 1$, conditional on the information available at time $t = 0$,

$$M_i = \text{pv}_{(i+1/2-i)}(X_i - f_i(ORP_i)) + \text{pv}_{(i+1-i)}(V_{i+1}(ORP_{i+1}) - V_{i+1}(ORP_i))$$

which, in case the expected cash flow replicating portfolio is selected as ORP, is given by

$$M_i = \mathbb{E}[R_i | F_{i+1}] - \mathbb{E}[R_i | F_i],$$

where $R_i$ denotes the discounted outstanding reserves at time $i$ defined in (2.10). From the mismatch random variables $M_i$ the future solvency capital requirements $SCR(i)$ for $i \geq 1$ are calculated via

$$SCR(i) = ES_{\alpha}[M_i].$$

The mismatches $M_i$ decompose according to the decomposition (2.18) of the portfolios into baskets,

$$M_i = \sum_b M^b_i,$$

where $M^b_i$ denotes the mismatch from the basket $L^b_i$.

Notice that we do not have to explicitly consider super-replicating portfolios for the SST at all in the following two cases:

- Business for which a deep and liquid market exists, or which can be replicated by instruments from such a market, or for which the basis risk can be assumed to be zero (such as, e.g., for the GMDB business, see below). The reason is that such business does not enter into the calculation of the $MVM(1)$, since the mismatch amounts $M_i$ are zero or approximately zero.
• Business for which the expected cash-flow-replicating portfolio is selected as ORP, since the calculation (2.22) of the mismatches $M_i$ does not involve other instruments besides the liabilities themselves.

Up to now, we have implicitly focussed on loss cash flows. There are, of course, further cash flows from the liabilities: premiums, external expenses, and internal expenses.

• **Premiums, external expenses**: For the new business $N_0$, these cash flows are modeled stochastically, assuming for simplicity that they are co-monotone with the losses. For run-off business we assume for simplicity that premiums and expenses are constant.

• **Internal administrative expenses**: The SST requires these to be considered and, in particular, that they be split into acquisition and maintenance expenses, where the former occur only for new business and not for run-off business. The proposed methodology to model internal expenses is described in Section 3.10, see pp. 100.

For the different types of insurance liabilities, we describe in the following the selection of the optimal replicating portfolios. For more details on life business, see Chapter 4 (pp. 104). More details on non-life business are found in Chapter 3 (pp. 41).

• **Non-life business, short-duration life business**: For these types of business, we select as ORP the expected cash-flow-replicating portfolio ERP. For short-duration life business, the conservative assumption is made that we have full information for the new business $N_0$ at $t = 1$ (“fluctuation risk”), but that reserve risk nonetheless remains for the run-off period.

• **GMDB business**: The optimal replicating portfolio consists of bonds, an asset index, and European and look-back put options. The precise derivation and composition is discussed in Section 4.7 (pp. 121). Since the basis risk appears to be small compared to other modeling uncertainties, it is assumed zero, and thus GMDB business is ignored in the calculation of the $MVM(1)$ (for a discussion of this assumption see also Section 2.3.2).

• **Life financial reinsurance**: For this type of business, insurance risk is considered immaterial, and only credit risk is considered.

• **Standard long-duration life business**: As described in the life liability model part, the cover period for such business is several (many) years. Similarly to short-duration business, it is assumed that the full information about any one-year cover period $i$ is available at the end of
that year. The change in the estimates from time \( t = i \) to \( t = i + 1 \) are separated in a stochastic component assuming that the underlying loss models are correct (“fluctuation risk”), and another component coming from the change in the underlying long-term assumptions of the model (“trend risk”). Fluctuation risk is considered as for the short-duration business, and trend risk is modeled by an unbiased ORP.

In the following, we provide a high level overview of our proposal to calculate the liability part for the three required quantities listed further above.

In order to calculate the enlarged capital \( AC_0(0) \) from (2.19), the value \( V_0^0(L_0) \) is given by the discounted future cash flows \( f_0(X_i) \) from \( ORP_0 \), i.e.

\[
V_0^0(L_0) = \sum_{i \geq 0} pv_{i+1/2-0}(f_0(X_i)),
\]

and for the special case \( ERP_0 \), we get the discounted best estimate of the reserves at time \( t = 0 \),

\[
V_0^0(L_0) = \sum_{i \geq 0} pv_{i+1/2-0}(E[X_i | F_0]) = E[R_0 | F_0].
\]

Similarly, to calculate the enlarged capital \( AC_0(1) \) from (2.20),

\[
V_1^0(L_1) = \sum_{i \geq 1} pv_{i+1/2-1}(f_1(X_i)),
\]

and for the special case \( ERP_1 \), we get the discounted best estimate of the reserves at time \( t = 1 \),

\[
V_1^0(L_1) = \sum_{i \geq 1} pv_{i+1/2-1}(E[X_i | F_1]) = E[R_1 | F_1].
\]

In the calculation of \( V_1^0(L_1) \), we explicitly distinguish the development of run-off business in year \( i = 0 \), which is described using a run-off model (see Section 3.9 for non-life business), and the new business \( N_0 \) from year \( i = 0 \), for which a new business model is used (see Section 3.3 for non-life business). For life business, see Chapter 4 (pp. 104); there, the risk pertaining to new business is called fluctuation risk, and risk in “run-off” business is called reserve risk.

Finally, we need to calculate the future mismatch amounts \( M_i \) for \( i \geq 1 \) from (2.21). Recall that we decompose the liability portfolios for \( i \geq 1 \) into disjoint modeling units called baskets,

\[
L_i = L_i^1 \cup L_i^2 \cup \cdots \cup L_i^n
\]
and that the mismatches $M_i$ decompose into basket mismatches accordingly,

$$M_i = \sum_{b=1}^{n} M_i^b,$$

with $M_i^b$ denoting the mismatch from the basket $L_i^b$.

Depending on the difference in speed of the reduction of uncertainty over time for the different baskets, the relative weight of a given basket $L_i^b$ within $L_i$ will change as $i = 1, 2 \ldots$ increases, in the sense that its contribution to the mismatch shortfall

$$SCR(i) = ES_\alpha[M_i]$$

will change with $i$.

For instance, a short-tail basket might have a high relative weight for small $i$ but will quickly be practically run-off, so that its weight will soon reduce to zero. On the other hand, the basket $L_i^b$ with the longest tail will eventually be the only remaining contributor to the mismatch, so that for large enough $i$,

$$ES_\alpha[M_i] = ES_\alpha[M_i^b].$$

Clearly, for any $b$, the basket “stand-alone” shortfall $ES_\alpha[M_i^b]$ provides an upper bound for the contribution of the basket $b$ to the shortfall $ES_\alpha[M_i]$.

In case the ORP selected is the expected cash-flow-replicating-portfolio, the mismatches $M_i$ given by (2.22) for the portfolios $L_i^b$ decompose into basket mismatches $M_i^b$ given by

$$M_i^b = E[R_0^b | F_{i+1}] - E[R_0^b | F_i],$$

where $R_0^b$ denotes the discounted outstanding reserves for basket $b$ at time $i$ defined in (2.10).

In order to calculate $SCR(i) = ES_\alpha[M_i]$ from the distributions of the basket mismatches $ES_\alpha[M_i^b]$, assumptions on the dependencies of the random variables $M_i^b$ have to be made. These dependencies might depend on $i$. Clearly,

$$ES_\alpha[M_i] \leq \sum_{b} ES_\alpha[M_i^b].$$

We propose to measure the speed of reduction of uncertainty for basket $L_i^b$ for $i = 1, 2 \ldots$ by the ratio

$$\xi_b^{(i)} := \frac{ES_\alpha[M_i^b]}{ES_\alpha[M_i]}.$$

Denote by $0 < \kappa_i \leq 1$ the “diversification benefit” for year $i \geq 1$ given by

$$SCR(i) = ES_\alpha[M_i] = \kappa_i \sum_{b} ES_\alpha[M_i^b].$$
The “stand-alone” shortfalls $ES_a[M^b_i]$ of the different baskets in this expression can be calculated for any $i = 1, 2, \ldots$, and it remains to estimate the “diversification benefits” $\kappa_i$, which in addition are impacted by the dependency assumptions between the $M^b_i$.

Our proposal is to suppose that the $\kappa_i$ for $i \geq 1$ are given as a function $f_n$ of the ratios from (2.23), with $n$ denoting the number of baskets,

$$
\kappa_i = f_n \left( \xi^{(i)}, \ldots, \xi^{(i)} \right).
$$

The function $f_n$ can then be calibrated at $i = 1$ by explicitly calculating $\kappa_1$ from expression (2.24) for $i = 1$ by calculating $ES_a[M_1]$, posing an explicit dependency structure between the mismatches $M^b_i$ for $i = 1$.

Denote an $n$-vector by $\underline{x} = (x_1 \ldots x_n)$. It seems reasonable to request the function $f_n$ to have the following properties:

1. For any $n \geq 1$, $f_n$ is continuous, and
   $$0 < f_n(\underline{x}) \leq 1.$$

2. For any 1-vector $\underline{x} = (x_1)$, there is no diversification, i.e.
   $$f_1(\underline{x}) = 1.$$

3. For a vector $\underline{x}$ with one zero entry, e.g. $\underline{x} = (0, x_2 \ldots x_n)$,
   $$f_n(0, x_2 \ldots x_n) = f_{n-1}(x_2 \ldots x_n).$$

4. The function $f_n$ is maximal if all entries of the vector are equal to $a$,
   $$\max_{\underline{x}} f_n(\underline{x}) = f_n(a \ldots a).$$

This reflects the supposition that diversification is maximal if all component shortfalls are of approximately equal size.

5. The function $f_n$ is symmetric, i.e. for any permutation $\pi$,
   $$f_n(x_1 \ldots x_n) = f_n(x_{\pi(1)} \ldots x_{\pi(n)}).$$

It follows from the second and the third condition above in particular that, for any vector $\underline{x}$ which has only one non-zero entry, there is no diversification, so

$$f_n(\underline{x}) = 1.$$ 

A candidate for such a function $f_n$ is given by

$$f_n \left( \xi^{(i)}_1, \ldots, \xi^{(i)}_n \right) := g_n \left( \frac{\xi^{(i)}}{\xi^{(i)}_1}, \ldots, \frac{\xi^{(i)}}{\xi^{(i)}_n} \right)$$
with \( \xi_b^{(i)} \) defined in (2.23), and

\[
\xi^{(i)} := \sum_{b=1}^{n} \xi_b^{(i)},
\]

where the function \( g_n \) is defined by

\[
g_n(x) = 1 - \prod_{b=1}^{n} (1 - x_b)^\beta,
\]

where \( \beta \) can be determined from \( \kappa_1 \).

Note that our proposed function \( f_n \) has an additional property not mentioned above: \( f_n \) is *linearly invariant*, that is, for any \( r > 0 \),

\[
f_n(rx) = f_n(x).
\]

### 2.2.3 Proposed Calculation of the Market Value Margin

In this section, we give a summary of our proposal to calculate the market value margin

\[
MVM(1) = \eta \sum_{i \geq 1} SCR(i),
\]  

(2.25)

with the required return \( \eta = 6\% \). According to our proposal, the solvency capital requirements \( SCR(1) \) for \( i = 1 \) in the above formula (2.25) is explicitly calculated (for \( \alpha = 1\% \)) by

\[
SCR(1) = ES_\alpha[M_1].
\]  

(2.26)

On the other hand, the \( SCR(i) \) for \( i \geq 2 \) in Equation (2.25) are not calculated explicitly but expressed in terms of the shortfalls for the individual baskets \( b = 1, \ldots, n \) by

\[
SCR(i) = ES_\alpha[M_i] = \kappa_i \sum_{b=1}^{n} \xi_b^{(i)} ES_\alpha[M_b^i].
\]  

(2.27)

In this expression (2.27), the basket shortfall ratios \( \xi_b^{(i)} \) are given by

\[
\xi_b^{(i)} = \frac{ES_\alpha[M_b^i]}{ES_\alpha[M_1^i]},
\]  

(2.28)

and \( \kappa_i \) denotes the “diversification factor.”

We propose not to explicitly calculate the basket shortfall ratios \( \xi_b^{(i)} \) for \( i \geq 2 \), but to use suitable proxies for them. The proposed proxies for the different basket are listed and commented upon in Section 2.2.4.
Further, we propose to calculate the “diversification factor” $\kappa_i$ by

$$\kappa_i = 1 - \prod_{b=1}^{n} \left(1 - w_b^{(i)}\right)^\beta \quad (2.29)$$

with the “weights”

$$w_b^{(i)} = \frac{\xi_b^{(i)}}{\sum_{l=1}^{n} \xi_l^{(i)}}. \quad (2.30)$$

The exponent $\beta$ in (2.29) is derived by setting $i = 1$ in (2.27), so $\beta$ is obtained by taking the logarithm of the following equation,

$$\left(\prod_{b=1}^{n} \left(1 - w_b^{(1)}\right)^\beta\right) = 1 - \frac{ES_\alpha[M_1]}{\sum_{b=1}^{n} ES_\alpha[M_1]^b}.$$

Consequently, to calculate the MVM according to our proposal, we need to calculate the shortfalls $ES_\alpha[M_1]$ of the basket mismatches $M_1^b$,

$$M_1^b = \mathbb{E}[R_b^1 | \mathcal{F}_2] - \mathbb{E}[R_b^1 | \mathcal{F}_1], \quad (2.31)$$

where $R_b^1$ denotes the discounted outstanding reserves for basket $b$ at time $i = 1$, and $\mathcal{F}_1, \mathcal{F}_2$ denote the information available at time $t = 1, t = 2$, respectively. Further, we need to calculate the shortfall

$$SCR(1) = ES_\alpha[M_1]$$

for the mismatch $M_1$ given by the sum of the basket mismatches $M_1^b$,

$$M_1 = \sum_{b=1}^{n} M_1^b$$

by proposing a dependency structure between the basket mismatches $M_1^b$ from (2.31).

### 2.2.4 Proposed Proxies for the MVM Calculation and Justification

It remains to find suitable proxies for the ratios $\xi_b^{(i)}$ for $i \geq 2$ of basket shortfall ratios given by (2.28),

$$\xi_b^{(i)} = \frac{ES_\alpha[M_1^b]}{ES_\alpha[M_1^1]}.$$

We propose to use the following proxies:
- **Non-life liabilities:** We propose to use as proxy for a basket the incurred or reported pattern by *calendar year* for this basket: The proxy for $\xi_h^{(i)}$ for year $i$ is proposed to be the *ratio of expected change in outstanding reported reserves in year $i$*, i.e. from time $t = i$ to time $t = i + 1$, divided by the *expected ultimate claims amount*.

- **Fluctuation risk for short duration life:** No proxy is needed since by assumption, the fluctuation risk (pertaining to new business) is fully manifested in year $i = 0$.

- **Reserve risk for short-duration life:** We use the same proxy as for non-life liabilities.

- **Fluctuation risk for long-duration life:** The proposed proxy is the ratio of the respective annual premiums.

- **Trend risk for long-duration life:** The proposed proxy is the ratio of the sum of the respective future premiums. This reflects the monotonously decreasing exposure to trend changes in the future.

- **Credit risk for life finance business:** The suggested proxy is the ratio of the sum of the respective future premiums. The sum of future premiums here is directly related to the remaining outstanding credit exposure.

Regarding the justification of the *proxy for non-life liabilities*, the use of the reporting (incurred) pattern for measuring the change of risk over time is a standard assumption we also use to derive the one-year changes for the new business model (see pp. 62), and use in the context of pricing capital allocation (see pp. 168). The basic idea is that the relative difference between the reported losses and the ultimate losses gives a measure of the uncertainty about the final outcome, and can thus be used as proxy for shortfall ratios. This proxy is necessarily rough. We plan to further evaluate the appropriateness of the proxy by analyzing reserve data.

### 2.3 Retro, Limitations, Comparison

#### 2.3.1 External Retro and Fungibility for MVM Calculation

In this section we summarize the treatment of *external retro* and *capital and risk transfer instruments* (internal retro, parental guarantees, dividend payments to parent, etc.) in the calculation of the market value margin, i.e. in the calculation of the solvency capital requirements $SCR(i)$ for $i = 1, 2, \ldots$.

The question is the following: Suppose that the legal entity Zurich has to go into assisted run-off at $t = 1$, will these instruments persist in the future years?
We propose the following treatment for the different instruments:

- **External retro in place in year \( i = 0 \):** In our balance sheet, this concerns mainly Nat Cat business. Since this business is short tail, we assume that the outcome is mainly known at \( t = 1 \), and since no more new business is written, we assume that all such external retro is discontinued.

- **Additional external retro purchased at \( t = 1 \) to cover run-off:** We assume that no such contracts are purchased.

- **Internal capital and risk transfer instruments (internal retro, parental guarantees, dividend payments to parent):** We assume that all these instruments are continued for the full run-off period. The potential impact of internal retrocession and parental guarantees on reserve risk of future years is simulated. But obviously, internal retrocession is negligible as it refers mainly to new business.

For more information on the treatment of external retrocession, see Section 6 (pp. 140). For information on capital and risk transfer instruments, see Section 24 (pp. 337).

### 2.3.2 Limitations of the Valuation Approach

In our proposal for calculating the market value margin \( MVM(1) \), we have to determine the future solvency capital requirements \( SCR(i) = ES_{\alpha}[M_i] \) for \( i = 1, 2, \ldots \). We suggest to decompose the full portfolio into baskets, corresponding to a decomposition of the mismatches

\[
M_i = \sum_{b=1}^{n} M_i^b,
\]

where the distributions of the \( M_i^b \) are assumed to be available. In order to calculate \( ES_{\alpha}[M_i] \), we in addition need dependency assumptions, for given \( i \), between the basket mismatches \( M_i^b \). We have simplified this by only posing dependencies for \( i = 1 \) and derive the \( SCR(i) \) for \( i \geq 2 \) by using an auxiliary function \( f_n \) expressing the diversification benefit. This, of course, does not do full justice to the changing dependency structure over time. However, a temporal, multi-period model of the cash flows including dependencies changing with time is currently not within reach.

In the modeling of GMDB contracts in the life liabilities, a replicating portfolio approach is proposed. There are two weaknesses of the proposed approach:

- The replicating portfolio does not perfectly replicate the GMDB cash flows but only approximately, so it is an optimal replicating portfolio
rather than a true replicating portfolio, and a certain basis risk exists between the liabilities and the ORP. However, the magnitude of this basis risk is considered small as compared to the other modeling uncertainties.

- The proposed ORP consists of bonds, the asset index, and European and look-back put options. By SST requirements, an ORP needs to consist of deeply and liquidly traded instruments, and it can be questioned whether this is indeed the case.

2.3.3 Valuation of Liabilities in SST and Pricing

In a reinsurance company, valuation of liabilities is required in two different contexts. In pricing, reinsurance contracts offered have to be valued at renewal to determine whether the offered price is sufficient and/or to determine the sufficient price. Our approach to valuing non-life liabilities in pricing is described in detail in the section about capital allocation (Section 8.3, pp. 168). On the other hand, the whole portfolio of liabilities has to be valued for the economic balance sheet in the context of the SST.

The valuations in pricing of non-life liabilities and in the SST are similar to a large extent. Both use the following assumptions:

- calculation of the market value margin, i.e., of the value of the basis risk, by a cost of capital margin approach with a constant required return $\eta$
- measuring the risk by the law-invariant risk measure $\rho$ given by the expected shortfall $ES_\alpha$
- using as optimal replicating portfolio the expected cash-flow-replicating portfolio $ERP_i$, so the mismatch $M_i$ in year $i$ is given by (2.12),

$$M_i = \mathbb{E}[R_0 | F_{i+1}] - \mathbb{E}[R_0 | F_i]$$

The main difference is that, while the SST follows the second strategy for covering the basis risk, in pricing we follow an approach closely related to the first strategy. In other words, the two approaches differ in the way shareholder capital is treated. In both cases, the total discounted capital costs at the start of year $i$ are given by

$$\eta \sum_{j \geq i} pv_{(j+1-i)}(\tilde{K}_j).$$

In the SST approach, the capital $\tilde{K}_j$ required in any year $j \geq i$ is given by the risk measure applied to the one-year change,

$$\tilde{K}_j = \rho(\mathbb{E}[R_0 | F_{j+1}] - \mathbb{E}[R_0 | F_j]).$$
In pricing, on the other hand, the required capital $\tilde{K}_j$ for $j \geq i$ is calculated for the ultimate risk in the sense of the sum of all future mismatches $M_l$ for $l \geq j$,

$$\tilde{K}_j = \rho \left( \sum_{l \geq j} \text{pv}(l-j)(M_l) \right) = \rho (R_0 - \mathbb{E}[R_0 \mid F_j]).$$

One of the issues here concerns shareholders capital. In the SST, there will be situations where, after a one-year period, there is no or not enough shareholder capital left to continue to write business or even to guarantee the run-off of the existing business. However, because of the way the value of liabilities has been defined, the super-replicating portfolio for the liabilities allows to pay out to a buyer the future required expected capital costs on the capital this buyer will have to put up for the run-off of the liabilities.

We have expressed the liquidity assumption that such a buyer can be found by introducing the purchase guarantees. The liquidity assumption is thus closely connected to the value of these purchase guarantees. In particular, one could consider the question whether this value is different for a company under more or less duress, and whether it is reasonable to assume that the required return is independent of these situations.
3

Non-Life Insurance Liability Model

3.1 Introduction

The goal of this chapter is to describe the (market-consistent) valuation of the non-life liabilities for the SST. To be more precise, the task is to construct a stochastic model of the difference between the value of the non-life liabilities at the start of one year and the value at the end of that year.

In our documentation we follow the paper by Christoph Möhr, Non-Life Liability Model (Möhr [2008]). The initial version is the internal document by Christoph Hummel, Internal liability model (non-life) (Hummel [2007b]) presented to the FOPI at the technical meeting.

In the following we define the types of non-life liabilities we are modeling, the relevant cash flows, and the data sources used to come up with the model.

Portfolio of Business under Consideration

The non-life liabilities considered in this chapter consist of the portfolio composed of all

- Inward reinsurance contracts
- Inward direct insurance contracts

in the balance sheet. Inward reinsurance contracts include facultative as well as treaty business.

Intragroup contracts (e.g. internal retrocession) between legal entities and/or branches, and outward (external) retrocession contracts are considered separately. Concerning intragroup contracts, the SST requires the relevant legal entities to be modeled separately. Intragroup contracts are then considered in the context of capital fungibility (see Chapter 24, pp. 337). See also pp. 37 in the section on valuation.
We distinguish within the non-life liabilities the new business and the run-off business:

- **Run-off business** is all business included in the initial balance sheet.
- **New business** denotes all business that is in the balance sheet at the end of the year but not in the initial balance sheet, i.e. all business written in that year.

The whole new business portfolio is usually not known in advance, and it is thus modeled using plan figures. New business is also valued when pricing contracts at renewal. Since this valuation is closely connected to the valuation for the SST, we describe the pricing valuation approach in this document in Section 8.3 (pp. 168). The SST and the pricing valuation approach are similar but not identical. A comparison of the two approaches for non-life business is given in Section 2.3.3 (pp. 39).

Note that we have to be more careful here and also have to distinguish written and **earned business**. In the balance sheet, written business, e.g. at time $t = 0$, is split into earned and unearned business (the latter balance sheet figure is called *unearned premium reserves*). In this document, we use the following definition:

- **Unearned business at time $t$** denotes business written before time $t$ for which we are still “under risk” after $t$ in the sense that we might incur losses from a loss producing event occurring after time $t$ (e.g. an earthquake).

With regards to earning of business, we make the simplifying assumption that

- all business written in a year $i$ is fully earned at the end of year $i + 1$.

We construct two separate models, a **new business model** and a **run-off model**. Conceptionally, the difference between the two is that the new business model describes business that can incur losses from loss producing events, and is thus “under risk” in this sense. Consequently, the unearned part of the business from prior years has to be modeled by the new business model and not by the run-off model.

For time $t = 0$, we have to consider three different models for the first year $i = 0$:

- **Earned new business**: The part of the business written in the current year which is earned in the current year. It is modeled by the new business model.
• **Unearned previous year business**: The part of the business written in the previous underwriting year which is earned in the current year. It is modeled by a new business model of the previous underwriting year business.

• **Earned run-off model**: The business written in prior underwriting years which is fully earned at time \( t = 0 \). It is modeled by the run-off model.

If the new business portfolio remains sufficiently stable from the previous year to this year, one can make the simplifying assumption that the earned new business together with the unearned previous year business is equal to the (full) new business.

At time \( t = 1 \), the SST assumes that no new business is written in the years \( i = 1, 2, \ldots \), so there are two different models:

- The part of the new business not earned in year \( i = 0 \)
- The new earned run-off business

Our proposal for the treatment of the different types of business is described in Section 3.9.5, pp. 100. With this proposal, it is no longer necessary to explicitly distinguish earned and unearned business.

**Stochastic Liability Cash Flows**

In order to value the liabilities, a model of the *stochastic liability cash flows* is needed. The cash flows of the liabilities consist of

- Premiums received (paid)
- Claims (losses) paid
- External expenses paid (brokerage, ceding commissions, etc.)
- Internal administrative expenses incurred from acquisition and maintenance of the liabilities

The *result cash flows* of the liabilities are thus given by

\[
\text{result} = \text{premium} - \text{losses} - \text{external expenses} - \text{internal expenses}.
\]

In a reinsurance contract, premiums and external expenses as well as the result are typically *monotone* functions, not necessarily linear, of the losses: higher losses lead to higher premiums and lower expenses, and to a lower result.

The stochastic cash flows of non-life liabilities are typically not modeled directly. Instead, one models the nominal ultimate amounts and uses *development patterns* to derive the cash flows.
To be more precise, a model is constructed for the nominal ultimate amounts given by the sums of the nominal cash flows for losses, premiums, and expenses, respectively.

Cash flows are then derived from the ultimates using patterns. A pattern for an ultimate amount expresses the expected cash flows in any one year as a percentage of the corresponding nominal ultimate amount. For more information on patterns, see Section 3.2.2.

Aggregation of Sub-Portfolio Models

Typically, stochastic models (for the ultimate amounts) will be given at sub-portfolio level and need to be aggregated to derive a model for a whole legal entity or for the whole portfolio. We call such a sub-portfolio, which is a modeling unit in our model, a basket. Note that the SST requires legal entities to be modeled separately. For this reason, the basket models need to be legal-entity specific.

In order to aggregate the basket models, dependency assumptions have to be made. Hence, the building blocks of a portfolio model are

- Basket models
- Dependency structure for the basket models.

We choose a hierarchical dependency structure as follows: The portfolio is decomposed into baskets. Baskets are grouped together to parent baskets. Dependencies are formulated between all baskets in one parent basket. The resulting aggregation of baskets in a parent basket results in a new collection of baskets consisting of all parent baskets. These baskets are again grouped to parent baskets, and we proceed as before. The resulting portfolio decomposition and dependency structure is called portfolio dependency tree. It is described in more detail in Section 8.1.

Data Sources – Pricing, Reserving, Planning

To calculate the liability model we use the following data sources:

- Historical data from the reserving and claims systems
- Contract distributions from pricing
- Planning data by Finance for the planned business

In our current approach, the run-off model is based on reserving data, and the new business model as well as the pricing capital allocation are based on the pricing distributions, using planning data.

The pricing information used for modeling the new business consists of distributions for losses, premiums, and expenses for every contract priced
in the pricing tool. Normatively, these distributions reflect the best estimate at the pricing date (usually, shortly before inception of the contract) of all possible outcomes of the contract with probabilities assigned to each outcome.

To come up with a distribution for the new business of a legal entity or the whole company, these contract distributions are aggregated using the portfolio dependency tree mentioned above (see Section 8.1 and following).

Typically, not all contracts written in a year are known and priced at the beginning of this year. The new business model thus uses plan figures from Finance together with available pricing data from a rolling year of business from the prior and the upcoming underwriting year, and adjusts the pricing data for the plan figures.

For the run-off model, reserving data is used to come up with the best estimate (i.e the expected value given the available information) of the outstanding liabilities, and with an estimate of the one-year change of this estimate. One-year change means the estimated difference at initial time between the best estimate at the end of the year and the initial best estimate. However, no full distribution is available from the reserving data, so a distribution assumption is required in order to make statements about small quantiles.

\[ M_i = \mathbb{E}[R_i \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_i \mid \mathcal{F}_i] = \mathbb{E}[R_0 \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_0 \mid \mathcal{F}_i], \quad (3.1) \]

3.2 Stochastic Liability Models for the SST

Recall the outline of the proposal to calculate the non-life liability components of the SST (Section 2.2.2, pp. 29). We denote by \( L_i \) the business in the economic balance sheet at the end of the year \( i - 1 \).

The liability portfolio \( L_i \) is decomposed into disjoint modeling units called baskets,

\[ L_i = L^1_i \cup L^2_i \cup \ldots \]

The SST requires legal entities to be modeled separately. Intragroup instruments between legal entities are considered at a later stage in the context of fungibility (see Chapter 24, pp. 337). For this reason, baskets need to be legal entity specific.

Recall that, for non-life insurance liabilities, we select as optimal replicating portfolio (ORP) the expected cash-flow-replicating portfolio ERP. Thus, the mismatches \( M_i \) \( (i \geq 0) \) between the actual liability cash flows and the cash flows from the ORP are given by the one-year change in the estimate of the ultimate loss from time \( t = i \) to \( t = i + 1 \),
conditional on the information available at time \( t = 0 \). In this expression, \( \mathcal{F}_j \) denotes the information available at time \( t = j \), and \( R_i \) denotes the discounted outstanding reserves at time \( t = i \) given by

\[
R_i := \sum_{j \geq i} \text{pv}_{(j+1/2-i)}(X_j)
\]

(3.2)

with \( X_j \) denoting the cash flows in year \( j \) assumed for simplicity to occur at time \( t = j + 1/2 \).

The mismatches \( M_i \) decompose according to the decomposition of the portfolios into baskets,

\[
M_i = \sum_b M^b_i,
\]

where \( M^b_i \) denotes the mismatch from the basket \( \mathcal{L}^b_i \).

For the SST, we need to calculate, conditional on the information available at time \( t = 0 \), for the enlarged capital \( AC_0(0) \) at time \( t = 0 \), the discounted best estimate of the reserves at time \( t = 0 \),

\[
V_0^0(\mathcal{L}_0) = \mathbb{E}[R_0 | \mathcal{F}_0],
\]

and, for the enlarged capital \( AC_0(1) \),

\[
V_1^0(\mathcal{L}_1) = \mathbb{E}[R_1 | \mathcal{F}_1],
\]

and, for the market value margin \( MVM(1) \), the mismatch amounts \( M_i \) for \( i \geq 1 \). These can be determined from the following quantities:

- For each basket \( b \), the discounted best estimate of the reserves at \( t = 0 \) from (3.3),

\[
\mathbb{E}[R^b_0 | \mathcal{F}_0].
\]

(3.4)

These figures can be derived from the balance sheet total reserves for all baskets, together with a percentage split of the total into different baskets from accounting data, by cash flow discounting using payment patterns.

- The distribution of the basket mismatches (cf. (3.1))

\[
M^b_i \quad \text{for} \quad i = 0, 1, 2 \ldots
\]

- The dependency structure of the basket mismatches \( M^b_i \)

\[\text{Note that the second equality in (3.1) holds because the cash flows } X_j \text{ for } j = 0 \ldots i-1 \text{ are known at times } t = i, i+1.\]
The approach to modeling the dependency structure between baskets is by means of a *portfolio dependency tree* as described in Section 8.1.

As outlined, we do not use as modeling units the whole portfolio but suitably defined sub-portfolios called baskets. Per se, it is not obvious why we do not directly model the whole portfolio (per legal entity), since we would then not have to worry about the dependency structure between baskets. The main reason for this decision is that we need the modeling units to be suitably *homogeneous* in their composition, meaning that they should consist of “similar” types of contracts. Homogeneity is relevant, for instance, in the following contexts:

- When using *historical data*, it is likely that the historical portfolios have to be adjusted for historical changes, such as changes in volume, inflation etc. However, these changes probably do not affect all contracts in the same fashion. For instance, different sub-portfolios might experience different volume growth. If we model sub-portfolios, homogeneity must guarantee that historical changes can be applied uniformly to the sub-portfolios.

- When using the *contract pricing models* and aggregating them to sub-portfolio models, homogeneity has to be ensured with respect to dependencies between the contracts in the sub-portfolio. Moreover, the sub-portfolio might have to be adjusted for growth, so homogeneity has to hold also with respect to the contract pricing models, since we might have to artificially duplicate (or remove) contracts to account for growth.

The requirement that modeling units be homogeneous thus means that the adjustments that might have to be applied to the basket affect all components of the basket in the same way, so that the adjustments can be applied uniformly to the basket.

Thus far, we have implicitly focussed on loss cash flows. There are, of course, further cash flows from the liabilities: premiums, external expenses, and internal expenses.

- **Premiems, external expenses:**
  - *New business* $N_0$: These cash flows are modeled stochastically, assuming for simplicity that they are *co-monotone* with the losses.
  - *Run-off business*: We assume for simplicity that premiums and expenses are constant.

- **Internal administrative expenses**: The SST requires these to be considered and, in particular, that they be split into acquisition and maintenance expenses, where the former occur only for new business and
not for run-off business. The proposed methodology to model internal expenses is described in Section 3.10 (pp. 100).

Overview of the Non-Life Liability Model

The description of the SST non-life liability model in the following sections is structured as follows. In the next Section 3.2.1 we summarize the data requirements. In Section 3.2.2 we outline how to derive stochastic cash flows from nominal figures using development patterns. Section 3.2.3 describes how to split the one-year claims development from the mismatches into a loss payment and a new reserve estimate.

Finally, Section 3.2.4 describes how, in order to calculate the one-year mismatches, the portfolio is decomposed into the following different types of business:

- New business
- Prior-year business
- Run-off business

For different types of business, different models are used to calculate the one-year mismatches. Recall in this context that we are using both pricing distributions and development triangles from reserving.

The primary idea is that, for more recent business, the pricing distributions are used to derive one-year mismatches, whereas, for older underwriting years, we use the reserving development triangles. This distinction also takes into account the difference between earned and unearned business.

The two primary types of models for the one-year mismatches are described in the following sections. Section 3.3, pp. 56, describes the new business model, based on pricing distributions, and Section 3.9, pp. 91, describes the run-off business model, based on reserving development triangles.

3.2.1 Data Requirements and Parameter

We summarize the requirements on the non-life insurance liability model for the SST.

- Definition of baskets of business for both new and run-off business. Baskets need to be
  - legal-entity specific,
  - gross of internal and external retro (except e.g. common account protections),
including business retroceded from legal entities which are not modeled.

For new and prior-year business, see the Section 3.3.2 (pp. 59), and for run-off business, Section 3.9.4 (pp. 99).

• For each basket $b$ we need
  
  – The discounted best estimate of the reserves at $t = 0$,
  
  $$\mathbb{E}[R^b_0 \mid \mathcal{F}_0]$$
  
  These figures can be derived from the balance sheet total reserves for all baskets, together with a percentage split of the total into different baskets from accounting data, by cash flow discounting using payment patterns.

  – Stochastic model for one-year changes in year $i = 0$, i.e. for the basket mismatches
  
  $$M^b_0 = \mathbb{E}[R^b_0 \mid \mathcal{F}_1] - \mathbb{E}[R^b_0 \mid \mathcal{F}_0]$$
  
  where, for $i \geq 0$,
  
  $$R^b_i = \sum_{j \geq i} pv_{(j+1/2-i)}(X^b_j)$$

  with $pv$ denoting risk-free present value.

• Method to calculate the future one-year changes for $i = 1, 2, \ldots$, by projection of the one-year changes derived above. In fact, we only need the shortfalls of the one-year changes

  $$ES_\alpha[M^b_i]$$

  for $i = 1, 2, \ldots$ and any $b$. Different methods are used for new and prior-year business, and for run-off business.

• **Aggregation methodology** for the basket one-year changes $M^b_0$, by specifying a dependency structure between baskets. For new and prior-year business, see Section 3.3.2 (pp. 59), and for run-off business, see Section 3.9.4 (pp. 99).

• For premiums and external expenses:
  
  – **New business**: Distribution of premium and external expense cash flows per basket, assumed *co-monotone* to the losses.

  – **Prior-year and run-off business**: Expected premiums and external expenses for each year $i = 0, 1, 2, \ldots$.  

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• **Dependency between new, prior-year, and run-off business:** This dependency is currently modeled at the level of the run-off business baskets, which have a lower granularity than the new business baskets. The latter are sub-portfolios of the former.

• **Internal expenses:** Given per legal entity considered, split into acquisition and maintenance costs, for the calendar years $i = 0, 1, 2 \ldots$ (see Section 3.10, pp. 100).

### 3.2.2 Development Patterns and Cash-Flow Discounting

Recall from Section 3.1 (pp. 41) that the stochastic cash flows of non-life liabilities are typically not modeled directly. Instead, a stochastic model for the **nominal ultimate amounts**, i.e. the sum of the nominal cash flows, is constructed. To derive the individual cash flows, different types of **development patterns** are used. In this section, we summarize these types of patterns and their data sources.

Development patterns are also used in another context, namely as proxies for the basket one-year mismatch shortfall ratios as described in Section 2.2.3. These patterns are summarized here as well.

#### Patterns and Discounting

To derive stochastic cash flow models from stochastic models for the nominal ultimate amounts, we use

- **Development patterns** for the **expected** percentage of the nominal ultimate amount which is paid out in each calendar year

We assume that all patterns start at January 1. Cash flows are then discounted using **yield curves** for risk-free government bonds in the appropriate currency of the cash flows.

Note that, when using patterns, the cash flows are derived using the **expected** percentages per year. Thus, we do not consider the volatility in the distribution of the ultimates over time. We believe this to be a justified simplification for the following reasons:

- There is usually insufficient data to construct a good stochastic model of the individual cash flows.

- The required data is often not available.

- The volatility in the ultimates is usually considered to be more relevant than the volatility in their distribution over time.

We distinguish

- **Incremental patterns**
Cumulative patterns

Incremental patterns provide the expected cash flow in one year as a percentage of the nominal ultimates, whereas cumulative patterns express the percentage of the sum of expected cash flows up to and including the year under consideration.

Further, we distinguish

- Underwriting-year patterns describing the expected cash flow distribution over time for all contracts (in a certain basket) written in a given underwriting year,

- Calendar-year patterns, where the cash flow distribution is for all contracts (in a certain basket) in the balance sheet, including business written in prior years.

Development patterns are typically derived from development triangles (e.g. of annual aggregate paid losses) by developing the triangles to ultimate values (using e.g. the chain ladder method) and calculating the required ratios from this. See Section 3.9.1 (pp. 93) for more on triangles and the different types of loss amounts.

New and Prior-Year Business

For new business, which by definition is written in one particular underwriting year, we need underwriting-year patterns.

For each new business basket, the following patterns are required:

- **Discounting of cash flows**: Underwriting-year patterns for
  - Paid losses
  - Paid (received) premiums
  - Paid expenses

- **Proxy to get from ultimate to one-year change**: Incremental underwriting-year patterns for the reported losses.

- **Proxy for the future mismatch shortfalls**: Not needed, because we base the future (new business) mismatch shortfalls on the run-off model.

These patterns can be derived from reserving development triangles based on the development of prior years’ business. Alternatively, one can use patterns calculated for each new business contract in pricing. In the aggregation application Phobos, these patterns are aggregated by basket to obtain basket patterns. Since pricing patterns start at the inception date of the contract, which is not necessarily January 1, they are shifted to January 1 before aggregation.

We propose to use the pricing patterns for the new business.
Run-Off Business

By definition, run-off business encompasses all business written in preceding underwriting years, and so calendar year patterns are relevant. That is, we need patterns for the whole outstanding reserves.

For each run-off business basket, the following patterns are required:

- **Discounting of cash flows:** Calendar year patterns for
  - Paid losses
  - Paid (received) premiums
  - Paid expenses

- **Proxy to get from ultimate to one-year change:** None are needed because the run-off models are already for the one-year change

- **Proxy for the future mismatch shortfalls:** Incremental calendar year patterns for the reported losses

These patterns are derived from reserving development triangles coming from the reserving database.

### 3.2.3 One-Year Claims Development and Split into Payment and Reserves

In this section, we consider nominal sums $S_i$ of future loss payments $X_j$ in calendar years $j = i, i+1 \ldots$ defined by

$$S_i := \sum_{j \geq i} X_j,$$

and denote by $\overline{S}_i$ the best estimate of the nominal reserves at time $t = i$,

$$\overline{S}_i := \mathbb{E}[S_i \mid \mathcal{F}_i],$$

conditional on the information $\mathcal{F}_0$ available at time $t = 0$. The *one-year claims development* $Q_i$ in year $i$ is defined as the sum of the payment $X_i$ in year $i$ and the reserves at the end of year $i$,

$$Q_i := X_i + \overline{S}_{i+1}, \quad (3.5)$$

so that the nominal one-year mismatch (compare with (3.1)) is given by the difference

$$Q_i - \overline{S}_i,$$

and the reserve $\overline{S}_i$ is a best estimate in the sense that

$$\mathbb{E}[Q_i \mid \mathcal{F}_i] = \overline{S}_i. \quad (3.6)$$
Since the split of the one-year claims development into payment $X_i$ and reserves $S_{i+1}$ is typically not available, we need a method to derive this split.

To this end, let the cumulative paid loss pattern (for all payments up to the end of year $i$) be defined by

$$\eta_i := \frac{E[\sum_{j=0}^{i} X_j | F_0]}{S_0} = 1 - \frac{E[S_{i+1} | F_0]}{S_0}, \quad (3.7)$$

and define the adjusted incremental paid loss pattern

$$\tilde{\eta}_i := \frac{\eta_i - \eta_{i-1}}{1 - \eta_{i-1}}. \quad (3.8)$$

We then get the following two identities:

$$E[X_i | F_0] = \tilde{\eta}_i \cdot E[Q_i | F_0] \quad (3.9)$$
$$E[S_{i+1} | F_0] = (1 - \tilde{\eta}_i) \cdot E[Q_i | F_0]. \quad (3.10)$$

The second equation follows from the first and the definition (3.5) of $Q_i$.

The first equation follows from

$$E[X_i | F_0] = (\eta_i - \eta_{i-1}) S_0,$$

and, using (3.6),

$$E[Q_i | F_0] = E[S_i | F_0] = (1 - \eta_{i-1}) S_0.$$

If we make the following two assumptions:

1. $X_i$ and $S_{i+1}$ are co-monotone.

2. The standard deviations of $X_i$ and $S_{i+1}$, respectively, are proportional to their expected values,

then the two equations (3.9) and (3.10) hold for the standard deviations in place of the expected values also.

For this reason, we propose to take the simplified approach setting

$$X_i = \tilde{\eta}_i \cdot Q_i \quad (3.11)$$
$$S_{i+1} = (1 - \tilde{\eta}_i) \cdot Q_i \quad (3.12)$$

with $\tilde{\eta}_i$ defined in (3.8).
3.2.4 New Business, Prior-Year Business, and Run-Off Business Models

Recall from the introduction, pp. 41, the difference between *earned* and *unearned* business. We have defined *unearned* business at time \( t \) to be business incepting before \( t \) for which we are still “under risk” at time \( t \) in the sense that we might incur losses from loss producing events occurring after time \( t \). In the balance sheet, this distinction is reflected by providing the “earned part” of the reserves only, and by accounting for the unearned part via the unearned premium reserves (UPR). With regards to unearned premium reserves, note that the SST considers an economic balance sheet, so the UPR have to be replaced by a best estimate of the corresponding liabilities.

As mentioned in the introduction, we make the simplifying assumption:

- Business written in a year \( i \) is fully earned at the end of year \( i + 1 \).

The distinction between earned and unearned business is relevant to us because we calculate the one-year mismatches \( M_i \) differently:

- For earned business, the mismatches are calculated using a “reserve model” based on the reserving development triangles.
- For unearned business, we use a “new business model” based on pricing distributions.

The reason for using the pricing distributions for unearned business is that, since this business is still “under risk”, catastrophic events might happen which result in an ultimate loss significantly different from the expected outcome. The corresponding volatility is captured in the pricing models by a strong focus on heavy-tailed distributions (see also Section 3.4, pp. 63).

As a consequence of having two different models for one-year mismatches, for instance at time \( t = 0 \), we would have to split the prior underwriting year \( i = -1 \) business into an earned and into an unearned part and derive two different models, since not all of this business is fully earned at \( t = 0 \).

This issue is related to the actuarial estimation of the reserves. Stochastic models of the reserves are often based on the chain ladder method (see pp. 93). However, in reality, the reserve calculation for the most recent years is typically not based on the chain ladder method, as that estimate is too uncertain. Instead, methods such as Bornhuetter-Ferguson are used, which utilize models from pricing to a certain extent.

Our proposed approach takes both of these issues into account. The basic supposition is the following:

At time \( t = i \),
• The reserve model for the underwriting years \( k = i - 1 \) and \( k = i - 2 \) is based on the pricing models.

• The reserve model for the underwriting years \( k \leq i - 3 \) is based on the chain ladder model applied to the reserving development triangles.

As a consequence of this assumption, the portfolio at time \( t = i \) splits into the following three parts:

• **Run-off business**: Business written in the years \( k \leq i - 3 \). To be modeled by the run-off model (see Section 3.9, pp. 91) using the chain ladder method. The best estimate of the reserves is given by the chain ladder reserves.

• **Prior-year business**: Business written in the year \( k = i - 2 \). To be modeled by the “new business” model based on pricing distributions (see Section 3.3, pp. 56). The best estimate for this business is equal to the expected value of the pricing distribution given the information at time \( t = i \) (which likely includes information about paid and reported losses).

• **New business**: Business written in the year \( k = i - 1 \). To be modeled by the “new business” model based on pricing distributions (see Section 3.3, pp. 56). The best estimate of the reserves at \( t = i \) is given by the expected value from the pricing distribution.

Observe that, using this approach, we no longer have to distinguish earned and unearned business.

Because we are using two different methods to calculate the best estimate of the reserves, an issue arises when we switch from one method to the other. For instance, at time \( t = 0 \), the reserves for the underwriting year \( k = -2 \) are calculated by the new business model, and, at time \( t = 1 \), the same reserves are estimated by the chain ladder run-off model. However, the one-year change estimated in the run-off model in Section 3.9 is based on the assumption that chain ladder is used to calculate the reserves both at time \( t = 0 \) and at time \( t = 1 \). To handle this problem we make the simplifying assumption that the reserve estimate from the new business model at time \( t = 0 \) agrees with the corresponding chain ladder estimate.

The reserve estimates at time \( t = 1 \) for the underwriting year \( k = -2 \) are thus best estimates in the sense that the expectation of these estimates given the information at time \( t = 0 \) is equal to the estimates at time \( t = 0 \).

As a consequence of the previous assumptions, there are the following different models to be used for the years \( i = 0, 1 \ldots \):

For the year \( i = 0 \) (i.e. from time \( t = 0 \) to \( t = 1 \)),
• **Run-off business model** for the underwriting years \( k \leq -2 \) with initial amount at \( t = 0 \) from the actual balance sheet and one-year mismatch from the chain ladder run-off model.

• **Prior-year business model** for the underwriting year \( k = -1 \) with initial amount from the balance sheet and one-year mismatch from the new business model.

• **New business model** for the underwriting year \( k = 0 \) with initial amount equal to the expected loss from the pricing model and one-year mismatch from the new business model.

For the year \( i = 1 \),

• **Run-off business model** for the underwriting years \( k \leq -1 \) with initial amount at \( t = 1 \) not needed and one-year mismatch from the chain ladder run-off model\(^2\).

• **Prior-year business model** for the underwriting year \( k = 0 \) with initial amount not needed and one-year mismatch from the new business model.

For the years \( i \geq 2 \),

• **Run-off business model** for the underwriting years \( k \leq 0 \) with initial amount not needed and one-year mismatch from the chain ladder run-off model.

### 3.3 Stochastic Model for New Business

In this section we describe the model for new business. By definition, the new business \( N_0 \) is the planned business to be written in year \( i = 0 \). Note that a contract which is renewed in year \( i = 0 \) is considered new business, even though it is not, strictly speaking, “new” business in the sense that this contract has not been written in the prior year. The new business model is also used for the pricing capital allocation (*see Section 8.3, pp. 168 in this document*).

As outlined in Section 3.2, the new business portfolio \( N_0 \) of business to be written in year \( i = 0 \) is decomposed into baskets

\[
N_0 = N_0^1 \cup N_0^2 \cup \ldots
\]

\(^2\)The initial amounts at times \( t = 1, 2 \ldots \) are not needed because the corresponding models are only used for the market value margin, and the calculation of the market value margin does not use the initial amounts but only the one-year mismatches.
with the dependency structure between baskets given by a portfolio dependency tree as described in Section 8.1. Note that, for ease of notation, we denote here by $M^b_i$ the basket mismatches specifically for the new business.

By definition, new business does not enter the calculation of the reserves at $t = 0$ from (3.4), and it remains to calculate the basket mismatches

$$M^b_i = \mathbb{E}[R^b_i | \mathcal{F}_{i+1}] - \mathbb{E}[R^b_i | \mathcal{F}_i] \quad \text{for } i \geq 0,$$

conditional on the information available at time $t = 0$, and to specify their dependency structure. See Section 3.3.2 (pp. 59) for the specification of the current dependency structure.

Our approach to calculate the one-year basket mismatches $M^b_i$ as described in Section 3.3.3, pp. 62, is to scale the “ultimate change”

$$R^b_0 - \mathbb{E}[R^b_0 | \mathcal{F}_0]$$

by a factor $0 \leq \kappa^b_i \leq 1$ measuring the fraction of the ultimate change we expect to be manifested in year $i$, i.e.

$$M^b_i = \kappa^b_i \cdot (R^b_0 - \mathbb{E}[R^b_0 | \mathcal{F}_0]).$$

The basket mismatches $M^b_i$ according to Equation (3.14) are calculated in three steps. First of all, using the pricing models, we obtain the distribution, conditional on the information available at $t = 0$, of the nominal ultimate change

$$S^b_0 - \mathbb{E}[S^b_0 | \mathcal{F}_0],$$

where $S^b_i$ for $i \geq 0$ denotes the nominal sum of the future payments $X^b_j$,

$$S^b_i := \sum_{j \geq i} X^b_j.$$

In a second step, the nominal sum $S^b_0$ is discounted to get the difference

$$R^b_0 - \mathbb{E}[R^b_0 | \mathcal{F}_0].$$

Finally, this difference is transformed into one-year changes by (3.14).

Considering the first step, the basket difference (3.15) is calculated by two different approaches, see Section 3.3.2 (pp. 59).

- Default basket model: The model for a basket is constructed by aggregating the individual treaty pricing models of all contracts in the basket priced in the pricing tool. Dependency between contracts is expressed by an implicit dependency model using a multi-dimensional copula. The resulting figures are adjusted for contracts not priced in the pricing tool. See Section 3.3.2 (pp. 59) for more on the default basket model.
• **Basket specific models**: In these cases, the basket model is determined directly using an explicit dependency structure. As a general rule, specific models are used for business with a high impact on the balance sheet.

Because pricing models thus are part of the SST calculation, we provide an overview of the pricing methodologies in Section 3.4, pp. 63. Note that the pricing models are for treaty business and not for facultative business. Facultative business is considered implicitly by adjusting the aggregated pricing models to the planning figures, which include facultative business. We believe this approach to be justified as the current volume of facultative business is small as compared to treaty business.

Considering the **second step**, to get discounted cash flows from nominal amounts, the cash flows are discounted using **development patterns**. Patterns express the *expected* cash flow in one year as a percentage of the nominal sum of all cash flows. More on patterns can be found in Section 3.2.2 (pp. 50).

Concerning the **third step**, to transform the “ultimate change” to one-year changes by (3.14), see Section 3.3.3 (pp. 62).

### 3.3.1 Data Requirements and Parameters for New Business

We summarize the requirements on the SST new business model.

- **Definition** of the new business baskets (see Section 3.3.2, pp. 59)
- **Treaty pricing models** for the new business for a rolling year within years $i = -1$ and $i = 0$ including losses, premiums, expenses, as well as **development patterns** (see Section 3.4, pp. 63).
- **Planning figures** for underwriting year $i = 0$ business, to calculate *growth factors for volume and inflation* to adjust the pricing models to planned volume. *Volume growth* means growth in number of accounts, while *inflation growth* means growth in (average) treaty size, or inflation.
- **Portfolio dependency tree** to express the dependency structure between baskets, including dependency parameters (see Section 3.3.2, pp. 59).
- Method (see Section 3.3.3, pp. 62) to transfer “ultimate changes”

$$R_0^b - \mathbb{E}[R_0^b \mid \mathcal{F}_0]$$

**to one-year changes**

$$M_0^b = \mathbb{E}[R_0^b \mid \mathcal{F}_1] - \mathbb{E}[R_0^b \mid \mathcal{F}_0]$$

- **Development patterns**: different types, see Section 3.2.2, pp. 50
3.3.2 Dependency Structure and Aggregation for New Business

The dependency structure for new business is expressed by a portfolio dependency tree as described in Section 8.1 (pp. 148). The dependency tree we are currently using\(^3\) is shown in Tables 3.1, 3.2. Note that each basket needs to be further split by legal entities.

Default Basket Model

In the default basket model, the basket distribution is derived by aggregation of the treaty pricing distributions. To express dependency between contracts in one basket, we use a multi-dimensional one-parametric Clayton copula with parameter \(\theta\) as described in Section 8.2.4.

Concerning the estimation of \(\theta\), we stress that we have by far insufficient data to estimate \(\theta\) statistically. We therefore rely on expert opinion to come up with an estimate of the shifted lower tail dependency \(\overline{TD}(q)\) as defined in 8.13 by expert opinion:

- Given that one contract in the basket at the end of next year is worse than 1 in 100 years, what is the probability that any other contract in the basket is worse than 1 in 100 years?

The treaty pricing models are then aggregated by drawing common samples of the contract distributions using Monte Carlo methods. Note that the contract losses, premiums, and expenses are co-monotone, so the same dependency assumptions can be used.

To reality-check the involved parameters derived by expert estimates, we study

- Historical events which resulted in simultaneous claims in various lines of business
- The results of scenario evaluations
- The shape of the resulting aggregated basket models, to compare them with historical basket outcomes and to “reality check” them by looking at different quantiles.

\(^3\)The dependency tree is implemented with the proprietary aggregation tool Phobos. For a more detailed description of this IT system we refer to Part VIII of this documentation.
The property related baskets and the structure of the portfolio dependency tree which are currently used and implemented in Phobos. For a basket, the $\theta$ value in brackets specifies the multi-dimensional Clayton copula expressing dependency between the children of the basket. The two top baskets “Property related” and “Not property related” (see Table 3.2) are aggregated with $\theta = 0.1607$.

Table 3.1: The portfolio dependency tree (property related baskets).
The *not property related* baskets and the structure of the portfolio dependency tree which are currently used and implemented in Phobos. For a basket, the $\theta$ value in brackets specifies the multi-dimensional Clayton copula expressing dependency between the children of the basket. The two top baskets “Property related” (see Table 3.1) and “Not property related” are aggregated with $\theta = 0.1607$.

Table 3.2: The portfolio dependency tree (*not property related* baskets).
Basket Specific Models

Basket specific models directly specify the basket distribution by an explicit dependency model. Currently, such basket specific models are available for Property Nat Cat business (see Section 3.8, pp. 80), and for basket consisting of only one large individual contract (e.g. GAUM, MDU), where the basket distribution is calculated directly by the responsible pricing actuary. Potential candidates for future specific models are Credit and Surety, and Aviation.

Aggregation of Basket Models

Further aggregation of the baskets along the portfolio dependency tree follows an approach analogous to the default basket model above, using a multi-dimensional Clayton copula.

3.3.3 One-Year Changes for New Business

In this section, we describe how to transform the “ultimate change” for basket $b$, $R_b^0 - \mathbb{E}[R_b^0 \mid \mathcal{F}_0]$ to the one-year change

$$M_b^i = \mathbb{E}[R_b^0 \mid \mathcal{F}_{i+1}] - \mathbb{E}[R_b^0 \mid \mathcal{F}_i]$$

for $i = 0$. In general, the relation between the random variables of ultimate and one-year change would be given by a copula.

As an approximation, we use a linear proxy $0 \leq \kappa_b^i \leq 1$ to get from the ultimate change to the one-year change, i.e.

$$M_b^i = \kappa_b^i \cdot \left( R_b^0 - \mathbb{E}[R_b^0 \mid \mathcal{F}_0] \right).$$

(3.16)

Since the one-year changes over all years $i \geq 0$ should add up to the ultimate change, the sum of the $\kappa_b^i$ for $i \geq 0$ should be equal to 1. Moreover, $\kappa_b^i$ should be largest for those years $i$ where we expect the most significant changes.

We propose to be consistent with the proposal of Section 2.2.3 (pp. 35) for the calculation of the market value margin, and thus to use as a proxy $\kappa_b^i$ for the ratio of ultimate change to one-year change for new business for a basket the reported pattern by underwriting year for this basket:

- The proxy $\kappa_b^i$ for the ratio of one-year change in year $i = 0$ to ultimate change for the new business from (3.16) is proposed to be the incremental underwriting year reported loss pattern for year $i = 0$, starting at January 1. Regarding patterns, see Section 3.2.2, pp. 50.
The rationale behind this approximation is that, for short-tail business, we expect the biggest one-year change to occur in the first year $i = 0$, especially with regards to natural catastrophes (Nat Cat). The ultimate risk is then very close to the one-year change for year $i = 0$, and the corresponding pattern is almost at 100%.

For new long-tail business, the risk of a one-year change in year $i = 0$ is small: It is very unlikely that new information for long-tail business will be revealed during the first year, and this perception is confirmed by our reserving data.

### 3.4 Treaty Pricing Models for New Business

As mentioned above, the SST model for new non-life business $N_0$ written in year $i = 0$ is based on treaty pricing models constructed by pricing actuaries and underwriters. Recall that new business here includes also renewed business and not just business not previously written.

According to the pricing guidelines, all treaty business (with the exception of very small treaties) has to be priced\(^4\).

The pricing workflow is set up in a way to guarantee that the treaty conditions assumed in pricing are consistent with the conditions specified in the underwriting system\(^5\).

The pricing criteria specify which contracts have to be either priced by a pricing actuary (or Nat Cat specialist), peer-reviewed by a pricing actuary, or which can be priced by an underwriter. Roughly speaking, the pricing criteria for actuarial pricing/review apply to proportional bouquets with large premium (our share), and to XL programs with large limit and/or large premium (both for our share), with more restrictive criteria for third party liability programs.

A similar set of peer review criteria specify which business has to be peer-reviewed by a senior pricing actuary. Adherence to these criteria is regularly monitored in the pricing profitability study and audited by group internal audit. Current figures for underwriting year 2007 as per August 27, 2007 show that 94.6% of the premium volume is priced in the pricing tool, and 84.5% of the premium volume is priced by pricing actuaries or Nat Cat specialists.

Roughly speaking, the general process for pricing a treaty consists of the following steps:

1. **Data requirements and analysis**: Ensure that the data supplied by the cedent (i.e. the reinsured insurer) corresponds to the data requirements for this particular type of business, and check whether the data

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\(^4\)In our proprietary pricing tool MARS. We refer to Part VIII.

\(^5\)The current underwriting system is GLOBUS.
is reasonable and, in particular, consistent with last year’s (and prior year’s) renewal information.

2. “As if” adjustments to the data: The submitted data is usually historical or from a certain snapshot in time, so it has to be adjusted to the upcoming treaty period. Adjustments include rate changes, loss inflation, volume adjustments, changes in the underlying portfolio, development to ultimate amounts etc. One particular adjustment is for incomplete years and/or incomplete most recent diagonal in a development triangle.

3. Loss model fitting: In this step, one or several loss models are derived from the adjusted data, typically by fitting a family of analytic distributions (e.g. log-normal, Pareto). Several loss models might be used for instance for different sub-lines of business covered. In particular, this step implies extrapolation of the available loss history to extreme bad outcomes which have not yet been observed but can potentially occur in the future. According to the pricing guidelines, at least one model has to use a heavy-tailed distribution. This is done to ensure that extrapolation gives sufficient weight to catastrophic outcomes, and particular emphasis is put on the appropriate modeling of the tail of these distributions.

4. Stress Testing and Reality Checks: In this crucial step, the selection of parameters underlying the loss model is varied to evaluate stability of the model and, if necessary, to refine it. Further, the resulting loss model is “reality-checked” by evaluating its reasonableness, for instance with regards to the past experience, the proposed reinsurance structure, and information from similar types of treaties. Since the loss models are typically based on taking averages of adjusted historical figures, it is crucial to check for trends remaining in the adjusted figures. Existence of such trends typically suggest that the selected adjustments do not fully capture changes to the business over time. There are various functionalities in the pricing tool available to assist these enquiries.

5. Application of the treaty structure to the loss model: In this section, samples from the loss models are drawn by Monte Carlo simulation. In sampling, it is possible to consider dependencies between the different loss models. Next, all relevant treaty and program features are applied to the loss samples. This includes limits and deductibles, and loss dependent premium features (reinstatements etc.) and external expense features (sliding scale commission, profit commission, etc.). Features that apply on program level, i.e. to the aggregated outcome from several treaties, can also be considered. Then, the cash flows
are discounted using default or customized quarterly patterns. The outcome of these simulations is a distribution of the net present value NPV of the contract result. This NPV distribution of the reinsurance results can be further “reality-checked.” See Section 3.4.5 (pp. 71) for more details.

6. Calculation of internal expenses and capital costs: This provides the final profitability estimate of the contract. The adequacy of the offered conditions such as premiums is measured by the so-called performance excess (Section 8.3, pp. 168).

As a general rule, the loss model constructed in pricing is a stochastic model of the nominal, ultimate losses to the cedent’s business subject to the reinsurance cover, in particular, prior to application of the reinsurance structure. Once such a “gross” loss model has been created, the specified reinsurance structure is automatically applied to the loss model in the pricing tool, including discounting of the cash flows using default or customized patterns, and calculation of corresponding internal expenses and capital costs. (Regarding patterns, see Section 3.2.2, pp. 50.) The calculation of the capital costs, the pricing capital allocation, is described in detail in Section 8.3 (pp. 168).

Note that the pricing models are usually gross of retrocession. Retrocession is only considered in pricing in case common account protection is in place for a specific program, so that the reinsurance premium can be reduced by the costs of this retrocession, and the loss model can be adjusted for the explicitly-given retrocession structure. Another potential situation could be in case the cedent also purchases Fac reinsurance to the benefit of the reinsurance treaty under consideration.

For treaty pricing, we distinguish

- Proportional contracts (e.g. quota share, surplus)
- Non-proportional contracts (XL programs, stop-loss)

Pricing models can be based on

- Historical treaty results (experience pricing)
- (Estimated) exposure information (exposure pricing)

We describe proportional pricing in Section 3.4.1, and non-proportional pricing in Section 3.4.2. There is usually a default approach available, and there might be specific approaches for certain lines of business. We consider all these different approaches. More specific exposure pricing approaches are described in specific sections for Property Nat Cat (Section 3.8, pp. 80),
3.4.1 Proportional Pricing

Proportional business comprises Quota Shares, Surplus contracts etc. Characteristically, the sharing of premiums, losses and expenses between cedent (i.e. the insurer) and reinsurer is on a proportional basis.

From a modeling point of view, proportional business is distinguished from non-proportional business in that, typically, any loss will impact the reinsurance treaty, whereas for non-proportional business, only sufficiently large losses need to be considered.

We therefore typically separately model the attritional or basic losses and the large losses. Both attritional and large losses might be modeled by one or several different loss models.

- An attritional loss model is expressed by the aggregate distribution of the loss ratio without large losses and/or extremal events.

- A large-loss model is a frequency/severity model, where the frequency is Poisson, Binomial, or Negative-Binomial distributed, and the severities follow a heavy-tailed distribution such as Pareto or Generalized Pareto.

The large-loss model is derived in the same way as the model for non-proportional treaties and is described in Section 3.4.2.

Attritional losses are losses which are observed regularly ("high frequency, low severity"), whereas large losses occur rarely, but have a high impact on the result ("low frequency, high severity"). By creating a separate large-loss model, the tail of the loss distribution can be evaluated more precisely. To a certain extent, the attritional loss amount is "more predictable," and thus reflects the "quality" of the insured business, whereas large losses reflect the "volatility," i.e. the random component.

According to the guidelines, at least one loss model has to be heavy-tailed. Typically, the following rule is applied: If the large losses are modeled separately, the attritional losses are modeled by a log-normal or normal distribution. In case no separate large-loss model is constructed (e.g. because large-loss information cannot be obtained), the loss ratios are modeled by a Czeledin distribution. The Czeledin distribution is a log-normal distribution up to a certain quantile, typically at 90%, and continued with a Pareto distribution in the tail. In contrast to the log-normal distribution, the Czeledin distribution is heavy-tailed, and thus allows for consideration of the tail.

The model for the large losses is derived in the same way as for non-proportional treaties and is described in the next Section 3.4.2. It is based
on experience pricing or, preferably, on exposure pricing.

In the remainder of this section we thus describe the *attritional loss model*. It consists of the three steps *data requirements*, “as if” *adjustments*, and *fitting an analytic distribution*.

**Data Requirements**

The following data is required for the attritional loss model:

- Development triangles by treaty years and development years of the annual aggregate amounts of
  - paid and outstanding or incurred (reported) losses
  - earned premiums
- Estimated written premium for the upcoming treaty year
- Large-loss information
  - ideally, with individual loss development
- Historical rate changes
- Information on the changes in the underlying portfolio/exposure over the history period considered

**“As-if” Adjustments**

In order to model attritional losses only, the large losses need to be deducted from the loss triangle, to avoid double counting. Since large losses might distort the triangle development, it is preferable to remove the whole development of the large losses. In case this information is not available, it is assumed that the large losses are ultimate values, and their development is inferred from the triangle development to remove them.

The triangles need to be *developed to ultimate amounts* for each treaty year. The default method used is the chain ladder method, but individual adjustments based on other methods are possible when appropriate.

One of the three assumptions of the Mack model for the chain ladder method is that different treaty years are independent. From a theoretical point of view, this implies that, before developing to ultimate amounts, the triangles have to be “as if” adjusted to the level of the upcoming treaty period. Such adjustments include rate changes, loss inflation, changes in portfolio composition etc. The resulting *adjusted estimated ultimate loss ratios* can then be interpreted as i.i.d. samples from one distribution, and distribution fitting can be applied, see below.
An alternative approach is used, for instance, for Credit and Surety business. This method consists in estimating unadjusted ultimate historical loss ratios by developing the unadjusted triangles. In many instances, the insurance cycle is visible in the time series of the unadjusted ultimate historical loss ratios, once large losses are removed from the triangle. The expected loss ratio for the upcoming treaty period can then be estimated by a naive forecast of the insurance cycle, taking into account economic developments etc.

Regardless of the method used, it is always helpful to compare the final loss ratio selection with the unadjusted historical loss ratios.

Fitting an Analytic Distribution

The families of analytic distributions available as a loss ratio model are the normal, log-normal and Czeledin distribution. As mentioned above, the Czeledin distribution is usually used when no separate large-loss model is constructed; otherwise, the other two options might be used.

In case the loss ratios are “as-if” adjusted as mentioned above, they can be understood as i.i.d. samples from one distribution, and thus a direct curve fitting can be done: The distribution of loss ratio samples is fitted to an analytic distribution family (e.g. log-normal) by a chi-square method taking into account model uncertainty\(^6\). The fit is visualized graphically.

In case the loss ratios are not adjusted, such direct fitting is not possible, and we then use the following method: The expected loss ratio is estimated as described above by naive forecast, and the standard deviation is estimated from the annual changes in historical loss ratios.

3.4.2 Non-Proportional and Large-Loss Pricing Model

As mentioned before, loss modeling for non-proportional treaties and for the large-loss model for proportional contracts is essentially the same, so we describe it at the same time. In both cases, the objective is to construct a frequency/severity model of losses exceeding a certain threshold.

There are two primary methods for constructing a large-loss model:

- *Experience pricing*, where the loss model is based on the historical large losses of the cedent
- *Exposure pricing*, which takes the actual exposures, i.e. the portfolio of risks, as a starting point

For experience pricing, two different methods are implemented in the pricing tool, one for short-tail and one for long-tail business. The two modules are described in Section 3.4.3.

\(^6\)The fitting method used is the same as the one used to fit severity distributions.
For exposure pricing, there is a standard method available based on the usual exposure curves. This method is described in Section 3.4.4.

Further, specific exposure models are used for:

- Aviation, see Section 3.5, (pp. 74)
- Credit and Surety, see Section 3.6 (pp. 76)
- Property natural catastrophe business, see Section 3.8 (pp. 80)

### 3.4.3 Non-Proportional Experience Pricing

In the pricing tool, two different modules exist for experience pricing non-proportional contracts:

- Short-tail module
- Long-tail module

The short-tail module is used for short-tail lines of business (e.g. Property non-cat, Marine); the underlying assumption in this module is that the number of historical losses and their severity do not have a large tail. The long-tail module is used for long-tail lines of business (Motor third party, liability etc.), where we expect further significant development for the past treaty years in the number of losses and in their size. In both modules, a frequency/severity model is constructed based on the historical losses of the cedent exceeding a certain reporting threshold.

To take into account changes in the portfolio over time, an exposure measure is required for the historical treaty periods, and an estimated figure for the upcoming treaty period. The exposure measure selected is often the relevant premium (SUPI = subject premium income, or GNPI = gross net premium income) received by the cedent for the reinsured business. Other exposure measures are used when premiums are not an adequate measure. Examples for other exposure measures are total sum insured for Property per risk; number of vehicle years or number of insured policies for Motor liability; and class equivalents for doctors and bed counts for hospitals in medical malpractice insurance etc.

If premiums are used as exposure measure, they must be developed to ultimate amounts if necessary, and adjusted for rate changes to bring them to the current rate level. This requires information on historical and planned rate changes. If available, premiums should be further adjusted for changes in the underlying risk and insurance coverage, such as changes in deductibles and limits.
Historical losses are “as if” adjusted to the level of the upcoming treaty period, taking into account loss inflation, potential loss caps etc. Stabilization clauses (index clauses) are applied to the layer if applicable. They then have to be developed to ultimate values if applicable. In the long-tail module, this development does not apply to individual losses but to the aggregate losses to a suitably selected layer, both in a triangle of the aggregate loss to layer and in a triangle of the number of losses to layer. To develop the triangle, chain ladder and cape cod methods are available.

Finally, the losses are adjusted for changes in the exposure measure, either by adjusting their frequency (“per risk”), or by adjusting their severity (“per event”), to take into account changes in the portfolio over time.

In the short-tail module, the expected frequency of losses exceeding a selected model threshold is calculated as a weighted average of the adjusted number of losses. This determines a Poisson distribution for the number of losses. By considering the Q-Panjer factor (the standard deviation divided by the expected value), it can be evaluated whether Poisson is a good model. Alternatively, a Binomial or Negative-Binomial distribution can be determined. The severity distribution is determined by fitting an analytic distribution to the adjusted losses exceeding the threshold by a chi-square method. Severity distributions used are heavy-tailed and typically Pareto, Generalized Pareto, or log-gamma. In particular, this leads to an extrapolation of the observed losses to higher loss amounts.

In the long-tail module, the expected frequency to calibrate a Poisson distribution is derived from the development of the triangle of the number of losses to layer. For the severity, a Pareto distribution is calculated from the expected frequency and the expected loss to layer (“burning cost”) obtained from the developed triangle of total loss to layer.

The pricing tool offers a number of visual helps to determine the goodness fit of the model to the data, the adequacy of projections to higher layers, and the detection of trends.

3.4.4 Standard Exposure Pricing

Standard exposure pricing is based on the risk profile of the cedent or, more generally, on the portfolio of insured risks. The risk profile combines the risks into sum insured bands (or limit bands for liability business): All risks whose sum insured is between minimum and maximum of a given band are assigned to this band.

The information required per band consists of

- The number of risks in the band
- The average sum insured of these risks
• Their total premium

The pricing tool is also able to handle layered profiles, i.e. profiles of layers. To derive a frequency/severity distribution from the risk profile, one has to select an exposure curve and estimate an expected loss ratio per band.

Given a risk profile, the risks in a band are automatically distributed within the band consistent with the average sum insured per band. From the exposure curve, the severity distribution of each risk is derived, see below. From the expected loss ratio and the total premium, the expected loss per band is derived, which in turn allows to calculate the expected loss frequency per band. The resulting individual model is approximated by a collective model in the form of a compound Poisson model, providing the desired frequency/severity model.

The exposure curves used for Property business and other lines are so-called deductible Rebate curves. These curves correspond to the loss degree distribution, that is, the distribution of the severity as a percentage of the sum insured. The underlying assumption when applying a Rebate curve to a portfolio of risks with different sums insured is that the distribution of the loss degrees is the same for all risks in the portfolio; in particular, the loss degree is assumed to be independent of the sum insured. Since this is not always correct (e.g. Property per risk), different curves might need to be applied to different bands.

For other types of business, primarily for third party liability business, other types of exposure curves are selected, such as increased limit factor (ILF) curves. The assumption underlying this model is that the absolute severity distribution is the same for all risks in the band, but that the severity is capped at the appropriate risk limit.

In the pricing tool, the risk profile can be adjusted in various ways (volume changes, inflation, insuring quota share or surplus); and a variety of exposure curves are available (Swiss Re Y-curves, MBBEFD, ISO, Riebesell, ILF, user-defined curves).

3.4.5 Calculating the Treaty NPV Distribution

In this section, we describe the derivation of the treaty NPV (net present value) distribution of the treaty results, i.e. the distribution of the present value of

\[ \text{premium} - \text{losses} - \text{external expenses} - \text{internal expenses}. \]

In view of the preceding sections, we have available one or several stochastic loss models of the cedent’s business subject to the reinsurance cover. Typically, these models provide the nominal ultimate losses. The task is then to apply all relevant reinsurance treaty and program features to these loss models, and to discount the cash flows.
In general, the structure of a reinsurance deal is given by a program consisting of one or several treaties. For instance, a non-proportional program might consist of different layers, and a proportional bouquet of several quota shares. Moreover, certain programs might consist of both proportional and non-proportional treaties. An example is given by a quota share treaty together with one or several XL layers on the retention of the quota share. Because a sufficiently large loss will affect both the quota share and the XL layers, it is crucial to model the two treaties together. More generally, profitability of a deal has to be evaluated on the program level. All structures mentioned above can be captured in our pricing tool; in particular, proportional and non-proportional treaties can be considered together.

To calculate the reinsurance result, a number of different loss models derived for the reinsurance program (as described in the previous sections) are available. The following types of loss models can be considered:

- **Aggregate distributions**, either as loss ratios or as absolute loss amounts, for the distribution families Normal, Log-Normal, Czeledin, Empiric (piecewise linear), and Discrete (step function).

- **Frequency/severity distributions** with frequency distribution families of Poisson, Binomial, Negative Binomial, Panjer, and Bernoulli; and as severity distributions, Log Normal, Czeledin, Pareto, Generalized Pareto, Gamma, Log Gamma, Empiric, Discrete.

- **Cat**: Event loss tables for major perils used to model natural catastrophe risks

- **Aviation** specific loss model for the Aviation market model.

The appropriate loss models are attached to the treaties in the program. Several treaties can use the same loss model, and several loss models can be used for one treaty.

The NPV distribution is determined by Monte Carlo simulation. The number of simulations can be selected manually and is typically around 1’000’000 iterations. The whole calculation for such a case usually takes a few seconds. The NPV distribution is determined in the following three steps applied for each individual simulation iteration:

- For each loss model, a loss sample is drawn. Dependencies between different loss models can be considered, if appropriate, by means of a multidimensional Clayton copula (see Section 161, pp. 161). This may enable more appropriate modeling of the tail.

- The quarterly cash flows of the (ultimate) loss samples are derived using quarterly patterns for premiums and losses, either using default patterns or patterns derived from cedent data.
• The relevant *terms and conditions* are applied to the loss cash flows to derive cash flows for losses after reinsurance, premiums, and expenses. A list of terms and conditions considered is found below. In this step, rational cedent behavior is taken into account, for instance for the case of a no claims bonus (NCB).

• Next, the derived cash flows are *discounted* to obtain the present value at treaty inception. This is done by selecting for each cash flow the yield for the appropriate maturity from default *yield curves* in the appropriate currency from Bloomberg (for government bonds in the respective currency).

After completion of all simulation iterations, *internal administrative expenses*, the *NPV distribution*, and the *allocated capital costs* are calculated. Profitability of the program is measured by the *performance excess* (see Section 8.3, pp. 168).

The relevant terms and conditions are usually imported from the underwriting administration system. The pricing tool supports *treaty features* such as

• Limits, retentions, AAD (annual aggregate deductibles), AAL (annual aggregate limits)

• Premium features, including reinstatements and swing rates

• External expense features such as fixed or sliding commissions, brokerage and other expenses, fixed or sliding profit commissions

• NCB (no claims bonus), common account protections, loss corridors

• Etc.

as well as *aggregate features* over several contracts in one program:

• AAD, AAL, NCB

• Profit commission (fixed or sliding), sliding scale commission.

In order to analyze the program results, *simulation results* are available for the whole program as well as for each treaty. They include

• Expected values and standard deviation for premium, loss, expenses, and NPV

• Expected *cash flow effect* to measure the impact of discounting of cash flows on the result, and on premiums and losses

• *Graphs of the cumulative distribution function* for losses, premiums, expenses, and NPV
• Measures of profitability such as performance excess, TRAC (time and risk adjusted capital, see Section 8.3, pp. 168), (stand-alone) expected shortfall of the NPV

• Risk transfer checks to ensure that the program fulfills risk transfer criteria

• Above figures allow plausibility checks such as
  – expected shortfall of NPV / expected premium ("how many times the premium is lost in a very bad outcome")
  – performance excess / TRAC ("excess return on capital")
  – probability of negative NPV ("probability that we lose money")

and further figures to allow checks for non-proportional business,
  – probability of loss > 0 ("probability of loss to layer")
  – probability of aggregate loss > 0 ("probability that one limit stretch is not sufficient")
  – probability of maximum aggregate loss ("probability of using up the whole capacity, including reinstatements").

Once the final pricing version of a program is determined, the corresponding treaty results are transferred to the underwriting administration system.

### 3.5 Exposure Pricing for Aviation

In the following we outline the exposure pricing approach for Aviation business. Experience pricing is normally not used because there is in general not enough loss experience to model large losses based on the client’s loss experience alone.

*General aviation business* is priced with the standard exposure pricing approach. Specific market exposure curves of the distribution of the loss as percentage of sums insured are available. The exposure information consists of client risk profiles by type of coverage (e.g. Hull, Liability) with number of policies, sums insured, and premiums per band.

*Direct airline business* a specific exposure pricing approach is used. The exposure information needed here is the *standardized aviation questionnaire* providing, in particular, detailed information about the client’s exposures on individual airlines/manufacturers (Non-US Airlines, US Airlines, Products). Our standard approach is to use our in-house aviation market model to simulate market losses, and to apply to these market losses the client’s market shares from the questionnaire information. The aviation market model is described in more detail in the next section.
3.5.1 Aviation Market Model

Our aviation market model developed in-house applies for direct airline business. We simulate market losses for Non-US Airlines, US Airlines and Products and apply the client’s shares to these simulated market losses. The model simulates market losses in excess of USD 100 million. For both Non-US Airlines and US Airlines, we simulate a random number of losses, corresponding severities, and the primary policies affected by the losses. Furthermore, with certain probabilities, related manufacturer’s losses are simulated and added to the pure Airline losses. Pure Product losses are simulated in addition. For each of the losses it is simulated if other airlines or manufacturers are involved (multi-losses). If yes, additional loss severities are simulated the same way.

Underlying Data

Our market model is based on historic market losses. The indexed market losses are analyzed in our pricing tool and fitted to heavy-tailed severity distributions. As exposure measures the market premiums are used, adjusted for market rate changes. Frequencies are simulated with a Negative Binomial distribution. The market premium and premium rate changes for Hull, Liability and Products come from different sources/brokers.

Application

We are mainly interested in the schedule A (Non-US Airlines), B (US Airlines) and C (Products) of the standardized aviation questionnaire, giving detailed information about the airlines/manufacturers exposures of the client.

We assume a market loss hits always a questionnaire listed airline (Non-US Airline, US Airline) or questionnaire listed manufacturer (Products). Please note that not all airlines are part of the questionnaire. Therefore one basic assumption is that the questionnaire is a representative sample, i.e. we assume the same behavior for non-listed airlines/manufacturers. Each airline respectively manufacturer has a certain probability assigned to be hit. This specific probability is calculated based on various exposure measures.

One major issue simulating market losses is that not each market loss can be a loss for each of the airlines respectively manufacturers, due to the given policy limit, in particular the very high market losses. Therefore one has to calculate the conditional probabilities that a specific airline or manufacturer has caused a given market loss. Based on the given policy limits we create baskets of all possible market losses with probabilities and possibly
affected airlines respectively manufacturers. In these baskets we calculate the conditional probabilities of these airlines respectively manufacturers.

Having simulated these market losses and allocated to the airlines respectively manufacturers in the way described above, the individual client shares on these losses can be calculated, based on the questionnaire information we received.

### 3.6 Credit and Surety Exposure Pricing

We outline here the standard exposure pricing approach for credit and bond business. Political risk and US Surety are excluded in this part.

The methodology is outlined by Bea Wollenmann, *Standard Approach to Pricing Credit and Bond Reinsurance Treaties* (Wollenmann [2007]). The following sections reproduce this paper.

#### 3.6.1 Data Requirements

For any type of reinsurance treaty covering trade credit or bond, the *list of granted credit limit per risk* and *bonded limits per risk* for “major” risks is indispensable. As exposure one uses for credit risks the granted limit, i.e. the limit of insured credits per risk set by the insurer, and the bonded limit per risk for bond exposures. The notion of granted limit must be consistent with the one agreed upon in the PML-study (Stahel and Jaggi [2003]). For proportional covers, we call a risk major, if the exposure to the treaty (=sum of the covered limits) is greater than 5% of the cedent’s GNPI. In case of an XL-cover, a risk is *major* if the exposure is greater than the priority of the smallest layer. Depending on inflation and expected growth, also risks which are currently not exposing the XL-programme might do so in the course of the contract period and need therefore be considered accordingly in pricing.

The cedent should report his internal ratings for all major risks as well. Moreover, a risk profile for non-major risk is highly desirable. Anticipated shifts in the risk profile between the reported profile and the expected profile during the covering period need to be reported as well.

In case of bond coverage on a risk attaching basis, a split by underwriting year is needed. However, there are so far only few cedents reporting on that basis. Ideally, the cedent’s report is split by types of bonds as well.

#### 3.6.2 Loss Model

For an exposure-based loss modeling we use in principle the individual model and the corresponding collective model obtained by compound Poisson approximation (see Rolski et al. [1999]). To this end, a probability of default and a loss-given-default-distribution (LGD-distribution) is associated with each risk.
The probability of default for a single risk is generally obtained from Moody’s KMV. In case the risk can be found in MKMV, then we use the 1-year EDF (Expected Default Frequency) for the company. In case a risk is not monitored in MKMV (e.g. non-public companies) or in case the model is based on a profile with unnamed risks in each band, then we select an appropriate EDF. To this end there are various approaches from which we select the one which seems most appropriate:

1. Mapping cedent’s internal ratings with EDF for those risks which are covered by MKMV using a regression analysis.
2. Taking average EDFs over an appropriate universe, e.g.
   (a) The intersection of cedent’s portfolio with the risks covered by MKMV.
   (b) Depending on the quality of the portfolio, select an EDF between the median and the 10% quantile of MKMV’s EDFs for country or industry sector.
   (c) Individually assessing a risk by comparing it with its peers in MKMV.

It remains to choose the appropriate LGD-distribution which is equivalent to choosing an exposure curve. There are essentially three cases to distinguish:

1. *Trade credit exposure*: We choose the LGD-distribution from the PML-study and distinguish between large and small insurers.
2. *Bond exposure*: This depends very much on the type of bond. If the cedent provides sufficient data of his historic losses, then we can construct an individual LGD distribution. Otherwise, we have historical data from a large European cedent which allowed us to deduce a rough LGD-distribution and which we used for similar business of other clients in Europe as well. Another approach is the selection of an MBBEFD-curve\(^7\). In general, \(E(LGD)\) should not be smaller than 35%.
3. *Mixture of bond and trade credit exposure*: We build a mixture of the corresponding LGD-curves for credit and bond. The mixture is done under the assumption of co-monotonicity between credit- and bond-LGD. Usually it suffices to split the portfolio into not more than five risk groups, namely a group with a negligible, small, medium, high bond exposure and a group with negligible credit exposure. The mixture of the LGD-curves is done for each group based on its average credit-bond-mix.

\(^7\)MBBDEF stands for Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac, three classes of analytical exposure curves derived from (and named after) the three major sorts of probability distributions used in statistical mechanics.
Note that for the exposure loss model we do not make use of the actual premium per risk. Instead, we calculate for each risk its EDF multiplied with its limit and the expected loss given default. This calculated figure is used in the exposure model as premium and equals the expected ground-up loss of the considered risk. In the usual exposure module, an implicit probability of loss is calculated from the premium and loss ratio information and the selected exposure curve. If we enter for the premium the figure described before and set the loss ratio to 100%, then the implicit probability of loss, deduced from this input, is again the EDF of the considered risk.

3.7 Credit and Surety Outlook: Portfolio Model

3.7.1 Modeling Approach

We receive from our clients sufficient information that it would in principle be possible to model the tail of the loss distribution for our whole credit and bond portfolio, and gain also explicitly the tail dependency between an individual contract and this total credit and bond portfolio.

To reach this goal, we would need to do a Monte Carlo simulation, selecting in each iteration the companies defaulting randomly but with the probability given by their EDFs. The loss given default could be selected co-monotone from the different LGD distributions for all contracts exposed to this specific defaulting risk. Finally, applying all contracts to the simulated losses exposing them in one iteration, we could obtain our loss distribution coming from these largest exposures and also measure the tail dependency between any two contracts or a contract and the rest of the portfolio.

However, if one looks closer to the details, there are several problems to be overcome to complete this task. While we have already resolved some of them and have set up our internal tools, especially also our accumulation control CrediEx, such that they will support such an approach, it would still be a major project to reach this goal.

3.7.2 Identification of the Risks

The risks for credit and bond are the individual companies. However, there is usually not a unique standard name for a specific company, but each cedent might spell its name in a slightly different way. Furthermore there are often highly complex legal and economic structures between holdings and their subsidiaries, and these structures change dynamically over time.

While we do not have the resources to model these complicated relations exactly, we nevertheless have built up a database supporting an adaptive, semiautomatic mapping between the different spellings of a company and its unique, internally standardized name, which is the name used in MKMV if
available. Additionally, in this database we can also define simple hierarchical structures over several levels of holdings, the subsidiaries they control, and the subsidiaries of the subsidiaries. This database, which is an integral part of our accumulation control system CrediEx, allows therefore a fast identification of the risks reported by the cedents and the accumulation of our exposures on the holding level.

3.7.3 Estimation of the True Exposure per Risk

In order to do a simulation as outlined in Section 3.7.1, we need for each contract of the considered portfolio and each risk exposing this contract information on the ground-up exposure, the EDF and the LGD distribution of this risk. While the EDF is a fixed number for each risk, the ground-up exposure and the LGD distribution depend on the contract and the type of credit or bond exposure. It is not straightforward how to set these figures and distributions.

- **Exposures:** Often the cedents report us their current limits per risk only on an accounting year basis, while the reinsurance contract works on a risk attaching basis. Therefore, taking the figures as given by the cedent would lead to an overestimation of our exposure from this contract in the first year and an underestimation in the subsequent years. We resolved this problem in CrediEx by estimating typical patterns for the average duration of different types of bonds. This allows us to have a more adequate estimate of our exposures from such contracts until the bonds mature.

- **EDF:** If a company can be found in MKMV, then we have its EDF available in CrediEx. For all other companies the way how to assign them an EDF would need to be defined. It could depend on EDF quantiles in MKMV for the country or the industry segment of the company or on the internal ratings of the company from some of our cedents.

- **LGD:** Except for pure trade credit there are so far no standard LGD distributions which could be applied directly, so they would need to be defined first. Then we would need to extend the functionality of CrediEx to assign them to the different types of bonds. If the exposure on a company comes from several types of bonds with different times to maturity, then the combined LGD distribution would need to change over time accordingly. This is not yet implemented in CrediEx, but is feasible since this tool has been designed already in view of such possible extensions.
3.7.4 Simulation

Currently it is not possible to run simulations in CrediEx, but would require a further extension of the functionality of this tool. A simulation for the whole credit and bond portfolio would require many iterations in order to get an adequate loss distribution also in its tail. Therefore efficient calculations would be key in order to run such a simulation within reasonable time.

Furthermore, one could also think of modeling dependencies between the defaults of different companies and/or the severity of their losses, or to include economic scenarios in the model.

3.8 Natural Catastrophe Modeling

SCOR Switzerland’s natural catastrophe modeling is described in the paper by Brigitte Pabst, Flavio Matter, *Natural Catastrophe Modeling* (Pabst and Matter [2008]). The following sections of our documentation are based on that paper.

3.8.1 General Approach

The analysis of SCOR Switzerland’s natural catastrophe (NatCat) exposure for property lines of business is primarily performed with NatCat modeling software that is widely accepted in the insurance market. The basic concept of modeling is very similar in all catastrophe models. The following section provides an overview on:

- Framework of probabilistic cat models
- Sources of uncertainty
- Sensitivity of model parameters and
- Clash potentials

The probabilistic approach comprises modules (Figure 3.1), which account for hazard, exposure, vulnerability and financial conditions to estimate insured losses.

The hazard is defined by a simulated set of events. This simulated event set is developed based on a statistical parametric approach or a numerical simulation. The common statistical approach derives the hazard information mainly from a historic event catalog that covers the range of observed events with respect to size, location and probability of occurrence. The more complex numerical models simulate time-dependent, three-dimensional structures. The potential impact of the hazard at each location is translated into damage through so-called vulnerability functions. Those sets of relationships describe the degree of loss to a structure resulting from exposure
Figure 3.1: Modules of the catastrophe model.

to a hazard of a given intensity. The models incorporate regional damage functions specified by structure and occupancy type.

The cedents’ exposure is captured geographically and broken down by type of business, policy and coverage. Given the cedents’ exposure, a loss can be calculated for each simulated event. SCOR Switzerland’s loss potential is then analyzed by overlaying the estimated loss for the cedent with the corresponding treaty structure. SCOR Switzerland models exposure for proportional as well as for non-proportional business. Although continuously revised, commercial models may not cover all geographical areas and perils of interest or reach a satisfactory level of accuracy. Therefore SCOR Switzerland developed in-house earthquake models for Switzerland, Romania, Israel and typhoon models for Taiwan and South Korea based on the same principal methods as commercial models.

In general, probabilistic cat models capture two types of uncertainty, commonly named primary (aleatory) and secondary (epistemic) uncertainty.

- The uncertainty related to the likelihood of a certain event to occur is termed the primary uncertainty.
- Secondary uncertainty is defined as uncertainty in the amount of loss, given that a particular event has occurred. Uncertainty is associated
with the modeling of the hazard, the level of vulnerability and the portfolio data.

– **Hazard uncertainty** for earthquake modeling arises mainly from the effect of soil types on the amount of ground shaking, and collateral hazards such as liquefaction or landslide. For windstorm hazard the sources of uncertainty are the turbulence of the wind-field by its nature, terrain effects, and the resulting wind speed at a particular location.

– **Vulnerability uncertainty** refers to the uncertainty in performance of the insured object. This uncertainty may come from the modeling of building characteristics, or from the inherent uncertainty in building performance response to the intensity and duration of the hazard.

– **Portfolio uncertainty** is linked to the level of detail of the input portfolio data as well as to the reliability of that information.

The extent to which secondary uncertainty is incorporated into the modeled results depends on the model itself as well as on the analysis mode. Taking into account secondary uncertainty implies uncertainty in the ground up loss and thus in all other financial perspectives. The following example refers to a technical documentation by RMS™, the *RMS™ Secondary Uncertainty Methodology, September 2007*.

RMS™ assumes that for each individual event the loss distribution follows a beta distribution with a probability density function (expressed in terms of the gamma function $\Gamma$):

$$f_{\alpha,\beta}(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1 - x)^{\beta-1}, \quad 0 < x < 1$$

The two parameters $\alpha$ and $\beta$ can be expressed in terms of the mean $\mu$ and the standard deviation $\sigma$,

$$\alpha = \frac{\mu^2 (1 - \mu)}{\sigma^2} - \mu$$

$$\beta = \frac{\alpha(1 - \mu)}{\mu}$$

The mean and the standard deviation, in turn, are estimated from the information in the event loss tables as follows:

$$\mu = \frac{{\text{Loss}}}{{\text{Exposure}}}$$

$$\sigma = \frac{{\text{Standard Deviation}}}{{\text{Exposure}}}$$
For detailed loss modeling (RiskLink DLM Japan) the basic input to the financial module is the individual location coverage value, the *event ground up mean damage ratio* and *associated coefficient of variation*. The mean damage ratio defines the percentage of total exposed value damaged in an event at the specific location coverage. The mean loss as a ratio to the exposure commonly refers to as the mean damage ratio (MDR). The secondary uncertainty in the size of loss is represented by the coefficient of variation, which equals the ratio of the standard deviation to the mean. The *portfolio statistics* are then built up from the location coverage statistics, event by event. The estimated probability distribution of damage at each location must then be combined with the distributions from all other locations. This is accomplished through a convolution process.

For calculations on the aggregated exposure (RiskLink ALM, Catrader) level, the concept is to match aggregate exposures with a precompiled database of loss ratios and associated coefficients of variation. This is done for different lines of business at predetermined aggregate geo-code levels. The standard deviation therefore is based on an *industry average* precompiled at an aggregate level, and not calculated directly from individual locations and summed as is done in detailed loss modeling.

A sound understanding of the uncertainties associated with the model’s key parameters is essential for the interpretation of the model outcome and thus for decision making. The model outcome describes a bandwidth of loss estimates and not a unique value. In order to identify and stress-test the key parameters, systematic sensitivity analyzes have to be conducted for each model release.

Another issue for cat modeling is the treatment of clash potentials of neighboring geographical zones, of perils or treaties. SCOR Switzerland has adopted a central monitoring system\(^8\) to manage the worldwide accumulations of catastrophe risk by peril and region. The accumulation control focuses on zones with a geographic concentration or peak exposures, such as US hurricane risk. This centralized analysis is essential for a global reinsurer, since business is written for the same region from more than one worldwide office. Great efforts were also undertaken to monitor clash potential, both from lines other than property catastrophe.

\(^8\)Global Cat Data Platform, GCDP, see Part VIII
3.8.2 Hazard Module

Wind

Windstorm modeling differs between atmospheric phenomena. The most relevant perils producing insurance losses are tropical cyclone, extratropical cyclone, tornado and hail.

Tropical cyclones, regionally also called hurricanes or typhoons, form and spend much of their life cycle in the tropical latitudes, which are characterized by large areas of relatively uniform pressure. Tropical cyclones tend to be structured, self-contained systems. These characteristics make the parameterized modeling approach an appropriate one. In modeling tropical cyclones, the most relevant parameters are central barometric pressure, radius of maximum windfield, and forward speed. Central pressure is the primary determinant of a tropical cyclone’s wind speed and, therefore, of its intensity. Minimum central pressure is the lowest sea level pressure measured at the centre of the tropical cyclone. Wind speed can be described as a function of pressure differential between the central pressure and the pressure outside the storm centre. For each of these model variables, the modeler fits theoretical probability distributions to the historical data, testing the goodness-of-fit.

A common approach for modeling the range of potential tropical cyclones is based on a set of stochastic storm tracks. This stochastic event set enables reliable estimation of events with large recurrence rates and for all geographical areas. This event catalog contains the pre-simulated physical parameters, location, and frequency for each storm. Under certain conditions, storms may transform into extra-tropical storms. Known as transitioning storms, these tropical systems interact with the jet stream and become more asymmetric with time. The strongest winds push to the right-hand side of the storm over a broader region than one finds in a hurricane. All such parameters have to be taken into account while generating the stochastic event set. A common approach is the generation of the event set with the help of a Monte Carlo algorithm simulating the random behavior of each single storm system. The stochastic-generated event set is then calibrated against historical track data.

While running an analysis the hazard module determines the peak-gust wind speed at each property location for every stochastic event that is likely to cause loss at that location. Beside the wind speed itself, another damage-causing component of hurricanes is surge. Most models include the option to model the height of surge for each stochastic storm and analyzed location. The impact of topography for both phenomena, wind speed and storm surge, is considered through surface roughness factors and a digital height
model.

**Extratropical cyclones**, such as European windstorms, are more complex than tropical cyclones. Atmospheric conditions in the mid-latitudes contain multiple high-pressure ridges, low-pressure troughs, fronts, and strong jet stream winds all interacting to develop complex storm systems. While a parameterized approach to modeling storm tracks works well for the relatively simple and symmetric horizontal structure of tropical cyclones, the approach falls short when it comes to modeling extratropical cyclones.

An alternative to the approach of predefined storm tracks offer numerical weather prediction models. Those use global environmental data such as sea surface temperatures, wind speed and pressure in conjunction with the physical equations that govern the atmosphere. Beginning with an initial snapshot of conditions at the surface and multiple upper atmospheric layers, the dynamic three-dimensional wind-field is derived from a set of partial differential equations governing fluid flow. Topographic effects such as ridging, sheltering and channeling are captured by gridded elevation and land use information. Depending on the spatial and temporal resolution, those models provide a relatively precise footprint of damaging winds associated with the storms.

Beside tropical and extratropical cyclone, **tornado and hailstorm** risk is assessed. Hailstorms are often accompanied by tornadoes. However, hailstorms occur also independently and are a significant hazard for almost all our countries, especially for the motor line of business.

Bibliography: Daily et al. [2007], RMS [2007b].

**Earthquake**

An earthquake is the result of a sudden release of energy in the earth’s crust that creates seismic waves. The ground shaking is typically measured by ground acceleration. Shaking and ground rupture are the main effects triggered by earthquakes. Damage may result directly from the ground shaking but also from secondary events like landslides, avalanches, flash floods, fires and tsunamis. A probabilistic seismic-hazard analysis accounts for the full range of possible earthquakes, their location, frequency of occurrence, size, and the propagation of earthquake motion from the rupture zone to the sites of interest. Uncertainty in each of these elements is taken into account. In the process of analyzing the historical seismicity and making projections about future seismicity, the main steps are the characterization of seismic source zones, the definition of an adequate ground-motion attenuation model and the calculation of the probability of an earthquake occurring.

A common modeling approach is to identify three-dimensional geograph-
ical source zones as fault, area or background seismicity zones. Important fault systems may be divided into segments, which are then individually parameterized. Area seismic sources are often defined where specific fault data are unknown, but seismicity does exist. Background sources are special cases of area sources that represent all seismicity that unexplained by other sources. Each seismic source zone is then commonly characterized by the Gutenberg-Richter law

$$\log_{10} N = a - bM$$

which expresses the relationship between the magnitude $M$ and total number of earthquakes $N$ for a given zone and time period. The constant $b$ describes the relative size distribution and is typically equal to 1. For instance, for every magnitude 4.0 event there will be 10 magnitude 3.0 quakes and 100 magnitude 2.0 quakes. The $a$-value indicates the total seismicity rate of the region. $N$ represents the cumulative number of events. The Gutenberg-Richter relationship applies globally and allows an extrapolation from the limited historical record to estimate a more complete picture of seismicity in an area. In order to enable reliable results for each defined seismic source zone the historic event set is extended to a consistent stochastically generated event set.

The ground motion at a site is usually described as a function of magnitude, the distance from epicentre, and the local geological and geomorphological conditions, which may amplify or reduce wave propagation. Subsurface geology layers and local soil conditions lead to soil amplification that contributes to the estimated ground-motion parameters on the surface. Therefore, region-specific ground-motion attenuation relationships have been developed. Commonly models calculate ground shaking using spectral acceleration, peak ground acceleration, or macro-seismic intensity (MMI or EMS scale).

Spectral acceleration is used to measure the response of a structure, depending on its frequency content, whereas the peak ground-motion value is the maximum value derived directly from ground motion records. Hence, the spectral acceleration parameter is closer related to the building behavior. However, for the peak ground acceleration a much larger global dataset is available. Both values are convertible into macro-seismic intensity for the use in vulnerability assessment.

The probability of an earthquake occurring within a specific time window is calculated using a time-independent (Poisson) or time-dependent recurrence distribution. Time-independent modeling means that the probability of an event is not influenced by the timing of the last event.

Bibliography: Campbell et al. [2000], Cornell [1968], Gutenberg and Richter [1944], Ishimoto and Iida [1939], RMS [2007a], Zbinden et al. [2003].

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River Flooding

In the insurance industry, models assessing river flood for certain European catchment areas have emerged in the last years (e.g. from EQECAT, RMS, SwissRe, Benfield). Loss modeling requires detailed information on location level and highly complex and data intense simulation models. The latest generation of models reflects the temporal-spatial patterns of flood-causing rainfall throughout the year. A dynamic, time-stepping rainfall-run-off model evaluates the amount of water that flows into the river systems, by considering detailed physical catchment characteristics, seasonality, elevation, temperature, or antecedent conditions. River flooding is not considered in the present analysis.

3.8.3 Geographic Value Distribution Module

The distribution of insured values across geographical zones is crucial for the analysis of NatCat risks. For instance, in risk modeling, vulnerability functions relate the site-specific hazard aspects (e.g. amount of ground shaking) with vulnerability parameters at a certain location. After catastrophie events, the linking of geo-coded exposure and loss data enable loss forecasting and monitoring as well as the recalibration of models against the incurred losses. Furthermore, the visualization of the geographical distribution of the exposure supports capacity management and data quality review.

Primary insurers usually provide aggregated exposure (e.g. on CRESTA or postcode level) split by coverage type. With increasing modeling capabilities, the market requests more detailed exposure information. For hazards, which are less impacted by local conditions, like windstorm and hail, geo-coding on postcode level already leads to realistic analysis results. Whereas, highly localized phenomena like flood, landslide or industrial accident, ask for data on address or even coordinate level. Detailed information regarding the geographical value distribution is also essential for multi-location policies. For primary insurers this is a challenging issue, as the split of the insured value per location is often unknown.

However, aggregated modules are appropriate for standard portfolios or if information is rudimentary. Aggregated modules calculate losses by applying assumptions (e.g. the RiskLink® aggregated loss module-profile) regarding policy structure, geographic distribution, construction inventory, and insurance coverage type for a specific peril for one line of business at a certain geographic resolution. For standard portfolios, some models are capable to distribute automatically unallocated exposures based on regional specific industry exposure assumptions.

For geographic modeling, critical coverages are moveable goods and time elements where various assumptions have to be made. Here, the
exposure retrieval and preparation, requires a good understanding of how vulnerability functions consider these coverages. For example, in RiskLink® or Catrader® time element loss is calculated based on the structural mean damage ratio of a given building coverage at a side.

For accurate **geo-coding on detailed basis**, the model requires certain combinations of address information. In the United States, for instance, to get a full street match, you need to know the house number, street name, zip code or city and state. As these combinations are not always intuitive to understand, documentation and support is required. Even if the underlying model does not take full advantage of high-resolution data, it is still beneficial to collect high-resolution data for **mapping and exposure analysis**.

Bibliography: Bertogg et al. [2000], RMS [2006], RMS [2004].

### 3.8.4 Vulnerability Module

The vulnerability module defines the relationship between the severity of the natural hazard (e.g. wind speed) and the resulting physical damage. The vulnerability is usually measured in terms of damage ratios. Thus, vulnerability functions are coverage-specific. Each coverage specific vulnerability function takes into account the uncertainty content of basic elements such as occupancy type, construction class, building height, or year of construction. The variability around the mean value is represented by the coefficient of variation. Interior non-structural and content damage is related to building damage. For detailed loss modeling, information on the structure or occupancy type is commonly provided in coded form, which can than be mapped against codes recognized by the model.

Beside the key coverage elements, secondary characteristics like roof type can be factored in. Detailed information may have a large impact on loss estimates, and usually reduces the uncertainty of loss estimates.

In cases where suitable data is not accessible like for aggregated insurance values or missing policy details, most models provide a synthesized vulnerability function based on regional building inventories.

In earthquake risk assessment, beside the estimation of the performance of the *average* building within a reference portfolio related to the Modified Mercalli intensity, other approaches are in use like the quantification of the damage by the building deformation. The deformation is defined as maximum horizontal displacement experienced by a building during an earthquake. Because each building has different mechanical characteristics and a different natural period, each will be subjected to a different seismic intensity (i.e., spectral displacement) and, hence, a different damage state. The adequate approach depends strongly on the data availability. Regardless of the applied methodology, the vulnerability relationships including the demand surge components have to be reviewed after each larger event. If necessary, the model has to be recalibrated against the occurred
3.8.5 Financial Module

After having estimated the ground up loss per event for each insured object, policy conditions are applied in order to calculate the insurer’s net loss. The resulting losses are then aggregated per event and saved as an **event loss table**. It stores for each event that generates a non-zero loss the annual rate, its mean loss, the corresponding standard deviation and the exposure value. It is the basic input for the construction of the exceedance probability curve.

The probability that the losses will exceed a certain amount is measured on either an occurrence basis or an aggregate basis. The **occurrence exceedance probability** is the probability that at least one event will occur that causes losses above a certain amount. The occurrence exceedance probability is calculated from the frequency and severity distributions that are generated directly from the event loss table. The **aggregated exceedance probability** shows the probability that aggregate losses in a year will be greater than a given loss threshold. It estimates the probability of activating or exhausting various aggregate-based structures such as stop loss treaties. In SCOR Switzerland’s modeling landscape the event loss table is the core instrument for pricing cat coverages, accumulation control and cat capacity steering.

All the cat models used in the calculation of SST deal with standard policy conditions like limit as percentage of sum insured, limit as fixed amount, deductible as percentage of sum insured/of loss, deductible as fixed amount and franchise deductible. Specific policy conditions like the one of California mini-policies or Japanese EFEI are also applicable. Limited applicability of primary insurance conditions is given in some cases for aggregated as well as for detailed exposure data with regard to the geographic value distribution and to the split by coverage. Thus, the way the insured values are provided is crucial for loss calculation.

3.8.6 SCOR Switzerland’s Model Landscape

The catastrophe portfolio has been analyzed using several model platforms. *Catrader® vs. 8.5.1* supports the analysis of all European countries where windstorm is a major natural peril. For assessing earthquake risk, the Catrader model is used for all modeled European countries except Switzerland and Romania. For the US and the Caribbean book, tropical cyclone including storm surge, extratropical cyclone, tornado and earthquake including fire following earthquake are modeled with Catrader for all relevant
areas. Catrader performs analyzes of loss probabilities using aggregated exposure data similar to RiskLink aggregated loss module. Like most cat models, AIR applies a parametric approach for earthquake and tropical cyclone modeling. For European extratropical cyclone, AIR has taken the lead in modeling the events based on a numerical simulation.

RiskLink® vs. 6.0 is applied for modeling Japanese typhoon and earthquake risk. Basically, RiskLink differentiates between two types of loss modeling: The detailed loss module (DLM) suits flexible modeling of individual risks whereas the aggregated loss module considers a variety of policy market assumptions for standard coverages (incl. proportional and first loss policies) as well as for market specific coverages like fire expense insurance (FEFI) or Zenkyoren for Japan. Where reasonable, earthquake risks have been modeled by location, whereas the sum insured for typhoon risks are generally analyzed aggregated on CRESTA-zone or Prefecture level.

Sharp is a probabilistic in-house modeling platform, which models the losses of a stochastic event set affecting a specific portfolio of exposed values. Sharp can handle either detailed or aggregated exposure. Detailed loss modeling can be performed down to a single location if sufficient address information is provided, otherwise loss modeling is performed based on total sums insured entered on an aggregated level (e.g. CRESTA) for given geographic zones. The result is an event-loss-table, from which the exceeding probability curve is derived. The event-loss-table can be directly imported into the in-house pricing (MARS) and accumulation (GCDP) tools. The event-loss-table has the same format as the one of RiskLink. Sharp is used for modeling earthquake risk in Switzerland, Romania and certain areas in Central and Latin America and South East Asia. For modeling wind risks, it is used for certain regions in South East Asia.

Table 3.3 depicts all loss potentials which are modeled stochastically.

Technical Documentation

AIR Earthquake Model Caribbean Region, Technical Document EQCAR-0206, June 2002
AIR Earthquake Models for Australia and New Zealand, Technical Document EQAZ-0405, May 2004
AIR Earthquake Model for the Mediterranean Region, Technical Document EQMED-0206, June 2002
AIR Earthquake Model for the United States, Technical Document EQUS-0508, August 2005
AIR Extratropical Cyclone Model, European Region, AIR Technical Document ETCEU-0607, July 2006
### Peril Regions

<table>
<thead>
<tr>
<th>Region</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe – windstorm</td>
<td>Catrader/AIR</td>
</tr>
<tr>
<td>Mediterranean region – earthquake</td>
<td>Sharp/SCOR</td>
</tr>
<tr>
<td>US – earthquake and windstorm</td>
<td>Catrader/AIR</td>
</tr>
<tr>
<td>Caribbean – earthquake and windstorm</td>
<td>Catrader/AIR</td>
</tr>
<tr>
<td>Mexico, Chile – earthquake</td>
<td>Sharp/SCOR</td>
</tr>
<tr>
<td>Japan – earthquake and windstorm</td>
<td>RiskLink/RMS</td>
</tr>
<tr>
<td>Australia, New Zealand – earthquake and windstorm</td>
<td>Catrader/AIR</td>
</tr>
<tr>
<td>Rest of Asia – earthquake and windstorm</td>
<td>Sharp/SCOR</td>
</tr>
</tbody>
</table>

Modeling regional Nat Cat portfolios. The commercial tools are Catrader, Risklink. The internally developed tools is Sharp.

### Table 3.3: The Nat Cat model landscape at SCOR Switzerland.

- AIR Tropical Cyclone Model Caribbean Region, Technical Document TCCAR-0311, November 2003
- AIR Tropical Cyclone Model for Australia, Technical Document TCAU-0209, September 2002
- AIR Winter Storm Model for the United States, Technical Document ETCUS-0508, August 2005
- RMS® Japan Earthquake Model Methodology, November 2005
- RMS® Secondary Uncertainty Methodology, September 2007

### 3.9 Stochastic Model for Run-Off Business

In this section we describe the stochastic model for run-off business. We consider the following types of cash flows:

- paid losses
- paid/received premiums
• paid external expenses (brokerage, commissions, etc.).

For simplicity, we assume that only the loss payments are stochastic, and use deterministic estimates for the premiums and the expenses.

Consequently, to model the stochastic loss payments, the objective is to determine the distribution of the basket mismatches

$$M^b_i = \mathbb{E}[R^b_0 | \mathcal{F}_{i+1}] - \mathbb{E}[R^b_0 | \mathcal{F}_i] \quad \text{for } i \geq 0,$$

conditional on the information $\mathcal{F}_0$ available at time $t = 0$, and to specify the dependency structure of the basket mismatches. To ease notation, $M^b_i$ here denotes the mismatch, in year $i$, for the run-off business only. See pp. 46 for the definition of the discounted outstanding reserves $R^b_i$ for basket $b$ at time $t = i$. Concerning the dependency structure of the run-off basket mismatches, see Section 3.9.4 (pp. 99).

Similarly as for the new business, we construct a model for the nominal basket mismatches

$$\mathbb{E}[S^b_0 | \mathcal{F}_{i+1}] - \mathbb{E}[S^b_0 | \mathcal{F}_i]$$

where $b$ denotes the basket and $S^b_i$ for $i \geq 0$ denotes the nominal sum of the future payments $X^b_j$,

$$S^b_i := \sum_{j \geq i} X^b_j.$$

In a second step, the nominal figures are transformed into cash flows and discounted using development patterns, see Section 3.2.2 (pp. 50).

With regards to the nominal basket mismatches from (3.17), these can be expressed using the notation introduced in Section 3.2.3, pp. 52. To ease notation, we drop the superscript $b$ denoting the basket in the following.

Since, using this notation,

$$\mathbb{E}[S_0 | \mathcal{F}_{i+1}] - \mathbb{E}[S_0 | \mathcal{F}_i] = (X_i + \overline{S}_{i+1}) - \overline{S}_i,$$

the nominal basket mismatches provide a model for the one-year claims development $Q_i$,

$$Q_i = X_i + \overline{S}_{i+1},$$

where the reserves are best estimates in the sense that

$$\mathbb{E}[Q_i | \mathcal{F}_i] = \overline{S}_i.$$

To split the one-year claims development $Q_i$ into payment $X_i$ and reserves $\overline{S}_{i+1}$, we use the proposal from Section 3.2.3. That is,

$$X_i = \tilde{\eta} \cdot Q_i$$

$$\overline{S}_{i+1} = (1 - \tilde{\eta}) \cdot Q_i$$
with \( \eta_i \) defined in (3.8).

The following sections describe the derivation of the nominal basket one-year mismatches from (3.17). In Section 3.9.1, we recall the definition of development triangles and the well-known Mack model for the chain ladder method. In Section 3.9.2, we outline our proposal for the calculation of the one-year change (3.17) for the first year \( i = 0 \). In Section 3.9.3, the projection of the first year mismatches to future years is described. Finally, Section 3.9.4 discusses the proposed dependency structure.

### 3.9.1 Development Triangles, Chain Ladder, Mack Method

In this section we describe the different types of loss amounts considered in the context of reserving, and we recall development triangles and the classical Mack model for the chain ladder method (Mack [1993]). In the following, all values are nominal. Typically, the calculations will be applied to suitably defined sub-portfolios of the portfolio of run-off business.

For convenience, we denote, in this section only, the start year not by \( i = 0 \), as in the rest of the document, but by \( i = I + 1 \), in order not to have to denote run-off underwriting years by negative numbers. The run-off portfolio thus consists of the run-off underwriting years \( i = 0, 1 \ldots I \).

The actuarial estimation of the reserves, at least for more mature years, is based on *loss development triangles*. With regards to these triangles, we can distinguish different components of the estimated ultimate losses at a given point in time:

\[
\begin{align*}
\text{estimated ultimate losses} & = \text{paid losses} + \text{case reserves} + \text{IBNR}, \\
\text{reported losses} & = \text{paid losses} + \text{case reserves}, \\
\text{estimated reserves} & = \text{case reserves} + \text{IBNR},
\end{align*}
\]

where

- *paid losses* are the nominal sum of all loss payments already booked,
- *case reserves* are estimates provided to us of the outstanding loss amounts for known losses (to be paid in the future),
- *IBNR*, short for *incurred but not reported* losses, denote our estimate of the difference between the reported and the actual ultimate losses.

Consequently, there are loss development triangles for paid, reported, and estimated ultimate losses.

As usual, *calendar year* \( i \) denotes the period from time \( t = i \) to time \( t = i + 1 \), and the first run-off underwriting year is \( i = 0 \). At the end of calendar year \( i = I \), we consider
• underwriting years \( k = 0, 1, \ldots, I \),

• development periods \( j = 0, 1, \ldots, I \).

The development period \( j \) of one year is considered relative to the underwriting year \( k \); consequently, \( i = k + j \) denotes the corresponding calendar year.

Denote by \( C_{k,j} \) the cumulative reported losses for the underwriting year \( k \) up to the end of calendar year \( k + j \), i.e. at time \( t = k + j + 1 \). Let \( T_j \) be the sum of the cumulative losses over all underwriting years in development period \( j \), i.e.

\[
T_j := \sum_{k=0}^{I} C_{k,j}.
\] (3.18)

We assume:

• The ultimate loss is fully reported after \( I \) development periods.

The ultimate loss for accident year \( k \) is thus equal to \( C_{k,I} \). The loss development triangle \( D_I \) at time \( I + 1 \) consists of all entries \( C_{k,j} \) known up to the end of calendar year \( i = I \), i.e. it is given by

\[
D_I := \{ C_{k,j} \mid k + j \leq I \}.
\]

\( D_I \) corresponds to the information on the losses available at \( t = I + 1 \). An estimator of the cumulative paid loss \( C_{k,j} \) is denoted by

\[
\hat{C}_{k,j},
\]

where the estimator is based on the information available at time \( t = I + 1 \).

For convenience, we introduce the following notation to describe entries in the most recent diagonal. Define the underwriting year \( j_\ast = j^I \) and the development period \( k_\ast = k^I \) relative to year \( I \) by

\[
\begin{align*}
 j_\ast & := I - j \\
 k_\ast & := I - k.
\end{align*}
\]

At time \( t = I + 1 \), \( j_\ast \) is the most recent underwriting year for which information is available about development period \( j \) and, similarly, \( k_\ast \) is the the largest development period for which information is available about underwriting year \( k \).

At the end of year \( i = I \), the sum \( T_j \) from (3.18) is estimated by

\[
\hat{T}_j := \sum_{k=0}^{j_\ast} C_{k,j} + \sum_{k=j_\ast+1}^{I} \hat{C}_{k,j}.
\] (3.19)
The basket mismatches (3.17)

\[ E[S_{0}^{b} \mid \mathcal{F}_{i+1}] - E[S_{0}^{b} \mid \mathcal{F}_{i}] \]

then correspond to the difference

\[ \hat{T}'_{I} - \hat{T}_{I} \quad (3.20) \]

where the prime "\(^{\prime}\)" denotes estimates at time \( t = I + 2 \), i.e. at the end of the following year \( I + 1 \).

To estimate this one year-change, in view of (3.19), we have to make assumptions about

1. the method used to estimate \( \hat{C}_{k,j} \)
2. a stochastic model for the random variables \( \hat{C}_{k,j} \),

The second point is considered in Section 3.9.2. Concerning the first point, one such model is the classical Mack model for the chain ladder method. The Mack model for the chain ladder method is based on the following three model assumptions:

1. Different underwriting years \( k \) are independent. More precisely, the random vectors \((C_{k,0}, C_{k,1}, C_{k,2}, \ldots)\) for different \( k \) are independent.
2. There exist positive development factors \( f_{j} \), called age-to-age factors or chain ladder factors, such that

\[ E[C_{k,j+1} \mid C_{k,0}, C_{k,1}, \ldots, C_{k,j}] = f_{j} C_{k,j}. \quad (3.21) \]
3. There exist positive parameters \( \sigma_{j} \) such that

\[ Var[C_{k,j+1} \mid C_{k,0}, C_{k,1}, \ldots, C_{k,j}] = \sigma_{j}^{2} C_{k,j}. \]

A few remarks about the Mack model:

- The Mack model is distribution-free: No assumption is made concerning the distribution type of the random variables \( C_{k,j} \). The only assumptions made concern the first two moments of \( C_{k,j} \). Consequently, it is likely that only statements about the first two moments of the estimation errors can be made.
- The first assumption in the Mack model, independency of different underwriting years, is questionable from a practical point of view, since, in reality, reserve increases are sometimes applied in one calendar year to all underwriting years. This makes sense intuitively because certain causes of dependencies (such as legal changes) apply to many underwriting years at the same time.
• Volatility in the Mack model comes from two sources: On the one hand, it comes from the deviation of the actual amount $C_{k,j+1}$ from the expected value $f_j C_{k,j}$ given the prior history (see (3.21)), which is called stochastic error or process variance. On the other hand, volatility comes from the fact that the chain ladder factors $f_j$ exist but are unknown and have to be estimated from $D_I$; this is the estimation error or parameter error.

• The assumption that the chain ladder factors exist but are unknown might seem odd (e.g. in view of (3.21)). An alternative approach is to use a Bayesian viewpoint, see Gisler [2006]. In such an approach, the chain ladder factors are themselves random variables $F_j$, and the available information $D_I$ can be used to assess the distribution of the $F_j$. When using a Bayesian approach, additional assumptions have to be made regarding the prior distribution of the $F_j$ and, in Gisler’s paper (Gisler [2006]), also regarding the distribution of the $C_{k,j}$.

• Consider the second Mack assumption (3.21). Per se, for a given underwriting year $k$, the expected value of the next amount $C_{k,j+1}$ given the prior history, i.e.

$$E\left[C_{k,j+1} \mid C_{k,0}, C_{k,1}, \ldots, C_{k,j}\right]$$

is a function of the prior history $C_{k,0}, \ldots, C_{k,j}$, where the function might depend on $k$ and $j$. To arrive at (3.21), we need to make three assumptions: The function depends on $C_{k,j}$ only; the function is linear in $C_{k,j}$; and the function depends on $j$ but not on $k$.

Given the Mack model, at time $t = I + 1$, the ultimate loss $C_{k,I}$ for underwriting year $k$ can be estimated by

$$E[C_{k,I} \mid D_I] = E[C_{k,I} \mid C_{k,0}, C_{k,1}, \ldots] = C_{k,k_+} \prod_{l=k_+}^{I-1} f_l. \quad (3.22)$$

The first equality above is a consequence of the first Mack model assumption, and the second follows from repeated application of the second assumption using the appropriate version of the equality

$$E[X \mid Z] = E[E[X \mid Y, Z] \mid Z].$$

Concerning again the first equality in (3.22), observe that this requires independency of the random vectors $(C_{k,0}, C_{k,1}, \ldots)$ and not just independency of the random variables $C_{k,j}$ for different $k$.

Moving to estimators of the involved quantities, the estimators $\hat{f}_j$ for $f_j$ given by

$$\hat{f}_j := \frac{\sum_{k=0}^{j-1} C_{k,j+1}}{\sum_{k=0}^{j-1} C_{k,j}}$$
are unbiased and uncorrelated, and an unbiased estimator of (3.22) is given by
\[ \hat{C}_{k,I} = C_{k,k} \prod_{l=k}^{I-1} \hat{f}_l. \]

Mack then derives an estimate of the mean squared error \( \text{mse}(\hat{C}_{k,I}) \) between the true ultimate \( C_{k,I} \) and the estimator \( \hat{C}_{k,I} \), given \( D_I \),
\[ \text{mse}(\hat{C}_{k,I}) := \text{Var}(\hat{C}_{k,I} - C_{k,I} | D_I) = \mathbb{E} \left[ (\hat{C}_{k,I} - C_{k,I})^2 | D_I \right]. \]
This error estimates the volatility of the “ultimate change,” i.e. the difference between the current estimate \( \hat{C}_{k,I} \) and the true ultimate \( C_{k,I} \). For the SST, however, we need an estimate for the one-year change from (3.20),
\[ \hat{T}'_I - \hat{T}_I, \]
i.e. the difference between the estimate at \( t = I + 2 \) and the estimate at \( t = I + 1 \). This problem is considered in the following section.

3.9.2 The One-Year Change for the Run-Off Business

In this section, we consider the question of how to estimate the one-year change from (3.20),
\[ \hat{T}'_I - \hat{T}_I. \] (3.23)
In terms of the development triangle, the task is to estimate how the ultimate loss estimate \( \hat{T}_I \) changes when an additional diagonal in the development triangle becomes known.

In the notation introduced in Section 3.9, the nominal one-year change is written (shifting back from time \( t = I + 1 \) to time \( t = 0 \) for convenience)
\[ Q_i - \overline{S}_i, \]
with the one-year claims development
\[ Q_i = X_i + \overline{S}_{i+1}. \]
In a first step, we consider the estimate of the mean square error, i.e. the standard deviation of the one-year change (3.23).

We then make the following distribution assumption:
- \( X_0 \) and \( \overline{S}_1 \) are log-normal distributed.
This assumption is standard for random variables for which extreme events have been removed.

Concerning the estimate of the mean square error, a method has been proposed by Merz and Wüthrich in Merz and Wüthrich [2007] to estimate
the mean-squared error for the one-year change (3.23). This method is based on a so-called time series model for the chain ladder method, which makes stronger assumptions than the Mack model and applies thus to a subset of the Mack model. As for the “ultimate change,” the variance naturally decomposes into a process variance (“stochastic uncertainty”) and an estimation error (“parameter uncertainty”), and estimators are provided for both quantities. The method was proposed to FOPI and has been approved.

Recently, an alternative approach has been developed by Markus Duerr in a currently unpublished paper. This approach does not assume the above time-series model but only the three Mack model assumptions.

We have compared the mean-squared estimates from the two methods on triangles from our reserving figures. As a general rule, the two figures are quite close, with the Duerr method giving a slightly larger estimate. However, for a reserving analysis it is crucial to be able to handle incomplete triangles and data errors. Unlike for the Merz-Wüthrich method, for the Duerr method there exists a mathematically precise derivation of the mean-square error also in case certain link ratios in the triangle are excluded - which more closely mirrors the way reserves are actually estimated by reserving actuaries. For these reasons we propose to use the Duerr method.

Further note that the derivations of the estimators for the Duerr method appear to be more straightforward and simpler than those in Merz and Wüthrich [2007]. Moreover, Duerr explicitly compares the estimators from the two approaches and provides arguments for why his method seems preferable.

3.9.3 Projection to Future One-Year Changes

To calculate the expected shortfalls of the future one-year changes, we propose to follow the approach outlined in Section 2.2.3 (pp. 35). That is, we specify a proxy for the shortfall ratio (for basket \(b\) and calendar year \(i \geq 2\)),

\[
\xi_b^{(i)} = \frac{\text{ES}_a[M^b_i]}{\text{ES}_a[M^b_1]},
\]

(3.24)

where ES denotes the expected shortfall. We propose (see also pp. 50):

- The proposed proxy for the shortfall ratios \(\xi_b^{(i)}\) from (3.24) is the incremental reported pattern by calendar year for basket \(b\).

Recall that the reported pattern measures the estimate of the ultimate amount not considering IBNR. Incrementally, this corresponds to the one-year change \(M^b_i\) of the reported loss amount. This pattern is usually longer-tailed than the pattern of the estimated ultimate loss including IBNR.
3.9.4 Dependency Structure and Aggregation for Run-Off Business

Since we do not have a detailed study of temporal dependencies available, we propose to use the same dependency parameters in the portfolio dependency tree for the run-off business as for the new business, see Section 3.3.2 (pp. 59).

This assumption might be questioned by arguing that the at least certain extremal dependencies are manifested in the first year $i = 0$, so that the dependency between run-off business baskets ought to be reduced. However, for long-tail business, only very little information about the ultimate outcome is assumed to be revealed in year $i = 0$, so that “catastrophic events” (such as changes in law) occur during the run-off period. This implies, in particular, that extremal dependencies for long-tail business are manifested primarily during run-off. This leaves the dependencies for short-tail business. However, for short-tail business, the shortfall contribution to run-off mismatches is expected to be small anyway, so that dependencies will not have a big impact.

Concerning the definition of the run-off baskets, the granularity will presumably not be selected as fine as for new business, in order to get reliable triangles. Currently, the following run-off baskets are used

- Agribusiness
- Aviation
- Credit and Surety
- Engineering
- Marine
- Property
- Liability
- Professional Liability (excluding MDU, which is modeled separately)
- Workers Compensation
- Non-life Personal Accident (PA)
- Motor
- Miscellaneous

For the legal entity Cologne, only Property, Engineering, Marine, Liability, Motor, and Non-Life PA are considered, and the remainder is included in the basket “Other.”
3.9.5 Data Requirements and Parameter for Run-Off Business

We summarize the requirements on the SST run-off business model.

- **Definition** of the run-off business baskets.
- **Reserving development triangles** of the reported/incurred losses by underwriting year and development period to calculate the one-year change.
- **Initial balance sheet reserves** and premiums receivable and expenses payable
- **Calendar-year earnings pattern**
- **Portfolio dependency tree** to express the dependency structure between run-off baskets, including dependency parameters, see Section 3.9.4 (pp. 99).

- Method to estimate future (shortfalls of) one-year basket changes $ES_a[M^b_i]$ for $i = 1, 2, \ldots$, see pp. 98. According to this specification, we need, for each run-off basket,
  - **Incremental development pattern for incurred/reported losses by calendar year.**
- **Development patterns for paid premiums and paid losses by calendar year** from reserving development triangles, see also Section 3.2.2 (pp. 50).

3.10 Model for Non-Life Internal Administrative Expenses

A **market-consistent** valuation of liabilities requires modeling the costs of doing business. In particular, we have to consider **administrative expenses**, also called **internal expenses**, since these are an integral part of the production costs of a liability, and are to be viewed as part of the value of the insurance liabilities. For this reason, we consider internal expenses both in the context of solvency assessment for the SST and for the pricing of contracts at renewal.

The approach presented here was developed by Christoph Hummel and described in the document *Cost model for SST* (Hummel [2007a]).
3.10.1 Acquisition and Maintenance Costs

In view of the one-year perspective inherent in the SST methodology, the internal expenses have to be split up into two classes:

- Acquisition costs
- Maintenance costs

*Acquisition costs* for a contract are the costs associated with the acquisition of the contract under consideration, and hence they are incurred only in the year the contract is written.

*Maintenance costs* are the costs for the administration of contracts already written. These costs persist throughout a potential run-off period of the contract and thus are incurred in all years starting from the year the contract is written up to the year the contract is closed.

Obviously, this *maintenance period* depends on whether the contract is short or long tail. Maintenance costs likely arise both from the payment of claims as well as from the reserving of claims. For a given year, it seems reasonable to measure the maintenance costs for claims payment by the percentage of the ultimate loss paid in this year, and the maintenance costs for claims reserving by the outstanding reserves as a percentage of the ultimate loss.

The situation now is the following: Administrative expenses are provided by the finance department for a financial, i.e. calendar year, separate for each legal entity. To simplify our exposition, we will in the following consider one specific legal entity but not specify the legal entity explicitly, and denote by $E_0$ the *total administrative expenses for the non-life business of the legal entity* under consideration. We must have available a breakdown of the total expenses $E_0$ into *acquisition costs* $A_0$ and *maintenance costs* $M_0$,

$$E_0 = A_0 + M_0.$$  

These expenses arise from the liability portfolio in the books in year $i = 0$, which consist of business written in the underwriting years $k = 0, -1, -2, \ldots$. The costs $E_0$ consist of that part of the internal expenses for each of these underwriting years $k$ which pertains to the year $i = 0$. Additional expenses will arise for the underwriting year $k$ business in the years $i = 1, 2, \ldots$. For this reason we have to make an assumption about the future administrative expenses $E_i$ for $i = 1, 2, \ldots$.

For the future financial years $i = 1, 2, \ldots$, we assume *going concern with constant volume*, which implies that the administrative expenses $E_i$ are constant except for inflation $a_i$, so

$$E_i = A_i + M_i \quad \text{with} \quad A_i = a_i A_0, \quad M_i = a_i M_0,$$
where we might take $a_i = (1 + \theta)^i$ for an average rate of inflation $\theta$.

Denote the administrative expenses from financial year $i$ pertaining to underwriting year $k$ by
\[ E_i^k = A_i^k + M_i^k. \]
Since acquisition costs only occur in the year a contract is written, we have
\[ A_i^k = A_i, \quad A_i^k = 0 \text{ for } k \neq i. \]

Concerning maintenance costs $M_i^k$, let
\[
\begin{align*}
p_i^k &= \text{percentage of ultimate loss for underwriting year } k \text{ paid in year } i \quad \text{(incremental paid loss pattern)} \\
r_i^k &= \text{percentage of ultimate loss for underwriting year } k \text{ reserved in year } i \quad \text{(reserve (case + IBNR pattern)} \\
\alpha &= \text{percentage of maintenance costs for claims reserving} \quad \text{(as opposed to claims payment)}
\end{align*}
\]
We then assume that the maintenance costs $M_i^k$ are given by
\[ M_i^k = \alpha M_i \frac{r_i^k}{\sum_{l \leq i} r_i^l} + (1 - \alpha) M_i \frac{p_i^k}{\sum_{l \leq i} p_i^l} \quad \text{for } k \leq i; \quad M_i^k = 0 \text{ for } k > i. \]
Assume for simplicity that administrative expenses pertain to mid year $i + 1/2$. Let $z_i(t)$ be the value at time $t \in \{0, 1\}$ of a zero coupon bond with face value 1 in the currency of the legal entity under consideration, which matures at time $i + 1/2$. We do not model interest rate risk in the framework of internal expenses, so we approximate $z_i(1)$ deterministically using the forward rates.

We can now calculate the internal expenses as a part of the value of the liabilities of the legal entity at time $t \in \{0, 1\}$. Indeed, the internal expenses $IC(t)$ are composed of expenses pertaining to the underwriting years $k \leq t$,
\[ IC(t) = \sum_{k \leq t} IC_k(t), \]
where the internal expenses $IC_k(t)$ for underwriting year $k$ that have to be part of the liabilities at time $t$ consists of the discounted expenses which arise in future financial years $i = t, t+1, t+2 \ldots$. Hence, they are given by
\[ IC_k(t) = \sum_{i \geq t} z_i(t) E_i^k. \]
3.10.2 Required Data and Discussion

Let us give an overview of the input data required for the administrative cost model:

1. Acquisition cost $A_{0, LE}$ and maintenance costs $M_{0, LE}$ for each modeled legal entity in the financial year $0$. For Cologne, we treat life and non-life separately.

2. For each legal entity, the aggregated paid loss pattern

3. For each legal entity, the reserve pattern (case + IBNR)

4. Expected average inflation ($\vartheta$) for years $1, 2, \cdots$

5. Risk-free yield curve in the balance sheet currency at $t = 0$.

6. A numerical parameter $\alpha \in [0, 1]$ which defines the portion of maintenance costs to be modeled on the expected reserves.

We have pointed out how this approach is based on a going concern assumption at constant volume for the financial years $i = 0, 1, 2, \cdots$, which makes sense from an economic perspective. Only in this way can we align the cost resulting from this valuation with the expenses allocated in pricing. It is also in line with the transfer approach taken in the SST. In this framework, we need to consider in the valuation of the liabilities the cost for capital and administration which a rational buyer would charge for taking over the corresponding liabilities. In our framework, the potential buyer would be an identical copy of our company.

Finally, we mention an important difference between internal expense allocation in SST and in pricing. In pricing, legal entities are not considered separately, so the group’s overall internal costs for the underwriting year $k = 0$ are allocated to the individual group’s reinsurance contracts in pricing, rather than allocating a legal entity’s cost to the legal entity’s reinsurance contracts. In this way we stick to our principle that the price for a reinsurance contract is the same regardless of which legal entity it is written.
Life and Health Methodology

Our description of the internal risk models for the Life and Health business is based on the document *Methodology Life and Health*, Gruetzner et al. [2007].

4.1 Structure of the Business

The L&H business is split in four lines of business (LoBs): Life, Health, Disability and Personal Accident. In addition to these lines we differentiate between the following types of business:

- GMDB business
- Financial reinsurance
- Short duration business
- Standard long-duration business

GMDB business

This is a portfolio of American variable annuities in run-off. Main exposure is to US-equity markets, i.e. asset risk, but being life insurance the portfolio is also to some extent exposed to more classical insurance risk.

Financial reinsurance

For our purposes financial reinsurance is described as long-duration insurance where financing aspects dominate. The classical insurance risk is limited by various mitigating measures. For this reason the main exposure is to cedent default risk. Of course this type of credit risk is highly correlated with general asset risk.
Standard long-duration business
In addition to the special two types mentioned above, standard long-duration business covers all the remaining long-duration business, i.e. mortality, disability or other classical covers with long-term rate guarantees.

Short-duration business
Here the main exposure is to losses from short-term insurance risk. This could be large-scale catastrophic events or high-concentration risk in one exposed area. The run-off is also exposed to reserve risk as it is known from non-life business.

Guided by the above we have the following risk-categories to consider:

Asset risk
Here we refer to these asset risks which arise on the liability side of the business only. Clearly the GMDB business suffers if the underlying funds fall as the difference between funds value and guarantee in case of the insured event represents a loss to SCOR Switzerland Group. In case of the standard long-duration business future losses are discounted less in situations with lower interest rates. This clearly leads to higher market values of losses in times of low interest rates.

Credit risk – cedents
The finance re business has risk characteristics similar to a loan of SCOR Switzerland Group to the cedent, which is amortized over time. Clearly, if the cedent defaults, SCOR Switzerland Group incurs a loss equal to the lost amortization.

Fluctuation risk – L&H insurance risk
SCOR Switzerland Group insures the usual biometric risks mainly mortality (including accidental death) and morbidity. If due to pure random fluctuation or a significant event affecting several insured risks negatively, e.g. a pandemic, the claims exceed the expected claims plus the safety margin, SCOR Switzerland Group incurs a loss.

Trend risk – L&H insurance risk
For some risks the insurance coverage is long term, i.e. much longer than one year and the premiums for this term is fixed in advance. In this case the insurer faces the fluctuation risks mentioned above as well as the risk that the probability distribution for future points in time differs substantially from the probability distribution assumed when pricing the risk. This risk is called trend risk, an example would be deteriorating mortality rates due to an increase to unhealthy behavior of insured persons.
4.2 GMDB

Definition and Exposure

The reinsured portfolio is retrocession of a portfolio of unit-linked contracts with Guaranteed Minimum Death Benefit guarantee (GMDB) sold in the USA. Decreasing funds prices and / or increasing mortality and / or more rational policyholder behavior cause increased claims. Guarantee types are return of premium, ratchets, roll-ups and resets. Ratchet- and reset-features imply path-dependency of the guaranteed pay-off. For purposes of risk management SCOR Switzerland Group has developed a stochastic cash flow model, which projects future cash flows using market-consistent asset scenarios from a Hull-White model calibrated to current financial market data. This model reflects all treaty conditions and dynamic lapse depending on in-the-moneyness (ITM) of the guarantees. Additionally product features like dollar-for-dollar ($for$) and partial withdrawal are reflected.

All actuarial assumptions have been checked and established in comprehensive studies and additional consultations of the cedents. The model is externally reviewed on a regular basis.

Risk factors influencing the GMDB exposure are

- Capital market risks
  - The development of policyholder’s fund and hence the GMDB-pay-off is influenced by equities, bonds, interest rate and volatility movements.
  - The GMDB-cover has characteristics of a derivative or option due to its asymmetric pay-off.

- Mortality risk
  - The portfolio is exposed to short-term fluctuation and trend risk: higher mortality implies higher notional of the embedded options and thus higher exposure.

- Policyholder behavior
  - Policyholders have the right to (partially) withdraw their money at any point in time, thus stopping to pay guarantee charges and therefore increasing the loss for SCOR Switzerland Group if the option is out of the money.
  - The pricing of the guarantee charges has been based on the assumption that a certain amount of policyholder lapses before the guarantee could come into effect, thus leading to reduced charges. If less policyholders lapse than planned in situations where the guarantee is in the money this also results in a loss for SCOR Switzerland Group.
Policyholders can increase volatility by choosing different underlying funds. If the level of volatility exceeds the planned level then this leads to a loss for SCOR Switzerland Group as the embedded options gain in value with increasing volatilities.

The GMDB portfolio is valued using a replicating portfolio approach. The replicating portfolio is a portfolio of assets closely matching the cash flows of the liabilities under all asset scenarios and consists of a set of financial instruments (candidate assets) which are easy to value, using closed form solutions. They are calibrated on a subset of the stochastic cash flow scenarios of the GMDB-liabilities. For a given set of financial scenarios the market value of the GMDB liability can be easily determined at each point in time using the replicating portfolio. This does not afford further Monte Carlo simulations and hence avoids the problem of stochastic-in-stochastic simulations.

The replication portfolio itself varies with mortality and dynamic policyholder behavior assumptions. For the calibration are best-estimate mortality assumptions and best-estimate dynamic policyholder behavior assumptions used. This allows reflecting all market risks precisely (e.g. development underlying equity funds, change of interest rates and change of volatility).

Not reflected using the replication portfolio approach are the fluctuation and trend risk from mortality and trend risk for lapses. These risk types are reflected by splitting market and mortality risk as follows. Mortality risk will be determined using a best-estimate asset-scenario. The resulting (mortality-)fluctuation risk is modeled and integrated into the fluctuation risk model, as described there. Trend risk from mortality is modeled in the trend risk model, as described there.

The risk of change in policyholder behavior is not modeled because of low materiality. An example: The impact of the worst case of withdrawal (every policyholder maximizes the loss for SCOR Switzerland Group) results in an increase of the market value of the liabilities of only USD 25 million. As of 2007 we model 5% withdrawal-rates and hence even a doubling of this rates only leads to an increase of only USD 1.25 million - and this is diversifiable risk.

Stochastic Modeling

The GMDB replicating portfolio is considered in detail in Section 4.7 of this documentation.
4.3 Finance Re

4.3.1 Definition and Exposure

First year financing strains of the cedents are financed by the reinsurer and the resulting debt is amortized over the lifetime of the treaty. SCOR Switzerland Group’s Finance Re portfolio is characterized by

- First year financing that exceeds the deferrable acquisition costs under statutory original terms
- An additional amortization margin contained in the reinsurance premium
- At least partial claw-back of paid financing for lapsed policies
- Termination of the treaty after amortization, measured by an experience account balance.

In order to qualify as reinsurance the treaties typically transfer some biometric or lapse risk. But due to these risk mitigation features the pure insurance risk is limited, e.g. a higher mortality or increased lapse rates result in a longer amortization period but the profitability remains nearly unchanged and the setup of these contracts is close to debt instruments.

This immateriality of the insurance risk factors was verified by evaluating the impact of insurance risk scenarios at the 1% quantile. E.g. mortality trend, being normally distributed with standard deviation 5%, was set to $112\% \cdot$ best estimate mortality. Lapse is the only risk factor where a contract exists that doesn’t amortize the financing, with an outstanding open balance of less than 5% of the financing. We regard this effect as immaterial.

The only remaining risk is a thus default of the cedent, i.e. pure credit risk.

4.3.2 Stochastic Modeling

The financial reinsurance portfolio of the SCOR Switzerland Group for life insurance business consists of three contracts with three different counterparties, defined in the economical sense, i.e. identifying all legal entities belonging to the same group. In order to measure the impact of a default, the outstanding cash flows at each point in time of each contract have been modeled individually, fully reflecting the terms and conditions of the underlying insurance and reinsurance treaties. All contracts with a single cedent are aggregated and modeled together, as a default is assumed to affect the whole group. This deterministic cash flow projection is produced based on best estimate assumptions and assuming no default. Hence the cash flows in the best estimate scenario are all positive, representing the payback of the initial financing. In case of default all future cash flows are set to zero.
That means the present value of the lost amortization cash flows represents the loss and we assume no recoveries.

The stochastic default scenarios are taken from the general credit risk model (see Part III of the documentation, pp. 303) so that they follow the same methodology as in Non-Life.

Consistent with the SST-approach, for both life and non-life we look at a one-year time-horizon only. When valuing this one-year risk, the rating can either remain unchanged, drop to default or change to another rating. In case of a default the realized loss is the net present value of all future cash flows, as mentioned above. In any other case the cash flows are currently kept on their original value. This approach might be improved by considering rating migration.

4.3.3 Dependency Structure

Because both market and credit information is included in the same scenario set, it is thus ensured that the dependency between asset and credit risk is explicitly modeled. As the cedents are insurance companies there will be a dependency between extreme insurance events and defaults of these companies, e.g., in case of a pandemic. Some extreme insurance events may also concern life and non-life business. Therefore this dependency between extreme insurance events and defaults is handled the same way for life and non-life business when combining insurance and asset risks. The approach used has already been discussed for the non-life business in this documentation (see the parts of this documentation on the Non-Life Liability Model).

4.4 Trend Risk

4.4.1 Definition and Exposure

Trend risk is defined as the risk that the assessment of initial best estimate assumptions changes over the one-year horizon. This could be due to incorrect initial estimation (e.g., parameter risk in statistical estimates) or due to systematic and unanticipated changes in the underlying risks (e.g., changes in incidence rates due to changing legal environment).

While this risk applies in principle to all insurance contracts, it has material consequences only in long-duration treaties. For this reason trend risk is only modeled for long-duration treaties within the L&H internal model.

SCOR Switzerland Group’s portfolio does not contain material treaties covering longevity, long-term care or critical illness and the exposure of long-duration treaties to health and personal accident is very limited.

Consequently the only material exposure to trend risk arises from risk factors concerning mortality, lapse and morbidity. Since Finance Re treaties
are almost not affected by changes in those risk factors, they are excluded from the trend risk calculations.

### 4.4.2 Stochastic Modeling

The approach is based on a simulation of cash flows per contract. For each contract its (deterministic) net present value ($NPV$) is a function of the actuarial assumptions. In case of mortality risk

$$NPV = NPV(\bar{q}_x)$$

where $\bar{q}_x$ denotes the mortality table used. Trend risk is taken into account by multiplying each $\bar{q}_x$ with a normal random factor $\epsilon$. There is no dependency of $\epsilon$ on $x$. Thus the net present value will be modified by applying a constant random factor to the relevant future assumptions. Taking again mortality as an example we will get a distribution of values

$$NPV = NPV((1 + \epsilon)\bar{q}_x)$$

with a *perturbation* of future mortality distributed according to a normal-distribution

$$\epsilon \sim \mathcal{N}(0, \sigma)$$

Due to the bottom-up stochastic modeling the NPV distribution will reflect non-linear features in the contracts, like e.g. profit commissions.

### 4.4.3 Choice of Parameters

The assumptions are based on the SST standard model for life insurers. The portfolio of the SCOR Switzerland Group neither has sufficient size nor sufficient historical data to justify a statistical estimation of trend risk on a stand alone basis.

For the relevant risk factors mortality, lapse and morbidity the volatility parameters are

<table>
<thead>
<tr>
<th>Risk factor</th>
<th>Volatility parameter $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mortality</td>
<td>5%</td>
</tr>
<tr>
<td>lapse</td>
<td>50%</td>
</tr>
<tr>
<td>morbidity</td>
<td>10%</td>
</tr>
</tbody>
</table>

Exactly as in the previous example volatility is to be understood as perturbation of the respective best estimate assumption. Note that we increased the volatility of the lapse perturbation from 25% in the SST standard model.
to 50%. We felt that an increase was advised to cover additional lapse risk
due to potential asset movements, which are not directly modeled.

The volatility parameters above are also consistent with the stress sce-
narios used by former SCOR Global Life to assess their capital requirements
within the SCOR Group.

4.4.4 Dependency Structure

When varying the risk factors we apply the same perturbation to every treaty
exposed to this risk. This means we assume no diversification within the
portfolio. As the portfolio covers a great variety of cedents in a large number
of locations, we consider this to be a somewhat conservative assumption.

We assume the change of each of the risk factors to be independent.
The result is then derived by adding up the cash flows, i.e. calculating the
convolution. The discounted best estimate will be derived by discounting
with appropriate yield curves. With the exception of GMDB and Finance
Re the specific treaties in our portfolio are known to exhibit no or only
negligible dependency on market risk. So that discounting will be done
independently from the values of the trend-risk factors.

Furthermore we assume independence between fluctuation risk and trend
risk.

4.4.5 Cash Flow Modeling

Almost all larger long-duration treaties are individually modeled. This
means the cash flows not only reflect the broader terms of the reinsured
portfolio but are also built on detailed assumptions about individual poli-
cies which are either directly based on cedent data or derived from bookings.

Those same models are also used to derive US GAAP and IFRS figures.
They are regularly updated and the models themselves as well as the under-
lying best estimate assumptions are regularly reviewed by external parties.

Smaller treaties, where it is economically unfeasible to have individual
models for each contract, are grouped into blocks of business with similar
characteristics, as for example product types or geography. Cash flows for
these blocks are derived from historical observations of duration and result
ratio.

4.5 Fluctuation Risk

4.5.1 Definition and Exposure

Fluctuation risk represents the uncertainty concerning losses deviating from
their expected value due to stochastic fluctuation within the one year time
horizon after valuation date.
Fluctuation risk should be seen in contrast to trend risk and reserve risk. For long-duration treaties fluctuations manifest themselves during the actual valuation year, while trends emerge in the actual year but impact all future contract years.

We assume also for short-duration treaties that fluctuation risk realizes itself and is completely paid during the one year time horizon. This is in contrast to reserve risk which exists only for run-off underwriting years.

The model refers to all treaties in-force during the valuation year. For simplicity reasons this portfolio was modeled by all short-duration treaties of actual underwriting year and all long-duration underwriting years in-force. In-force short-duration treaties with premiums unearned in the actual balance year arising from former underwriting years were not taken into account. Consistent with that unearned premiums of the actual underwriting year were included.

In principle all long- and short-duration treaties are exposed to fluctuation risk but because of the special contractual features of financing, fluctuation risk is not material and hence excluded from modeling.

Risk factors that determine the fluctuation risk are mortality, morbidity and accident incidences. Because of the one-year time horizon lapse is no substantial risk driver. Market risk drivers as interest rate changes or similar are not material.

The exposure of the GMDB portfolio to fluctuation risk is modeled at best estimate assumptions regarding financial markets parameters and furthermore in consistence with the modeling of remaining US death cover.

4.5.2 Stochastic Modeling

As individual modeling on contract level is not available for the short-duration treaties, the portfolio exposed to fluctuation risk is separated into clusters of treaties having similar risk profile. For a definition of risk clusters see next section. The general approach is then to

- Model each risk cluster’s joint loss distribution
- Define and parameterize dependencies between risk cluster
- Aggregate the cluster distribution to a joint loss distribution of the Life fluctuation risk

The loss distribution of a single risk cluster is obtained by a collective model. The collective model generally assumes that a realization of the random variable total loss is given by a number of claims and the amount of loss of each claim.

Our interpretation of the collective model further assumes that the number of claims is determined by the number of events and the number of claims arising from a single event.
This approach provides an approach of parameterization of the distribution of event separated from the distribution of the number of total claims.

Expressed in formulas the random amount of a cluster’s total loss $S$ is thus defined as

$$S = X_1 + X_2 + \cdots + X_N$$

where $X_i$ denotes the variable of single loss per event. Those severity variables are assumed to be mutually independent and identically distributed according to a severity distribution. For further discussion of the $X_i$ see the section on parameter selection on p. 117. $N$ denotes the random variable of the number of claims per event arising from the set of contracts.

We assume the frequency distribution $N$ for the number of claims to be

$$N = M_1 + M_2 + \cdots + M_{\tilde{N}}$$

where $\tilde{N}$ denotes the numbers of events happening during the year and $M_i$ the random variable of number of claims per event. Again all $M_i$ are assumed to be mutually independent and identically distributed. The number of events $\tilde{N}$ is modeled as a Poisson distribution having mean value $\lambda$. The distribution of the number of claims per event $M$ is either a zero-truncated negative binomial $tnb$ or a mixture of the $tnb$ distribution and a Poisson distribution that gives additional weight to a predefined worst case scenario $mtnb$.

The unmixed frequency distribution $tnb$ assigns very little probability to high numbers of claims per event. As an example (only illustrative since e.g. a pandemic produces higher mean of total claims) the following exhibit shows probabilities below 0,01% of the negative binomial distribution with mean 1,8:
In case of extreme events like pandemic high numbers of deaths occur with a small but still considerable probability (say 1% if pandemic is assumed to happen every 100 years). This is translated into the distribution of claims per event by adding a second distribution

\[
mtnb = (1 - \mu) \cdot tnb + \mu \cdot f
\]

which carries the weight of the probability of occurrence of the extreme scenario and has the mean value of expected number of claims in this extreme event. If \( f \) is chosen as a Poisson distribution the mixed distribution \( mtnb \) has a fatter tail as can be seen from the following picture (Probabilities below 0.01%, negative binomial distribution with mean 1.8 plus Poisson distribution with mean 18 and \( \mu = 1 \)): 
The distributions of number of claims per event are characterized by the following generating functions

\[ tnb(z) = \frac{[1 - \beta(z - 1)]^{-r} - (1 + \beta)^{-r}}{1 - (1 + \beta)^{-r}} \]

with free parameters \( \beta, r \), and

\[ mtnb(z) = (1 - \mu) \frac{[1 - \beta(z - 1)]^{-r} - (1 + \beta)^{-r}}{1 - (1 + \beta)^{-r}} + \mu \cdot e^{\eta(z-1)} \]

with free parameters \( \beta, r, \eta, \mu \).

The dependencies between the individual risk cluster \( C_1, \ldots, C_n \) is modeled by a n-dimensional Clayton copula

\[ C_{C_1, \ldots, C_n}(u_1, \ldots, u_n) = (u_1^{-\theta} + \cdots + u_n^{-\theta} - 1)^{-1/\theta} \]

This copula defines a joint loss distribution together with the marginal distribution \( S_i \) of the risk cluster. We assume independence between fluctuation risk and trend risk and between fluctuation risk and reserve risk.

### 4.5.3 Determination of Risk Clusters

The Life portfolio exposed to fluctuation risk is decomposed into risk homogenous parts, i.e. groups of contracts with similar risk profile. The common risk profile of a cluster is determined by various features
For the classification of contracts into risk clusters SCOR Switzerland Group conducted an analysis of its business to assess the individual exposure with respect to the above categories.

This analysis revealed that geography is an important indicator because extreme scenarios are often defined regionally. Also for global scenarios the impact is modeled and evaluated by country (see description of pandemic scenario below).

Risk factors (mortality, morbidity, ...) translate into SCOR Switzerland Group’s lines of business Life, Health, Disability and Accident. Although risk factors are not explicitly modeled, the distinction of clusters by line of business is useful because related extreme scenarios are different. The exposure to lapse and financial market risk is within the one-year time horizon is negligible.

The risk assessment analysis further revealed that distinct contractual features do not lead to substantially different risk profiles (at least when seen from an aggregated perspective). The portion of non-proportional treaties is low.

There are parts of the portfolio where exclusions reduce substantially the risk exposure. Health treaties in Middle East mostly exclude epidemic events which reduces the exposure to pandemic\(^1\).

The distinction of individual sold policies and group covers is important (hence used for the risk clusters) because of the degree of local risk concentration.

Individual information on Underwriting guidelines, limitation of contracts or accumulation control was studied and did not define distinctions in risk classes. This information was partly used for the estimation of severity distribution, partly for assessing the impact of extreme events.

\(^1\)These pandemic exclusions in the Middle East are watertight. There is an explicit and unequivocal clause in those treaties.
<table>
<thead>
<tr>
<th>Region</th>
<th>Duration</th>
<th>LoB</th>
<th>Premium Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle East</td>
<td>Short</td>
<td>Health</td>
<td>13%</td>
</tr>
<tr>
<td>Middle East</td>
<td>Short</td>
<td>Non-Health</td>
<td>3%</td>
</tr>
<tr>
<td>Central Europe</td>
<td>Long</td>
<td>Life</td>
<td>21%</td>
</tr>
<tr>
<td>Central Europe</td>
<td>Short</td>
<td>Life</td>
<td>16%</td>
</tr>
<tr>
<td>Italy</td>
<td>Short</td>
<td>Accident</td>
<td>6%</td>
</tr>
<tr>
<td>Sweden</td>
<td>Short</td>
<td>Disability</td>
<td>5%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Long</td>
<td>Disability</td>
<td>10%</td>
</tr>
<tr>
<td>USA, IRP treaties</td>
<td>Long</td>
<td>Life</td>
<td>7%</td>
</tr>
<tr>
<td>USA, GMDB</td>
<td>Long</td>
<td>Life</td>
<td>1%</td>
</tr>
<tr>
<td>USA, other</td>
<td>Long</td>
<td>Life</td>
<td>9%</td>
</tr>
<tr>
<td>Latin America</td>
<td>Long</td>
<td>Life</td>
<td>4%</td>
</tr>
<tr>
<td>Far East</td>
<td>Long</td>
<td>Life</td>
<td>5%</td>
</tr>
</tbody>
</table>

### 4.5.4 Choice of Parameters

The model of each risk cluster is calibrated to the expected loss of its portfolio. This expected loss is derived from:

- Information provided by individually treaty models for the long-duration segments Central Europe, Netherlands and IRP treaties
- Pricing data for the Swedish disability contracts
- The known number of policies and estimated incidence rates in case of several short-duration segments
- An estimation of risk premium based on the US GAAP premium

The US GAAP premium used for calibration is the premium estimated for the actual balance year for long duration and for the actual underwriting year for short-duration business. For simplicity reasons premiums unearned in the actual balance year arising from former underwriting years were not taken into account, unearned premiums of the actual underwriting year were included.

Further input data are the number of risks and the average incidence rate of each cluster. The number of risks was deducted from the known portfolio or reported on cedents accounts or estimated from information like premium, expected loss or limitations per risk.

The choice of parameters of the various distributions comprises the following steps:
• Determination of the frequency of events (parameter $\lambda$ of the Poisson variable $\hat{N}$)

• Determination of the frequency of claims per event (parameter $\beta$ and $r$ of the number of claims distribution)

• Determination of the severity distributions $M_i$

• Calibration of the single free parameter $\theta$ of the Clayton copula

The free parameters of the frequency distribution are determined by the following conditions:

• The joint loss distribution has to reproduce the mean expectation of loss supplied by pricing data (risk premium).

• The frequency of claim distribution (for some cluster the joint loss distribution) should reproduce the impact of predefined extreme scenarios.

• The modeled number of claims per event should correspond to a quantity observed in the portfolio or judged reasonable.

From the equation for expected values

$$E(N) = E(M)E(\hat{N})$$

it follows that the free parameter $\lambda$ of the Poisson distribution of the number of events is given by the expected number of total claims (which is estimated by portfolio data as average incidence rate times number of policies) and the mean value of the distribution of the number of claims per event. As described above the latter function is either a regular or an extended negative binomial distribution.

The regular case that is applied to the non-pandemic dominated cluster is characterized by the parameters $\beta$ and $r$. The parameters were chosen as follows:

- $r$ is set to 1 for all clusters.
- $(1 + \beta)$ equals the mean value of number of claims per event. $\beta$ was chosen individually for each cluster by Underwriting information (low for individual insurance, high for group insurance).

For the clusters dominated by the pandemic extreme scenario the frequency per event was modeled by the extended distribution $mtnb$. The free parameters $\beta, r, \eta, \mu,$ were determined as follows:

- $r$ as above.
• \((1 + \beta)\) equals the mean values of number of claims per event, as above.
• \(\mu\) corresponding to the probability of occurrence of the extreme scenario.
• \(\eta\) corresponding to the excess mortality of pandemic scenario.

The determination and evaluation of extreme scenarios for each cluster represents the main tool of model calibration. For each cluster we defined a principal scenario and additional scenarios. The principal scenarios are the following:

<table>
<thead>
<tr>
<th>Region</th>
<th>Duration</th>
<th>LoB</th>
<th>Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle East</td>
<td>Short</td>
<td>Health</td>
<td>Industry accident</td>
</tr>
<tr>
<td>Middle East</td>
<td>Short</td>
<td>Non-Health</td>
<td>Pandemic</td>
</tr>
<tr>
<td>Central Europe</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>Central Europe</td>
<td>Short</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>Italy</td>
<td>Short</td>
<td>Accident</td>
<td>Earthquake</td>
</tr>
<tr>
<td>Sweden</td>
<td>Short</td>
<td>Disability</td>
<td>No scenario</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Long</td>
<td>Disability</td>
<td>Pandemic</td>
</tr>
<tr>
<td>USA, IRP treaties</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>USA, GMDB</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>USA, other</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>Latin America</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
<tr>
<td>Far East</td>
<td>Long</td>
<td>Life</td>
<td>Pandemic</td>
</tr>
</tbody>
</table>

The impact of pandemic on the portfolio was modeled in accordance with the approach used for the FSAP scenarios. The excess mortality applied to regional business is the following:

<table>
<thead>
<tr>
<th>Region</th>
<th>Excess mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>100%</td>
</tr>
<tr>
<td>Americas</td>
<td>60%</td>
</tr>
<tr>
<td>Middle East</td>
<td>300%</td>
</tr>
<tr>
<td>Far East</td>
<td>800%</td>
</tr>
</tbody>
</table>

As secondary extreme events we considered basically two scenarios within areas of high population density:

• Terror attack (minor: explosion destroying huge building, similar to WTC or major: Anthrax attack, spread of deadly poison among urban area)
4.5.5 Technical Modeling

The generation of the loss distributions of the risk clusters as well as the aggregated loss distribution is based on MatLab code.

Once the free parameters of the frequency distribution are set by the calibration process and the severity distribution is determined the MatLab program calculates the joint loss distribution of each cluster using a Fast Fourier algorithm.

The modelling of the risk cluster is based on actual portfolio data. Information on cluster volumes are based on the US GAAP estimated premium.

4.6 Dependency Structure of the Risk-Categories

In order to manage the approach, we make the following assumptions on the dependency structure:

- There is no dependency between the trend risks and all other risk-categories.
- There are dependencies between asset and credit risks – these are reflected in the central asset and credit risk model.
- There is a (tail) dependency between all insurance risks (including non-life) and asset risks. The underlying assumption is that major insurance events can influence the financial markets. This effect is reflected on the level of the overall integration.
- There will be a tail dependency between L&H insurance risks and credit risks. The underlying assumption is that major L&H insurance events could lead to the insolvency of L&H insurers and thus to losses in SCOR Switzerland Group L&H finance Re portfolio. This effect is again reflected on the level of the overall integration.

We can determine the loss distribution for the different areas separately, because

- The fluctuation risks and trend risks are regarded as independent
- All potential asset risk dependencies are reflected by using the central asset scenarios where necessary, i.e. for the GMDB (market risks) and Finance Re portfolio
The GMDB portfolio is split into a market risk, a fluctuation risk and a trend risk part (see the section on GMDBs, pp. 121).

Table 4.1 summarizes all partial models and their aggregation.

Table 4.1: Life business: Partial models and their aggregation.

4.7 The GMDB Replicating Portfolio

The liability cash flows generated by GMDB (Guaranteed minimum death benefit) contracts (Life Insurance) are narrowly matched by a portfolio consisting of European put options, European look-back put options, bonds and equities (Erixon and Kalberer [2005]).

The GMDB cash flows could equally be replicated by futures and swaps, which are traded in a deep and liquid market. This would lead to a minor lack of precision with respect to calibration of the model to market values. The precise choice of financial instruments (European look-back put options) in the replicating portfolio is for purely technical reasons: Instruments with good replication properties and closed valuation formulas were chosen.

The scenarios used for the GMDB model are fully consistent with the bootstrapping-the-economy model (Chapter 10). Market consistent scenarios are used for determining the market value. While calibration of the replicating portfolio is (up to statistical error) independent of the economic
scenarios used, the same economic scenarios are used for valuation of the replicating portfolio as for any other assets.

The composition of such theoretical portfolios is determined by the SCOR Switzerland’s Actuarial Life and Health team in Cologne based on stochastical projections of the future cash flows and numerical methods (Moore-Penrose inverse). The replicating portfolio allows an evaluation of GMDB in stochastic scenarios via usual option pricing formulas.

A GMDB is a Guaranteed Minimum Death Benefit based on the concept of variable annuities. Payments are variable with respect to their level and the exact time at which payments occur. The premium paid by the policyholder is being invested in equity funds, bond funds, mixed funds. Contingent on the specific policy the insured person may choose among a variety of funds and may even switch funds during the runtime of the treaty. The GMDB product in this sense is not so much an income support for retirement; primarily it offers the opportunity of tax-deferred financial investments. Any policyholder will hold an account of assets in equities, bonds and the money market.

The risk which SCOR Switzerland accepts is the guaranteed value in case of death, the GMDB, during the deferment period. SCOR Switzerland covers the difference between the account value and the GMDB in the moment of death of the insured person. The capital at risk is the difference between the account value and the GMDB. In case of death of the insured individual during the surrender charge period, SCOR Switzerland might have to pay surrender charge additionally. SCOR Switzerland receives as a reinsurance premium a percentage of the current account value. Types of guarantees include ROP, ratchet and roll-up. With the ROP, Return of premium, at minimum the premium will be paid out in case of death. The one-year ratchet annually adjusts the death benefit to the account value in case this is higher than the death benefit up to then, so cash flows depend on the maximum asset value reached at previous policy anniversaries. Roll-up means that the GMDB annually grows by a percentage independent of the account value until a ceiling is reached. Proportional withdrawals do not affect the ratio of GMDB to account value while dollar-for-dollar withdrawals will. Dollar-for-dollar withdrawals reduce the GMDB and the account value by the same nominal amount.

GMDB treaties are long duration contracts. The projection length of the stochastic scenarios is 30 years. For the purpose of the stochastic model the mortality, the partial withdrawals and any lapses are deterministic. The only stochastic variable is the performance of the assets. The calculation of net present values (NPVs) is based on cash flows. Premiums received by SCOR Switzerland are a percentage of the account value. Given the projected cash flows from the comprehensive stochastic model, a so-called
replicating portfolio consisting of a set of financial instruments is calibrated and then used to integrate GMDB into SST.

How to replicate GMDB liabilities by market portfolios is described in Erik Erixon and Tigran Kalberer, *REPLICATING PORTFOLIO* (Erixon and Kalberer [2005]). The document introduces the replication method. It does specify the classes of assets to be used under the replication. Numerical illustrations and practical examples which the document presents shall merely serve as a demonstration of the method’s applicability. They also illustrate how the calibration is done. It is understood that the implementation of the model under real conditions required hundreds of options with an almost continuous range of strike prices when the GMDB document (Erixon and Kalberer [2005]), in order to elucidate the basic mechanisms, was bound to select among a narrowly confined amount of merely ten or twenty of these assets.

In the following we will describe the procedures and methods used to set up and calibrate a replicating portfolio that replicates the cash flows of the GMDB business. The GMDB business by SCOR Switzerland consists of several cedents and more than one million reinsured and retroceded policies. These can through methods of condensing be modeled accurately using approximately 150'000 model-points that are representative of the business. In order to simulate the behavior of the market value and the mean NPV\(^2\) significant speed requirements are posed on the model. Projecting one stochastic-within-stochastic scenario for one time period, i.e. projecting the business forward one period and then calculating a new market value using Monte Carlo simulations, takes around 30 hours using the current seriatim GMDB model. This is not a feasible approach. Below we will show that a portfolio of assets can be found, which replicates the total cash flows of the portfolio accurately enough and which can be valued using closed-form solutions. This replicating portfolio can then replace the real portfolio for all simulation purposes which will increase computation speed significantly but also will reduce the impact of potential calibration errors of the economic scenarios used for valuation purposes. The difficult part of the replicating portfolio approach is to determine a set of assets which may be combined to constitute the replicating portfolio. Essentially assets need to be found that have the same characteristics and dependencies as the GMDB portfolio itself. In this section we are going to describe methods of finding this set of assets and calibrating a replicating portfolio to the real portfolio. In order to find the appropriate set of assets we need to have a closer look at the contracts and the definition of the policyholder benefits.

\(^2\)The mean NPV is defined as the average of the net present values of 1'000 real world scenarios, discounted at a rate currently set to 5%.
4.7.1 Determining a Set of Replicating Assets

Return of Premium and Roll-up GMDBs

The most basic GMDB covers are the return of premium (ROP) and the roll-up GMDB covers. We can assume that the pay-off of such GMDB is strictly dependent on the asset index (AI) at the time of the pay-off, assuming that the policyholder behavior is a known deterministic factor, independent from the past behavior of the assets. The pay-off of a ROP contract in one time period \( t \) is calculated as

\[
Claim_t = \max(0, GMDB_t - AV_t) \left( \frac{l_{x+t-1} - l_{x+1}}{l_x} \right)
\]

where

\[
GMDB_t = \sum_{i=issue}^t P_i
\]

is the sum of the premiums paid until time \( t \) and

\[
AV_t = \sum_{i=issue}^t P_i \left( \frac{AI_t}{AI_i} \right)
\]

is the account value at time \( t \). Since there are very few premiums paid into the account value (AV) currently, we can assume that there will be no future premiums after valuation. Thus, the account value for each contract (or model-point) can be rewritten as

\[
AV_t = \sum_{i=issue}^{valuation} P_i \left( \frac{AI_t}{AI_i} \right) = f(AI_t)
\]

where the function \( f \) symbolizes that the only stochastic variable is the asset index \( AI_t \) at time \( t \), all other variables are already fixed. Certainly lapses and partial withdrawals will also affect the account value, although for this purpose it shall be assumed that these are deterministic assumptions. Premiums are also payable to SCOR Switzerland by the cedents, these are strictly dependent on the account value at every point in time\(^3\)

\[
Premium_t = Rate \cdot AV_t
\]

\(^3\)For some products there are maximum premium limits, an amount in USD after which the premiums do not increase by the account value.
The rate is a fixed rate for each contract and as described above, the account value is a strict function of the asset index at time $t$. Thus the total cash flow ($TCF_t$) can be written as a function of the account value

$$ TCF_t = \sum_{\text{policies}} f(policy, AV_t) \equiv g(AV_t) $$

A very important observation is that due to the large number of policies involved and the wide dispersion of the parameters (GMDBs, ages etc.) $g$ is approximately a smooth function of the account value. The function $g$ is also strictly increasing. The cash flows of all ROP and roll-up GMDB contracts have been plotted as a function of the AV in Figure 4.1, excluding partial withdrawals to illustrate this observation.

![Figure 4.1: GMDB cash flows, ROP and roll-up policies.](image)

As can be seen in the graph, there is for each account value only one corresponding value of the cash flow. This proves the above conclusion, that the cash flow can be written as a function strictly dependent on the asset index. It is now obvious, that it is sufficient to know the shape of these functions in order to model the behavior of the whole liability function. Theoretically it would be sufficient to determine the values of these functions for a sufficiently large number of account values, covering a wide enough range, and interpolate between them to determine the functions for simulation purposes. Because the number of account values for which function values that
need to be evaluated is small compared to the number of model-points involved, this would imply that the determination of the cash flows for each scenario requires at most two calculations instead of 160’000 (approximately the number of model points). This is a significant improvement.

But we want to go a step further. We want to approximate the cash flow function by the sum of functions which represent the cash flows of derivatives that have closed-form solutions to the market value. The portfolio consisting of these derivatives then replicates our liability portfolio and its value can be determined using closed-form solutions which is extremely helpful for a lot of relevant applications. In order to determine this replication portfolio it is in fact only necessary to evaluate the cash flow function over a sufficiently wide range of account values, it is not necessary to use account values, which have been produced by scenarios. Using appropriately calibrated scenarios just ensures that areas of account values which are more frequently hit by scenarios are evaluated with a higher granularity than other areas of less importance. Thus it is not important whether the scenarios used for fitting a replication portfolio are accurately reflecting current market conditions. It is of higher importance that the area of possible account values is sufficiently spanned by the scenarios in order to prevent extrapolation errors. By inspection of the graph we decide to choose the following assets to determine the replication portfolio:

- An asset with the pay-off of the underlying account value itself
- A fixed cash flow (zero bond) and
- European put options on the asset index with different strike prices

The asset paying the account value itself will then replicate the premiums and the European put options will replicate the claims. The slope of the put options has to be smoothed into the cash flow curve. Such a slope can be created by combining several European put options with different strikes. It is very important to ensure that the replication portfolio is well behaved for very small and very large account values. Otherwise scenarios which produce very small or large account values would not be valued appropriately which could cause substantial distortions. In our case the asymptotic behavior of the replication assets and the portfolio is the same, which ensures that this problem will not arise (as can be seen in the graph).

A complicating matter for the ROP and roll-up GMDBs is the dollar-for-dollar withdrawals, which do not affect the account value and the GMDB in a proportional manner as normal partial withdrawals do. While the account value decreases by a percentage which is equal to the rate of partial withdrawal, the GMDB amount will be decreased by a proportionally different amount. This creates a dependency on the path of the asset index until time $t$. In Figure 4.2 the cash flows which include the effect of partial withdrawals have been plotted as a function of the account value.
As can be seen, the premiums are still strict functions of the account value while the claims are now also dependent on another variable, not visible in this graph, and are therefore not a function strictly dependent on the account value. Although the claims are somewhat scattered, the basic shape of the dependency can still be clearly observed. We have produced an example of how the GMDB cash flows can be replicated using European put options\(^4\) in Table 4.2. The pay-offs of the options are plotted in Figure 4.3.

As can be seen, the claims are not perfectly replicated, the dollar-for-dollar withdrawals being the main reason for the differences. It can also be observed that the pay-offs of the replicating portfolio are piecewise linear, the reason being the limited number of options used to replicate the cash flows. Since there are closed-form solutions available for the European put options the market value and the mean NPV can easily be calculated while projecting the business forward. The premiums can be replicated using a similar method.

\(^4\)For the purpose of this example we have excluded the premiums. Replicating the premiums by an appropriate volume of investments into the underlying account value should be a straightforward exercise.

Figure 4.2: GMDB ROP and roll-up, including partial withdrawals.
<table>
<thead>
<tr>
<th>Asset</th>
<th>Strike</th>
<th>Number of assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>European put option 1</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>European put option 2</td>
<td>1</td>
<td>208583</td>
</tr>
<tr>
<td>European put option 3</td>
<td>1.111</td>
<td>-216250</td>
</tr>
<tr>
<td>European put option 4</td>
<td>1.667</td>
<td>-167369</td>
</tr>
<tr>
<td>European put option 5</td>
<td>2.222</td>
<td>-228732</td>
</tr>
<tr>
<td>European put option 6</td>
<td>2.778</td>
<td>-40322</td>
</tr>
<tr>
<td>European put option 7</td>
<td>3.333</td>
<td>-45794</td>
</tr>
<tr>
<td>European put option 8</td>
<td>3.889</td>
<td>-41517</td>
</tr>
<tr>
<td>European put option 9</td>
<td>4.444</td>
<td>-37240</td>
</tr>
<tr>
<td>European put option 10</td>
<td>10</td>
<td>5532</td>
</tr>
</tbody>
</table>

Table 4.2: GMDB: Replication of the *roll-up* and *ROP* pay-off

Figure 4.3: GMDB: pay-offs of options.

CF by account value after ten years. ROP and roll-up GMDB policies.
Ratchet Policies

Ratchet GMDBs are dependent on the maximum of the asset index as well as the asset index at the time of the cash flow. This means that the claims will be written as a function of the maximum of the asset indices, $AI_{max}$, up to time $t$ and the account value at time $t$.

$$Cash\ flow_t = f (AI_t, AI_{max})$$

So these liabilities are dependent on two factors instead of one. But the basic approach will be the same as for the ROP GMDBs. When actually modeling ratchet GMDBs there are two features which also need to be considered. The ratcheted GMDB amount is set to the maximum of the GMDB cover of the previous period and the asset index only once per year. To enable condensing of policies with different quarter of ratchet, each policy is split into four different parts which are ratcheted once per year on the four calendar quarters respectively. This means that the claim amount for a single policy will be dependent on the asset index at the time of the payout and the maximum of the asset index of the four quarters (referred to as $AI_{maxqX}$ for quarter $X$) separately.

$$Cash\ flow_t = f (AI_t, AI_{maxq1}, AI_{maxq2}, AI_{maxq3}, AI_{maxq4})$$

Several of the ratchet products have a maximum age after which the GMDB amount is no longer increased. After that age the GMDB amount equals the amount as at that date. For a policy where the ratchet cover expired at a prior time $z$ the cash flow can be written as

$$Cash\ flow_t = f (AI_t, AI_{maxq1}, AI_{maxq2}, AI_{maxq3}, AI_{maxq4})$$

Figure 4.4 shows a surface plot of the cash flows from one ratchet GMDB policy by the asset index and its maximum. The policy simulated only includes one of the four quarter parts described above, the third quarter, and the maximum asset index, plotted on the x-axis of the graph, takes into account the maximum ratchet age as described above. Dollar-for-dollar partial withdrawals have also be set to zero in order to single out a policy that is strictly dependent on the asset index and its maximum.

As can be seen above, the plot shows a surface strictly dependent on the maximum of the third-quarter asset index, the asset index at valuation and the maximum ratchet age. This proves that the conclusions made above are correct. In the remainder of this section we will add on the features described above to show which of these are material and which are not. Figure 4.5 displays the graph of a policy also run with dollar-for-dollar partial withdrawals.

The surface is now somewhat distorted by the dependency on the path of the asset index. If the quarterly parts 1, 2 and 4 are also included the result is Figure 4.6.
Figure 4.4: Surface plot of GMDB cash flows (ratchet policy)

Figure 4.5: Surface plot of GMDB cash flows including dollar-for-dollar withdrawals.
Although still being distorted, the shape and the angle of the surface is still clearly visible. As a last test, the consideration of the maximum ratchet age is removed, meaning that the cash flows are plotted by the maximum asset index without a time limit and the asset index. Figure 4.7 shows how the ratchet policies behave in the VB GMDB model.

As can be seen in the above graph, the dispersion is significant. The payoff is clearly not a function of the asset index and the maximum asset index alone any more. From the above graphs it is concluded that the maximum ratchet age should be considered in the replicating assets. The effects of the other distortions, the modeling of four parts corresponding to the calendar quarters and the dollar-for-dollar withdrawals, are small in comparison. The path dependency of the replicating portfolio can be introduced through look-back put options, which pay the difference between the maximum of the asset index and the asset index itself. There exist closed-form solutions to the market value of look-back put options and they therefore have the advantage of easy simulation of the market value as well. The dependency on the maximum ratchet age can be introduced by modeling of partial look-back put options. There exist closed-form solutions to these assets.

4.7.2 Calibration of the Replicating Portfolio

For the purpose of demonstrating how the replicating portfolio can be calibrated towards the GMDB cash flows a replicating portfolio consisting of the following assets shall be used:
Figure 4.7: GMDB with no maximum for the ratchet age.

- 10 European put options with strikes of\(^5\) 0.8, 1, 1.11, 1.67, 2.22, 2.78, 3.33, 3.89, 4.44, 10
- Zero coupon bonds with par 1
- The asset index itself
- 10 look-back put options with strikes of\(^6\) 1, 2, \ldots, 9 and 10

Denote the cash flow of the replicating asset number \(n\) (numbered in the order they are described above) under scenario \(s\) by \(RPCF_{s,n}\), the cash flows of the GMDBs of scenario \(s\) by \(GMDBCF_s\) and the volume (nominal) of replicating assets of number \(n\) by \(NR_n\). Then the following term has to be minimized by choosing an appropriate vector \(NR\) and the least squares measure \(|·|\) for the deviation

\[
\left| \begin{array}{ccc}
RPCF_{1,1} & \cdots & RPCF_{1,22} \\
RPCF_{2,1} & \cdots & RPCF_{2,22} \\
\vdots & \cdots & \vdots \\
RPCF_{S,1} & \cdots & RPCF_{S,22}
\end{array} \right| \left( \begin{array}{c}
NR_1 \\
\vdots \\
NR_{22}
\end{array} \right) - \left( \begin{array}{c}
GMDBCF_1 \\
\vdots \\
GMDBCF_S
\end{array} \right)
\]

\(^5\)Given a nominal asset index of 1.
\(^6\)The look-back put option strikes cannot be lower than the asset index at valuation, otherwise these would immediately be ratcheted to that strike. However, it can be that the strike is higher than the current asset index due to a previous ratcheted amount.
4.7.3 Checking and Bias

There will be an element of bias introduced into the model if the replication portfolio is determined using scenarios which in turn are used to determine the goodness of fit. The choice of the assets used and the number of scenarios used to replicate the cash flows of the GMDBs should be analyzed so that the error of the replicating portfolio is kept at a minimum; over-determination by using too many assets should be avoided. The error of a given replicating portfolio should be measured on a set of scenarios that were not used to actually calibrate the replicating portfolio in the first place. The reason for using a new set of scenarios is that if the calibration is compromised, it might be that the replicating assets are simulating the scenarios rather than the GMDB portfolio. We have suggested a few different measures of how to quantify and visualize the error and the bias of a replicating portfolio.

Comparing Replication Portfolio Cash Flows with Original Cash Flows

If the replicating portfolio cash flows are plotted by the original cash flows of the portfolio a straight line should be the result if there is no difference between the replicating portfolio and the GMDB cash flows (y equals x type of dependency). If there are differences between the replicating portfolio cash flows and the GMDB cash flows the line will be spread, where the deviation is the error. An example of such a scatter is shown in Figure 4.8.

In order to further clarify the differences the GMDB cash flows have also been plotted within the graph, those are visible as a straight line.

Standard Deviation of Error

The standard deviation and the mean of the error is a good way to visualize the error for different times \( t \). If the standard deviation of the error is significantly larger during certain periods this might be a reason for looking into the range of scenarios that are used to calibrate the replicating portfolio of the period covered. Due to the spreading-out effect of stochastic simulations, it can be that a wider range and thus a larger number of scenarios are required to calibrate a replicating portfolio with a long maturity than for one with a short maturity. Figure 4.9 provides an example where it is clearly visible that the standard deviation of the error is significantly higher during certain years than for others.

In the above example only 50 scenarios were used to calibrate the replicating portfolio. The 50 scenarios are clearly not adequate to calibrate the portfolio during year 9 (quarters 36 to 39), because the full set of 1’000 scenarios moves through ranges not covered by the first 50 scenarios, leading to extrapolation error.
Replicating portfolio cash flow versus original cash flow

Figure 4.8: Scatter plot of GMDB cash flows

Figure 4.9: GMDB, standard deviation of error.
4.7.4 Projecting the Replicating Portfolio

The cash flows of the replicating portfolio can be projected straightforward using the asset index itself, as described in the section on checking and bias. The market value of the replicating portfolio can be calculated using closed-form solutions and observable market variables, such as the yield curve and a parameter for the volatility. However, if the market value is calculated using this approach several approximations are introduced:

- The market value of an option is dependent on the volatility, which in the Black and Scholes framework is a fixed parameter of the projection. However, the distribution of the asset index may not exactly be normally distributed due to the basket like nature of the asset index. For example, if equities develop strongly compared to bonds, the relative proportion of equity in the asset index increases. This results in a higher volatility of the asset index and a skewed distribution. This is not considered in the standard Black and Scholes option formula where only one fixed volatility parameter is used.

- The replicating portfolio is not calibrated on a market value basis but rather on a cash flow basis. The maximum asset index of the look-back put options is a continuous maximization. This will cause a difference between the closed-form solution valuation and the stochastic valuation using the GMDB model.

Closed-form solution of discrete look-back options can be derived, but the expressions will not be easily handled for five or more updating times, the longest look-back put option of the replicating portfolio is updated 120 times. Instead of deriving closed-form solutions it is possible to apply numerical approximations, it can be shown\(^7\) that the following adjustment can be made to the closed-form solution of the continuous price to derive an estimation of the discrete price

\[
MV_{m}^{Put}(S, X, T, \sigma) = e^{-B_1 \sqrt{\frac{T}{m}}} MV^{Put} \left( Se^{B_1 \sigma \sqrt{\frac{T}{m}}, X, T, \sigma} \right) + \left( e^{-B_1 \sigma \sqrt{\frac{T}{m}}} - 1 \right) S
\]

Where \(MV_{m}^{Put}\) is the market value of the look-back put option with discrete updating and \(MV^{Put}\) is the market value of its continuous counterpart. \(B_1\) is a constant, that can be shown to be approximately 0.5826\(^8\). In Table 4.3 market values of the replicating portfolio have been calculated using both closed-form solutions and Monte Carlo simulations. The closed form solutions have been calculated both using the standard look-back put option


formula both including and excluding the adjustment for discrete updating. The market value calculated using Monte Carlo simulations is calculated using 10,000 market-consistent scenarios to avoid any sampling errors. The idea of the table is to give the reader a sense of the magnitude of the error that is introduced when approximating the market value of discrete look-back options by that of the continuous counterpart and to show what the impact of the constant volatility assumption is.

<table>
<thead>
<tr>
<th></th>
<th>European put options</th>
<th>Look-back put options</th>
<th>Bonds &amp; equities</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation of replicating portfolio using 10'000 market consistent scenarios:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating asset mix</td>
<td>-50</td>
<td>-7</td>
<td>-22</td>
<td>-79</td>
</tr>
<tr>
<td>Fixed asset mix</td>
<td>-49</td>
<td>-7</td>
<td>-22</td>
<td>-78</td>
</tr>
<tr>
<td>Valuation of replicating portfolio using closed-form solution:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% standard deviation</td>
<td>-51</td>
<td>-10</td>
<td>-22</td>
<td>-84</td>
</tr>
<tr>
<td>14% standard deviation</td>
<td>-49</td>
<td>-9</td>
<td>-22</td>
<td>-81</td>
</tr>
<tr>
<td>15% standard deviation including adjustment for discrete look-back put options</td>
<td>-51</td>
<td>-7</td>
<td>-22</td>
<td>-80</td>
</tr>
</tbody>
</table>

Market value of replicating portfolio as of 31 September 2005 (USD million).

Table 4.3: Market value of GMDB replicating portfolio.

Changing the standard deviation to 14% results in a value which is approximately USD 3 million lower than the value estimated using 15% standard deviation. This should be seen as a sensitivity of the market value towards the volatility. Figure 4.10 is a graph of the standard deviation measured from 10'000 market consistent scenario with floating asset mix.

As can be seen the error caused by using one parameter for the standard deviation for all future periods can not greatly influence results as the fluctuations between years are small. Furthermore, it is also clear that the standard deviation lies within a range of 14.9% to 15.4%, which discards the 14% sensitivity as being unrealistic. The conclusion is the main source of error comes from approximating discrete look-back options by continuous look-back options, and it is therefore recommended to use the adjustment for discrete updating as described above.
Graph of the standard deviation measured from 10’000 market consistent scenario with floating asset mix.

Figure 4.10: GMDB scenario with floating asset mix.
Dependency Structure of the Life & Health Insurance Risk with the Non-Life Insurance Risks

It is important to clarify the dependency structure between the non-life and the life-insurance risks in order to determine the group SST-target capital.

We have to consider fluctuation risks within L&H only when assessing this dependency, as the trend risks are assumed to have no dependency to any other risk category.

The dependency structure between asset and credit and Non-Life risks implies an indirect dependency between Life and Non-Life risks, but the discussion of the method to determine this relationship is outside the scope of this section.

A tail dependency between non-life and the life-insurance risk categories could arise if major events affect both life and non-life treaties negatively.

As such events have been very rare and the current historical data most likely do not reflect the occurrence of such events adequately, we use an approach involving scenario techniques and copulas to reflect dependencies between these risk categories:

1. We determine a list of possible events which could influence both life and non-life treaties substantially. This is a collaborative effort of L&H and Non-life experts and is based on the Risk and Exposure study which lists all relevant such events.

2. We determine scenarios, covering these events and their range of impact adequately, assessing the probability using actuarial judgement and underwriting-know-how.
3. We evaluate the impact of these scenarios on L&H and Non-Life.

4. The marginal distributions without those events of the Life and Non-Life insurance risks are assumed to be given and independent.

5. A Monte Carlo simulation is used to combine the distributions with each other and with the extreme scenarios (see the section on fluctuation risk, p. 111, for a detailed description of this approach) to get an empiric distribution of the combined risk.

6. A Clayton copula between the non-life and life risks is calibrated to the result, reflecting the empirical dependency structure as far as possible (see section on fluctuation risk for a detailed description of this approach).

7. This copula is then used to combine the marginal distributions of life and non-life risks (now we refer to the distributions including the extreme scenarios in a smoother way) (see section on fluctuation risk).
Mitigation of Insurance Risk

This section describes the risk (mitigation) modeling approach regarding insurance risks that have been retroceded in the past (i.e. reinsurance assets including the balances due from retrocessionaires for paid and unpaid losses and loss expenses, ceded unearned premiums and ceded future life benefits\(^1\)) or are being retroceded in the current year. The credit risk due to the reinsurance assets is addressed in Part III (pp. 297) of this documentation under the credit risk model (Müller [2007b]). Amounts recoverable from the retrocessionaires are estimated in a manner consistent with liabilities associated with the reinsurance contracts\(^2\). Our internal risk models treat the gross losses as stochastic variables modeled by line of business. Retroceded claims from past business are modeled the same way as the gross reserves (see Chapter 3). Regarding retrocessions planned for the current year, we differentiate their modeling depending on the potential recovery amounts and contract structure. Especially the cat retro structure is of major importance for the risk mitigation of SCOR Switzerland (13% retroceded catastrophe premiums in 2007). Small recovery amounts and/or pure quota share retrocession are modeled in a simplified way using regression techniques and/or the corresponding fraction of the modeled gross loss. For large retrocession contracts the (individual) retroceded losses are modeled by applying the explicit reinsurance contract structure on the individual gross losses. Hence, for these lines of business frequency-severity loss models are required. The amounts to be recovered from retrocessionaires we call the retroceded losses or recoveries, designated by \(\hat{Z}\), in order to distinguish them from the gross losses which we denote by \(Z\). In the following, both \(Z\) and \(\hat{Z}\) are to be regarded as losses and recoveries by line of business, respectively\(^3\). So, for

\(^{1}\)For the definition see Converium annual report 2006, p. 72.

\(^{2}\)Converium annual report 2006, p. 54.

\(^{3}\)The aggregation tool used for this purpose is Phobos, an IT component which allows to collect the various lines of business into so called baskets. These baskets are organized in a hierarchical tree of sub-portfolios and thereby reflect the dependency structure among sub-portfolios which is central in the analysis of the liability model. In this section, line of business.
large standard excess of loss contracts for instance, individual recoveries $\hat{Z}_i$ are computed considering limits $L$, deductibles $D$ and individual gross losses $Z_i$ via

$$\hat{Z}_i = \min(L, \max(0, Z_i - D))$$

Regarding small recoveries the central assumption is that, for the purpose of stochastic simulations, the aggregate retroceded losses $\hat{Z}$ are linearly dependent on the aggregate gross losses $Z$ via the equation

$$\hat{Z} = \alpha + \beta Z$$  \hspace{1cm} (6.1)

(How to take into account the fraction to be recovered from retrocessionaires for small recoveries is outlined in a document by Michael Moller (Moller [2007]).) The parameters $\alpha$ and $\beta$ we shall determine for each line of business from the following data which, for each line of business, are assumed to be known beforehand.

- The retention $\delta$ defines the retroceded premiums $\hat{P}$ as a portion of the written premiums $P$

$$\hat{P} = P(1 - \delta)$$

- The retrocession loss ratio $\nu$ gives us the expected retroceded losses $\mathbb{E}(\hat{Z})$ contingent on the retroceded premium $\hat{P}$

$$\mathbb{E}(\hat{Z}) = \nu \hat{P}$$

- The reinsurance risk loading $\kappa$ is known together with the expected retroceded losses $\mathbb{E}(\hat{Z})$ and the expected gross losses $\mathbb{E}(Z)$. This allows to express the standard deviation of retroceded losses $\sigma_{\hat{Z}}$ via the standard deviation of gross losses $\sigma_Z$ in the form

$$\sigma_{\hat{Z}} = \sigma_Z \frac{\mathbb{E}(\hat{Z})}{\mathbb{E}(Z)} + \kappa \mathbb{E}(\hat{Z})$$

- The linear correlation of gross losses and retroceded losses is estimated from past data$^4$.

$$\text{Corr}(Z, \hat{Z}) = h$$

\footnote{business shall therefore mean a line of business as specified in Phobos. For more details on Phobos and other IT tools used for the SST process we refer to Part VIII, pp. 408. The relevant SST process modules are described in the process landscape document, Skalsky and Bürgi [2008], incorporated into this document as Part VII, pp. 353.}

$^4$Currently it is assumed by experts that the linear correlation is roughly about 0.7. A statistical investigation for further backing this assumption will be done in short-term.
We compute the two parameters which govern the linear Equation (6.1)

\[ \beta = \frac{\sigma_{\hat{Z}}}{\sigma_{Z}} \text{Corr}(Z, \hat{Z}) \]

\[ \alpha = \mathbb{E}(\hat{Z}) - \beta \mathbb{E}(Z) \]
7

Limitations of the Liability Model

7.1 Life Business

The main and most material limitation is the almost complete absence of data. This problem is most clearly visible:

- For the main mortality risk driver \textit{pandemic}
- For all trend risks

Furthermore it is obvious that given this lack of data for the marginals it is even less possible to reliably specify a dependency structure. This problem is not specific to the SCOR Switzerland Group but affects all (re)insurers. Likewise it cannot be solved by the SCOR Group on its own but poses a general challenge to any internal model.

In our specific internal model we tried to address those difficulties by:

- Making very conservative choices (e.g. no diversification in trend risk)
- Relying on data provided by the regulator in the first place (FSAP pandemic scenario and trend risk distributions)

All other model limitations seem to be much smaller in comparison.

7.2 Non-Life Business

Limitations of the non-life liability model have been described implicitly in the corresponding chapter. We mention here some of the main points.

With regards to the liability models constructed, we have separated the \textit{new business} and the \textit{run-off} model. This is unsatisfactory in that new
business written in the first year $i = 0$ becomes run-off business at $t = 1$, so the two models are not independent. Therefore, it would be preferable to have a consistent approach, combining new and run-off business in one consistent model.

In particular, concerning the portfolio dependency tree, it is unsatisfactory that the new and the run-off business dependency is formulated at the top level. Especially for long-tail business, risk factors that impact the dependency between different contracts probably have an impact on both new as well as run-off business contracts. For this reason, new and run-off business for one type of business should probably be joined at a lower level in the portfolio dependency tree.

Concerning the models of new and run-off business, the model for the new business is considerably more sophisticated than the run-off business model. This holds true for the dependency structure within the models as well as for the respective basket models. We now address shortcomings of the two models separately.

### 7.2.1 New Business Model

To begin with, the new business model initially models nominal ultimate amounts. For the SST, however, models for the one-year changes are required. Thus, we have to select proxies to derive one-year changes from ultimate changes, and it would clearly be preferable to directly have a model for the one-year changes available. This applies both to the basket models as well as to the dependency structure in the portfolio dependency tree.

For the new business baskets, we use both default models (with implicit dependency structure) as well as basket-specific models. Ideally, default models should be replaced as much as possible by basket-specific models, which are explicit dependency models. However, very likely there will never be a specific model for every type of business. This because, first of all, the required data for the contracts will often not be available (recall the requirements from Section 8.2.1, pp. 153), and, secondly, because there might not be (market-wide) data available to construct and parameterize such a model. The most promising candidates for new basket-specific models currently available are Aviation and Credit and Surety.

Another area for improvements is the default model itself. As described, the aggregation of the contract models to an (end) basket model is currently using a one-parametric multidimensional Clayton copula with the property that the dependency structure is the same between any two contracts in the basket. A potential improvement would be to use a copula family with more than one parameter.

Considering improvements in the dependency structure, we mention the following points:

- The structure of the portfolio dependency tree has to be updated.
The values of the dependency parameters have to be reconsidered.

The whole structure of the portfolio dependency tree is an implicit dependency model. It is desirable to try to work out at least qualitatively the underlying explicit dependency model, i.e. to look for the risk factors causing the different types of dependencies. This could be done directly or by creating a list of (realistic disaster) scenarios where such (extremal) dependencies are manifested.

It would be useful to classify dependencies. For instance, some dependencies arise while a contract is “under risk” (e.g. earthquakes, hurricanes, etc.), while others occur during run-off (e.g. legal changes).

7.2.2 Natural Catastrophe Modeling

This section is based on Pabst and Matter [2008]. Revised (commercial and in-house) catastrophe models often lead to significant changes in model outcome. Those changes reflect the uncertainty and/or sensitivity of parameters mainly associated with the hazard and vulnerability modules.

The understanding of the hazard’s mechanism and its mathematical representation are generally difficult for scientists. Major areas of research are for instance the projection of future event probabilities from the past, and thus the completeness and comparability of data. An important aspect in hazard modeling is the correlation of hazards. (The occurrence probability of a severe loss amount increases with the consideration of collateral hazards.)

Usually, vulnerability functions for insured objects are derived from claims data and engineering analysis. They are often critical with respect to

- Regional specific reconstruction costs
- Representation of specific structure types
- Consideration of prevention measures (like dams)

The calibration of vulnerability functions is a process of continuous improvements and enhancements as more and more detailed descriptions and loss data are available.

Exposure data are one of the model components where uncertainty can be reduced most easily. Critical exposure data are mainly replacement value, construction type, occupancy type, and the geo-reference of the exposure. Significant data quality issues exist in particular for commercial property.
For multi-location policies the quality of the loss estimates depend strongly on the description of each single location with regard to the assignment of the location limits and replacement values as well as the occupancy and building type.

The today’s generation of catastrophe models focuses on exactly locatable property values. Thus modeling business interruption and marine business remains challenging.

At present vendor catastrophe models do not take into account the accumulation of **attritional losses** for instance stemming from hail, thunderstorm or torrential rain. Hence, models underestimate those loss components. In addition aggregate excess of loss contracts covering attritional losses cannot be evaluated properly.

For the accumulation control or for pricing of individual contracts, we thus frequently apply adjustments to the models reflecting commercial structures, non-modeled perils or collateral hazards.

Vendor models run the risk being biased by current market perspectives not necessarily reflecting scientific views.

Catastrophe modeling technology is not static and the models themselves continue to evolve in terms of detail, realism, and accuracy.

### 7.2.3 Run-off Business Model

Consider the **run-off basket models** as described in Section 3.9 (pp. 91). Unlike for new business, these models are already modeling the one-year change. However, for run-off business, no full distribution for the baskets is derived. Instead, only the expected value and the standard deviation are calculated, and a distribution assumption (currently log-normal) has to be made. In particular, the estimated standard deviation is used together with the distribution assumption to estimate very small quantiles. It would be preferable to derive the full distribution directly.

The method used to estimate the standard deviation for the one-year change is based on the **Mack model assumptions**. These, in particular, assume that the ultimate amounts in the triangle are calculated using the **chain-ladder method**. This is potentially unrealistic especially for the more recent years, where the reserve estimates are often based on the pricing figures or on a combination of the pricing figures and the triangles, such as the Bornhutter-Ferguson method.

Further, the Mack method assumes that **different underwriting years are independent**. This assumption is again questionable, as, especially for long-tail business, risk factors for one calendar year leading to notable reserve increases for run-off business might reasonably have an impact on business
from all underwriting years at the same time. Thus, calendar year dependencies cannot currently be considered.

Finally, the dependency structure for run-off business should be reconsidered, as the proposed structure was derived for new business, capturing ultimate changes.
8

Appendix

8.1 Portfolio Dependency Tree

In this section, we introduce the portfolio dependency tree, which is the basic structure we use to capture dependencies in the non-life liability portfolio\(^1\). The portfolio dependency tree is a suitable decomposition of the portfolio into a hierarchy of sub-portfolios. We call these sub-portfolios *baskets*. Ultimately, the baskets are contracts or collections of contracts. The portfolio dependency tree is needed in two contexts: for the *bottom-up aggregation* and for the *top-down allocation*. Bottom-up aggregation is used for the ALM, in particular for the SST, and also for the pricing capital allocation. Top-down allocation is used mainly for the pricing capital allocation.

*Bottom-up aggregation*: Stochastic models will typically be given at the level of sub-portfolios (possibly down to individual contracts), and a stochastic model for the whole portfolio results from dependent aggregation of these sub-portfolio models. The required dependency structure is captured in the portfolio dependency tree.

Next, typically, a risk measure is applied to the distribution of the whole portfolio to give the overall amount of required capital.

*Top-down allocation*: The overall required capital is allocated top-down to the baskets of the portfolio dependency tree and ultimately down to individual contracts. The capital allocated to an individual contract gives rise to capital costs, which determine the risk loading for that contract in pricing.

8.1.1 Suitable Portfolio Decomposition

Let \( \mathcal{Z} \) denote a portfolio of liabilities (e.g. of a whole group or of a legal entity in the group). At this point, we do not distinguish whether the portfolio

\(^1\)The IT system currently used to model portfolio dependencies is *Phobos*. We refer to Part VIII in this document.
contains new business or run-off business or both.

To build a stochastic model for the portfolio \( Z \), we inevitably need to split the portfolio into suitable disjoint sub-portfolios, since stochastic models will be available at the sub-portfolio level. We call the sub-portfolios which are not further decomposed the end baskets. These end baskets need to be suitably homogeneous for several reasons, see Section 3.2.

When aggregating the models of the sub-portfolios, dependencies between the sub-portfolios need to be taken into account. Additionally, for pricing, as described in capital allocation (Section 8.3, pp. 168), we need a model of the dependency of an individual contract to the whole portfolio.

If the number of sub-portfolios is large, and since any sub-portfolio might depend on any other sub-portfolio, the dependency structure might become very complicated. Moreover, as mentioned in Section 8.2, constructing high-dimensional dependency structures is not an easy task because constructing arbitrary high-dimensional copulas is not easy. For instance, assume that the portfolio consists of three end baskets, and that we specify the dependency between the first and the second basket, between the first and the third, and between the second and the third basket. Then, in general, there will not always be a copula between the three baskets satisfying these specifications.

For this reason, the basic idea is to aggregate the end baskets by means of a hierarchical dependency structure, and to use simple multidimensional copulas to express dependency within a hierarchy level.

Put differently, the portfolio is decomposed into disjoint sub-portfolios, some of which are further decomposed, etc. In this way we obtain a hierarchical portfolio decomposition. We call sub-portfolios in this decomposition baskets. So, in a first step, we decompose the portfolio \( Z \) into baskets

\[
Z = Z_1 + Z_2 + \cdots + Z_n
\]

Then, we further decompose some of the baskets \( Z_i \) into baskets

\[
Z_i = Z_{i,1} + Z_{i,2} + \cdots + Z_{i,n_i}
\]

and so on.

Recursively, we arrive at a hierarchical decomposition of \( Z \), which we view as a tree with root \( Z \), see Table 8.1.

To describe the tree structure, we introduce the following definitions:

**Definition 8.1** The hierarchy level \( l \) of a basket is defined as the number \( l \) of indices of the basket in the notation \( Z_{i_1, \ldots, i_l} \). The whole portfolio has hierarchy level 0. A basket \( X \) is called descendent of a basket \( Y \) in a given portfolio decomposition, denoted

\[
X \preceq Y,
\]

if \( X \) is a sub-portfolio of \( Y \) in this decomposition. A child is defined to be a descendent in the portfolio tree of the next lower level, and a parent
Hierarchical decomposition of $Z$ into sub-portfolios.

Table 8.1: The portfolio dependency tree.

is defined analogously. Baskets that are not further decomposed into sub-portfolios (i.e. that have no descendents) are called end baskets of the tree.

Dependencies in the portfolio tree are always specified between children of a basket.

In order to ensure that the portfolio decomposition is applicable for our purposes, the decomposition needs to fulfill an additional condition we call suitability. To define this condition, denote by $X$ a random variable derived from the cash flows of the sub-portfolio $\mathcal{X}$ (typically, the ultimate nominal losses or the net present value of the result) at a future point in time $t$, usually $t = 1$ or $t = \infty$.

**Definition 8.2** A portfolio dependency tree of $Z$ is defined to be a suitable portfolio decomposition, where a portfolio decomposition is called suitable if the following holds: Given baskets $\mathcal{X} \preceq \mathcal{Y}$ and a basket $\mathcal{S}$ which is not a descendent of $\mathcal{Y}$, the random variables $X$ and $S$ are independent given $Y$, that is, for any $x, s, y \in \mathbb{R}$,

$$
P[X \leq x, S \leq s \mid Y = y] = P[X \leq x \mid Y = y] \cdot P[S \leq s \mid Y = y].$$

(8.1)

Equivalently, suitability can be expressed by

$$
P[X \leq x \mid S \leq s, Y = y] = P[X \leq x \mid Y = y].$$

(8.2)

**8.1.2 Why Suitability is Needed**

The property of suitability (8.1) is essential for two reasons:

1. In the bottom-up aggregation process, it allows to decentralize the modeling of certain baskets.
2. For the top-down allocation process, it enables a consistent risk allocation along the branches of the tree.

Regarding the first point, for a given basket, we would like to aggregate the descendents of a basket without having to consider what happens at other parts of the dependency tree.

Regarding the second point, given baskets \( \mathcal{X} \preceq \mathcal{Y} \preceq \mathcal{S} \), we would like that the dependency between \( \mathcal{X} \) and \( \mathcal{S} \) is determined by the dependency between \( \mathcal{X}, \mathcal{Y} \) and between \( \mathcal{Y}, \mathcal{S} \).

As an illustration of the first point, consider a basket \( \mathcal{S} \) consisting of three baskets \( \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3 \), so \( \mathcal{S} = \mathcal{X}_1 + \mathcal{X}_2 + \mathcal{X}_3 \). Setting \( \mathcal{Y} = \mathcal{X}_1 + \mathcal{X}_2 \), we would like to define the dependency between \( \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3 \) by means of a hierarchical tree structure. That is, we want the dependency between \( \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3 \) to be determined by the dependency between \( \mathcal{X}_1, \mathcal{X}_2, \) and between \( \mathcal{Y}, \mathcal{X}_3 \).

For this to work, we have to be able to derive the dependency between, e.g., \( \mathcal{X}_1 \) and \( \mathcal{X}_3 \) in term of the given dependencies. In fact, conditioning on \( \mathcal{Y} \),

\[
P[\mathcal{X}_1 \leq x_1, \mathcal{X}_3 \leq x_3] = \int P[\mathcal{X}_1 \leq x_1, \mathcal{X}_3 \leq x_3 \mid \mathcal{Y} = y] dF_Y(y).
\]

With no assumption on suitability, this can be rewritten

\[
P[\mathcal{X}_1 \leq x_1, \mathcal{X}_3 \leq x_3] = 
\int P[\mathcal{X}_1 \leq x_1 \mid \mathcal{Y} = y, \mathcal{X}_3 \leq x_3] P[\mathcal{X}_3 \leq x_3 \mid \mathcal{Y} = y] dF_Y(y).
\]

Using suitability (8.2), the right hand side can be expressed in terms of the dependencies between \( \mathcal{X}_1, \mathcal{Y} \) (i.e. between \( \mathcal{X}_1, \mathcal{X}_2 \)) and between \( \mathcal{Y}, \mathcal{X}_3 \), and suitability is necessary for this to hold.

Further, an expression for the copula \( C_{\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3} \) between \( \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3 \) can be derived as follows (see Section 8.2.2 for more on copulas). Using the definition of the copula \( C_{\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3} \) and conditioning on \( \mathcal{Y} \),

\[
C_{\mathcal{X}_1,\mathcal{X}_2,\mathcal{X}_3}(u_1, u_2, u_3) = \int_0^1 P \left[ X_1 \leq F_{\mathcal{X}_1}^{-1}(u_1), X_2 \leq F_{\mathcal{X}_2}^{-1}(u_2), X_3 \leq F_{\mathcal{X}_3}^{-1}(u_3) \mid \mathcal{Y} = F_{\mathcal{Y}}^{-1}(v) \right] dv. \tag{8.3}
\]

Since \( \mathcal{Y} = \mathcal{X}_1 + \mathcal{X}_2 \), we must have above that \( v \) is no larger than

\[
v_* \equiv v_*(u_1, u_2) := F_{\mathcal{Y}}(F_{\mathcal{X}_1}^{-1}(u_1) + F_{\mathcal{X}_2}^{-1}(u_2)), \tag{8.4}
\]

and provided this holds, the expression in the integral in (8.3) can be written

\[
P \left[ F_{\mathcal{Y}}^{-1}(v) - F_{\mathcal{X}_2}^{-1}(u_2) < X_1 \leq F_{\mathcal{X}_1}^{-1}(u_1), X_3 \leq F_{\mathcal{X}_3}^{-1}(u_3) \mid \mathcal{Y} = F_{\mathcal{Y}}^{-1}(v) \right].
\]
To this probability, suitability (8.1) can be applied, and using that, e.g.,
\[ P[X_3 \leq F_{X_3}^{-1}(u_3) \mid Y = F_{Y}^{-1}(v)] = \partial_1 C_{Y,X_3}(v, u_3), \]
we finally get for the copula
\[ C_{X_1,X_2,X_3}(u_1, u_2, u_3) = \int_{v^*}^{u_3} \left( \partial_2 C_{X_1,Y}(u_1, v) - \partial_2 C_{X_1,Y}(F^{-1}_X(v) - F^{-1}_X(u_2)), v \right) \cdot \partial_1 C_{Y,X_3}(v, u_3) dv, \]
with \( v^* = v^*(u_1, u_2) \) as defined in (8.4). The expression in distribution functions and their inverse above evaluated is equal to 0 at \( v = 0 \), and equal to \( u_1 \) at \( v^* \).

Concerning the second point, the top-down allocation process, the requirement expressed in terms of copulas is that, for a portfolio dependency tree with baskets \( X \preceq Y \preceq S \), the copula \( C_{X,S} \) can be calculated from the copulas \( C_{X,Y} \) and \( C_{Y,S} \); in fact,
\[ C_{X,S} = C_{X,Y} \circ C_{Y,S}, \]
where the composition of two copulas \( C_1, C_2 \) is defined as
\[ (C_1 \circ C_2)(u, w) := \int_0^1 \partial_2 C_1(u, v) \partial_1 C_2(v, w) dv. \]
To show this, note that suitability implies that the random variables \( X \) and \( S \) are independent given \( Y \). The claim then follows by conditioning on \( Y \) in the equality
\[ C_{X,S}(u, w) = P[X \leq F^{-1}_X(u), Z \leq F^{-1}_S(w)], \]
and using that, e.g.,
\[ P[X \leq F^{-1}_X(u) \mid Y = F^{-1}_Y(v)] = \partial_2 C_{X,Y}(u, v). \]

Because of the requirement of suitability, a portfolio decomposition needs to be selected with great care to ensure that we indeed get a portfolio dependency tree.

### 8.2 Modeling Dependencies

In this section, we first introduce the difference between explicit and implicit dependency models. We give a short introduction into copulas and
present different scalar measures of dependency measures including linear correlation, and sketch some of the well-known fallacies related to linear correlation. Next, we present the important family of Archimedean copulas, and its member the Clayton copula, which we use as a default assumption in our models, see for instance in Section 3.3.2 (pp. 59) for the default new business model.

Most of the contents of this section follow McNeil et al. [2005] and Nelsen [1999], where the proofs of most statements not given below can be found.

8.2.1 Explicit and Implicit Dependency Models

To begin with, we distinguish two different types of dependency models for random variables: explicit and implicit models. For implicit dependency models, dependency between random variables is expressed by specifying the dependency structure of the random variables directly, for instance by a correlation matrix or, preferably, by a copula between the random variables. More details about copulas are provided in Section 8.2.2 below.

Explicit dependency models, on the other hand, are causal in the sense that the underlying risk factors that are giving rise to dependencies are available as random variables, and the dependency between the given random variables is expressed through these risk factors. That is, given the risk factors, the random variables are independent. The risk factors might again depend on each other in an implicit fashion, or they might be independent. The model is explicit because the causes of the dependencies between the given random variables are modeled.

As an illustration, two random variables $X$ and $Y$ might depend explicitly on each other through a common random variable $\theta$ in the sense that

\[ \text{given } \theta, \ X \text{ and } Y \text{ are independent.} \]

Explicit dependency models are used, for instance, in the modeling of natural catastrophe contracts. More information about this can be found in this document in Section 3.8 (pp. 80). The basic approach is the following: A natural catastrophe event is drawn stochastically out of an event catalog according to the event’s probability (e.g. an earthquake in a certain region, a certain tropical cyclone, etc.). The drawn event comes with its relevant specifications (ideally, geographical distribution of the “severity” of the event over time). The covered risks are specified by their geographical location (typically constant over time), their value, and their susceptibility to damage caused by an event as a function of the event’s severity. The latter is usually measured by so-called vulnerability functions and expresses loss or damage degree, i.e. loss as a percentage of value. The relationship between event intensity and loss degree in our models is usually stochastic.

The previous relationships provide the pre-insurance, or ground-up loss to the risk from the event. Applying the insurance structure to the pre-
insurance losses gives the insured losses, and applying the reinsurance structure then gives the reinsured loss per contract and event.

In order to create an explicit dependency model for reinsurance contracts, we thus typically need

1. A multivariate distribution of the risk factors including dependencies (for instance, a collection of scenarios, that is, common samples of the risk factors with sample probability)

2. A collection of covered risks (“exposure information”),

3. A function (either deterministic or stochastic) assigning a pre-insurance loss amount to a risk and a sample of risk factors,

4. The relevant insurance and reinsurance structures (as functions of the pre-insurance and insured loss, respectively).

In this case, dependencies between reinsurance contracts can come from several sources (where below, dependencies mentioned early have an impact also on the later ones):

- Dependencies between risk factors
- Dependencies between pre-insurance losses to different risks due to their dependency on the same risk factors
- Dependencies between insured losses to different insurance policies due to their covering the same risks
- Dependencies between reinsured losses to different reinsurance contracts due to their covering the same insurance policies.

In our non-life liability model, we use both implicit and explicit dependency models. In case no explicit dependency model is available, we currently use as default model an implicit model expressing dependency by means of a multidimensional, one-parametric Clayton copula. Arguments for this choice are given below in Section 8.2.4.

### 8.2.2 Copulas

In this section, we introduce the concept of copulas as a general approach to capture dependencies between random variables, and list some of the main properties\(^2\).

\(^2\)The Clayton copula, which is the Archimedean copula we currently use to model implicit dependencies (see 3.3.2, pp. 59) will be introduced in Section 8.2.4.
A natural approach to assess implicit dependency between random variables is by means of rank vectors: Draw common samples from the random variables to get a collection of sample vectors. Assign each entry in a vector its rank among the corresponding entries of the other sample vectors (e.g. the largest sample of the first random variable, the second largest sample of the third random variable, etc.) to obtain a collection of rank vectors. For instance, one rank vector might consist of the second largest sample of the first random variable together with the tenth largest sample of the second random variable, etc. Plotting these rank vectors in a hypercube, we get an example of a discrete copula. The density of points in a particular region of the cube expresses dependency with respect to this region (for instance, whether large values of one random variable go together with large values of another random variable).

More formally, a \textit{d-dimensional copula}

$$C : [0,1]^d \to [0,1]$$

is defined to be a distribution function on \([0,1]^d\) with standard uniform marginal distributions (margins).

To a copula \(C\) belongs a probability measure \(\mu = \mu_C\) on \([0,1]^d\) such that

$$C(u_1, \ldots, u_d) = \mu([0,u_1] \times \cdots \times [0,u_d]).$$

Explicitly, a function \(C : [0,1]^d \to [0,1]\) is a copula if and only if it satisfies the following three criteria:

1. \(C(u_1, \ldots, u_d)\) is increasing in each component \(u_k\)
2. \(C(1, \ldots, 1, u_k, 1, \ldots, 1) = u_k\) for any \(k\)
3. The rectangle inequality holds: If \(\mu\) is the probability measure belonging to \(C\), then the measure of any rectangle in \([0,1]^d\) has to be non-negative. This condition can be expressed in terms of the function \(C\), which we do not explicitly give here.

For multivariate distributions, copulas are very useful because they allow to separate the dependency structure from the behavior of the univariate margins. This is summarized in the following fundamental theorem of Sklar.

\textbf{Theorem 8.3} (Sklar’s theorem): Let \(F\) be a distribution function with margins \(F_1 \ldots F_d\). Then, there exists a copula \(C : [0,1]^d \to [0,1]\) such that the function \(F\) can be written as

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)) \text{ for every } -\infty \leq x_k \leq \infty. \quad (8.5)$$

The copula \(C\) is unique if the margins are continuous. Conversely, \((8.5)\) defines a multivariate distribution function if \(F_1 \ldots F_d\) are univariate distribution functions and \(C\) is a copula.
Moreover, the copula $C$ is given by

$$C(u_1, \ldots, u_d) = F(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)) \quad \text{for every } 0 \leq u_k \leq 1,$$

(8.6)

where the generalized inverse $G^-$ of a distribution function $G$ is defined by

$$G^-(v) := \inf\{y \mid G(y) \geq v\}.$$

(8.7)

Note that the copula is in general not unique if the margins are not continuous. For instance, if $F$ has margins $F_1$ and $F_2$, where $F_k$ has an atom at $x_k$ of weight $d_k > 0$ for $k = 1, 2$ (i.e. $F_k$ has a jump of size $d_k$ at $x_k$), then the copula is not unique on the rectangle

$$[F_1(x_1), F_1(x_1) + d_1] \times [F_2(x_2), F_2(x_2) + d_2],$$

and can, moreover, be arbitrarily chosen on the rectangle, provided that the projections to the sides of the rectangle are appropriately uniformly distributed.

A crucial property of copulas is their invariance under strictly increasing transformations of the margins: If the random vector $(X_1, \ldots, X_d)$ has continuous margins, and if $T_1, \ldots, T_d$ are strictly increasing functions, then the random vectors $(X_1, \ldots, X_d)$ and $(T_1(X_1), \ldots, T_d(X_d))$ have the same copula (given by (8.6)).

This statement is intuitively clear if we think of a copula as a collection of rank vectors as sketched above: Since each of the transformations $T_k$ is strictly increasing, the ranks of the samples of the random vectors $(X_1, \ldots, X_d)$ and $(T_1(X_1), \ldots, T_d(X_d))$ are identical.

The copula corresponding to independent random variables is the independence copula $\Pi$ given by

$$\Pi(u_1, \ldots, u_d) = \prod_{k=1}^{d} u_k.$$

(8.8)

Every $d$-dimensional copula $C$ satisfies the following so-called Frechet-Hoeffding bounds

$$\max\left\{ \sum_{k=1}^{d} u_k + 1 - d, 0 \right\} \leq C(u_1, \ldots, u_d) \leq \min\{u_1, \ldots, u_d\}.$$

(8.9)

These bounds have interpretations in terms of dependency structures: The upper bound on the right hand side above is assumed for the co-monotonicity copula $M$ and represents the dependency structure of perfectly dependent random variables. For $d = 2$, the lower bound above is assumed by the counter-monotonicity copula $W$, representing perfect negative dependency; for $d > 2$, there is no copula assuming the lower bound.
Define the *concordance order* $\prec$ for copulas $C_1$ and $C_2$ by

$$C_1 \prec C_2 \text{ if } C_1(u_1, \ldots, u_d) \leq C_2(u_1, \ldots, u_d) \text{ for all } 0 \leq u_k \leq 1.$$  \hspace{1cm} (8.10)

Then, the bounds (8.9) state that, for any 2-dimensional copula $C$,

$$W \prec C \prec M.$$  

In the final part of this section we collect properties of copulas which are desirable for our default dependency model. These properties put restrictions on the possible types of copulas that should be considered for such a default dependency model.

In our default basket model, we need to aggregate large numbers of contracts in one basket. To obtain a reasonably simple dependency model, it makes sense to assume that, for the dependency structure, the contracts are exchangeable in the sense that it does not matter which contract is considered to be the first one, which one the second one etc. (Of course, exchangeability can be seen as a drawback, because the dependency structure does not distinguish between components of the random vector. In our portfolio dependency tree, however, this problem can be circumvented by creating more baskets.)

Moreover, we would like not to have to recalculate the parameters for the dependency structure when a new contract is added, and to ensure that we are consistent with the structure before adding the additional contract. These two requirements on the copula are reflected in the following two definitions.

A copula is called *exchangeable* if it is the copula of an *exchangeable* random vector $X$, where the latter means that, for any permutation $\pi$,

$$X \equiv (X_1, \ldots, X_d) \text{ is distributed like } (X_{\pi(1)}, \ldots, X_{\pi(d)}),$$

so for the copula $C$,

$$C(u_1, \ldots, u_d) = C(u_{\pi(1)}, \ldots, u_{\pi(d)}). \hspace{1cm} (8.11)$$

A bivariate copula $C$ is called *associative* if, for any $u_1, u_2, u_3 \in [0,1]$,

$$C(u_1, C(u_2, u_3)) = C(C(u_1, u_2), u_3). \hspace{1cm} (8.12)$$

Associativity is closely related to Archimedean copulas, see Section 8.2.4. Archimedean copulas are, moreover, exchangeable.

For solvency considerations, we are primarily interested in evaluating extreme occurrences, so we should focus on dependencies between extreme events, that is, on the part of the copula describing such dependencies. Let’s assume that extreme events correspond to large negative values of the random variables. Then, for a bivariate random variable $(X_1, X_2)$ with
continuous margins $F_1$ and $F_2$ and exchangeable copula $C$, define the lower threshold copula $C_0^\nu$ for $0 < \nu \leq 1$ as the copula describing the dependency of $X_1$ and $X_2$ conditional on their both being below their $\nu$-quantile. (The upper threshold copula is defined analogously.) That is,

$$
C_0^\nu(u_1, u_2) = \frac{C(F_1^\nu(u_1), F_2^\nu(u_2))}{C(\nu, \nu)} \quad \text{for } u_1, u_2 \in [0, 1],
$$

where $G^-$ denotes the generalized inverse of $G$ as defined in (8.7), and $F(\nu)$ is the conditional distribution

$$
F(\nu)(u) = \mathbb{P}[X_1 \leq F_1^\nu(u) \mid A_\nu] = \frac{C(u, \nu)}{C(\nu, \nu)} \quad \text{for } 0 \leq u \leq \nu
$$

conditional on the event $A_\nu$ of both $X_1$ and $X_2$ being below their $\nu$-quantile,

$$
A_\nu = \{X_1 \leq F_1^\nu(\nu), X_2 \leq F_2^\nu(\nu)\}.
$$

It is then of interest to consider the limit of the copula $C_0^\nu$ as $\nu \to 0$. Such a limit copula $C$ is called limiting lower threshold copula, and needs to possess the stability property that, for any $0 < \nu \leq 1$,

$$
C_0^\nu(u_1, u_2) = C(u_1, u_2).
$$

In particular, the Clayton copula used in the default model is such a limiting lower threshold copula, and it is an important attractor for a large class of underlying exchangeable copulas.

Lastly, it is convenient to work with a parameterized family of copulas. For parameterized copulas, we would like to be able to parameterize the full range of dependency structures as expressed by the Frechet-Hoeffding bounds (8.9), so the copula family should interpolate from the counter-W to the co-monotonicity copula $M$, passing through the independence copula $\Pi$. Such a family of copulas is called comprehensive. The Clayton copula family (see below in Section 8.2.4) is comprehensive. Since we rarely encounter negative dependency, it would generally be sufficient to interpolate between independence and co-monotonicity copula.

In case the copula family is one-parametric with parameter $\theta$, we would like the family of copulas to be positively (or negatively) ordered in the sense that, for $\theta_1 \leq \theta_2$ (or $\theta_1 \geq \theta_2$), we have $C_{\theta_1} \prec C_{\theta_2}$ for the concordance order defined in (8.10).

### 8.2.3 Linear Correlation and other Dependency Measures

In this section we recall the definition of Pearson linear correlation, rank correlation, and coefficients of tail dependency. The main objective here is to have a scalar measure to express the nature of the dependency as captured in the copula. Moreover, we recall limitations of these dependency measures.
measures and, in particular, the fallacies connected with linear correlation.

The well-known linear correlation between two random variables \( X, Y \) is defined as
\[
\rho(X, Y) := \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X) \text{var}(Y)}} \in [-1, 1],
\]
with
\[
\text{cov}(X, Y) = E[(X - E[X])(Y - E[Y])], \quad \text{var}(X) = \text{cov}(X, X).
\]

Linear correlation has the property that it is equal to 0 for independent random variables; the converse, however, is not true – a correlation of 0 does not imply independence. A simple, rather extreme example is given by a normally distributed \( X \) and \( Y := X^2 \): the correlation is zero although we clearly have a strong dependency.

A linear correlation of \( \pm 1 \) corresponds to perfect (positive or negative) linear dependency, meaning that \( Y = \alpha + \beta X \). Correlation is independent under strictly increasing linear transformations – but not under arbitrary strictly increasing non-linear transformations. The latter point is problematic because it implies (since copulas do have the full invariance property) that correlation cannot depend on the copula alone, but additionally also depends on the margins; consequently, correlation does not separate dependency structure from margins.

Another problem is that correlation is obviously only defined for random variables with finite variance, which might pose a problem for heavy-tailed distributions.

Next, we sketch three well-known fallacies related to linear correlation. They show that certain properties one would intuitively assume correlation to possess are in reality not fulfilled. The fallacies are discussed in more detail, including explicit counterexamples, in Embrechts et al. [1999] and in McNeil et al. [2005].

The first fallacy is that the joint distribution is uniquely determined given the (continuous) marginal distributions and the pairwise correlations (uniqueness). The main point here is that it is in general possible to construct different copulas giving rise to different joint distributions having the same correlation. What is more, the VaR of a sum of dependent random variables is not uniquely determined from the marginal distributions of these random variables and their correlation matrix. The third fallacy below presents an additional pitfall when working with correlation and VaR.

The second fallacy is that, for given marginal distributions, it possible to attain any correlation in \([-1, 1]\) for a corresponding joint distribution (existence). In fact, using (8.9), one can show that the set of attainable correlations form a closed interval in \([-1, 1]\) containing 0. Since a correlation of \( \pm 1 \) corresponds to perfect linear dependency as defined above, the interval is, with the exception of the latter case, not the full interval \([-1, 1]\).
The third fallacy pertains to correlation as well as to VaR. It states that the VaR of two risks is at its worst if the two risks have maximal correlation, i.e. if they are co-monotonic.

We focus now on scalar measures of dependency which only depend on the copula and not on the margins: the two rank correlations Kendall’s tau and Spearman’s rho, and the coefficients of (upper and lower) tail dependency. From a practical point of view, they can be used to calibrate copulas to empirical data.

The rank correlation Kendall’s tau $\rho_\tau$ is defined as

$$\rho_\tau(X,Y) := \mathbb{E}\left[\text{sign}\left((X - \tilde{X})(Y - \tilde{Y})\right)\right] = 2\mathbb{P}(X - \tilde{X})(Y - \tilde{Y}) > 0 - 1 = 4\int_0^1 \int_0^1 C(u, v) \, dC(u, v) - 1 \in [-1, 1],$$

where $(\tilde{X}, \tilde{Y})$ is an independent copy of $(X, Y)$, and the latter expression shows that Kendall’s tau indeed only depends on the copula.

The rank correlation Spearman’s rho $\rho_S$ is defined in terms of the linear correlation $\rho$ by

$$\rho_S(X,Y) := \rho(F_X(X), F_Y(Y)) = 12\int_0^1 \int_0^1 (C(u, v) - u v) \, du \, dv \in [-1, 1].$$

The second fallacy above no longer applies to the two types of rank correlation as opposed to linear correlation. Clearly, the first fallacy remains relevant, which would be expected since one scalar should not be sufficient to fully specify a dependency structure.

Lastly, we consider the coefficients of tail dependency. These provide a measure of extreme dependency: Given that one random variable takes an extreme value, what is the probability that the other random variable also takes an extreme value?

Indeed, for two random variables $X$ and $Y$, the coefficient of lower tail dependency $\lambda_l$, and the coefficient of upper tail dependency $\lambda_u$ are defined as the limits

$$\lambda_l := \lim_{q \to 0^+} \mathbb{P}[X \leq F_X(q) \mid Y \leq F_Y(q)] = \lim_{q \to 0^+} \frac{C(q, q)}{q},$$

$$\lambda_u := \lim_{q \to 1^-} \mathbb{P}[X > F_X(q) \mid Y > F_Y(q)] = \lim_{q \to 0^+} \frac{\hat{C}(q, q)}{q},$$

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provided that the respective limits \( \lambda_l, \lambda_u \in [0, 1] \) exist, where the expressions in terms of the copula hold for continuous margins. In the expression for the upper tail dependency, \( \hat{C} \) denotes the survival copula

\[
\hat{C}(u, v) = u + v - 1 + C(1 - u, 1 - v).
\]

Two random variables are said to show upper (lower) tail dependency if \( \lambda_u > 0 \) (\( \lambda_l > 0 \)); otherwise, they are said to be asymptotically independent in the upper (lower) tail.

While tail dependency expresses dependency in the limit, it might make more sense for practical purposes to study tail dependency at a certain quantile. We would then address questions such as: Given that one random variable is worse than its 1%-quantile, what is the probability that the other random variable is also worse than its 1%-quantile?

To this end, we define the shifted (lower) tail dependency at the \( q \)-quantile \( TD(q) \) for \( q \in [0, 1] \) by

\[
TD(q) := P[X \leq F_X^{-1}(q) | Y \leq F_Y^{-1}(q)] - P[X \leq F_X^{-1}(q)] = \frac{C(q,q)}{q} - q,
\]

where we shift the quotient to have \( TD(q) = 0 \) for independent random variables.

It is helpful, for a given copula, to study the function

\[
h: (0, 1] \rightarrow [0, 1], \ q \mapsto \frac{C(q,q)}{q} = P[X \leq F_X^{-1}(q) | Y \leq F_Y^{-1}(q)].
\]

For \( q = 1 \), \( h \) takes the value \( h(1) = 1 \). As \( q \) moves to 0, \( h(q) \) converges to the coefficient of lower tail dependency \( \lambda_l \), provided the corresponding limit exists. Plotting the function \( h \) from empirical data might thus give an indication of whether a certain dependency structure exhibits lower tail dependency, and if so, what value the tail dependency might take. Also, studying the graph of \( h \) might give an indication of what kind of copula should be used to model a certain dependency structure.

### 8.2.4 Archimedean Copulas, Clayton Copula

In this section we introduce the Clayton copula, which is the copula we currently use to model implicit dependencies, see for instance Section 3.3.2 (pp. 59). The Clayton copula belongs to the class of Archimedean copulas. This latter class has a number of desirable properties, and we will highlight some of them.

In particular, to model dependencies in our portfolio dependency tree, we need higher dimensional copulas. However, constructing copulas of dimensions \( d > 2 \) is not easy, and might even be one of the main open problems concerning copulas. Recall to this end the three required properties (8.2.2) a
function $C: [0, 1]^d \to [0, 1]$ has to fulfill in order to be a copula (in particular, the third property).

A naive approach to construct, e.g., three-dimensional copulas would be to couple two two-dimensional copulas $C_1$ and $C_2$ by defining

$$C(u_1, u_2, u_3) := C_2(C_1(u_1, u_2), u_3). \quad (8.14)$$

Unfortunately, this procedure fails in general (see Nelsen [1999]). For instance, for the special copulas $W$, $\Pi$, and $M$, the formula (8.14) defines a copula in the following cases:

- If $C_1 = M$, then $C_2$ can be any copula, and if $C_2 = M$, then $C_1$ has to be $M$.
- If $C_1 = W$, then $C_2$ has to be $\Pi$, and if $C_2 = W$, then $C_1$ has to be $M$.
- If $C_2 = \Pi$, then $C_1$ can be any copula.

These results already indicate that formula (8.14) will not most of the time produce a copula. However, in the class of Archimedean copulas, the procedure often succeeds.

One hint to construct Archimedean copulas of arbitrary dimensions comes from rewriting the independency copula $\Pi$ from (8.8) as follows

$$\Pi(u_1, \ldots, u_d) = \prod_{k=1}^d u_k = \exp \left( -\sum_{k=1}^d (-\log(u_k)) \right).$$

Following this, let $\phi: [0, 1] \to [0, \infty]$ be a continuous, strictly decreasing, convex function with $\phi(1) = 0$. We call $\phi$ an Archimedean copula generator, and strict if $\phi(0) = \infty$. The pseudo-inverse $\phi^{-1}(t)$ at $t \in [0, \infty]$ is defined to be the usual inverse $\phi^{-1}(t)$ for $0 \leq t \leq \phi(0)$ and 0 otherwise. The idea is then to define a d-dimensional copula $C^d$ by

$$C^d(u_1, \ldots, u_d) := \phi^{-1}\left( \sum_{k=1}^d \phi(u_k) \right).$$

For dimension $d = 2$, this construction always succeeds. For higher dimensions $d > 2$, it succeeds for strict generators $\phi$ whose inverse $f \equiv \phi^{-1}$ is completely monotonic, that is,

$$(-1)^k \frac{d^k}{dt^k} f(t) \geq 0 \text{ for any } k = 0, 1, 2 \ldots$$

Any bivariate Archimedean copula $C \equiv C^2$ satisfies the following algebraic properties
1. $C$ is symmetric, and thus also exchangeable as in (8.11). I.e., for any $u_1, u_2 \in [0, 1]$, 
\[ C(u_1, u_2) = C(u_2, u_1) \]

2. $C$ is associative, as defined in (8.12)

3. for any $u \in [0, 1]$,
\[ C(u, u) < u. \] (8.15)

Moreover, the latter two properties characterize Archimedean copulas: Any bivariate copula which is associative and satisfies (8.15) is Archimedean.

Regarding property (8.15), note that $C(u, u) \leq u$ always, and that $C(u, u) = 0$ implies 
$\mathbb{P}[X \leq F_X(u) \mid Y \leq F_Y(u)] = 1$.

Recalling the proposed coupling construction (8.14), we have, for Archimedean copulas with the same generator $\phi$,
\[ C^d(u_1, \ldots, u_d) = C^2(C^{d-1}(u_1, \ldots, u_{d-1}), u_d). \] (8.16)

Consequently, higher dimensional Archimedean copulas can be derived from their bivariate counterparts by successive coupling.

An important family of Archimedean copulas are LT-Archimedean copulas, whose generators are inverses of the Laplace-Stieltjes transform of a distribution function $G$ on $[0, \infty)$ satisfying $G(0) = 0$; that is,
\[ \phi^{-1}(t) = \int_0^\infty e^{-tx} dG(x). \] (8.17)

An appealing aspect of LT-Archimedean copulas is the fact that they have an underlying explicit dependency model. Let $V > 0$ be a random variable. Denote its distribution function by $G$, and let $\phi$ be defined by (8.17). If the random variables $U_1, \ldots, U_d$ on $[0, 1]$ are independent given $V$ with conditional distributions given $V$ given by
\[ \mathbb{P}[U_k \leq u \mid V = v] = \exp(-v \phi(u)) = (\exp(-\phi(u)))^v, \]
then the distribution function of the random vector $(U_1, \ldots, U_d)$ is the LT-Archimedean copula with generator $\phi$.

In particular, the dependency between the random variables $U_1, \ldots, U_d$ is explicitly given by the random variable $V$. Moreover, the above immediately leads a procedure to sample from LT-Archimedean copulas by Monte Carlo methods.
The \textit{d-dimensional Clayton copula} is an LT-Archimedean copula with one parameter $\theta$ given by
\[ C^{Cl}(u_1, \ldots, u_d) := \left( u_1^{-\theta} + \cdots + u_d^{-\theta} \right)^{-1/\theta}. \]

For $d = 2$, the parameter $\theta$ can take any value in $[-1, \infty) \setminus \{0\}$ with $\theta = -1$ giving $W$, $\theta \to 0$ equal to $\Pi$, and $\theta \to \infty$ giving $M$. For $d > 2$, we must have $\theta > 0$.

The bivariate Clayton copula has no upper tail dependency but lower tail dependency for $\theta > 0$ with coefficient of lower tail dependency equal to
\[ \lambda_l = 2^{-1/\theta}. \]

The generator $\phi$ of the Clayton copula is
\[ \phi(t) = \frac{1}{\theta} \left( t^{-\theta} - 1 \right), \]
and the Clayton copula is an LT-Archimedean copula for a Gamma distributed random variable $V$ with
\[ V \sim Ga(1/\theta, 1), \]
where the Gamma distribution $Ga(\alpha, \beta)$ for $\alpha, \beta > 0$ has density
\[ f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x). \]

We use the Clayton copula consistently as a default assumption in our internal model and in the capital allocation used for pricing to capture dependency between the children of a basket in the portfolio dependency tree.

In the following we summarize the desirable properties of the Clayton copula family:

- It can be defined for arbitrary dimensions $d$.
- It is an LT-Archimedean copula, so there exists an underlying explicit dependency model, and it is easy to draw samples for Monte Carlo simulations.
- It is exchangeable and associative.
- Because of the property (8.16), if a random vector has a $d$-dimensional Clayton copula with parameter $\theta$, the \textit{sub-random vector} composed of any two entries of the random vector is bivariate Clayton with the same parameter $\theta$. 

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• It is upper semi-comprehensive in that it allows to interpolate between the independency copula Π (in the limit \( \theta \to 0^+ \)) and the co-monotonicity copula \( M \) (for \( \theta \to \infty \)).

• It is positively ordered\(^3\), i.e. for \( \theta_1 \leq \theta_2 \), we have \( C_{\theta_1} \leq C_{\theta_2} \).

• It has lower tail dependency, which corresponds to the observation that, in insurance and reinsurance, random variables are often almost independent in normal situation but dependent in extreme situations (i.e. if it goes bad, then all contracts go bad).

• It is a natural choice to express dependency in extreme situations, since it is a limiting lower threshold copula: For a large class of copulas, the part of the copula expressing dependency between extremes converges to the Clayton copula (as shown by Juri and Wüthrich [2002]).

8.2.5 Explicit Dependency between Economic Variables and Liabilities

The stochastic liability model as explained in Section 3.2 offers full distribution functions of aggregated claims. For some lines of business these distributions include economic risk factors. An example is Credit & Surety where the aggregated claim size is partially driven by the credit cycle, in addition to idiosyncratic risk factors arising from individual treaties and insured companies or securities. The stochastic liability model normally covers all these risk factors, but there may be no technical way to separate the influence of an economic variable from that of other risk factors.

Therefore economic risk factors are reintroduced in the internal model in the form of an explicit dependency. Let the original loss be a stochastic variable \( X \) from a distribution which originates from the liability model with mean \( \bar{X} \). In normal cases the distribution of \( X \) has a finite variance \( \sigma^2_X \).

The economic risk factor \( Y \) usually comes from the Economic Scenario Generator of the internal model. It describes the impact of the economic risk factor on the specific liability with loss \( X \) and may be a function of an economic variable rather than the variable itself. This function should be chosen so that \( Y \) models the actual impact of the economic variable. If only extreme values of the variable have an impact, for example, we choose a function with \( Y \approx 0 \) for all average values of the economic variable and \( |Y| \gg 0 \) only for extreme variable values. In this example we shall observe a tail dependency between the economic risk factor and the loss corrected by \( Y \).

\(^3\)This is proved in Nelsen [1999] for \( d = 2 \) and follows for higher dimensions \( d > 2 \) from (8.16) and the \( d = 2 \)-case and component-wise monotonicity of copulas.
The two stochastic variables \( X \) and \( Y \) are independent as they originate from two entirely unrelated models. Mean \( \bar{Y} \) and variance \( \sigma_Y^2 \) of \( Y \) are known as they can easily be computed by the model simulation tool\(^4\). The impact of \( Y \) modifies the original loss \( X \) and leads to the corrected loss \( Z \) in a stochastic simulation. We always can and should define \( Y \) in a way that its impact on \( Z \) is positive. This means that increases in \( X \) and in \( Y \) affect the resulting corrected loss \( Z \) in the same positive direction.

For modeling the dependency we distinguish three cases with different values of the additionality parameter \( \alpha \):

1. The distribution of losses \( X \) fully contains the impact of the risk factor \( Y \), so the only task is to make the hidden impact of \( Y \) on the losses explicit. The corrected loss has the same variance as the original one, \( \sigma_Z^2 = \sigma_X^2 \). This is the normal case. There is no additional risk, so \( \alpha = 0 \).

2. The distribution of losses \( X \) does not contain the impact of the risk factor \( Y \), possibly because that risk factor has been arbitrarily omitted in the calculation method of \( X \). We mark the full additionality of the new risk factor by setting \( \alpha = 1 \). This is not the case in our current liability modeling methodology, but here we offer a way to cover the case of an additional risk factor, too.

3. The distribution of loss \( X \) partially contains the impact of the risk factor \( Y \). This case can be treated by choosing a parameter \( 0 < \alpha < 1 \). As in case 2, the corrected loss has a larger variance than the original one, \( \sigma_Z^2 > \sigma_X^2 \).

In all the three cases the mean loss stays constant,

\[
\bar{Z} = \bar{X}
\]  

(8.18)

The dependency method never modifies the original loss expectation.

The following explicit dependency model defines the corrected stochastic loss \( Z \) of a simulation scenario as a linear combination of loss \( X \) and risk factor \( Y \):

\[
Z = \bar{X} + \beta (X - \bar{X}) + \gamma (Y - \bar{Y})
\]  

(8.19)

This definition immediately satisfies Equation (8.18), whatever the values of the parameters \( \beta \) and \( \gamma \). Both \( \beta \) and \( \gamma \) are positive or zero (in the case of \( \gamma \) because we defined \( Y \) accordingly). The variance of \( Z \) is

\[
\sigma_Z^2 = \beta^2 \sigma_X^2 + \gamma^2 \sigma_Y^2
\]  

(8.20)

\(^4\)Currently the stochastic simulations are written in *Igloo*. For an overview of IT systems and technologies, we refer the reader Part VIII in this documentation.
using the independence of \( X \) and \( Y \). Using this independence again, we can compute the linear correlation coefficient \( \rho \) between the risk factor \( Y \) and the corrected loss \( Z \):

\[
\rho = \frac{\gamma \sigma_Y}{\sigma_Z} = \frac{\gamma \sigma_Y}{\sqrt{\beta^2 \sigma_X^2 + \gamma^2 \sigma_Y^2}}
\]

In practice we choose \( \rho \) as the dependency parameter and use it to determine the parameter values \( \beta \) and \( \gamma \). Notice that \( \rho^2 \) is the ratio between the variance of the correction term \( \gamma(Y - \bar{Y}) \) and the variance of the corrected loss \( Z \). Thus the new risk factor controls a share of \( \rho^2 \) of the risk whereas the original insurance risk of \( X \) keeps a share of \( 1 - \rho^2 \). In case of a considerable correlation of, say, \( \rho = 0.3 \), the new risk factor controls only 9\% of the risk, and 91\% of the original risk variance are maintained after the correction.

In typical applications, \( \rho \) tends to be small as \( Y \) is just a minor economic risk factor modifying a dominating insurance risk. Since we defined \( Y \) with a correct sign, \( \rho \) is confined between 0 (no impact of \( Y \)) and 1 (where \( Y \) converges to being the sole risk driver).

The parameters \( \beta \) and \( \gamma \) of Equation (8.19) can now be determined as functions of two more intuitive parameters, namely the additionality \( \alpha \) and the correlation \( \rho \):

\[
\beta = \sqrt{1 - \rho^2 (1 - \alpha)} \quad (8.21)
\]

and

\[
\gamma = \frac{\beta \sigma_X}{\sigma_Y} \frac{\rho}{\sqrt{1 - \rho^2}} \quad (8.22)
\]

Reinsertion of \( \beta \) and \( \gamma \) in Equation (8.20) shows that the definitions lead to a variance with the desired properties:

\[
\sigma_Z^2 = \sigma_X^2 \frac{1 - \rho^2 (1 - \alpha)}{1 - \rho^2}
\]

In particular, the corrected variance \( \sigma_Z^2 \) equals the original one, \( \sigma_X^2 \), in the frequently used case \( \alpha = 0 \).

Let us summarize the procedure for obtaining a dependency model of stochastic liabilities on an economic risk factor.

1. Confirm or correct the assumption \( \alpha = 0 \). (The modeled risk factor does not add any new risk, it just explains part of the already known variance.) Assess the correlation \( \rho \) between the economic risk factor and the total loss variance. Both parameter estimates, \( \alpha \) and \( \rho \), rely on the (possibly model-based) input of experienced actuaries and other experts.

2. Use Equations (8.21) and (8.22) to compute the model parameters \( \beta \) and \( \gamma \).
3. Use Equation (8.19) to compute the corrected loss \( Z \) in each trial of the stochastic simulation.

This dependency model offers a simple and efficient algorithm in the framework of a stochastic simulation. It avoids the introduction of indirect methods such as copulas and thus has the advantage of never creating circular or contradictory dependency conditions in complex models.

On the other hand there are possible weaknesses: (a) the additive model of Equation (8.19) may cause negative losses in some cases, (b) the distribution of \( Z \) may differ from that of \( X \) even if they have the same mean and variance and (c) the original dependency of \( X \) on other liability types is diluted by a factor \( 1 - \rho^2 \). All these problems become negligible as long as the impact of the risk factor is small, i.e. \( \rho^2 \ll 1 \). As soon as the economic risk factors start to dominate, we are using different modeling techniques, one example being our model for Guaranteed Minimum Death Benefit (GMDB) liabilities.

An important example for applying the dependency model of this section is the modeling of Credit & Surety losses. The credit level as explained in Part III takes the role of the risk factor \( Y \) in that case.

### 8.3 Pricing Capital Allocation

We describe the calculation of the **sufficient premium** for an insurance or reinsurance contract or policy implemented in the pricing tool. More precisely, we describe the **risk loading**, which is based on capital (cost) allocation, where the risk of the whole portfolio is measured by a spectral risk measure, which is currently the **expected shortfall**.

The allocated capital is calculated\(^5\) by general principles equivalent to the **Euler principle**. Dependencies within the portfolio are modeled by means of copulas in a hierarchical dependency structure we call the **portfolio dependency tree**. We derive expressions for the allocated capital in terms of copulas. The impact of **diversification** is expressed by the transformation of an initial risk function specifying the selected spectral risk measure into transformed risk functions specifying the risk contribution of a sub-portfolio of the whole portfolio.

Further, the **temporality** of risk capital (the fact that risk capital is tied down by a policy until there is no longer uncertainty about the outcome) is considered by means of a time factor measuring the **duration** of the policy.

In this way, the individual risk of a policy, the diversification with the whole portfolio, and the duration of the policy are taken into account in the calculation of the sufficient premium.

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\(^5\)For more details on the IT tools for pricing, dependency modeling and capital allocation, we refer to Part VIII, pp. 408.
8.3.1 Introduction

It is well known that the required premium for an insurance or reinsurance policy or contract has to be larger than the sum of expected loss and expected external and internal (i.e. administrative) expenses. The additional amount can be considered as a risk loading, and to calculate it, actuaries have traditionally used so called premium principles, such as expected loss principle or standard deviation principle.

For ease of notation, in the following, we take policy to mean either policy or contract.

Intuitively, when comparing the risk loading for different policies, the risk loading should have the following desirable properties: The risk loading should be larger

- The higher the risk (downside potential) of the policy
- The longer there is uncertainty about the final outcome
- The less the risk diversifies the overall portfolio of the (re)insurer

The most simple of the classical premium principles is the expected loss principle, where the risk loading is taken to be proportional to the expected loss of the policy. The problem with this principle is, of course, that none of the above desirable properties are satisfied. In particular, the risk (or volatility) of the policy is not taken into account. Other premium principles such as the standard deviation principle or the exponential principle do take the risk into account, but usually do not account for diversification. In addition, the standard deviation principle charges not just for downside deviations but also for upside deviations and, moreover, is not well suited to reinsurance loss distributions which are often heavy tailed.

To overcome these shortcomings, one approach is to account for risk and diversification in the above principles by the proportionality factor. That is, the portfolio is split into sub-portfolios, so that, within each sub-portfolio, the selected premium principle provides a reasonable measure. The resulting loading is then multiplied by a sub-portfolio specific proportionality factor. For instance, the sub-portfolios could be defined by line of business and type of policy (e.g. for reinsurance treaties we could split into proportional and non-proportional treaties).

Obviously, this approach – in theory – allows to account for all three desirable properties above. It applies, more generally, to factor based models. For instance, the risk loading could be calculated as the sum of a factor applied to the premium plus another factor applied to the reserves, where the factors are specified by line of business and type of policy.

However, the approach has numerous drawbacks. To start with, it is highly questionable that the required split into sub-portfolios is possible. As an example, for sub-portfolios composed of all policies of a certain line of
business and type of business, the expected loss principle will hardly be an adequate measure of the risk of all policies in a given sub-portfolio, so a much finer split would be needed. The problem of accounting for the desirable properties is then transferred to the determination of the proportionality factors. Thus, a method is required to determine these factors correctly – how would such a quantitative method look like? This then raises the question of consistency: how can we ensure comparability of the risk loading for policies from different sub-portfolios? This is certainly desirable if we want to have a true measure of expected profitability of policies, in order to be able to optimize the overall net result of the portfolio.

More generally, in order to have a generic quantitative method to measure profitability which guarantees comparability, it seems reasonable to require that the method to calculate the risk loading rely as much as possible only on the stochastic properties of the net cash flow of the policy over time, and not on additional features of the policy.

We now formulate the idea of risk loading in a more economic way: Equity capital provided by the shareholders of the (re)insurance company is exposed to the risk that the actual result might be worse than the expected result, and for this reason investors expect a minimal annual spread above the risk free return on their investment.

This excess return gives rise to an overall annual capital cost amount $cc(Z)$ to be earned by the whole portfolio $Z$ of the (re)insurer. Given a risk $X$ (a policy or a sub-portfolio) in the portfolio $Z$, the task of capital (cost) allocation is to determine the fraction of the overall capital costs that has to be earned by the risk $X$. In the case of a policy $X$ this fraction has to be earned by the required premium of the policy in addition to the sum of expected loss and expected expenses.

Assume that the risk of the whole portfolio $Z$ is measured by a risk measure $\rho(Z) \in \mathbb{R}$. Denote by $K(X, Z)$ the contribution of the risk $X$ to the overall risk $\rho(Z)$. Then it seems reasonable to allocate the capital costs proportionally to the risk contribution, i.e. the capital costs $cc(X)$ allocated to $X$ are given by

$$cc(X) = cc(Z) \cdot \frac{K(X, Z)}{\rho(Z)}.$$

In particular, if we identify the risk $\rho(Z)$ with the capital amount supporting the portfolio $Z$ on which the required return $\eta$ has to be earned, i.e. if $cc(Z) = \eta \cdot \rho(Z)$, we get

$$cc(X) = \eta \cdot K(X, Z),$$

that is, the capital costs to be earned by $X$ are given by the required return $\eta$ applied to the risk contribution.

We denote by temporality of risk capital the fact that the total capital for a policy $X$ has to be set aside at inception of the policy and can
be gradually reduced as more information reduces the uncertainty about
the final outcome of the policy. However, some fraction of the capital is tied
down until the ultimate outcome of the policy is known.

In order to capture temporality of risk capital we need a model for the
reduction of uncertainty over time. We choose as a proxy the incurred (or
reported) pattern, which measures the fraction of ultimate losses that is
incurred, i.e. known, up to some point in time. From this, we calculate a
time factor $\tau_X$ for a policy $X$, which measures the duration of the policy.
Incorporating temporality of risk capital, the capital costs to be earned by
$X$ are given by

$$\tilde{cc}(X) = \eta \cdot \tau_X \cdot K(X, Z).$$

In our model, we use as risk measure the expected shortfall at a given
quantile, applied to the net present value of the result

$$\text{premium} - \text{losses} - \text{expenses},$$

where future cash flows are discounted using the risk free yield curve for the
_corresponding_ currency, and the respective maturity of the cash flows.

In other words, the risk measure is given by the average of the discounted
worst outcomes, weighted with the probabilities of each of these outcomes.
The selected quantile for the expected shortfall expresses the risk tolerance
of the company.

Note that we deliberately apply the risk measure to the _net present
data of the result_ as a random variable and not to the losses. This
is important for instance for reinsurance contracts, where the premiums
charged and the expenses paid to the reinsured often are loss sensitive:
Higher losses lead to higher premiums and/or lower expenses. Examples of
this are sliding scale and profit commissions, reinstatements etc. This loss
sensitivity of premiums and expenses leads to a reduction of the volatility of
the result of the contract, and would not be taken into account if we would
only consider the losses. In addition, the loss sensitivity of premiums and
losses is often non-linear (e.g. for reinstatements).

Given the risk measure expected shortfall, we use the Euler principle
to calculate the risk contribution of a risk to the whole portfolio. That is,
$K(X, Z)$ is defined as the directional derivative of the risk measure $\rho$ at the
whole portfolio $Z$ in the direction of the risk $X$.

There are various justifications for calculating the risk contribution by
the Euler principle. Tasche [2000] argues by means of portfolio steering/
performance measurement: The risk contribution should be calculated in
such a way that it rewards policies with a positive contribution to the overall
result, and punishes policies with a negative contribution. Denault [2001]
uses cooperative game theory: The allocation of the overall capital costs
to the policies has to be _fair_, which means that no _coalition_, i.e. _no_ sub-
portfolio of policies, would be _better off_ on their own (i.e. the allocation is
in the core of the cooperative game).
Further approaches to capital allocation include Kalkbrener [2005], where an axiomatic approach to capital allocation is used, formulating desirable properties. Another very general approach to capital allocation using different methods is found in Denneberg and Maass [2006].

Note that the capital allocation methodology takes the risk of a policy into account as well as the dependency between the outcome of the policy and the outcome of the whole portfolio. Thus, diversification is taken into account.

For this reason, the method requires that we model the net present value of the result of the whole portfolio and its dependency with the net present value of each policy. To do this we need a method to aggregate risk in the portfolio taking into account the dependencies between the risks. This aggregation of the portfolio is done in Phobos. It is also used to model the new business in the liability part for the SST and described in the corresponding SST documentation.

8.3.2 Preliminaries

To begin with, we fix some notation. We denote by $Z$ the portfolio under consideration. The letters $S, X, Y$ are used to describe sub-portfolios of $Z$. Usually, $X$ refers to policies, and $Y$ to portfolios of lines of businesses. Note that these variables stand for financial instruments. Monetary amounts corresponding to these financial instruments are in general random variables. In particular, we denote by $Z, S, X,$ and $Y$ the random variables of the corresponding net present values of the result

$$\text{premium} - \text{losses} - \text{expenses}$$

The present values of future cash flows are always discounted using the risk free yield curves of the corresponding currency, and the respective maturity of the cash flows. The random variables are defined on a suitable probability space with probability measure $P$. By definition, bad outcomes correspond to large negative amounts.

For a random variable $X$ we denote by $F_X$ its distribution function

$$F_X : \mathbb{R} \to [0, 1], \quad x \mapsto P[X \leq x],$$

and by $F_X^{-1} : [0, 1] \to \mathbb{R}$ the “inverse” $q \mapsto \inf \{ x \in \mathbb{R} \mid F_X(x) \geq q \}$. For simplicity, we assume, unless stated otherwise, that the random variables under consideration are continuous and strictly increasing.

In this article we are not considering how to model losses and expenses, and for this reason we always assume that the random variable losses-expenses is given for any risk we consider. In other words, we assume implicitly that we have a stochastic model for losses and expenses. However, we deal with the question of how to set the premium for a risk $X$ in the portfolio $Z$, or, more precisely, the premium as a function of losses and expenses.
8.3.3 Risk Measure, Capital Allocation, Euler Principle, Risk Function

In this section we describe capital allocation in the axiomatic framework of Kalkbrener [2005] and the extension of this approach to spectral measures of risk due to Overbeck [2004]. Very similar results, using a different approach, have been derived by Denneberg and Maass [2006]. As a special case of spectral risk measures, we consider the expected shortfall.

For spectral risk measures there does not seem to exist a systematic investigation of when the Euler principle holds, i.e. when the capital allocation is given by the directional derivative of the risk measure. Below, we give a result for a special case which should suffice for practical applications.

Finally, we express the formula for capital allocation in terms of the copula between the whole portfolio and a risk in the portfolio. This permits a representation of capital allocation in terms of the risk function.

To start with, we recall the well-known definition of coherent risk measure (see for instance Artzner et al. [1999]). A mapping \( \rho \) assigning a real number \( \rho(X) \in \mathbb{R} \) to a random variable \( X \) is called a coherent measure of risk if the following conditions are satisfied

- **positive homogeneity:** \( \rho(aX) = a\rho(X) \) for \( a \geq 0 \) (8.23)
- **sub-additivity:** \( \rho(X + Y) \leq \rho(X) + \rho(Y) \) (8.24)
- **monotonicity:** \( \rho(X) \geq \rho(Y) \) if \( X \leq Y \) (8.25)
- **translation invariance:** \( \rho(X + a) = \rho(X) - a \) for \( a \in \mathbb{R} \) (8.26)

An example of a coherent risk measure is the expected shortfall at safety level \( \alpha \),

\[
\rho(X) = ES_\alpha(X) = -\frac{1}{\alpha} \int_0^\alpha F_X^{-1}(u) \, du,
\]

where the random variable \( X \) describes, e.g., the net present value of the result of a financial instrument, so bad outcomes correspond to large negative values of \( X \), and a high risk is a large positive number \( \rho(X) \). Unlike the expected shortfall, the value at risk, \( VaR_\alpha(X) = -F_X^{-1}(\alpha) \), is not a coherent risk measure, since it is in general not sub-additive, so it does not always account for diversification.

The property (8.23) of positive homogeneity has been criticized since, for instance, for large values of \( a \), liquidity risk may arise. As an alternative, Föllmer and Schied [2002] define convex measures of risk by replacing in the above the properties (8.23) and (8.24) by

- **convexity:** \( \rho(aX + (1 - a)Y) \leq a\rho(X) + (1 - a)\rho(Y) \) for \( 0 \leq a \leq 1 \).

Using the notion of risk measures we can now describe capital allocation. Consider the company’s portfolio \( \mathcal{Z} \), and decompose it into disjoint risks
\((X_i)_{i=1,\ldots,m}\), so for the net present values,
\[
\sum X_i = Z.
\]

The portfolio \(Z\) is supported by some capital \(K = \rho(Z)\), where \(\rho\) is a given risk measure. A minimum annual spread \(\eta\) over the risk free interest rate has to be generated on \(K\). The problem of determining the premium for \(X\) is reduced to the problem of allocating the capital \(K\) to the risks \(X_i\).

In this framework, the task of capital allocation is to find risk contributions \(K(X_i, Z)\) such that
\[
\sum K(X_i, Z) = \rho(Z).
\]

We then say that the amount of capital \(K(X_i, Z)\) is allocated to the risk \(X_i\).

In the following we assume that the capital allocated to a risk \(X_i\) depends only on this risk and not on the decomposition of the rest
\[
Z - X_i = \sum_{k \neq i} X_k.
\]

Consequently, a capital allocation with respect to a risk measure \(\rho\) can be considered as a map
\[
K : V \times V \to \mathbb{R}, \quad \text{with } K(X, X) = \rho(X) \quad \forall X \in V,
\]
where \(V\) is some linear subspace of the space of equivalence classes of real valued random variables on the selected probability space.

Kalkbrener [2005] shows that, given a risk measure \(\rho : V \to \mathbb{R}\) which is positively homogeneous and sub-additive, i.e.
\[
\rho(aX) = a\rho(X) \quad \forall a \geq 0, X \in V,
\]
\[
\rho(X + Y) \leq \rho(X) + \rho(Y) \quad \forall X, Y \in V,
\]
there exists a capital allocation \(K : V \times V \to \mathbb{R}\) which is linear and diversifying, i.e.
\[
K(aX + bY, Z) = aK(X, Z) + bK(Y, Z) \quad \forall a, b \in \mathbb{R}, X, Y, Z \in V,
\]
\[
K(X, Y) \leq K(X, X) = \rho(X) \quad \forall X, Y \in V.
\]

Conversely, if a linear and diversifying capital allocation exists with respect to \(\rho\), then \(\rho\) is positively homogeneous and sub-additive.

Moreover, if a linear and diversifying capital allocation \(K\) for \(\rho\) is continuous at some point \(Z \in V\), i.e.
\[
\lim_{s \to 0} K(X, Z + sX) = K(X, Z) \quad \forall X \in V,
\]
\[
(8.27)
\]
then $K(\cdot, Z)$ is given by the directional derivative

$$K(X, Z) = \lim_{s \to 0} \frac{\rho(Z + sX) - \rho(Z)}{s} \quad \forall X \in V.$$  

In other words, the risk contribution $K(X, Z)$ is given by the Euler principle.

Kalkbrener (Kalkbrener [2005]) proves these results by considering, for given $Z \in V$, sub-tangents $g_Z \in V^*$, where $V^*$ denotes the dual space of linear functions from $V$ into $\mathbb{R}$, satisfying

$$g_Z(Z) = \rho(Z), \quad g_Z(X) \leq \rho(X) \quad \forall X \in V.$$  

Existence of the sub-tangents can be proved using the Hahn-Banach theorem, or using theorems on sub-gradients of convex functions. Provided these sub-tangents $g_Z$ exist for any $Z \in V$, the capital allocation $K(X, Z)$ is defined as

$$K(X, Z) := g_Z(X) \quad \forall X \in V. \quad (8.28)$$

As shown by Overbeck [2004], the above results of Kalkbrener (Kalkbrener [2005]) can be applied to so called spectral risk measures. We formulate this approach in a way which will be useful for us later on.

First, consider the special case where the risk measure is given by the expected shortfall,

$$\rho(Z) = \text{ES}_\alpha(Z) = -\frac{1}{\alpha} \int_0^\alpha F^{-1}_Z(u) \, du$$

$$= -\int_0^1 F^{-1}_Z(u) h_0^{(\alpha)}(u) \, du \quad \text{(8.29)}$$

$$= -\mathbb{E}[Z \cdot h_0^{(\alpha)}(F_Z(Z))], \quad \text{(8.30)}$$

where

$$h_0^{(\alpha)}(u) = \frac{1}{\alpha} 1_{\{u \leq \alpha\}} \quad \text{(8.31)}$$

with $1_A$ denoting the indicator function of the set $A$.

For continuous random variables, $\rho(Z) = \text{ES}_\alpha(Z) = -\mathbb{E}[Z \mid Z \leq F_Z^{-1}(\alpha)]$.

According to Kalkbrener (Kalkbrener [2005]), the capital allocation $K(X, Z) = K_\alpha(X, Z)$ is given by (8.28) with

$$K_\alpha(X, Z) := g_Z(X) = -\mathbb{E}[X \cdot h_0^{(\alpha)}(F_Z(Z))] = -\mathbb{E}[X \mid Z \leq F_Z^{-1}(\alpha)], \quad (8.32)$$

where the last equality again holds for continuous random variables only.

For the extension of the approach to spectral risk measures, the idea is to replace the function $h_0^\alpha$ in (8.31) by a more general function $h_0$. To this end, let

$$h_0 : [0, 1] \to \mathbb{R}$$
be a bounded, monotonously decreasing function. Define the associated risk measure $\rho$ by

$$\rho(Z) := -\int_0^1 F^{-1}_Z(u) h_0(u) \, du. \quad (8.33)$$

The interpretation is that such a risk measure is given by a weighted mean, where the $u$th quantile of $Z$ is weighted by $h_0(u)$. In the special case of the expected shortfall, the weight function $h_0(u)$ is given by (8.31), which means that all $u$th quantiles with $u \leq \alpha$ are weighted with $1/\alpha$, while all other quantiles get weight zero.

We can rewrite (8.33), using

$$h_0(u) = h_0(1) - \int_0^1 \beta h_0^{(\beta)}(u) \, dh_0(\beta) \quad (8.34)$$

and (8.29), and changing the order of integration, to get

$$\rho(Z) = -h_0(1) E[Z] - \int_0^1 \beta E S_\beta(Z) \, dh_0(\beta). \quad (8.35)$$

Now define the candidate $K(X, Z)$ for the capital allocation by

$$K(X, Z) := -E[X \cdot h_0(F_Z(Z))]. \quad (8.36)$$

Using (8.34) and (8.32), and switching expected value and integration, (8.36) can be rewritten as

$$K(X, Z) = -h_0(1) E[X] - \int_0^1 \beta K_\beta(X, Z) \, dh_0(\beta). \quad (8.37)$$

Since $h_0$ is decreasing, $-dh_0(\beta) \geq 0$. Thus, from (8.35) and (8.37), using the corresponding properties of the expected shortfall and expected value, it then follows that $\rho$ is a positively homogeneous, sub-additive risk measure, and that $K$ defines a linear, diversifying capital allocation for $\rho$. Note that $h_0$ decreasing is needed to ensure sub-additivity of the risk measure and the diversification property of the capital allocation.

The remaining more difficult part is to show that the capital allocation $K(X, Z)$ is continuous at $Z \in V$ in the sense of (8.27), which implies by Kalkbrener’s results (Kalkbrener [2005]) mentioned above that $K(X, Z)$ is given by the Euler principle, i.e. by the directional derivative of the risk measure,

$$K(X, Z) = \lim_{s \to 0} \frac{\rho(Z + sX) - \rho(Z)}{s}. \quad (8.38)$$

In view of (8.36), it has to be proved that

$$\lim_{s \to 0} E[X \cdot (h_0(F_{Z+sX}(Z + sX)) - h_0(F_Z(Z)))] = 0. \quad (8.38)$$
The main technical problem here seems to be that the random variable $F_{Z+sX}(Z+sX)$ might have an atom at a point where $h_0$ is discontinuous. Equation (8.38) has been checked under the following (non-necessary) assumptions:

**Assumption 8.4** *(Capital allocation by the Euler principle)*

1. The random variable $Z$ is continuous and strictly increasing.
2. The random variable $X$ is continuous with existing first and second moment.
3. The function $h_0: [0, 1] \to \mathbb{R}$ is decreasing, bounded, and continuous in all but finitely many points. In the discontinuity points, both left and right limits exist.

As a next step, we express the formula (8.36) using the copula $C = C_{X,Z}$ between $X$ and $Z$,

$$ C: [0, 1] \times [0, 1] \to [0, 1], \quad F_{X,Z}(x, z) = C(F_X(x), F_Z(z)), $$

where $F_{X,Z}$ is the distribution function of $(X, Z)$, and $F_X$, $F_Z$ are the marginal distributions of $X$, $Z$, respectively.

To this end, define $H_0$ to be the integral of $h_0$,

$$ H_0(u) := \int_0^u h_0(t) \, dt \text{ for } u \in [0, 1], $$

and assume additionally that

$$ H_0(1) = 1. $$

Using the copula $C = C_{X,Z}$ we get from (8.36) with the definition of the expected value,

$$ K(X, Z) = -\int_{\mathbb{R}^2} x h_0(F_Z(z)) \partial_x \partial_z C(F_X(x), F_Z(z)) \, dx \, dz. \quad (8.39) $$

Integrating in (8.39) first with respect to $x$ and collecting terms not depending on $z$, we get

$$ K(X, Z) = -\int_{\mathbb{R}} x \partial_z (H \circ F_X)(x) \, dx, \quad (8.40) $$

with

$$ H(u) := \int_{\mathbb{R}} \partial_z C(u, F_Z(z)) \, h_0(F_Z(z)) \, dz. \quad (8.41) $$

Substituting $F_Z(z)$ in (8.41) by $v$,

$$ H(u) = H_{X,Z}(u) = \int_0^1 \partial_z C_{X,Z}(u, v) \, dH_0(v). \quad (8.42) $$

Formulas (8.40) and (8.42) can be found in Theorem 5.2 in Denneberg and Maass [2006].
Definition 8.5 The composition $C \ast H$ of a copula $C$ with a function $H : [0, 1] \to [0, 1]$ is defined by

$$(C \ast H)(u) := \int_0^1 \partial_2 C(u, v) \, dH(v). \quad (8.43)$$

Lemma 8.6 Let $C$ be a copula, and let $H : [0, 1] \to [0, 1]$ be a monotonously increasing function satisfying

$$H(0) = 0, \quad H(1) = 1. \quad (8.44)$$

Then, the composition $C \ast H$ is a monotonously increasing function

$$C \ast H : [0, 1] \to [0, 1] \quad \text{with} \quad (C \ast H)(0) = 0, \quad (C \ast H)(1) = 1.$$

Proof 8.7 (Proof.) Let $U, V$ be uniformly distributed random variables on $[0, 1]$ such that

$$C(u, v) = P[U \leq u, V \leq v].$$

Then, the function

$$u \in [0, 1] \mapsto \partial_2 C(u, v) = P[U \leq u \mid V = v] \in [0, 1]$$

is increasing with $\partial_2 C(0, v) = 0$ and $\partial_2 C(1, v) = 1$ for any $v \in [0, 1]$. Since $H$ is increasing, we have $dH(v) \geq 0$ for every $u \in [0, 1]$, and using (8.44), the claim follows.

Then, we arrive at the following expression for the capital allocation $K(X, Z)$ in terms of the copula between $X$ and $Z$,

$$K(X, Z) = -\int_{\mathbb{R}} x \, d(H_{X,Z} \circ F_X)(x), \quad (8.45)$$

with $H_{X,Z} = C_{X,Z} \ast H_0$. \quad (8.46)

Moreover, substituting $u$ in (8.33) by $z := F_Z^{-1}(u)$, we obtain for the associated risk measure

$$\rho(Z) = K(Z, Z) = -\int_{\mathbb{R}} z \, d(H_{Z,Z} \circ F_Z)(z), \quad (8.47)$$

for

$$H_{Z,Z} := H_0,$$

and the Euler principle holds

$$K(X, Z) = \lim_{s \to 0} \frac{\rho(Z + sX) - \rho(Z)}{s}.$$

As an interpretation of these formulas, consider the risk measure $\rho$ to be represented by the function $H_0$, which we can view as the initial risk
function. The impact of diversification, by considering a risk \( X \) as part of a whole portfolio \( Z \), leads to a transformation of the initial risk function \( H_0 \) by the copula \( C_{X,Z} \) into a transformed risk function \( H = C_{X,Z} \ast H_0 \).

Furthermore, the equation for the risk contribution if a risk \( X \) in a portfolio \( Z \) from (8.45) can be rewritten by a change of variables to give

\[
K(X, Z) = - \int_0^1 F_X^{-1}(u) h_{X,Z}(u) \, du, \tag{8.48}
\]

where \( h_{X,Z} \) denotes the derivative of \( H_{X,Z} \), assuming it exists. In words, the risk contribution is calculated by "weighting" the quantiles \( F_X^{-1}(u) \) with the derivative \( h_{X,Z} \) of the transformed risk function \( H_{X,Z} \).

**Definition 8.8** A function \( H : [0, 1] \to [0, 1] \) with \( H(0) = 0, \, H(1) = 1 \)

is called an admissible initial risk function if \( H \) is concave, increasing, and continuously differentiable in all but finitely many points, and in those points, both left and right derivatives exist.

Consequently, the preceding formulas hold for any initial risk function \( H_0 \) which is admissible.

In the case where the risk measure is given by the expected shortfall at safety level \( \alpha \), the derivative \( h_{0}^{(\alpha)} \) of the initial risk function \( H_{0}^{(\alpha)} \) is given by (8.31),

\[
h_{0}^{(\alpha)}(u) = \frac{1}{\alpha} 1_{\{u \leq \alpha\}},
\]

and the transformed risk function from (8.46) becomes

\[
H_{X,Z}^{(\alpha)}(u) = \left( C_{X,Z} \ast H_{0}^{(\alpha)} \right)(u) = \frac{1}{\alpha} C_{X,Z}(u, \alpha). \tag{8.49}
\]

In this special case, consider again formula (8.48)

\[
K(X, Z) = - \int_0^1 F_X^{-1}(u) h_{X,Z}^{(\alpha)}(u) \, du, \tag{8.50}
\]

where \( h_{X,Z}^{(\alpha)} \) denotes the derivative of \( H_{X,Z}^{(\alpha)} \), that is, from (8.49),

\[
h_{X,Z}^{(\alpha)}(u) = \frac{1}{\alpha} \partial_1 C_{X,Z}(u, \alpha) = \frac{1}{\alpha} \mathbb{P}[Z \leq F_Z^{-1}(\alpha) \mid X = F_X^{-1}(u)]. \tag{8.51}
\]

As an interpretation of these formulas, recall that the risk measure expected shortfall only considers those states \( \omega \in \Omega \) of the world in which the whole portfolio distribution \( Z \) is worse than or equal to its \( \alpha \)-quantile \( F_Z^{-1}(\alpha) \). Consider now the formula for the risk contribution \( K(X, Z) \) of one risk \( X \)
in the portfolio. According to (8.51), the weight of one particular quantile $F_X^{-1}(u)$ of $X$ in the risk contribution formula (8.50) is then given by the normalized probability of the shortfall condition $Z \leq F_Z^{-1}(\alpha)$ given that $X$ is equal to this quantile $F_X^{-1}(u)$.

In particular, if $X$ takes discrete values $x_k$, we can write

$$K(X, Z) = -\sum_k x_k \mathbb{P}[Z \leq F_Z^{-1}(\alpha) \mid X = x_k] \mathbb{P}[X = x_k],$$

so the weight of the value $x_k$ of $X$ in the risk contribution is given by its probability to occur multiplied by its “contribution” to those states of the world for which the whole portfolio is bad.

Similarly to formula (8.37), we can express the transformed risk function $H_{X,Z}$ for initial risk function $H_0$ with derivative $h_0$ in terms of the transformed risk functions $H_{X,Z}^{(\beta)}$ for the initial risk function $H_0^{(\beta)}$ of the expected shortfall. In fact, applying partial integration to (8.46), and using (8.49), we get

$$H_{X,Z}(u) = h_0(1) u - \int_0^1 \beta H_{X,Z}^{(\beta)}(u) dh_0(\beta)). \quad (8.52)$$

It follows from (8.52) for instance that the transformed risk function $H_{X,Z}$ is concave if every transformed risk function $H_{X,Z}^{(\beta)}(u)$ is concave.

8.3.4 Capital Allocation for a Portfolio Dependency Tree

As we describe in more detail in the documentation of the liability model, in order to capture the dependency structure of the whole portfolio $Z$, we decompose the portfolio into disjoint sub-portfolios, which are in turn further decomposed into sub-portfolios, etc. In this way we obtain a hierarchical decomposition of the portfolio, which we view as a tree with root $Z$ and call the portfolio dependency tree. This portfolio dependency tree is needed for aggregation of sub-portfolios in the decomposition and for allocating risk capital along the branches of the tree.

Note that sub-portfolios here can be true sub-portfolios in the sense of containing at least two policies, but can also be single policies. Sub-portfolios which are not single policies are also called baskets.

The goal is to express the formulas from the previous section in the framework of a portfolio dependency tree. In particular, given a sub-portfolio $\mathcal{Y}$ which is “in between” a policy $\mathcal{X}$ and the whole portfolio $Z$ (such as a line of business sub-portfolio), we would like to express the allocated capital $K(X, Z)$ in a way that separates the dependency between $\mathcal{X}$ and $\mathcal{Y}$ from the dependency between $\mathcal{Y}$ and $Z$. The resulting formula is (8.57) below, which we first prove for the risk measure expected shortfall and then generalize to spectral risk measures in the sense of the prior section.
In order to be able to decentralize the modeling of certain sub-portfolios and to guarantee a consistent capital allocation along the branches of the portfolio dependency tree, the portfolio decomposition has to be suitable. To define suitability we need the following definition.

**Definition 8.9** A sub-portfolio $X$ is a descendent of a sub-portfolio $Y$ in a given portfolio decomposition, denoted

$$X \preceq Y,$$  

if $X$ is a sub-portfolio of $Y$ in this decomposition. Moreover, a child is defined to be a descendent in the portfolio tree of the next lower level, and a parent is defined analogously.

**Definition 8.10** A portfolio dependency tree of $Z$ is defined to be a suitable portfolio decomposition, where a portfolio decomposition is called suitable if the following holds: Given sub-portfolios $X \preceq Y$ and a sub-portfolio $S$ which is not a descendent of $Y$, the random variables $X$ and $S$ are independent given $Y$, that is,

$$P[X \leq x, S \leq s \mid Y = y] = P[X \leq x \mid Y = y] \cdot P[S \leq s \mid Y = y] \quad \forall x, s, y \in \mathbb{R}.$$  

In the present context, given $X \preceq Y \preceq Z$, suitability implies that the random variables $X$ and $Z$ are independent given $Y$. Hence, conditioning on $Y$ in the equality

$$C_{X,Z}(u, w) = P[X \leq F_X^{-1}(u), Z \leq F_Z^{-1}(w)],$$

and using that, e.g.,

$$P[X \leq F_X^{-1}(u) \mid Y = F_Y^{-1}(v)] = \partial_2 C_{X,Y}(u, v),$$

we obtain, for a portfolio dependency tree and sub-portfolios $X \preceq Y \preceq Z$,

$$C_{X,Z} = C_{X,Y} * C_{Y,Z},$$  

(8.54)

where the composition of two copulas $C_1$, $C_2$ is defined as

$$(C_1 * C_2)(u, w) := \int_0^1 \partial_2 C_1(u, v) \partial_1 C_2(v, w) \, dv.$$  

One checks that the composition $*$ is associative in the sense that, for $H: [0, 1] \to [0, 1]$,

$$(C_1 * C_2) * H = C_1 * (C_2 * H).$$  

(8.55)

Moreover, the comonotonicity copula $C_M$ is the neutral element with respect to the composition $*$, in the sense that, for any copula $C$ and function $H$,

for $C_M(u, v) := \min(u, v)$, $C * C_M = C = C_M * C$, $C_M * H = H$.  

(8.56)
Now assume that $H_0$ is an arbitrary admissible initial risk function defining the associated risk measure according to (8.47), and denote its transformed risk functions by, e.g., $H_{X,Z}$.

Given sub-portfolios $X \preceq Y \preceq Z$ in a portfolio dependency tree, we claim that

$$H_{X,Z} = C_{X,Y} \ast H_{Y,Z}.$$  \hfill (8.57)

Here, $\preceq$ defines descendants according to definition (8.53).

In fact, (8.57) is a consequence of (8.46), (8.54), and (8.55), because

$$H_{X,Z} = C_{X,Z} \ast H_0 = (C_{X,Y} \ast C_{Y,Z}) \ast H_0 = C_{X,Y} \ast (C_{Y,Z} \ast H_0) = C_{X,Y} \ast H_{Y,Z}.$$  

Formula (8.57) is useful because, for the calculation of the allocated capital $K(X,Z)$, we can separate the dependency between $Y$ and $Z$ in the risk function $H_{Y,Z}$ from the dependency between $X$ and $Y$ in the copula $C_{X,Y}$. Moreover, we get a consistent step wise allocation of the risk capital along the dependency tree:

**Remark 8.11** Allocation of risk capital along the portfolio dependency tree.

1. Start with the initial risk function $H_0 = H_{Z,Z}$.

2. Use the copula $C_{Y,Z}$ between $Z$ and a child $Y$ of $Z$ to derive the transformed risk function $H_{Y,Z}$ for $Y$ according to (8.57),

$$H_{Y,Z} = C_{Y,Z} \ast H_{Z,Z}$$

3. Calculate the allocated capital $K(Y,Z)$ by (8.45),

$$K(Y,Z) = - \int_{\mathbb{R}} x d(H_{Y,Z} \circ F_Y)(x).$$

4. For a child $X$ of $Y$, use $H_{Y,Z}$ and the copula $C_{X,Y}$ to get $H_{X,Z}$ and thus $K(X,Z)$, etc.

So, to calculate the allocated capital $K(X,Z)$ for a sub-portfolio or single policy $X$, where $X \preceq Y \preceq Z$ for a portfolio dependency tree, we need

1. the risk function $H_{Y,Z}$

2. the copula $C_{X,Y}$ and

3. the distribution $F_X$ of the net present value of the result of $X$

Consider now the situation when we are pricing a policy $X$ in the portfolio $Z$. In order to apply the method above, the problem is that we need the copula $C_{X,Y}$ (or, alternatively, the risk function $H_{X,Z}$) for every policy $X$.
This might be feasible in case an explicit model for the dependency structure is given, but, in the general case, a simpler approximation might be desirable. The approach we propose is the following: Instead of the copula $C_{X,Y}$ we assume we are given the copula $C_{X_0,Y}$

between $Y$ and a very small policy $X_0$ not contained in $Y$, which we call the distinguished policy.

The above topic is related to what we call the volume effect: Since the policy $X$ is in the portfolio $Z$, the dependency in $H_{X,Z}$ is between $X$ and the portfolio $Z$ containing $X$. We can decide to write a bigger or smaller share on the policy $X$, and obviously, then, the size of $X$ has an impact on the dependency between $X$ and the portfolio $Z$. In other words, denoting $Z_s := Z + sX$, we want that

$$\frac{1}{s} K(sX, Z_s)$$

increases with $s > 0$.

To describe the proposed approach, let $X$ be a policy with parent bucket $Y \preceq Z$. Because of (8.57), $K(X, Z)$ can be calculated from $H_{Y,Z}$ and $C_{X,Y}$.

**Assumption 8.12** The size of a policy $X$ with $X \preceq Y \preceq Z$ has an impact only on the dependency between $X$ and $Y$, and not on the dependency between $Y$ and $Z$. (Thus, $H_{Y,Z}$ is fixed.)

Now note that, for random variables $S, T$ with copula $C_{S,T}$, we have

$$C_{S,T}(s, t) = \int_0^s \partial_1 C_{S,T}(r, t) \, dr = \int_0^s P[T \leq F_T^{-1}(t) \mid S = F_S^{-1}(r)] \, dr,$$

which implies, setting $S = X$ and $T = Y + X$, that

$$C_{X,Y+X}(u, v) = \int_0^u \partial_1 C_{X,Y}(r, F_Y(F_{Y+X}^{-1}(v) - F_X^{-1}(r))) \, dr.$$  

In this formula, the dependency between the policy $X$ and the portfolio including the policy has been removed from the copula on the right hand side. Moreover, for $s > 0$ arbitrarily small, $C_{X,Y} = C_{sX,Y}$. Hence, it seems reasonable to approximate

$$C_{X,Y} \approx C_{X_0,Y},$$

where $X_0$ denotes the distinguished policy, which is a very small policy “not contained” in $Y$. Therefore,

$$C_{X,Y+X}(u, v) \approx \int_0^u \partial_1 C_{X_0,Y}(r, F_Y(F_{Y+X}^{-1}(v) - F_X^{-1}(r))) \, dr. \quad (8.58)$$

To determine the copula $C_{X,Y+X}$ approximatively in an efficient way, we calibrate it by its tail dependency at some point $\beta \in (0, 1)$. 183
Definition 8.13  For a copula $C$, the tail dependency quantile $TdQ_\beta$ (for $\beta \in (0,1)$) is defined as

$$TdQ_\beta := \frac{C(\beta, \beta)}{\beta} - \beta. \quad (8.59)$$

Since $F_{Y+X}(\beta) \leq F_Y^{-1}(\beta)$, and using (8.58), we can estimate $C(\beta, \beta) = C_{X,Y+X}(\beta, \beta)$ in (8.59) by

$$C_{X,Y+X}(\beta, \beta) \leq \int_0^\beta \partial_1 C_{X_0,Y}(t, F_Y(F_Y^{-1}(\beta) - F_X^{-1}(r)) \ dr.$$

To calculate the tail dependency quantile numerically, given a decomposition $0 = u_0 < u_1 < \cdots < u_n = \beta$ of the interval $[0, \beta]$, we decompose the above integral into a sum of integrals

$$\int_{u_{j-1}}^{u_j} \partial_1 C_{X_0,Y}(t, F_Y(F_Y^{-1}(\beta) - F_X^{-1}(r)) \ dr,$$

which we approximate by upper and lower bounds

$$Z_j^\pm := \int_{u_{j-1}}^{u_j} \partial_1 C_{X_0,Y}(t, F_Y(F_Y^{-1}(\beta) - F_X^{-1}(u_j^\pm)) \ dr,$$

where $u_j^+ := u_{j-1}$ and $u_j^- := u_j$.

Formula (8.59) for $TdQ$ is then approximated by

$$TdQ \approx \frac{1}{2\beta} \sum_{j=1}^n (Z_j^+ + Z_j^-) - \beta.$$

This formula allows to determine a one-parametric copula approximating $C_{X,Y+X}$.

In conclusion, to calculate the capital $K(X, Z)$ allocated to a policy $X$ with $X \preceq Y \preceq Z$, taking into account the volume effect, we need

1. the risk function $H_{Y,Z}$
2. the copula $C_{X_0,Y}$ between $Y$ and the distinguished policy $X_0$
3. the distribution function $F_Y$ and
4. the distribution function $F_X$ of the net present value of the result of $X$. 

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8.3.5 Temporality of Capital, Capital Costs

Consider a policy \( X \) in the whole portfolio \( Z \) consisting of all policies written in a given underwriting year. For this policy we determine a measure for its “duration.” By duration, we do not mean here the time span until all cash flows related to the policy are paid. Instead, duration refers to the, in general, shorter time the policy is “under risk” in the sense that uncertainty remains about those cash flows. This is because, as soon as the precise amounts of future cash flows are known, no risk remains. Economically, duration corresponds to the time span capital has to be put aside for the respective policy.

More precisely, what we are interested in here is the sequence of risk capitals

\[ K_0, K_1, K_2, \ldots \]

that have to be set aside for the policy \( X \) in the years \( i = 0, 1, 2 \ldots \) after inception. We then have to guarantee that, on the level of expected values, the corresponding capital costs can be provided on the risk capitals \( K_i \) by the policy premium. Note that the required return on capital will be given.

A main issue here is what to do with excess premium, i.e. in case we get more premium than needed, how is this excess premium changing the sequence of risk capitals?

Recall that \( X \) denotes the net present value of the ultimate result

\[ \text{premium} - \text{losses} - \text{expenses}. \]

Since \( Z \) consists of all policies written in a given underwriting year, and because we measure risk by the expected shortfall at safety level \( \alpha \), we want to guarantee that, in the average worst \( \alpha \) percent of the outcomes of the whole underwriting year portfolio \( Z \), we are able to fulfill our obligations to the policyholders. This means that we have to set aside the “ultimate” capital amount

\[ \rho(Z) = \sum_{X \in Z} K(X, Z) \]

at the beginning of the underwriting year under consideration. As information about the outcome of the policies increases (and as losses are paid), uncertainty decreases, which means that we can free up a certain amount of capital. Some fraction of the capital remains tied down until there is no uncertainty left.

Consequently, we have for the sequence of risk capitals of the policy \( X \),

\[ K_0 = K(X, Z) \geq K_1 \geq K_2 \geq \ldots, \]

where the capital amount \( K_i \) is tied down by the policy \( X \) from \( t = i \) until \( t = i + 1 \). On this capital, the return \( \eta \) has to be provided at \( t = i + 1 \), and
consequently, the total capital costs to be earned by the policy $\mathcal{X}$ are given by

$$
\sum_{i \geq 0} \text{pv}_{(t=i+1\rightarrow t=0)} (\eta K_i),
$$

where $\text{pv}$ denotes present value taken using the risk free yield curve of the currency and the appropriate maturity.

We can measure the reduction of uncertainty over time by the knowledge pattern $(\zeta_i)_i$ with

$$
\zeta_i := \frac{K_i}{K_0}.
$$

The knowledge pattern satisfies

$$
1 = \zeta_0 \geq \zeta_1 \geq \zeta_2 \geq \ldots \quad \text{with} \quad \lim_{i \to \infty} \zeta_i = 0.
$$

A main question here, as mentioned before, is how to treat excess premium. That is, let $\pi > 0$ be an additional premium amount (for the same risk $\mathcal{X}$), which we suppose for simplicity is received in cash at $t = 0$. Then the risk contribution $K(X, Z)$ satisfies

$$
K(X + \pi, Z + \pi) = K(X, Z) - \pi,
$$

which suggests that the excess premium $\pi$ can be used to reduce the required ultimate capital. In view of Equation (8.60) for the capital costs, this further suggests that no "capital costs" need to be paid on the excess premium, unlike on the capital. While this holds for the year $i = 0$, as $\pi$ would not be considered part of the capital at $t = 0$, for the following years $i = 1, 2 \ldots$ this is not so clear since it might be argued that the excess premium becomes capital at $t = 1$ and thus has to contribute to the capital costs.

The question could be formulated as: Who owns the excess premium? We come back to this in the next section.

As a proxy for the reduced uncertainty at time $t = i$ we use the incurred (or reported) pattern of that policy. That is, we approximate the knowledge pattern by

$$
\zeta_i \approx \frac{\text{ultimate losses} - \text{incurred losses up to start of year } i}{\text{ultimate losses}} \quad \text{for } i = 0, 1, 2 \ldots
$$

8.3.6 Sufficient premium, Performance excess, RAC, TRAC

In this section we apply the preceding analysis to the pricing of an individual policy and sub-portfolios of policies. We define the sufficient premium for a given policy, and the performance excess for the policy, which is a monetary amount measuring the profitability of the policy. Further, we introduce normalized measures RAC and TRAC of the required capital for a policy.
By slight abuse of language, we will in the following mean by a policy $X$ either an individual policy or a sub-portfolio of policies in the whole portfolio $Z$.

As before, denote by $X$ the net present value of the result

\[ \text{premium} - \text{losses} - \text{expenses}. \]

Write $K_X = K(X, Z)$ for the capital allocated to $X$ in $Z$.

We call the premium for a policy $X$ sufficient if the expected value of the net present value of the result pays for the cost of capital allocated to $X$ from (8.60), in particular, considering temporality of capital.

More precisely, the premium for a policy $X$ is defined to be sufficient if

\[ E[X] \geq \sum_{i \geq 0} \text{pv}(t = i + 1 \rightarrow t = 0) \left( \eta K_i \right), \quad (8.61) \]

where $\text{pv}$ denotes present value taken using the risk free yield curve of the currency and the appropriate maturity, and the hurdle rate $\eta$ is the required annual return on capital (in the appropriate currency). The premium is called technical in the case of equality above.

We define the risk adjusted capital $RAC_X = RAC(X, Z)$ to be the capital $K_X$ allocated to a policy $X$ in case the premium is technical, so in such a case, (8.61) becomes

\[ E[X] = \sum_{i \geq 0} \text{pv}(t = i + 1 \rightarrow t = 0) \left( \eta RAC_i \right), \quad (8.62) \]

where $RAC_i$ denotes the required capital for year $i$ if the premium is technical, and $RAC_0 = RAC_X$.

The technical premium is the natural premium since it covers the expected losses and, on the level of expected values, allows to pay out the capital costs. In this sense, the technical premium can be said to be that part of the premium which without question belongs to the policy.

For this reason we define the knowledge pattern $(\zeta_i)_i$ with respect to the capital amounts required for the technical premium, so, for $i \geq 0$,

\[ \zeta_i := \frac{RAC_i}{RAC_0}. \]

The time factor $\tau_X$ as a measure of the “duration” of the policy is then defined by

\[ \tau_X := \sum_{i \geq 0} \text{pv}(t = i + 1 \rightarrow t = 0) \left( \zeta_i \cdot 1 \right), \]

where 1 is the unit in the currency of the policy. Using the time factor, we get at the technical premium from (8.62) that

\[ E[X] = \eta \tau_X RAC_X. \quad (8.63) \]
The time factor $\tau_{X_1+X_2}$ of the sum of two policies $X_1$, $X_2$ is given by the weighted sum

$$\tau_{X_1+X_2} = \frac{RAC_{X_1}}{RAC_{X_1} + RAC_{X_2}} \tau_{X_1} + \frac{RAC_{X_2}}{RAC_{X_1} + RAC_{X_2}} \tau_{X_2}.$$  \hspace{1cm} (8.64)

We now turn to the issue mentioned in the previous section on how to treat excess premium, i.e. an amount $\pi > 0$ of premium in excess of the technical premium. For simplicity we suppose the excess premium is received in cash at $t = 0$. The question is relevant when we want to measure the resulting excess profit, which is done in terms of the performance excess to be defined below.

As mentioned before, the risk contribution $K_X = K(X, Z)$ satisfies

$$K(X + \pi, Z + \pi) = K(X, Z) - \pi,$$

which suggests that the additional premium $\pi$ can be used to reduce the required capital, implying that no capital costs need to be paid on the excess premium.

The issue then boils down to the question: Who “owns” the excess premium, the shareholder or the policy? This question has an impact on the capital amounts $K_i$ on which the required return $\eta$ has to be earned. There are three possible approaches to this:

1. The full excess premium is considered capital from the end of year $i = 0$. Then, the capital amounts $K_i$ subject to capital costs become

$$K_0 = RAC_0 - \pi, \quad K_1 = RAC_1, \quad K_2 = RAC_2, \ldots$$

2. The full excess premium is never subject to capital costs. Thus, we get

$$K_0 = RAC_0 - \pi, \quad K_1 = RAC_1 - \pi, \quad K_2 = RAC_2 - \pi, \ldots$$

3. The portion of the excess premium not subject to capital costs is proportional to the knowledge pattern $\zeta_i$. Thus,

$$K_i = RAC_i - \zeta_i \pi = \zeta_i (RAC_X - \pi) = \zeta_i K_X,$$

where $K_X$ denotes the risk contribution at the premium given by the sum of the technical premium and the excess premium.

Our approach is the third option above\textsuperscript{6}.

\textsuperscript{6}A justification for this approach can be given as follows: The starting point is to suppose that the excess premium $\pi > 0$ is used to provide a higher return on capital $\bar{\eta}$ than the required return $\eta$. The required return $\eta RAC_i$ is provided disregarding the...
So if the premium is technical, an excess premium $\pi$ applied to (8.63) leads to

$$E[X] + \pi = \eta \tau_X K_X.$$  \hfill (8.66)

As a consequence, we can reformulate the definition (8.61): The premium for a policy $X$ is *sufficient* if

$$E[X] \geq \eta \tau_X K_X.$$  \hfill (8.67)

Notice that in (8.67) we deliberately do not write $E[X]/K_X \geq \eta \tau_X$; that is, we do not express profitability in terms of the return on the allocated capital. The latter is not equivalent to (8.67) since $K_X$ could be zero or negative.

Definition (8.67) of the sufficient premium can be expressed in terms of the expected value with regards to a *distorted probability measure*, providing a link to the work of Denneberg (Denneberg [1989]), and Wang (Wang [1996]). From a theoretical point of view, this has the nice feature that sufficiency is no longer expressed using a separation between the expected result and an additional “loading” quantity.

In fact, from (8.67) and (8.45), we find that the premium for $X$ in the portfolio $Z$ is sufficient if and only if

$$\int_{\mathbb{R}} x \, d(G_{X,Z}^0 \circ F_X)(x) \geq 0$$

for $G_{X,Z}^0(u) := u + \eta \tau_X H_{X,Z}(u)$. In order to get a probability measure $G$, we need $\lim_{x \to \infty} G(x) = 1$, which means that we have to divide above by $1 + \eta \tau_X$. This then leads the following “premium principle” for a risk $X$:

The premium for the risk $X$ is *sufficient* if and only if

$$\tilde{E}_{X,Z}[X] := \int_{\mathbb{R}} x \, dG_{X,Z}(x) \geq 0,$$

excess premium. Let $\pi_i$ be the part of the excess premium not subject to capital costs in year $i$, and assume that $\pi_0 = \pi$. Then $\pi_i$ allows to reduce the required capital in year $i$ to $RAC_i - \pi_i$, and so a higher return $\tilde{\eta}_i$ can be provided according to

$$\tilde{\eta}_i (RAC_i - \pi_i) = \eta RAC_i.$$  \hfill (8.65)

(This equation is valid for $\pi_i < RAC_i$.) We now impose that the higher return $\tilde{\eta}_i = \tilde{\eta}$ be the same for all years $i$. Setting $i = 0$ in (8.65), we find that the return $\tilde{\eta}$ is given by

$$\tilde{\eta} = \eta \frac{RAC_X}{RAC_X - \pi},$$

and using this information in (8.65) for $i \neq 0$, we find that, for any $i$,

$$\pi_i = \pi \frac{RAC_i}{RAC_0}.$$
where the probability measure $G_{X,Z}$ is defined by

$$G_{X,Z}(x) := \frac{F_X(x) + \eta \tau_X H_{X,Z}(F_X(x))}{1 + \eta \tau_X}.$$  

This corresponds to a quantile based premium principle; however, in the above set up, the distorted probabilities differ from policy to policy, and depend on the overall portfolio $Z$! This has to be expected if the premium principle is supposed to take into account diversification explicitly and not just the stand-alone risk.

The profitability of a policy $X$ is measured by the performance excess $PE = PE(X, Z) \in \mathbb{R}$, which is defined as the constant amount by which the net present value of the result can be reduced (positive $PE$) or has to be increased (negative $PE$) so that the adjusted premium becomes technical. From (8.66), it follows that the performance excess $PE$ solves the equation

$$E[X] - PE = \eta \tau_X (K_X + PE).$$  

Thus,

$$PE = \frac{E[X] - \eta \tau_X K_X}{1 + \eta \tau_X}.$$  

One checks that a given premium is sufficient if and only if the performance excess is non-negative.

The performance excess has the useful property of being additive: The performance excess of a portfolio of policies is equal to the sum of the performance excesses of the policies in the portfolio. This allows to assess in an easy way the profitability of a program composed of several contracts, or of a whole client relationship – some of the client’s policies might not be profitable, but others are, and summing up the performance excesses of each policy gives the performance excess of the client relationship. Additivity of the performance excess follows from (8.68) due to additivity of the expected value $E$ and the risk contribution $K_X$, and the definition of the time factor of the sum $X_1 + X_2$ from (8.64).

The risk adjusted capital $RAC_X = RAC(X, Z)$ was defined above, and can be written as

$$RAC_X = K_X + PE.$$  

The time and risk adjusted capital $TRAC_X = TRAC(X, Z)$ is defined by

$$TRAC_X := \tau_X \cdot RAC_X$$  

While the capital $K_X$ can become zero or negative, $RAC_X$ and $TRAC_X$ are always positive (unless $X$ is constant), so they are more suited to calculating return on capital than $K_X$. Note also that

$$E[X] \geq \eta \tau_X \cdot K_X \iff E[X] \geq \eta \tau_X \cdot RAC_X,$$

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which is a consequence of the fact that a given premium is sufficient if and only if the performance excess is non-negative.

Finally, $TRAC_X$ is related to the capital $RAC_3 = RAC_3(X, Z)$ (in the notation of the financial modeling team) given by

$$RAC_3 := \mathbb{E}[X] + K_X,$$

which results in the case of the risk measure expected shortfall if the shortfall is measured as the deviation from the expected net present value instead of the deviation from zero. In fact, we have

$$TRAC_X = \frac{\tau_X}{1 + \eta \tau_X} RAC_3.$$ 

This equation follows by adding $K_X + PE$ to both sides of Equation (8.68) and using the definition of the involved quantities.
II

Market Risk
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9

Introduction

9.1 Economic Risks in Reinsurance

In insurance risk models, liability risk is generally well examined and understood. Investment risk, however, is often neglected or simplified\(^1\). To be able to assess these risks and their interaction SCOR Switzerland’s financial modeling team has developed an economic scenario generator\(^2\) which makes predictions for a set of key risk factors in the most important economic zones. It provides realistic scenarios of possible future behaviors and consistent estimates of risks in terms of both volatility and dependency.

The fortune and the risk of a business venture depends on the future course of the economy. This is particularly true for the insurance and reinsurance industry. There is a strong demand for economic forecasts and scenarios that can be applied to planning and modeling. The main application of economic scenario generation is asset-liability management (ALM)\(^3\) in a reinsurance context which require models for all assets and liabilities of a firm and thus a comprehensive dynamic model for all economic variables that determine asset and liability values. ALM requires forecasting economic development. By analyzing the risk exposure of the company,

\(^1\)In this section we extensively quote from P. Blum, M. Dacorogna, U. Müller, Å. Wallin, Using the past to predict economic risks, Helix brochure, Converium, August 2004, also from M. Dacorogna, Å. Wallin, Economic risks in reinsurance, Helix brochure, Converium, November 2004, and from the abstract and the introduction of (Müller et al. [2004])

\(^2\)Economic scenario generation is a process step in the ALM and SST process. We refer to the process landscape documentation, Part VII, pp. 353. The ESG itself is an IT system. For an overview of this system we refer to Part VIII.

\(^3\)Central to the asset-liability management is the determination of the risk based capital (RBC) which is required to sustain a certain risk exposure. The capital intensity thereby determined per line of business guides insurance business planning. The ALM and SST processes are documented in Part VII. The ALM process optimizes the company’s overall risk return profile. It thus comes into play for the investment risk side also through strategic asset allocation. The ALM process dynamically steers the company’s asset and liability portfolio. The entire ALM rests on a correct identification of all relevant risk factors.
with an emphasis on the dynamics of invested assets, the company’s ALM process automatically provides the basis for the *Swiss Solvency Test* (SST). The SST demands from a company to assess the given risk exposure via the calculation of the *target capital* (TC).

Invested assets are obviously contingent on the evolution of key economic or financial indicators such as yield curves, stock indices and foreign exchange rates and these also affect the performance of a reinsurer’s liabilities. SCOR Switzerland’s economic scenario generator is a concept to model and simulate the world economy (see also Part VIII in this documentation). Within the ALM it enables to examine the various investment strategies and to predict their behavior. The large number of future economic scenarios finds its basis in the past known experience which allows to predict the expected performance and risks of assets by reproducing the *historically observed* relationships between economic variables. The ESG explicitly captures *joint risks*. The asset and liability risks are thus more accurately modeled, leading to more realistic investment strategies. The use of economic scenarios derived from historical market data combined with internal risk models gives a more cautious picture of the investment portfolio’s risks. The method has several advantages. The use of historical observations reduces the number of assumptions to be made. Any *simultaneous dependence*, such as between consumer price indices (CPI) and interest rates, are preserved.

Certainly the future may hold surprises which have not been observed in the past or are otherwise not part of the data sample. As a partial compensation for this lack of historical data the economic scenario generator introduces *tail corrections* which provide for an additional stochastic variability.

The methods behind the economic scenario generator (ESG) are documented in Müller, Blum, Wallin, *Bootstrapping the economy – a non parametric method of generating consistent future scenarios* (Müller et al. [2004]).

### 9.2 The Economic Scenario Generator

Scor Switzerland uses an *economic scenario generator*\(^4\) (ESG) to model the risks emanating from the financial markets. The stochastic model (Müller et al. [2004]) behind the ESG allows to simulate the future development of *interest rates* (IR), *gross domestic products* (GDP), *foreign-currency exchange rates* (FX), *consumer price indices* (CPI), *equity indices*, *hedge fund indices*, *real estate indices*, *mortgage based securities* (MBS) and the credit risk levels of entire currency zones (*credit cycles*). Historical time series of economic indicators are collected, analyzed, adjusted and finally processed. The historical *innovations* are arranged into the future time series of inno-

\(^4\)For IT systems in connection with the SST, see Part VIII in this documentation.
vations by means of statistical resampling. The notion of innovation emphasizes the unanticipated movements that occur over time. An innovation is the deviation of the value of a quantity from what was previously expected. The historical time series cover a time span of about ten years. For a typical economic variable the innovations are calculated quarterly, resulting in a sample size of 40. The sample of innovations is thus indexed by around 40 discrete time values. Due to the small size of this sample the model must provide for additional stochastic variation which it does in the form of one additional random variable, Pareto distributed, acting on each innovation as a weight factor.

Innovation events at different times are thought to occur independently. By contrast the innovations across the various variables which occur in parallel at a specified point in time epitomize the aspect of statistical dependence. The model preserves the dependencies between the variables. Arranging a scenario of future innovations means drawing from the sample of historical innovations repeatedly. The method is bootstrapping: resampling with replacement from the original sample. In each simulation step the ESG randomly selects among the forty available index times and thus specifies an entire vector of simultaneous innovations. These are concurrent innovations in all the variables under consideration. Simultaneous dependencies among the economic quantities are therefore respected by this method.

At each simulated time step the simulated value of a variable is obtained by adding up the simulated innovation with the market forecast. The bootstrapping step in simulations is done upon a transformation of the variables in order to attain unlimited additivity of innovations. Stochastic trends are removed beforehand. Mean reversion effects are understood as corrections to the market forecast. The autoregressive behavior of the volatility of innovations is modeled by a GARCH process. To make the simulation of extreme risks realistic the resampled innovations are being weighted with a random factor. This will impose the desired heavy tail behavior to the probability distributions of innovations, without changes to their variance.

Interest rates (IR) are modeled in terms of forward rates of IR future contracts. The model requires a non linear mapping and a soft bound on negative rates. The mapping incorporates assumptions on the volatility at different interest rate levels and on the mean reversion of interest rates which further constrain long-term development of interest rate volatility.

Inflation is modeled using consumer price indices. Quarterly inflation numbers are subject to a seasonal trend. The innovations are appropriately defined for deseasonalized and smoothed inflation data. The dependence between inflation and interest rates is in the form of an adjusted interest rate which is the difference between interest and inflation. Target inflation and target interest rates are proportional to the deviation of the adjusted interest rate from its mean. Mean reversion is introduced to the model when these targets enter the formulation of market forecasts.
The model of foreign exchange rates (FX) is essentially specified by an assumption on how market expectations will be affected by the mutual deviations of purchase power parities of different currency zones from each other. A simplified approach transforms the FX scenarios into a stochastic model of the company’s overall FX risk (Dacorogna [2007a]).

For equity indices and gross domestic products (GDPs) the market expectation heavily relies on the latest attainable value together with the average historical growth.

ESG simulations in the wider sense must be thought to comprise the generation of scenarios for variables not directly included in the economic scenario generator. Among these additional variables are option prices. Any statistical sample originally drawn by the ESG will deterministically translate into a scenario for the price of an option via an option pricing formula. The ESG thus extends to novel variables. Scenarios for structured notes (Iannuzzi [2007]) and European look-back put options (Erixon and Kalberer [2005]) demonstrate the adequacy of the method.

ESG scenarios generally isolate the market risks. Nevertheless they affect the study of the remaining components of risks which are interrelated. As an example the credit cycles of currency zones are modeled like economic variables in the ESG (Müller [2007b]). They express an overall level of credit risk inherent to the corporate bonds within a currency zone and thus constitute an essential aspect of credit risk (Müller [2007b]).

Asset models like those for equity and bond portfolios (Müller [2006a], Müller [2007a]) clearly depend on the stochastic generation of economic scenarios (ESG). In our model, there is a very clear separation between those modules which control the economic risks to which the assets are exposed (ESG), and those which mimic the policy of holding certain assets, like the bond portfolio management which is modeled separately (Müller [2007a]). On the side of liabilities, the economic scenario generator is no less essential: Yield curves, together with other variables, are needed in the valuation (discounted best estimate) of future cash flows, therefore these economic variables will inevitably appear in the liability model in the form of stochastic risk factors. A cash flow manager is then needed to account for the cash flows between assets and liabilities (Müller [2006c]).

The most prominent among the economic risks to which SCOR Switzerland is exposed is interest rate risk. The stochastic economic scenario generation therefore effectively translates to a stochastic model for the interest rate hedges (Iannuzzi [2007]) as these are composed of mainly swaptions and structured notes whose valuation strictly depends on the risk factors modeled by the ESG.

Economic scenario generation, following a bootstrap approach, is laid down in Müller et al. [2004]. This comprehensive paper follows a straight path from the model’s overall principles to the concrete mathematical relations which specify economic variables and their dependencies. It explains
the details needed in any concrete implementation of the model. The fol-
lowing chapter will present, in a slightly adapted manner, the core sections
of the bootstrapping paper (Müller et al. [2004]), a document that is clearly
the foundation on which other parts of the market risk model shall rest. It
allows for a reliable judgment on the level of accuracy and the consistency
with which the market risk is modeled at SCOR Switzerland.
Bootstrapping the Economy

10.1 A Non Parametric Method of Generating Consistent Future Scenarios

In this section, a concept to model and simulate the world economy is presented. The economy is represented by a few key variables such as interest rates (yield curves), inflation, Gross Domestic Product (GDP) and equity indices, all of these for several currency zones, plus the foreign exchange (FX) rates between these zones. The goal is to generate scenarios that, in their entirety, represent the space of likely future developments. These scenarios can be used for simulating anything that depends on the economy.

Our main application is Asset-Liability Management (ALM) in a reinsurance context. ALM (see e.g. Ziemba and Mulvey [1998]) and Dynamic Financial Analysis (DFA, see Casualty Actuarial Society [1998] or Blum and Dacorogna [2003]) require models for all assets and liabilities of a firm and thus a comprehensive, dynamic model for all economic variables that determine asset and liability values. Our Economic Scenario Generator (ESG) has been developed to fulfill this requirement. Partial models for a restricted set of economic variables cannot do this, no matter how sophisticated they are, because of the complex dependencies between the variables.

Our method is bootstrapping, also called resampling. Initially, bootstrapping was a non-parametric method for limited tasks such as assessing confidence limits of models estimated on finite data samples (Efron and Tibshirani [1993]). Barone-Adesi et al. [1999] then applied bootstrapping to portfolio risk assessment, followed by Zenti and Pallotta [2000], Barone-Adesi et al. [2002] and Marsala et al. [2004]. The historical returns of certain assets became objects of resampling in simulations.

Bootstrapping constitutes the core of our model rather than being an additional tool. Our basis is a set of historical time series of economic key variables. The returns or innovations of all economic variables as observed in a randomly selected historical time interval are taken and used for the simu-
lation of future time intervals. While there is an ongoing debate on modeling economic scenarios, the bootstrapping approach has several advantages. It can be implemented in a straightforward way and relies on past behaviors of real markets rather than debatable assumptions on models and parameters. Empirical distribution functions and simultaneous dependencies between economic variables are automatically captured. Bootstrapping belongs to the family of non-parametric methods. Like other non-parametric models, our method still needs some parameters in order to define the method in a useful way, which ultimately makes the model semi-parametric. Another advantage of bootstrapping is flexibility. We can easily add more economic variables, which typically leads to large, comprehensive models.

Bootstrapping also has some disadvantages. Random trends may be continued to the future indefinitely, serial correlations are disrupted by the random selection of past intervals, and the statistical variety of behaviors may be too small in the historical time series, which implies that the probability of extreme events may be underestimated. These problems are solved by adding some preprocessing algorithms to the bootstrapping method. The following aspects have to be considered: choice and transformation of variables, data frequency, dependence (serial and between variables), fat tails of distributions, the treatment of trends and mean reversion, and an arbitrage-free consistency of the resulting scenarios. Some of these refinements are not new. Barone-Adesi et al. [1999] already found that some variables should preferably be resampled in a mapped rather than raw form, so they developed the method of filtered bootstrapping (Barone-Adesi et al. [2002]; Marsala et al. [2004]; Zenti and Pallotta [2000]). This paper offers a wide set of bootstrapping refinements, based on economic principles and facts. These refinements eliminate the major pitfalls of bootstrapping and turn this technique into a reliable generator of realistic scenarios.

A major difficulty for any parametric or non-parametric simulation model is to determine reasonable expectations for economic variables such as inflation or the growth of GDP or equity indices. Empirical means based on available samples of, say, ten years have stochastic errors or are biased by long-term economic cycles. Long samples are not available in some cases (e.g. for the Euro, where synthetic data are needed). If they are available, they may be useless, such as foreign exchange data from before 1973, when currencies were under a different regime. Our ESG is based on historical data and naturally takes empirical means as expectations, but these can be modified on the basis of expert opinion or special long-term studies such as Dimson et al. [2003].

The quality of economic scenarios and forecasts based on bootstrapping has to be measured. Typical time horizons of economic scenarios are measured in years and quarters, so we have a limited number of historical observations that can be used for backtesting. An out-of-sample backtesting study based on a Probability Integral Transform (PIT) confirms the validity
of our approach.

The part of our document where we deal with the bootstrapping method is organized as follows: After a general introduction of the bootstrapping method, some generic steps of the method are presented in Section 10.2. The implementation of these general bootstrapping steps demands a lot of specific treatment of individual economic variables in Section 10.3, where the Sections 10.3.2-10.3.6 deal with the particularly complex case of interest rates. Some resulting scenarios and out-of-sample backtesting results are shown and discussed in Section 10.4. Section 10.5 concludes.

This simplified diagram shows the bootstrapping method. We start with a historical series of data vectors containing different economic variables. Then we compute the innovations (≈ returns) of the (mapped) economic variables and store them in a series of historical innovation vectors. The simulated scenarios start from the last available data vector and continue by adding innovations, which are taken from randomly resampled innovation vectors.

Figure 10.1: The bootstrapping method.
10.2 Bootstrapping – the Method and its Refinements

10.2.1 The Idea of Bootstrapping

Our concept of bootstrapping is presented in Figure 10.1 in a schematic, simplified form. Before introducing methodological details or economic variables, we discuss the bootstrapping method by means of a simple example.

We start from a sample of historical data, that is a set of time series with historical observations over a certain time period. There is a regular time sequence \( t_i \) with time steps of size \( \Delta t \):

\[
t_i = i \Delta t
\]  (10.1)

The corresponding time series values are \( X_i = X(t_i) \) (e.g. an equity index) and \( Y_i = Y(t_i) \) (e.g. the GDP figures of the same country). The observations of all the series are synchronous and cover the same historical period (e.g. the last ten years).

The last available values (the values now) are \( X_n \) and \( Y_n \). Our task is to simulate future values at times \( t > t_n \): the vectors \((X_{n+1}, Y_{n+1}), (X_{n+2}, Y_{n+2}), \ldots,\) where the future values are in the same regular sequence, i.e. \( t_{n+k} = (n + k)\Delta t \). The basic idea of resampling is randomly picking an old time \( t_i \) of the sample and assuming the same set of observations for a future time of a scenario, e.g. for \( t_{n+1} \).

This is bootstrapping in its raw form, which will be modified in several respects. If we applied direct bootstrapping to the observations \( X_i \) and \( Y_i \), the simulated values would never leave the range given by historical values. A GDP figure could never grow to a yet unobserved value. Therefore, our main concept is to bootstrap innovations in economic variables rather than the variables themselves. These innovations will be resampled and added to old variable values at each simulation step in a cumulative way.

A simple definition of innovations might be first differences of variables. When cumulating randomly resampled first differences, the simulated variable may become negative, which is not appropriate for positive definite economic variables. Returns are usually better than first differences. Logarithmic returns are an obvious choice. We can first transform the economic variable by taking the logarithm and then take first differences. In the general case, we first introduce a variable transformation,

\[
x(t_i) = x_i = F(X_i, I_i) = F[X(t_i), I(t_i)]
\]  (10.2)

While the method relies on regular historical input data for bootstrapping, an algorithmic enhancement allows for starting a simulation from an irregular time point. We do not have to wait for the end of a quarter to produce up-to-date scenarios based on quarterly data.
where $F$ can be a logarithm or a more complex function, which may depend not only on $X_i$ but also some simultaneous values of other economic variables such as $Y_i$ or, in general, the information set $I_i$ available at time $t_i$, which includes earlier values of the considered variables. The function $F$ should be invertible to determine $X$ from $x$; its choice will be discussed for different variables. The innovation is defined in terms of $x_i$ rather than $X_i$, for example as the first difference $x_i - x_{i-1}$. Most suitably, the innovation is defined as the deviation of $x_i$ from its expectation $^2E_{i-1}[x_i]$ that the market had at the previous time point $t_{i-1}$:

$$I_i = x_i - E_{i-1}[x_i]$$  \hspace{1cm} (10.3)

The innovation $I_i$ can be negative as well as positive. It constitutes the unanticipated element of surprise in a new value $x_i$ and is thus unrelated to the market conditions at $t_{i-1}$. In case of the martingale hypothesis, if the expectation of $x_i$ made at $t_{i-1}$ was $x_{i-1}$, $I_i$ would indeed be the first difference of $x_i$. In reality, the market often has a slightly different expectation $E_{i-1}[x_i]$ at $t_{i-1}$, so the innovation somewhat differs from the first difference. The market expectation $E_{i-1}[x_i]$ depends on the economic variable. For FX and interest rates, it is a forward rate. We agree with James and Webber [2000], Section 1.4.1, that forward rates are not particularly good predictors of spot rates, because the innovations $I_i$ are large and unanticipated. Yet, an appropriate definition of $E_{i-1}[x_i]$ matters for long-term simulations, where seemingly weak modifications sum up to substantial effects. In Section 10.3, there are formulas for different economic variables, sometimes including some weak mean-reversion effects in $E_{i-1}[x_i]$.

The bootstrapping method will produce realistic results only if the $I_i$ values are independent over time and identically distributed (i.i.d.) with zero mean. It should be impossible to reject the i.i.d. hypothesis, given the empirical sample of historical innovations. Then the expectation of $I_i^2$ is independent of current market conditions, in sufficient approximation. The mapping function $F$ of Equation (10.2) has to be chosen accordingly. There is however the empirical phenomenon of volatility clustering which violates the independence of $I_i^2$: a large $I_{i-1}^2$ tends to be followed by a large $I_i^2$ with increased probability. In Section 10.2.9 this problem is solved.

In the course of simulation, the resampled innovations are used to modify the simulated, future $x$ values. For a future time $t_j$, we randomly pick a historical index $i$ and the innovation $I_i$ of $t_i$ to obtain the new simulated

\footnote{$^2E_{i-1}[x_i]$ is used as a shortcut for the correct notation $E[x(t_i) \mid I_{i-1}]$.}
value:\[ x_j = E_{j-1}[x_j] + I_i \hspace{1cm} (10.4) \]

This is an iteration. The next simulation time \( t_{j+1} \) will be treated the same way, picking a new historical index \( i' \) and re-using Equation (10.4) to obtain \( x_{j+1} \). After a few iterative simulation steps, the resulting \( x \) value will contain an accumulation of many resampled innovations \( I_i \). The variable \( x \) can drift to any value and will not observe any range constraints. Most original economic variables \( X_i \), on the other hand, are positive definite. The logarithmic function transforms a positive definite variable to an unlimited real variable and is thus a standard choice for the mapping function \( F \) of Equation (10.2).

A main strength of the bootstrapping method is preservation of dependencies and correlations between variables. If the innovations \( I_i[x] \) and \( I_i[y] \) (the corresponding innovation of the variable \( Y_i \)) exhibit some dependence in the historical sample, the simulated variables \( x_j \) and \( y_j \) will be characterized by the same dependence structure. This is due to the fact that the resampled innovations \( I_i[x] \) and \( I_i[y] \) are always taken from the same historical time \( t_i \) within a simulation step. The simulated mapped values \( x_j \) can be transformed back to standard values \( X_j \) by applying the function \( F^{-1}(.,I_j) \) inverse to \( F(.,I_j) \), see Equation (10.2).

10.2.2 General Overview of the Bootstrapping Method

An economically meaningful bootstrapping procedure requires a set of well-thought steps in addition to the simple bootstrapping principle as outlined in the previous section. The general sequence of analysis steps is as follows:

- Start from a complete and representative sample of historical economic time series for several economic variables, regularly updated to the newest values.

- Transform the economic variables (see Equation (10.2), sometimes with deseasonalization, see Section 10.2.5), in order to attain unlimited additivity of innovations.

- Compute the market’s expectations of variables at each time \( t_{i-1} \) for time \( t_i \) (e.g. a forward rate as market predictor for a foreign exchange spot rate), including some weak, long-term mean-reversion effects.

---

3 The first simulation step starts at the last regular time \( t_{j-1} = t_n \) and leads to \( x_j \) at time \( t_j \). Sometimes, there is information available at an irregular time \( t_{irreg} \) after the last regular historical time \( t_n \). In order to include this information in the first simulation step, the resampled innovation \( I_i \) can be modified to \( I_{modified} = I_{irreg} + [(t_j - t_{irreg})/\Delta t]^{1/2}I_i \), where \( I_{irreg} \) is the historical innovation from \( t_{j-1} \) to \( t_{irreg} \).
• Compute the innovations of variables as the differences between the current variable values and their previous market expectations, see Equation (10.3).

• Remove stochastic trends by forcing a zero mean of innovations, to avoid arbitrary trends in later simulations, see Section 10.2.6.

• Treat autoregressive conditional heteroskedasticity (clusters of volatility) of innovations by fitting a GARCH process, leading to GARCH-corrected innovations, see Section 10.2.9.

After this preparation, we are able to simulate future scenarios. We start by initializing all the variables (including auxiliary ones) to the latest historical values (the values now). The following sequence of steps describes one time step into the simulated future, which can be iteratively repeated.

• Do the central bootstrapping step, taking a vector of past GARCH-corrected innovations, all from the same randomly picked historical time interval.

• Multiply all these innovations by a random tail correction factor, thus injecting some rare shocks or stress scenarios that are not present in the initial data sample.

• Re-transform the GARCH-corrected innovations to the actual innovations to be used, and update the GARCH volatility equation.

• Compute the simulated variable values as sums of previous market expectations and innovations, see Equation (10.4).

• Compute the market expectations of variables for the next simulation step.

• Compute the simulated values of economic variables in their original definitions by doing transformations inverse to Equation (10.2) (reestablishing seasonality, if needed).

Notice that this sequence of steps mirrors the initially taken analysis steps in reverse order.

This elaborated methodology applies to all economic variables, but the details of each step may look different for them. More details and problems of the method are described below. The special treatment of different economic variables follows in Section 10.3.

10.2.3 Time Steps: Using High-Frequency Observations?

The size of the bootstrapping time steps depends on the application. Commercial simulation and planning tools may have yearly time steps, but the
generation of the underlying economic scenarios should be done in shorter steps. We can take quarterly steps and only use every fourth set of variable values, resulting in a yearly simulation.

When using a past of ten years, we have a basis of only 40 quarterly time intervals. This is better than 10 (the number for using yearly steps), but still rather low. A resampled year will consist of four quarters, each having the randomly selected innovations of a historical quarter. Thus there will be a wide variety in the behavior of simulated years: $40^4$ (more than 2 million) possible sequences of quarterly innovations.

Of course, we can further increase the number of historical intervals by taking monthly, weekly or daily time steps. For some variables such as GDP, high-frequency observations are not available. The clustering of volatility has been found to be stronger for high-frequency data in the literature, so the GARCH analysis (see Section 10.2.9) becomes more important. In our actually implemented economic scenario generator, we are always using quarterly time steps.

10.2.4 Noise

For some economic variables, the available data exhibit some noise. Here we mean mean-reverting short-term noise rather than the natural volatility of economic variables. Noise affects the innovation values computed by Equation (10.3) and leads to an increased variance of innovations. This increase is spurious because it reflects mean-reverting movements rather than true drifts, so it may lead to a too high volatility of results simulated over several time steps.

When using reliable data sources, this phenomenon is restricted to those variables whose definition is sensitive to such noise. In practice, this means the innovations of inflation and quarterly forward interest rates. The noise in consumer price index (CPI) figures is reinforced when computing inflation (a kind of first difference of the logarithmic CPI) and a second time when computing inflation innovations (which are similar to second differences of the logarithmic CPI). Quarterly forward interest rates have some almost inevitable noise due to small interpolation errors in the rather coarse grid of maturities supported by the yield curve data.

In these cases of noise, some smoothing techniques such as averaging are recommended in order to avoid spurious volatility in simulation results.

10.2.5 Seasonality

A variable recorded in a time series is called seasonal if its values or its first differences (or returns) have a seasonal pattern. This means that averages sampled at certain regular time intervals (e.g. second quarters of each year)
significantly deviate from averages sampled at shifted intervals (e.g. third quarters).

Prices in liquid markets such as FX, fixed income or equity hardly exhibit any significant seasonality, as empirical studies have shown. (Otherwise, these could be exploited in a systematic way.) Other financial variables such as inflation and GDP may be seasonal as there is no investment strategy to exploit seasonality. Quarterly inflation rates (first differences of logarithms of the CPI) indeed exhibit some seasonality aside from the noise discussed in Section 10.2.4.

In order to use a seasonal variable for bootstrapping, we have to deseasonalize its historical observations before computing the innovations. The simulation results will be reseasonalized at the end. This is further discussed in Section 10.3.7.

10.2.6 Detrending

In our simulations, we use innovations according to Equation (10.4) in an iterative way, thereby cumulating the innovations. Innovations are defined as deviations from prior market forecasts. If the market forecasts are reasonable\(^4\), we expect a mixture of positive and negative innovations in the long run, but the empirical mean of innovations within a historical sample may slightly (stochastically) deviate from zero. In that case, we risk introducing a trend into the simulated future.

Generating such a trend is not justified even if it existed as a random phenomenon in the historical data. Therefore we force the innovations to have a zero mean:

\[
I_i = \sqrt{\frac{n}{n-1}} \left( I_{raw,i} - \frac{1}{n} \sum_{j=1}^{n} I_{raw,j} \right) \tag{10.5}
\]

Each raw innovation \(I_{raw,i}\) is corrected by subtracting the sample mean. When doing so, we implicitly minimize the variance of \(I_i\) about zero by using one degree of freedom. Therefore we need the correction factor \(\sqrt{n/(n-1)}\) to restore the expected variance of innovations. Equation (10.5) is used for the correction of all innovations of the algorithm.

10.2.7 Mean Reversion Effects

When cumulating our detrended innovations, we obtain a stochastic random walk of the resulting variable, similar to a Brownian motion. Such motions do not exhibit any mean reversion. For most variables such as equity indices,

\(^4\)This does not mean free of trends. Some economic variables such as equity indices or some FX rates (Peso effect) have a natural trend that we have to model in the market forecast.
this behavior conforms to theory and empirical findings. For other variables such as interest rates, however, there is a weak mean-reverting force which makes sure that interest rate levels do not drift to arbitrarily high (or low) values, even after decades and centuries. Another law with mean-reverting character is purchasing-power parity (PPP). FX and inflation rates observe this law only hesitantly, with time lags of several years (see the article by Cheung in Chan et al. [2000]).

In our bootstrapping algorithm, a natural place to implement the small mean-reverting correction is the market forecast $E_{i-1}[x_i]$ of Equation (10.3). Mean reversion is a known phenomenon rather than an innovative surprise, so it belongs to the market forecast in the form of a small correction of the purely technical market forecast. Although such corrections are small, they may persist over years and exert a decisive force in real markets as well as in our simulations.

The mathematical nature of the small correction differs between economic variables and will be discussed in Section 10.3. In many cases of mean reversion, we use varying target values rather than constant means. Some of these are moving averages. This can be implemented in the form of an exponentially weighted moving average (EMA), which has the advantage of a very simple iteration formula for updating:

$$EMA_i[x; \Delta t_{\text{range}}] = \mu EMA_{i-1}[x; \Delta t_{\text{range}}] + (1 - \mu) x_i$$  \hspace{1cm} (10.6)

with

$$\mu = e^{-\Delta t/T}$$  \hspace{1cm} (10.7)

where $x_i$ stands for any variable to be averaged over time, and the time constant $\Delta t_{\text{range}}$ is the range (= centre of gravity of the weighting kernel) of the EMA. There are more complex moving averages and iterations, see Dacorogna et al. [2001], but the simple mechanism of Equations (10.6) and (10.7) is certainly good enough to describe the behavior of means that are only used to make weak corrections. At the beginning, each EMA has to be initialized, using a sufficiently large sample of $x_i$. We use the best estimate for the EMA at the very beginning of the historical sample and iteratively use Equation (10.6) through the whole historical sample.

When modeling weak mean reversion, we stay simple and pragmatic, otherwise the method would become a full-fledged econometric modeling effort rather than remaining a bootstrapping algorithm. Mean-reversion effects often involve several variables with different volatility levels. In this case, we often prefer applying the mean-reversion correction to the high-volatility variable, where the low-volatility variable acts as a sort of dragging anchor. In the example of purchasing-power parity (PPP), the high-volatility FX rate is weakly anchored by the low-volatility consumer price indices (CPI) of the two currency zones.
10.2.8 Dependence

Simultaneous innovations in different time series often depend on each other. Equity indices in different countries, for example, rarely move in different directions. The bootstrapping method captures these dependencies very well, as all innovations of a simulation step are resampled from the same historical time interval. Contemporaneous dependencies found in historical data are thus preserved.

Other forms of economic dependency pose some problems. Dependencies do not only exist for innovations, but also for the original and simulated variables. This can often be described as a mean-reversion effect such as purchasing-power parity (PPP) and has already been discussed in Section 10.2.7.

Serial dependence of innovations would mean that new innovations are partially anticipated by older ones. This is not the case here, since we define innovations as unpredictable in Equation (10.3).

If the serial dependence is in the volatility of an economic variable rather than the variable itself, we talk about autoregressive conditional heteroskedasticity. This is treated in Section 10.2.9.

10.2.9 Heteroskedasticity Modeled by GARCH

Heteroskedasticity means a variation in the volatility of a variable over time. This is only a useful concept if a model for this volatility can be formulated. One way to model future volatility would be using implied volatility from option markets. For market variables such as FX, IR and equity indices, this is feasible as long as such volatility data are available (which is not the case for the long time horizons of certain ALM studies). We do not pursue this idea here.

Autoregressive conditional heteroskedasticity (clustering of volatility) is a well-known phenomenon (Bailie et al. [1996]; Bollerslev [1986]; Dacorogna et al. [2001]; Engle [1982]) in finance. For economic high-frequency data such as daily market prices, this is very significant. However, the effect is weaker for the data frequency of economic scenarios (such as quarterly or yearly). We model the effect by approximately assuming a GARCH(1,1) process (Bollerslev [1986]). We fit a GARCH(1,1) process to the innovations as computed from the raw data.

The GARCH(1,1) process for the observed innovations $I_t$ is

$$ I_t = \sigma_t \varepsilon_t, $$

$$ \sigma_t^2 = \alpha_0 + \alpha_1 I_{t-1}^2 + \beta_1 \sigma_{t-1}^2 $$

with three positive parameters $\alpha_0$, $\alpha_1$ and $\beta_1$. The variable $\varepsilon_t$ is identically and independently distributed (i.i.d.), with mean 0 and variance 1. The GARCH process is stationary with finite variance if $\alpha_1 + \beta_1 < 1$. 

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Calibrating the parameters of a GARCH process to the innovations $I_i$ is not a trivial routine task, although its feasibility with the help of commercial software may lead a user to that assumption. The usual quasi-maximum-likelihood method poses some problems in practice, such as convergence to non-stationary solutions (especially if the GARCH process is misspecified), local maximums of the likelihood function and other convergence problems. In some cases, Zumbach [2000] finds a maximum of the likelihood function for a GARCH process whose unconditional variance is about ten times the empirical variance of the data sample. The reasons are misspecification and limited sample size. Zumbach [2000] even finds such effects for data generated by a GARCH(1,1) process. Finite sample sizes pose problems for GARCH fitting that tend to be underestimated in the literature.

Our historical innovation series based on low-frequency data definitely constitute small samples. Yet we need a reliable, robust GARCH calibration algorithm for the repeated analysis of dozens of economic variables without any human intervention. We warn against a careless use of standard GARCH fitting method. Our robust GARCH calibration is described in Appendix 10.6 and follows Zumbach [2000] with a special emphasis on avoiding local optima and a careful build-up procedure. ARCH(1) and white noise are embedded as valid solutions of GARCH(1,1). The white noise solution means that there is no GARCH correction and the original innovations $I_i$ are kept.

We apply GARCH corrections to all innovations, with one exception. The forward interest rate innovations of all maturities have a common GARCH model which we calibrate for a weighted sum of these innovations. This sum has positive weights and approximately stands for a first principal component. Balocchi et al. [1999] have shown that the first principal component of the term structure of forward interest rates is the only component with significant autoregressive conditional heteroskedasticity. This finding supports our GARCH modeling approach for interest rates.

After calibrating the GARCH process, we assume all the innovations $I_i$ to be products of the volatility $\sigma_i$ as resulting from the GARCH process and normalized innovations $J_i$ which can be seen as the historically determined $\varepsilon_i$ values of Equation (10.8). We obtain $J_i$ by dividing the original innovations $I_i$ by $\sigma_i$. The normalized innovations $J_i$ are the final results of our preprocessing of economic variables. We can also call them GARCH-filtered innovations, using an analogy to the filtered bootstrap by Barone-Adesi et al. [1999].

In the simulation of the future time $t_j$, we resample a $J_i$ and compute the volatility $\sigma_j$ by using Equation (10.8), initially starting at the last computed historical GARCH variance $\sigma^2_n$. The newly constructed innovations $I_j = \sigma_j J_i$ will be used. The sequence of these $I_j$ will have the desired property of volatility clustering, unlike the randomly resampled $J_i$ values.
10.2.10 Fat Tails of Distribution Functions

All the economic variables and their innovations have their empirical distributions as determined by the historical data. When using quarterly observations over ten years, we have 40 innovations. This is a small sample size for detailed statistics.

From the literature (Dacorogna et al. [2001]; Embrechts et al. [1997]), we know that many financial variables exhibit fat tails in their distribution functions, if studied with enough data, using high frequency or very long samples. Typical tail indices of high-frequency foreign-exchange data are around $\alpha = 3.5$ (see Dacorogna et al. [2001]). For the internal model with a one-year horizon we arrive at the optimal choice $\alpha = 4$.

Our economic scenarios are made for studying risk as well as average behaviors. We use tail-based risk measures such as value at risk (VaR) and, more importantly, the expected shortfall, see Artzner et al. [1997]. The simulation of extreme events (such as the 1 in 100 event should be realistic. How is this possible based on only 40 quarterly innovations for bootstrapping? Pure bootstrapping will underestimate risks, except for the unlikely case that the most extreme historical observation substantially exceeds the quantile that can reasonably be expected for the maximum in a small sample.

Some risk and ALM specialists rely on a few arbitrary stress scenarios, that is some stylized extreme events. Here we propose a more consequent way to include a rich variety of many possible stress scenarios. When doing the simulations, we add some stochastic variation to the resampled innovations to attain a more elaborated tail behavior. We do not really change the tail behavior, we just add some small stochastic variability on both sides, increasing or decreasing an original innovation. Technically, this can be done without increasing the overall variance. The stochastic variation of historically observed innovations is small and harmless, except for very rare, extreme tail events. We explain the method for an economic innovation $I_i$. If the GARCH analysis of Section 10.2.9 is made, we apply the tail correction to the GARCH-corrected innovations, so $I_i$ actually stands for the normalized innovation $J_i$.

$I_i$ has an unknown distribution function with mean 0. We assume a tail index $\alpha > 2$ in both tails. A suitable value for many economic variables might be $\alpha = 4$. Now we define an auxiliary, Pareto-distributed random variable $\eta$ to modify the original, resampled innovations $I_i$ in a multiplicative way:

$$I_i' = \eta I_i$$  \hspace{1cm} (10.9)

The new variable $\eta$ is defined to have the same tail index $\alpha$:

$$\eta = A + B (1 - u)^{-1/\alpha}$$  \hspace{1cm} (10.10)
where \( u \) is a uniformly distributed random variable in the range between 0 and 1. Thus \( \eta \) is confined:

\[
\eta \geq \eta_{\text{min}} = A + B \quad (10.11)
\]

This minimum corresponds to \( u = 0 \). We always choose \( A + B > 0 \), so \( \eta \) is positive definite. The inverse form is

\[
u = 1 - \left(\frac{\eta - A}{B}\right)^{-\alpha} \quad (10.12)
\]

This is the cumulative probability distribution of \( \eta \), where \( u \) is the probability of \( \eta \) being below the specific \( \eta \) value inserted in Equation (10.12). This is indeed a Pareto distribution with tail index \( \alpha \).

We choose the parameters \( A \) and \( B \) in a way that \( \eta \) is normally close to 1, so the modified variable \( I'_i = \eta I_i \) is similar to the original, resampled value \( I_i \), and the overall character of the bootstrapping method is maintained. However, the modified innovation \( I'_i \) based on the random variable \( \eta \) and the independently chosen resampling index \( i \) will exhibit a fat tail in simulations. The larger the number of simulations, the denser the coverage of this fat tail will be. Tail observations of \( I'_i \) will occur if two unlikely events coincide: very large values of both \( |I_i| \) and \( \eta \).

The resulting tail index\(^5\) of \( I' \) is \( \alpha \), as assumed for \( I \). Thus we do not make the tail fatter than it should be, we just introduce enough variation in the tail for realistic simulations.

The parameters \( A \) and \( B \) must be defined in a suitable way. We have to keep the original variance of innovations unchanged. This is important when using the GARCH correction of Section 10.2.9. GARCH is a variance model, so we should not modify the unconditional variance in our simulations here. The condition is

\[ \mathbb{E}[I'^2_i] = \mathbb{E}[I^2_i] \]

Considering Equation (10.9) and the independence of \( \eta \), this implies the condition

\[
\mathbb{E}[\eta^2] = A^2 + \frac{2}{\alpha - 1} A B + \frac{\alpha}{\alpha - 2} B^2 = 1 \quad (10.13)
\]

which is the result of an integration over the distribution of \( \eta \), using Equation (10.10). In order to keep the variance \( \mathbb{E}[\eta^2] \) finite, we need \( \alpha > 2 \), which turns out to be well satisfied by empirical economic data. The second equation to determine \( A \) and \( B \) is given by Equation (10.11): \( A + B = \eta_{\text{min}} \). Solving this equation together with Equation (10.13), we obtain

\[
B = \frac{1}{2} \left[ \sqrt{\eta_{\text{min}}^2 (\alpha - 2)^2 + 2 (\alpha - 1) (\alpha - 2) (1 - \eta_{\text{min}}^2)} - \eta_{\text{min}} (\alpha - 2) \right] \quad (10.14)
\]

\(^5\)A closer tail analysis shows that \( \eta \) should be based on a tail index infinitesimally larger than \( \alpha \), otherwise the resulting tail index of \( x' \) is infinitesimally less than \( \alpha \). This theoretical consideration does not matter in practice.
and

\[ A = \eta_{\text{min}} - B \]  \hfill (10.15)

We still need to choose the minimum \( \eta_{\text{min}} \) of the correction factor \( \eta \). We argue that the tail correction should neither be too timid nor too strong (which would mean to destroy the character of the bootstrapping method). We allow it to be just strong enough to fill the gap between the largest and the second largest historical innovation. In reality, the empirical values of these innovations are subject to wide stochastic variations. Just for the sake of a reasonable definition of \( \eta_{\text{min}} \), we assume them to be regular quantiles here. We locate the largest observation of \( I_i \), called \( I_{\text{max}} \), at a cumulative probability between 1 - 1/n and 1, in fact in the middle of this range, at 1 - 1/(2n). Assuming a Pareto behavior at the tail around \( I_{\text{max}} \) with tail index \( \alpha \), we obtain the heuristic approximation

\[ I_{\text{max}} \approx (2c n)^{1/\alpha} \]  \hfill (10.16)

where the constant \( c \) stays undetermined. Following the same logic, the second largest value of \( I_i \) can be associated to the cumulative probability range between 1 - 2/n and 1 - 1/n. The probability value 1 - 1/n separates the expected domain of \( I_{\text{max}} \) from the domain of the second largest value. The \( I \) value corresponding to this separating limit is

\[ I_{\text{limit}} \approx (c n)^{1/\alpha} \]

By applying the tail correction of Equation (10.9), the largest observation can be reduced to \( \eta_{\text{min}} I_{\text{max}} \), but not more. We identify this reduced value with the limit \( I_{\text{limit}} \):

\[ \eta_{\text{min}} I_{\text{max}} \approx (c n)^{1/\alpha} \]  \hfill (10.17)

Equations (10.16) and (10.17) can be solved for \( \eta_{\text{min}} \). The unknown constant \( c \) cancels out. We obtain the following recommended choice:

\[ \eta_{\text{min}} = 2^{-1/\alpha} \]  \hfill (10.18)

This result is independent of \( n \) and always < 1. For an \( \alpha \) of 4, we obtain \( \eta_{\text{min}} \approx 0.841 \), which is rather close to 1. Our definition of \( \eta \) is complete now and consists of Equations (10.10), (10.14), (10.15) and (10.18).

Eventually, the tail correction will be made for all resampled innovations, not only for one variable \( I_i \). When doing it for all innovations in a multi-dimensional setting, two issues have to be addressed:

- Do we use the same tail index \( \alpha \) for all economic variables? This is not necessary. Detailed statistical studies of all variables may lead to specific \( \alpha \) values. In a simpler approach, we can use a general assumption such as taking \( \alpha = 4 \) for all economic variables.
Do we use the same random variable \( u \) for all economic variables? In the case that we also take the same \( \alpha \) (which is not necessary, see above), this implies using the same \( \eta \) for all variables. Using different \( u \) values for different variables adds some noise and blurs the dependence in the tails. Using the same \( u \) or \( \eta \) leads to an emphasis on the dependence in the extreme tails of all those variables that simultaneously have extreme observations. Some findings (Dacorogna et al. [2001]) indeed indicate that dependencies between variables are larger in the tails than under less extreme circumstances. In a parametric model, this effect could be modeled through copulas. In our bootstrapping approach, we obtain a conservative, risk-conscious effect by assuming the same \( u \) for all variables. At the same time, this reduces the number of computations per simulation step.

Using the proposed method, we can successfully reconcile the bootstrapping method with the requirement of realistic tail simulations. There is some room for human judgement. If conservative users have reasons to believe that future behaviors will be more extreme than historical behaviors, they can decrease the assumed tail index \( \alpha \).

10.3 Bootstrapping of Different Economic Variables

10.3.1 Choice of Economic Variables

The set of economic variables to be modeled depends on the availability of raw data and the needs of the model user. There are interactions between economic variables (e.g. weak mean reversion effects) that can only be modeled if a sufficiently large set of variables is chosen.

The following economic variables are included in a reasonable implementation of an economic scenario generator based on bootstrapping:

- Interest rates (IRs). These have different maturities. We have to deal with whole yield curves. The interest rate model is the heart of any comprehensive economic model.

- Foreign Exchange (FX) rates between the supported currencies of the generator.

- Equity indices. It is possible to include several indices per currency zone, e.g. different sector indices, real-estate fund indices or hedge fund indices. We prefer total-return indices which include reinvested dividends, because these indices are directly related to investment performance. However, the bootstrapping technique also works for price indices.
Inflation, in the form of a Consumer Price Index (CPI). It is possible to add other indices, e.g. wage inflation or medical inflation.

- Gross Domestic Product (GDP).

The variables have different levels of volatility. We can roughly sort them, from low to high volatility: real gross domestic product (GDP), consumer price index (CPI), interest rates, inflation (which is a temporal derivative of the CPI), FX rates, equity indices.

All the variables are modeled for several major currency zones. Major currencies should be included as well as those minor currencies that are relevant for an application. We are using the currencies USD, EUR, JPY, GBP, CHF and AUD.

The lists of variables and currencies can be varied. One of the advantages of the bootstrapping method is that adding or removing an economic variable from the model is technically easy. As an example, we may include rating-dependent credit spreads as a new variable to simulate the behavior of corporate bonds.

Other economic variables such as the values of certain bonds, including mortgage-backed securities with their special behavior, can be derived from the simulated values of primary variables such as interest rates in sufficiently good approximation.

In the following sections, the treatment of different variables is discussed in detail. For each of them, the steps of the bootstrapping method as outlined in Sections 10.2.1 and 10.2.2 take different forms.

10.3.2 Interest Rate Forwards and Futures

When modeling interest rates, we refer to risk-free market interest rates as extracted from different, liquid financial instruments, which are issued by governments or institutions of the highest ratings. Such interest rates, for different maturities, can be summarized in the form of a zero-coupon yield curve, or just yield curve, such as the fair-market yield curves composed by Bloomberg.

An interest rate (IR) as quoted in a yield curve has a complex dynamic behavior. Interest rates for different maturities are available at the same time, with a complicated dependence structure. Long-term interest rates have maturity periods of many years, over which the economic conditions can be expected to change. The dynamic behavior of an IR with constant maturity period is characterized by the fact that this period is continuously moving over time. The IR thus refers to a moving target.

A way to disentangle the complex dynamics and dependencies of interest rates – both in market practice and in modeling – is using forward interest rate or IR futures. Using IR futures is the most consequent solution, as these future contracts always focus on the same future time interval, for
example from 15 March 2007 to 15 June 2007. For such a fixed, well-defined period, the price-finding process in the market is more efficient than for large, heterogeneous, moving time intervals. This fact helped to make IR futures the most liquid financial instrument in the IR market for maturities from three months to about two years. We shall see that IR futures have similar advantages in modeling, too. A major advantage is arbitrage-free consistency. If all IR-based financial instruments are constructed from the same forward IRs and thus the same market prices of IR futures, there is no way to generate riskless profits, no matter how sophisticated the IR portfolio composition.

There is a rich literature on interest rate modeling; we use James and Webber [2000] as a main reference. The basics of yield curve mathematics can be found in Section 3.1 of that book. We transform the information contained in a yield curve and package it as an equivalent set of forward interest rates. The yield curve consists of annualized interest rates $r(T)$ as a function of the time interval to maturity, $T$. We use interest rates $R$ in logarithmic form,

$$R(T) = \log \left(1 + \frac{r(T)}{100}\right)$$ (10.19)

This has the advantage of transforming the multiplicative compounding of interest rates to simple additive compounding. Now we regard the forward interest rate $\varphi(T_1, T_2)$ for the interval between the future time points $T_2 > T_1$. From elementary interest compounding rules, we derive

$$T_2 R(T_2) = T_1 R(T_1) + (T_2 - T_1) \varphi(T_1, T_2)$$

which is additive due to the logarithmic transformation of Equation (10.19). We solve for $\varphi(T_1, T_2)$:

$$\varphi(T_1, T_2) = \frac{T_2 R(T_2) - T_1 R(T_1)}{T_2 - T_1}$$ (10.20)

When starting from a yield curve, this equation serves as a definition and computation formula for the empirical forward rates $\varphi(T_1, T_2)$, where $T_1$ and $T_2$ are neighboring maturities of the yield curve. In practice, $R(T_1)$ and $R(T_2)$ are often interpolated values from a more coarsely defined yield curve. We need a good yield curve interpolation formula, but even an excellent formula may lead to small interpolation errors which are reinforced by building the difference of Equation (10.20). This problem requires an additional smoothing procedure later in the bootstrapping algorithm.

---

6There is also a disadvantage when using futures. IR futures markets require a collateral margin account which leads to a small deviation between the values of forward rate agreements and futures, called the convexity adjustment (see Section 5.5 of James and Webber [2000]). We assume that our basic curves are fair-market yield curves where the convexity adjustment is accounted for when they are constructed from futures prices.
The forward rate of an infinitesimally small maturity interval, from \( T \) to \( T + dT \), is denoted by \( \varrho(T) \). The logarithmic interest rate \( R(T) \) can be written as

\[
R(T) = \frac{1}{T} \int_0^T \varrho(T') \, dT'
\]  

(10.21)

In fact, \( R(T) \) is the average forward IR as measured over the whole maturity axis from 0 to \( T \).

At the end of a simulation step, the resulting set of forward interest rates can be retransformed to a yield curve, following the notion of Equation (10.21).

10.3.3 The Innovations of Forward Interest Rates

Setting up a satisfactory bootstrapping algorithm for forward interest rates is a complex task. For the sake of completeness, we formulate in this section an intuitive direct approach to resampling forward rates. However, this approach leads to problems, so we shall need a more sophisticated method as described in Sections 10.3.4 and 10.3.5. At the end of Section 10.3.5, the steps for bootstrapping interest rates are summarized.

First we add the dimension of time \( t \), using the regular time points of Equation (10.1). We write \( \varrho_i(T) \) for the forward rate at time \( t_i \), named \( \varrho(T, T + \Delta t) \) in Section 10.3.2. For bootstrapping, we are only interested in rates with a forward period of the size of the basic time step \( \Delta t \) (= three months for quarterly steps) and a time to maturity \( T \) that is an integer multiple of \( \Delta t \). For the corresponding spot rate with maturity \( \Delta t \), we write \( R_i \) (\( = \varrho_i(0) \)). How do forward rates \( \varrho_i(T) \) evolve over time? At first glance, we might consider the behavior of the forward rate \( \varrho_i(T) \) for a fixed maturity period \( T \). However, the time \( t_i + T \) of the maturity would move in parallel with time \( t_i \). The value of \( \varrho_i(T) \) would therefore refer to changing time points with changing market conditions, which makes the assessment difficult.

Instead, we focus on the forward IR for a fixed time interval in the future. This is exactly the point of view of IR futures markets. The price of an IR future reflects the current market consensus forecast \( \varrho_i(T) \) of the underlying interest rate. When the futures contract reaches maturity, at time \( t_i + T \), we can directly read the value \( R_{t_i+T/\Delta t} \) of this interest rate from the yield curve. In other words, \( \varrho_i(T) \) is the market’s forecast\(^7\) of \( R_{t_i+T/\Delta t} \). There is a stream of unanticipated news that leads to innovations in this forecast. At the earlier time \( t_{i-1} \), the market forecast for \( R_{t_{i-1}+T/\Delta t} \) was \( \varrho_{i-1}(T + \Delta t) \); at \( t_i \) it is \( \varrho_i(T) \). We observe the following innovation from \( t_{i-1} \) to \( t_i \):

\[
I_i[\varrho(T)] = \varrho_i(T) - \varrho_{i-1}(T + \Delta t)
\]  

(10.22)

\(^7\)This statement will be qualified twice: first in Section 10.3.6, due to the asymmetry in interest rates, then in Section 10.3.6, where a small mean-reverting correction term is added.
This is Equation (10.3) applied to forward interest rates. The innovation \( I_i[\varrho(T)] \) can be resampled and cumulated in our bootstrapping method. However, such a direct procedure may lead to negative interest rates in the simulation and some other shortcomings as shown below. We need a deeper analysis of \( \varrho(T) \) and a more sophisticated method.

### 10.3.4 Mapping and the Asymmetry of Interest Rates

Three problems arise when directly using \( I_i[\varrho(T)] \) from Equation (10.22) for resampling:

1. Innovations can be negative as well as positive. When cumulating \( I_i[\varrho(T)] \) values from randomly resampled historical time points \( t \), the resulting \( \varrho \) values in some scenarios may drift to a value less than zero after some simulation steps. Such a behavior cannot be accepted as it violates an economic principle which states that no increase of profit can be reached at zero risk. As soon as an IR (or forward IR) is negative, a risk-free profit can be made by storing money physically instead of investing it in a deposit. In historical data, we hardly find any negative interest rates.

2. Interest rates are more volatile on a high level than on a low level close to zero. The same innovation value \( I_i[\varrho(T)] \) may produce high volatility in the context of low \( \varrho \) values and low volatility when resampled in a high-interest regime. This is against the bootstrapping principle. A resampled innovation should always model approximately the same force on the market, regardless of the current economic condition.

3. The empirical forward rate \( \varrho_i(T) \) as determined by the market is a forecast with uncertainty rather than a simple quantity. Market participants know that the distribution is skewed: Negative values of \( R_{t+T/\Delta t} \) are unlikely while the positive part of the distribution is unlimited. Under normal conditions, they will thus agree on a forward rate \( \varrho_i(T) \) exceeding the expected median of \( R_{t+T/\Delta t} \) by an amount that is related to the term premium.

All these problems are related to the asymmetry or skewness of interest rate distributions. There is a mathematical method that solves all of them at the same time: non-linear mapping of short-term interest rates \( R_i \), for which we simply write \( R \) here. We define a mapped variable \( z \):

\[
z = z(R) = \begin{cases} 
\sqrt{R + \varepsilon} - \sqrt{\varepsilon} & \text{for } R \geq 0 \\
A R & \text{for } R < 0
\end{cases}
\]  

(10.23)

with a small offset \( \varepsilon \approx 0.01 \) and a large factor \( A \approx 1000 \). The idea behind the mapping of Equation (10.23) is to eliminate the asymmetry of interest
rates. At time \( t_i \), the distribution expected for the rate \( R_{t_i+T/\Delta t} \) at maturity time \( t_i + T \) is asymmetric with a variance depending on the value of \( g_i(T) \).

In contrast, we define \( z \) in a way to fulfill two working hypotheses: (1) the distribution of the \( z \) value expected for time \( t_i + T \) is symmetric around a mean \( \bar{z} \); (2) the variance \( \sigma^2_z \) of \( z \) is independent of the \( \bar{z} \) level. If the parameters of Equation (10.23) are chosen appropriately, both hypotheses should hold in sufficiently good approximation. The working hypotheses are no firm claims, they are just used to motivate and introduce our method of bootstrapping interest rates.

Historical interest rates are rarely negative. In simulations, the large parameter \( A \) will cause a sort of soft boundary for interest rates below zero. This boundary is not as absolute as in Section 6.4.3 of James and Webber [2000]. The function \( z(R) \) is continuous and has a pronounced kink at \( R = z = 0 \), which is natural for a quantity for which the limit \( R = 0 \) plays an important role.

In the upper part \( (R \geq 0) \), \( z \) approximately grows with the square root of \( R \). This is in agreement with the Cox-Ingersoll-Ross (CIR) model of interest rates (which is very different in other aspects, see Section 3.3.2 of James and Webber [2000]). The CIR model assumes the volatility of interest rates to be proportional to the square root of the current IR value. Our mapping implies a similar behavior by assuming a fixed distribution of \( z \) and translating the behavior of \( z \) back to the behavior of interest rates \( R \). The square-root law is modified by adding a constant \( \varepsilon \) to \( R \) in Equation (10.23). This makes the volatility at very low interest rates less aberrant and more similar to that of higher IR levels, a behavior we have observed for Japanese interest rates. The very low Japanese rates since the late 1990s have given us some useful hints on how to model low levels realistically. Our model based on Equation (10.23) is robust for a wide range of different IR levels, using the term robustness as in Section 1.5.2 of James and Webber [2000] and relating to the discussion of Section 6.4 of the same book.

The function \( z(R) \) is strictly monotonic and can thus be inverted:

\[
R = R(z) = \begin{cases} 
\left(\frac{z + \sqrt{\varepsilon}}{A}\right)^2 - \varepsilon & \text{for } z \geq 0 \\
\frac{z}{A} & \text{for } z < 0 
\end{cases} \quad (10.24)
\]

\( A \) is a very large parameter, so \( R \) will be very close to zero even if \( z \) is distinctly negative, as shown in Figure 10.2. This is a first reason why the simulation will never produce strongly negative interest rates. If it ever produces negative IRs, these are so close to zero that they can be rounded to zero in most practical applications.

Equation (10.23) relates the new variable \( z \) to the short-term interest rate \( R \). In order to use \( z \) in practice, we need to define its relation to observable forward rates \( \varphi \). This follows from the distribution function of \( z \).
The solid curve shows the mapping of interest rates in the inverse form of Equation (10.24), $R = R(z)$. In the region of $\bar{z} < 0$, the curve is not horizontal, but has a tiny positive slope of size $1/A$. The dotted curves show forward interest rates $\varrho$ as functions of the bootstrapping variable $\bar{z}$, following Equation (10.27) and assuming different variances $\sigma_z^2$ of $z$ about $\bar{z}$. The values $\sigma_z = 0.05, 0.1$ and $0.15$ approximately represent three maturity periods: $\frac{1}{2}$ year, 2 years and the long-term limit. In the case of the solid line, the maturity and the variance $\sigma_z^2$ are zero, $z = \bar{z}$, and $\varrho$ stands for the spot interest rate $R$.

Figure 10.2: Interest rate mapping.
which we approximately assume to be normal with mean \( \bar{z} \) and variance \( \sigma_z^2 \):

\[
\tilde{z}_{i+T/\Delta t} \sim N[\bar{z}_i(T), \sigma_z^2(T)]
\]  

(10.25)

This ensures mathematical tractability. Now we express \( \varrho_i(T) \) as the expectation value of \( R_{i+T/\Delta t} \). Taking the expectation value is justified if the values of simple IR-based portfolios at time \( t_i + T \) are linear functions of \( R_{i+T/\Delta t} \) and risk aversion effects are negligible. In good approximation, this is the case for efficient markets with low to moderate rate levels, where risk aversions of large lenders and borrowers are low, act in opposite directions and approximately cancel out. Using Equation (10.25), the expectation value of \( R_{i+T/\Delta t} \) is

\[
\varrho_i(T) = \frac{1}{\sqrt{2\pi \sigma_z}} \int_{-\infty}^{\infty} R(z) e^{-\frac{(z-\bar{z}_i)^2}{2\sigma_z^2}} \, dz
\]  

(10.26)

where \( R(z) \) is defined by Equation (10.24). This means averaging \( R \) with a Gaussian weighting kernel. The integral can be solved:

\[
\varrho_i(T) = \varrho(\bar{z}_i, \sigma_z^2) = \Phi \left( \frac{\bar{z}_i}{A} - \left( \bar{z} + \sqrt{\varepsilon} \right)^2 + \varepsilon - \sigma_z^2 \right) + \frac{\sigma_z}{\sqrt{2\pi}} e^{-\frac{\bar{z}_i^2}{2\sigma_z^2}} \left( \bar{z}_i + 2\sqrt{\varepsilon} - \frac{1}{A} \right) + (z + \sqrt{\varepsilon})^2 - \varepsilon + \sigma_z^2
\]  

(10.27)

where \( \Phi(\cdot) \) is the cumulative standard normal distribution function. Whenever a simulation produces a value of \( \bar{z}_i(T) \), Equation (10.27) is used to transform it to a forward rate \( \varrho_i(T) \) which then can be used to construct a simulated yield curve. Figure 10.2 shows forward rates \( \varrho \) as functions of \( \bar{z} \) for several values of \( \sigma_z^2 \) according to Equation (10.27). What happens if \( \bar{z} \) is drifting in the negative region in a simulation? The corresponding \( \varrho \) values will stay close to zero for quite some time. This can be a real behavior, as we have seen for Japanese rates over the last few years.

The variance \( \sigma_z^2 \) has to be known in order to fully establish the link between \( \varrho \) and \( \bar{z} \). In our model\(^9\), \( \sigma_z^2 \) only depends on \( T \) and is independent of the current \( \bar{z} \) level; this was one of the goals when we introduced the variable \( z \). When \( z \) is normally distributed and innovations in \( \bar{z} \) are assumed to be unexpected (caused by news) and independent, we can describe its dynamics in terms of a Brownian motion of \( \bar{z} \). At maturity (\( T = 0 \)), \( \sigma_z^2 = 0 \),

\(^8\)We cannot investigate \( z \) empirically here, because \( \bar{z} \) is not yet defined as a function of observable forward rates.

\(^9\)Again, we cannot use an empirical variance of \( z \) here, because we are still in the process of defining \( \bar{z} \) as a function of observable variables. As soon as the model is complete, we can verify and calibrate it. In the further course of the algorithm, we are using a GARCH model for the variance of innovations of \( \bar{z} \), see Section 10.2.9. That sophisticated volatility model should not be confused with the simple \( \sigma_z^2 \) model introduced here for the sole purpose of defining a suitable variable \( \bar{z} \).
as no uncertainty on the outcome remains. The longer the maturity period, the more unexpected news may increase the uncertainty. For a Brownian motion, we obtain \( \sigma_z^2 \propto T \). However, \( \sigma \) does not grow to infinity with increasing \( T \). Historical interest rate plots over several decades or even centuries (e.g. Figures 1.1, 1.2 and 17.2 of James and Webber [2000]) show that interest rate levels hardly drift to very extreme values (such as –0.5% or 40%) and never stay extreme for a long time. We rather observe a weak mean reversion\(^{10}\) of IR levels that brings these levels back to a certain range in the long run. Thus our \( \sigma_z^2 \) will not infinitely grow but rather converge to a finite value at very long maturities \( T \). The variance behaves as follows, approximately:

\[
\sigma_z^2 = \sigma_z^2(T) = b \frac{T}{T_{rev} + T}
\]  

This is just one possible function to model \( \sigma_z^2 \). The proposed function has two interesting properties. First, we look at short maturities and obtain

\[
\sigma_z^2 \approx b \frac{T}{T_{rev}} \quad \text{for} \quad T \ll T_{rev}
\]

This is indeed proportional to \( T \). A reasonable choice of the constant \( T_{rev} \) is around three years. Now we also look at very long maturities:

\[
\sigma_z^2 \approx b \quad \text{for} \quad T \gg T_{rev}
\]

The constant \( b \) is the asymptotic value which defines the maximum volatility. Values roughly around \( b \approx 0.02 \) lead to realistic models.

### 10.3.5 The Innovation of Mapped Forward Interest Rates

Now we are finally able to define \( \bar{z} \) as a function of the observed forward interest rate \( \varrho \). The variable \( \bar{z} \) is the variable that satisfies Equation (10.27). This is the definition:

\[
\bar{z}(T) = Z[\varrho(T), \sigma_z^2(T)]
\]  

where \( Z(., .) \) is the inverse function of \( \bar{\varrho}(., .) \), with

\[
Z[\bar{\varrho}(\bar{z}, \sigma_z^2), \sigma_z^2] = \bar{z}
\]  

and \( \sigma_z^2(T) \) is the result of Equation (10.28). There is no analytical formula for the function \( Z(., .) \), so we have to invert Equation (10.27) numerically. This is not a large problem as \( \bar{\varrho}(., .) \) and \( Z(\varrho, .) \) are monotonic functions for a constant \( \sigma_z^2 \). There is always exactly one finite solution of each function, given a finite argument.\(^{10}\)Mean reversion effects in the long run are explicitly discussed in Section 10.3.8. At the moment, we are only interested in the behavior of \( \sigma_z^2 \).
For our bootstrapping algorithm, we shall use \( \bar{z}_i(T) \) from Equation (10.29) instead of the unsuitable variable \( \varrho_i(T) \). Now we can define the innovations in the sense of Equation (10.3): \[ I_i[\bar{z}(T)] = \bar{z}_i(T) - \bar{z}_{i-1}(T + \Delta t) \] (10.31)

where both \( \bar{z}_i(T) \) and \( \bar{z}_{i-1}(T + \Delta t) \) result from Equation (10.29). This replaces the insufficient definition of Equation (10.22). In Section (10.3.8), this definition of innovations will be slightly modified as we correct the expectation \( \bar{z}_{i-1}(T + \Delta t) \) of \( \bar{z}_i(T) \) by a small mean-reversion term. The historically determined innovations \( I_i[\bar{z}(T)] \) will not necessarily be normally distributed. We include the fat-tail correction of Section 10.2.10 and the GARCH correction of Section 10.2.9 in simulations, which implies a further deviation from the normal distribution. The empirical distribution of \( z_{i+T/\Delta t} - \bar{z}_i(T) \) may also deviate from a theoretical normal distribution as assumed by Equation (10.25). The deviations should however be limited, in general and also under specific market conditions. This is a guideline when calibrating the four parameters of the \( \bar{z} \) definition: \( \varepsilon, A, T_{rev} \) and \( b \). Another useful study is to test if the historical innovations \( I_i[\bar{z}(T)] \) are serially independent, as they should be for resampling.

The bootstrapping of interest rates is rather complex. This is why we summarize the steps in the following list, which is just a specific implementation of the general list of Section 10.2.2:

- Compute all historical forward rates \( \varrho_i(T) \) for a wide range of maturities \( T \) from 0 to 30 years (or more) as integer multiples of the basic time step \( \Delta t \), using Equation (10.20) with an interpolation formula for interest rates \( R \) on the yield curve.
- Map all forward rates \( \varrho_i(T) \) to \( \bar{z}_i(T) \) by using Equations (10.29) and (10.28).
- Compute all innovations \( I_i[\bar{z}(T)] \) through Equation (10.31), including the small mean-reverting correction of Section (10.2.7).
- Apply detrending and a GARCH analysis of innovations as specified in Sections (10.2.6) and 10.2.9.

A simulation step, which can be repeated arbitrarily often, is done as follows:

- Resample the innovations of a randomly selected historical index \( i \), apply standard tail and GARCH corrections.
- Add the resampled innovations \( I_i[\bar{z}(T)] \) to \( \bar{z}_{j-1}(T + \Delta t) \) (or a more complex expectation of \( \bar{z}_j(T) \), including the small mean-reverting correction of Section 10.3.8) to obtain \( \bar{z}_j(T) \).
• Convert all \( \bar{z}_j(T) \) to the simulated forward rates \( \varrho_j(T) \), using Equation (10.27).

• Construct the simulated yield curve by averaging the obtained forward rates \( \varrho_j(T) \) in the sense of Equation (10.21).

Drift-free yield curves based on constant \( \bar{z} \): These stylized yield curves are based on fixed, constant values of the mapped interest rate \( \bar{z} \) and represent standard forms. Simulated yield curves resulting from the bootstrapping method fluctuate around these standard forms and exhibit a wider variety of different forms.

![Drift-free yield curves (normal forms)](image)

Figure 10.3: Drift-free yield curves.

10.3.6 Interest Rate Mapping and the Form of the Yield Curve

In this small section, we demonstrate that our asymmetric mapping of forward interest rates is closely related with the typical form of yield curves.

Let us imagine a static yield curve, free of drift and surprise, with constant values of \( z = \bar{z} \) over the whole maturity axis, unchanged over time. The corresponding yield curve is not flat, in spite of the constant value of \( \bar{z} \). This is due to the variance \( \sigma_z^2 \) which grows with increasing maturity \( T \), following Equation (10.28). We can compute the forward rates as functions of \( T \) by inserting that equation in Equation (10.27) and average them to build the conventional yield curve of interest rates \( r = [\exp(R) - 1] \cdot 100\% \) as functions of \( T \). This has been made in Figure 10.3.
The yield curves of Figure 10.3 indeed look familiar, they have a standard or classical form, see e.g. James and Webber [2000], Section 1.3.1. A curve starts at a low value for very short maturities, has a positive slope, becomes flatter with increasing maturity and reaches an asymptotic value at long maturities $T \gg T_{rev}$. Values at longer maturities exceed the short rates by a positive amount which can be called the term premium. The curves with $\tilde{z} < 0$ look slightly different with an almost horizontal tangent at $T = 0$. This is similar to the form of the Japanese yield curve in the years 1998 to 2007.

The fact that the drift-free yield curves have familiar forms confirms the suitability of our interest mapping. In reality and in our simulations, $\tilde{z}$ is not constant across the maturity axis, and its values are affected by varying innovations, so the resulting yield curves will not be drift-free. Real yield curves make complex movements centered around the normal forms of Figure 10.3 and exhibit many different forms, as in Figures 1.3 and 1.4 of James and Webber [2000]: sometimes with steeper slopes, sometimes flatter, sometimes with inverted (negative) slopes, sometimes with humped forms. The variety of simulated yield curve forms will be shown in Section 10.4.2, Figure 10.4.

10.3.7 Processing Inflation

Historical values of the Consumer Price Index (CPI) are the basis of our inflation processing. The CPI is the nominal value of a representative basket of consumer goods and results from rather sophisticated statistical methods. There is a debate on these methods, and there are alternative price indices that might be used instead of or in addition to the CPI.

We take logarithms of the CPI and define inflation as a first difference:

$$x''_i[\text{Infl}] = \log CPI_i - \log CPI_{i-1}$$ (10.32)

For quarterly data, we obtain quarterly inflation figures that are not annualized. Annualized inflation in the usual sense can be computed as

$$[\exp(4x''_i[\text{Infl}]) - 1] \cdot 100\%.$$

Inflation exhibits serial correlation and behaves more like Brownian motion than like white noise. It is highly correlated with the IR level. Therefore, we resample innovations of inflation, which are correlated with IR innovations, rather than innovations of the CPI.

Inflation figures $x''_i[\text{Infl}]$ as computed by Equation (10.32) are not yet suitable for bootstrapping. Inflation exhibits seasonality: Values in winter are typically higher than in summer. We should not resample winter innovations to simulate summers directly. Our solution is to deseasonalize inflation values. A simple way of deseasonalizing quarterly inflation figures
is to subtract the mean of all historical inflation figures that occurred in the same quarter, for example the second quarter of a year. Let us denote the deseasonalized inflation by $x_i'[\text{Infl}]$.

Inflation is a difference in Equation (10.32) and is highly sensitive against small changes in the underlying CPI data\textsuperscript{11}. Inflation innovations are like second differences of log(CPI) and are thus more affected by small changes than inflation itself. In order to prevent spurious volatility of these innovations due to noise, we need some smoothing of the deseasonalized $x_i'[\text{Infl}]$ values. An obvious way of doing this is to take a short-term moving average of $x_i'[\text{Infl}]$ in the sense of Equation (10.6):

$$x_i[\text{Infl}] = EMA[x_i'[\text{Infl}]; \Delta t_{\text{smooth}}]$$

(10.33)

We choose a moderately short smoothing time constant $\Delta t_{\text{smooth}}$. There is a trade-off: a large $\Delta t_{\text{smooth}}$ will cause better smoothing, but then $x_i[\text{Infl}]$ will no longer be up to date.

The innovations of the deseasonalized and smoothed inflation are computed by Equation (10.3). This requires a formula for the expectation of inflation. The formula is given in Section 10.3.8, where the interaction of inflation and interest rates is described.

The bootstrapping algorithm leads to simulated values of the deseasonalized and smoothed inflation $x[\text{Infl}]$. We have to reseasonalize $x[\text{Infl}]$ to obtain simulated inflation values $x''[\text{Infl}]$ and cumulate these results to obtain simulated values of log(CPI). Undoing the smoothing of Equation (10.33) is also possible by adding some artificial noise, but this is probably useless in practice.

10.3.8 Interaction and Expectation of Interest Rates and Inflation

In the long run, neither interest rates nor inflation are freely drifting out of their usual range. This fact is implemented in the form of weak mean-reverting forces. We use constant target values, where high precision is not required, as the mean reversion is a very weak force. For inflation, we choose a target $x_{\text{target}}[\text{Infl}]$, for mapped forward interest rates $\bar{z}$ a target $\bar{z}_{\text{target}}$, both based on long-term historical experience.

Empirical research shows that a slightly stronger force holds IR levels and inflation together. In other words, real interest rates ($\approx$ IR minus inflation) have a lower volatility and a stronger mean reversion than IRs or inflation alone. This effect affects IRs of all maturities, but we choose a rather short-term reference maturity $T = m \Delta t$ with a low integer number $m$ to model

\textsuperscript{11}Due to the complex computation procedure with debatable assumptions, CPI figures have natural uncertainties, are computed with a delay and may be modified by posterior corrections. Sometimes, we are forced to extrapolate the most recent CPI value in order to have a complete set of historical data.
it. We define an adjusted forward IR,

$$\varrho_{\text{adj},i} = \varrho_i(m\Delta t) - x_i[\text{Infl}]$$

where $x_i[\text{Infl}]$ results from Equation (10.33). This adjusted rate is similar to a real interest rate, except for the timing: $\varrho_i(m\Delta t)$ refers to a time interval after $t_i$ whereas $x_i[\text{Infl}]$ is the inflation of the interval before $t_i$. It has a sample mean $\bar{\varrho}_{\text{adj}}$ which we estimate from historical data and possibly modify on the basis of expert opinion or a specialized study. At time $t_{i-1}$, the adjusted rate $\varrho_{\text{adj},i-1}$ probably deviates from the mean $\bar{\varrho}_{\text{adj}}$, so we model a force reverting to that mean. We obtain a corresponding target for inflation:

$$x_i'[\text{Infl}] = x_i[\text{Infl}] - \mu [\bar{\varrho}_{\text{adj}} - \varrho_{\text{adj},i}]$$  

(10.34)

with a positive constant $\mu \approx 0.4$ which is discussed below. The target for $\varrho_i(m\Delta t)$ is

$$\varrho_i'(m\Delta t) = \varrho_i(m\Delta t) + (1 - \mu) [\bar{\varrho}_{\text{adj}} - \varrho_{\text{adj},i}]$$  

(10.35)

where $1 - \mu \approx 0.6$ is also positive. The mean-reverting force due to the interaction of interest rates and inflation acts on both variables, but there are empirical indications for a slight lead-lag effect. Inflation affects IR levels slightly more than the other way around. This can be modeled by choosing $\mu < 1 - \mu$. The difference of the two target values of Equations (10.35) and (10.34) is the mean adjusted rate $\bar{\varrho}_{\text{adj}}$, as it should be.

For inflation, we arrive at the following formula for the expectation of $x_i[\text{Infl}]$:

$$E_{i-1}[\text{Infl}_i] = x_{i-1}[\text{Infl}] + \varepsilon_{\text{infl}} \{x_{\text{target}}[\text{Infl}] - x_{i-1}[\text{Infl}]\}$$

$$+ \varepsilon_{\text{adj}} \{x_i'[\text{Infl}] - x_{i-1}[\text{Infl}]\}$$

For mapped forward interest rates, we obtain the following modified version of the expectation of $\tilde{z}_i(T)$:

$$E_{i-1}[\tilde{z}_i(T)] = \tilde{z}_{i-1}(T + \Delta t) + \varepsilon_{\text{IR}} \{\tilde{z}_{\text{target}} - \tilde{z}_{i-1}(T + \Delta t)\}$$

$$+ \varepsilon_{\text{adj}} \sqrt{\frac{m\Delta t}{T}} \{Z(\varrho_{i-1}(m\Delta t), \sigma_z^2(m\Delta t)) - \tilde{z}_{i-1}(T + \Delta t)\}$$

where the function $Z(., .)$ of Equation (10.30) is used to convert a mean-reversion target from an unmapped rate to mapped one. The small factor $\varepsilon_{\text{adj}}$ determines the mean-reversion effect due to the adjusted IR and is slightly larger than the tiny constants $\varepsilon_{\text{infl}}$ and $\varepsilon_{\text{IR}}$. The factor $\sqrt{m\Delta t/T}$ is used to modify the corrections for maturities $T$ other than $m\Delta t$. The choice of this function as well as the diverse $\varepsilon$ parameters and $\mu$ should be made on
the basis of a study of the behavior of IRs and inflation. The resulting expectation $E_{i-1}[\tilde{z}_i(T)]$ is used to compute the innovations of mapped forward interest rates:

$$I_i[\tilde{z}(T)] = \tilde{z}_i(T) - E_{i-1}[\tilde{z}_i(T)]$$

(10.36)

This is the corrected version of Equation (10.31) to be used in the bootstrapping algorithm.

### 10.3.9 Foreign Exchange Rates

Foreign exchange (FX) rates can be treated in a simpler way than interest rates. An FX rate such as EUR/USD is defined as the value of a unit of an exchanged currency (exch, here EUR) expressed in another currency (expr, here USD). We take the logarithm as our mapping function in the sense of Equation (10.2), which is the only function that leads to an equivalent treatment of inverted FX rates (USD/EUR in our example):

$$x_i[FX] = \log FX_i$$

The market forecast of a spot FX rate $x_i[FX]$ at time $t_i$ is the forward FX rate at time $t_{i-1}$. A forward FX rate depends on the difference of interest rates of the two involved currency zones, see Equation (2.2) of Dacorogna et al. [2001]:

$$E'_{i-1}[FX_i] = x_{i-1}[FX] + (R_{\text{expr},i-1} - R_{\text{exch},i-1}) \frac{\Delta t}{\text{year}}$$

(10.37)

where $R_{\text{expr},i-1}$ and $R_{\text{exch},i-1}$ are logarithmic interest rate at time $t_{i-1}$ for the maturity period $\Delta t$ as defined by Equation (10.19), for the exchanged currency (exch) and the currency in which the FX rate is expressed (expr), respectively. Here we express all FX rates in USD, so the index expr always refers to the US market. Equation (10.37) is also known as covered interest parity.

There is no static mean reversion for FX rates, but there is purchasing-power parity (PPP). If the values of consumer good baskets (Section 10.3.7) in two different currency zones strongly deviate, there is a reverting market force which should be added to the forecast $E'_{i-1}[FX_i]$. This force is weak (see the article by Cheung in Chan et al. [2000]) and only matters for long-term simulations. PPP can be modeled as a force towards a target level of the FX rate:

$$x_{\text{ppp},i} = EMA_i[x[FX] - \log CPI_{\text{expr}} + \log CPI_{\text{exch}}; \Delta t_{\text{ppp}}]$$

$$+ \log CPI_{\text{expr},i} - \log CPI_{\text{exch},i}$$

where the exponential moving average of Equation (10.6) is used. PPP is a slow effect, and the time constant $\Delta t_{\text{ppp}}$ should be long enough to cover more
than one PPP cycle (many years). An extremely large $\Delta t_{ppp}$ would not be appropriate, because the consumer price indices (CPI), as values of baskets of varying composition, may not be entirely consistent in two countries over many decades.

The technical forward rate $E'_{i-1}[FX_i]$ is now modified by a weak force towards $x_{ppp,i-1}$ which is proportional to the distance between the two values:

$$E_{i-1}[FX_i] = E'_{i-1}[FX_i] + \varepsilon_{ppp} [x_{ppp,i-1} - E'_{i-1}[FX_i]] \tag{10.38}$$

The force is weak, so $\varepsilon_{ppp}$ is a small constant. The innovations of FX rates can now be computed by Equation (10.3), using the expectation $E_{i-1}[FX_i]$.

### 10.3.10 GDP

The modeling of the Gross Domestic Product (GDP) is rather simple. We use real (= deflated or inflation-corrected) GDP figures\footnote{Due to the complex computation procedure, GDP figures are computed with a delay and may be modified by posterior corrections. Sometimes, we are forced to extrapolate the most recent GDP value in order to have a complete set of historical data.} rather than the nominal GDP, as this is less volatile. There are reliable, yearly OECD data that can be used to verify the quarterly data. The mapped real GDP is logarithmic:

$$x_i[GDP] = \log GDP_i$$

The volatility of the real GDP is modest as compared to the fluctuations of other variables such as equity indices or FX rates. Thus, normal applications (with no particular emphasis on GDP) do not need a sophisticated model for the market expectation of GDP growth. We simply take the sample mean of historical growth:

$$E_{i-1}[GDP_i] = x_{i-1} + \frac{1}{n} \sum_{j=1}^{n} x_j[GDP] \tag{10.39}$$

This is our expectation of $x_i$ made at time $t_{i-1}$ for any $i$, in all historical and simulated cases, independent of the market situation. This may be improved if an elaborated GDP model is available. Following Equation (10.3), the GDP innovation is

$$I_i[GDP] = x_i[GDP] - E_{i-1}[GDP_i]$$

The further steps of the algorithm follow the standard procedure as described in Sections 10.2.1 and 10.2.2. If we need a nominal GDP figure historically or in a simulation, we can always compute it as

$$NominalGDP = c \, GDP \cdot CPI = c \, e^{x[GDP]+x[CPI]} \tag{10.40}$$

where a constant $c$ determines the basis of the nominal GDP.
10.3.11 Equity Indices

Many applications focus on the performance of equity investments. Thus we prefer modeling total-return (or gross) indices that are based on the assumption that dividends are immediately and fully reinvested. An obvious choice is to take standard indices such as the MSCI gross indices which are available for all major currency zones. It is possible to have several equity indices per currency zone in the same scenario generator. We can technically treat price indices (excluding dividends), sector indices, hedge-fund indices or real-estate indices like the main total-return index.

Equity indices have a high volatility which may lead to extreme drifts in long-term simulations based on pure resampling. Is there a mean-reverting force to prevent this? In the long run, the stock exchange economy cannot arbitrarily drift away from the real economy as expressed by the GDP. Indeed, the equity-to-GDP ratio does not show a large trend over time. In the 1920s, before the Black Friday, the ratio of the Standard & Poors 500 index to the US GDP reached similar levels as in the late 1990s. This notion leads us to choosing the equity-GDP ratio for resampling. The mapping is

\[ x_i[\text{Equity}] = \log \text{EquityIndex}_i - (\log \text{GDP}_i + \log \text{CPI}_i) \]

The equity index has no inflation correction. Therefore we need the nominal GDP for our equity-GDP ratio, using the CPI as in Equation (10.40).

Now we compute the market expectation\(^{13}\) of \(x_i[\text{Equity}]\). The simplest model is taking the mean growth within the historical sample as a constant expectation, similar to Equation (10.39):

\[ \mathbb{E}_{t-1}[\text{Equity}_t] = x_{i-1}[\text{Equity}] + \frac{1}{n} \sum_{j=1}^{n} x_j[\text{Equity}] \]

A first refinement is to add a bias to this market expectation of growth. If an external model gives rise to a certain assumption on future growth, this can be incorporated in the formulation of the market expectation. We are doing this in our scenario generator. For long-term simulations, we need a mean reversion of our equity-GDP ratio. The value of \(x_i[\text{Equity}]\) slowly reverts to a static long-term mean (a good assumption for price indices) or to a slow growth path (for total-return indices). This mean reversion can be implemented as in the case of FX rates, similar to Equation (10.38).

Equity returns have been found to have a negatively skewed distribution. Rare crashes imply few strongly negative returns, whereas long boom phases generate many moderately positive returns. This skewness of historical stock returns will be reflected by the resampled innovations and thus maintained

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\(^{13}\)We do not use market-implied forecasts extracted from market prices of derivatives because these are generally not available for the time horizons we are interested in (especially in the case of long-term simulations).
by the bootstrapping method, at least when looking at one-step returns, i.e. quarterly returns in our implementation. The historical correlation between stock and bond returns is also respected. Returns of long-term bond portfolios are mainly driven by changes in IR levels (rather than the IR levels themselves). Since IR innovations values are resampled along with equity index innovations, a realistic correlation between bond and equity returns will be reproduced by the simulation. This is important for the application to asset allocation, where most portfolios contain bonds as well as equity.

As an example for simulation results of the economic scenario generator (ESG), US yield curves are shown. The bold, solid US yield curve of the end of 2002 marks the starting point of the simulation. The thin yield curves are simulated for the end of 2003, based on the information set available at the end of 2002, representing nine different simulation scenarios. The bold, dotted curve is the yield curve actually observed at the end of 2003. We can compare this curve to the set of simulated curves. Such comparisons are a basis for backtesting. In usual simulation studies and in backtests, we use thousands of scenarios rather than just nine of them.

Figure 10.4: Simulation of US yield curves.

10.4 Results and Testing

10.4.1 Calibration

As a non-parametric method, the pure bootstrapping algorithm has a natural advantage over parametric models: There is no need for any calibration.
Historical behaviors are reproduced in the simulated future. If past behaviors provide a reliable guidance for future behaviors (which is not always an accurate assumption), the simulated results will automatically be in line.

On a subordinated level, our refined bootstrapping method has some parametric elements such as some mapping formulas, GARCH and small corrections of market expectations, mainly the weak mean-reversion effects of some variables. The models of these refinements rely on a few parameters that were calibrated by some special studies during the development phase. Some calibrations are done as a preparatory step at the beginning of each new scenario generation, based on the same historical data that are also used by the bootstrapping method. This is the case for the GARCH calibrations as presented in the Appendix 10.6.

At the end of our development process, some experienced finance specialists looked at many graphs with first simulation results. Given the complexity of setting up a comprehensive set of calibration and testing procedures, the value of judgments by human experts on real-life results should not be underestimated.

Although a calibration procedure may lead to an in-sample quality measure of a model, it does not provide an independent, reliable quality test. At the end, an out-of-sample test is necessary to finally assess a scenario generator and its results.

### 10.4.2 Example of Results: Yield Curve Simulation

An economic scenario generator produces a wealth of results: many complete scenarios for many economic variables, simulated over many time steps. In Figure 10.4, we show a tiny part of these results for the example of simulated US yield curves. The historical curve for the end of 2002 marks the starting point of the simulations. The nine simulated curves are for the end of 2003, based on four quarterly simulation steps. Each curve belongs to a complete, consistent scenario for all economic variables, including interest rates with times to maturity up to 30 years, yield curves for all currencies and many other variables. Although the simulation period is just one year, we see a variety of yield curve forms on different levels. Simulation 3 has an inversion for maturities up to two years, simulation 6 has a more humped form.

The true US yield curve at the end of 2003 is also plotted in Figure 10.4 as a bold, dotted curve. It lies slightly above the curve of 2002. The simulated curves show a wider range of deviations. This indicates a considerable IR risk at start of 2003; the actual move in 2003 was rather small in comparison. The overall IR level increased, which is in line with the majority of simulated curves. The very low short rate at the end of 2003 is below most of the simulated values, indicating a particular low-rate policy of the US Federal Reserve.
10.4.3 Out-of-Sample Backtesting

Our quality testing method is backtesting, see e.g. Christoffersen [2003]. Comparisons between historical variable values and their prior scenario forecasts, as in Figure 10.4, are a basis for backtesting. In Chapter 8 of Christoffersen [2003], backtests are proposed for three different types of forecasts: (1) point forecasts for the value of a variable, (2) probability range forecasts (e.g. the value at risk [VaR] which is the projected quantile at a certain probability, often 1%) and (3) forecasts of the complete probability distribution. Such distribution forecasts are the most comprehensive type as they imply range forecasts and point forecasts (using the mean or median of the distribution, for example).

Scenarios produced by a scenario generator are no forecasts in the usual sense. In typical studies, we produce many thousands of scenarios. Each of these scenarios has its own forecast value for a certain variable at a certain future time. All the scenario values together define an empirical distribution for the variable. Hence we have distribution forecasts rather than just point or range forecasts.

Our task is comprehensive out-of-sample backtesting of distribution forecasts. Even the limited task of testing specialized models such as an interest rate model is difficult, as discussed in Section 1.5.2 of James and Webber [2000]. Here we propose a methodology based on the Probability Integral Transform (PIT). Diebold et al. [1998, 1999] have introduced the PIT (also known as Lévy or Rosenblatt Transform) as a method for testing distribution forecasts in finance. The whole test is described in detail in Blum [2004]. This is a summary of the steps:

1. We define an in-sample period for building the bootstrapping method with its innovation vectors and parameter calibrations (e.g. for the GARCH model). The out-of-sample period starts at the end of the in-sample period. Starting at each regular time point out-of-sample, we run a large number of simulation scenarios and observe the scenario forecasts\(^\text{14}\) for each of the many variables of the model.

2. The scenario forecasts of a variable \(x\) at time \(t_i\), sorted in ascending order, constitute an empirical distribution forecast. In the asymptotic limit of very many scenarios, this distribution converges to the marginal cumulative probability distribution \(\Phi_i(x) = \mathbb{P}(x_i < x | I_{i-m})\) that we want to test, conditional to the information \(I_{i-m}\) available up to the time \(t_{i-m}\) of the simulation start. In the case of a one-step forecast, \(m = 1\). The empirical distribution \(\hat{\Phi}_i(x)\) slightly deviates from

\(^{14}\)Our main test is for one-step forecasts where the simulation is for one time step (a quarter in our case). Multi-step forecasts can be tested using the same methodology, but the number of available independent observations with non-overlapping forecast intervals will be distinctly smaller, given the same out-of-sample period.
this. The discrepancy $\Phi_i(x) - \hat{\Phi}_i(x)$ can be quantified by using a formula given by Blum [2004]. Its absolute value is less than 0.019 with a confidence of 95% when choosing 5000 scenarios, for any value of $x$ and any tested variable. This is accurate enough, given the limitations due to the rather low number of historical observations.

3. For a set of out-of-sample time points $t_i$, we now have a distribution forecast $\hat{\Phi}_i(x)$ as well as a historically observed value $x_i$. The cumulative distribution $\hat{\Phi}_i(x)$ is used for the following Probability Integral Transform (PIT): $Z_i = \hat{\Phi}_i(x_i)$. The probabilities $Z_i$, which are confined between 0 and 1 by definition, are used in the further course of the test. A proposition proved by Diebold et al. [1998] states that the $Z_i$ are i.i.d. with a uniform distribution $U(0, 1)$ if the conditional distribution forecast $\Phi_i(x)$ coincides with the true process by which the historical data have been generated. The proof is extended to the multivariate case in Diebold et al. [1999]. If the series of $Z_i$ significantly deviates from either the $U(0, 1)$ distribution or the i.i.d. property, the model does not pass the out-of-sample test.

Testing the hypotheses of $U(0, 1)$ and i.i.d. can now be done by using any suitable method from statistics. We pursue two approaches here:

1. An approach which we call non-parametric is suggested by Diebold et al. [1998, 1999]. It consists of considering histograms in order to detect deviations from the $U(0, 1)$ property, and correlograms of the $Z_i$'s and their low integer powers to detect deviations from the independence property. We complement these graphical evaluations by the usual $\chi^2$ test for uniformity, and by Kendall-Stuart bounds for the significance of the autocorrelations.

2. Chen and Fan [2004] suggest another approach, which we call parametric. It relies on the assumption that the $Z_i$’s form a Markov chain with stationary distribution $G^*(\cdot)$ and copula $C^*(\cdot, \cdot)$ for the dependence structure of $(Z_t, Z_{t-1})$. One can then select some model for $G^*(\cdot)$ which contains $U(0, 1)$ as a special case, and some model for $C^*(\cdot, \cdot)$ which contains the independence copula as a special case. The joint null hypothesis of independence and uniformity can then be tested by standard likelihood ratio or Wald procedures. We specifically use the Farlie-Gumbel-Morgenstern copula as a model for dependence structure and the $\beta$-distribution as a model for the marginal distribution. In a semi-parametric variant of this procedure, no model for $G^*(\cdot)$ is chosen, but the empirical distribution of the $Z_i$’s is plugged in instead. This allows to test for the isolated hypothesis of independence, irrespective of the marginal distribution.
Out-of-sample backtesting: uniform distribution of PIT-transformed variables. The frequency of empirically found probabilities $Z_i$ (results of the Probability Integral Transform, PIT) is plotted. A model is rejected if such a histogram significantly deviates from a uniform distribution, corresponding to a low p-value of the $\chi^2$ test ($p < 0.05$). The left histogram is based on all economic variables, whereas some short- and medium-term interest rates are excluded from the computation of the other histograms. The dashed lines indicate a 95% confidence range for the individual frequencies.

Figure 10.5: Out-of-sample backtesting.

A rejection by one of these tests does not necessarily mean that a model is valueless. It means that the model does not live up to the full predictive potential indicated by the data or that there is a structural difference between the in-sample and out-of-sample periods.

When applying the tests to our ESG results, the limited number of historical observations poses a problem. For a few economic variables, we have decades of historical data, but we are more or less restricted to the period after September 1993 when constructing our comprehensive ESG with many variables and many currencies. This leaves little space for defining a reasonable out-of-sample period.

Here we report a study made in summer 2004. We plan to make similar studies from time to time in order to profit from growing sample sizes. In order to increase the size of the out-of-sample period, we reduced the in-sample period (which normally covers ten years) to eight years, from end of September 1993 to September 2001. We obtained an ESG with only 32 quarterly innovations, which implied a less stable behavior than the production version with 40 innovations. This reduced ESG was tested out of sample. The out-of-sample period started at the end of September 2001 and ended in June 2004, which allowed for testing 11 one-step forecasts, i.e. 11 observations of PIT-transformed values $Z_i$ per economic variable. This is a low number for any statistical test. However, we obtained a sizable total number of $Z_i$ observation by considering all the economic variables for all the currencies. Our tested variables were equity index (MSCI gross), FX
rate against the USD, CPI and GDP. We added four interest rates to this set of variables, namely the extremes on our maturity scale, the three-month and the 30-year rates, and two intermediate rates with times to maturity of two years and ten years. Thus we obtained eight variables for each of the six currency zones (USD, EUR, JPY, GBP, CHF, AUD). We linked the small $Z_i$ series of all variables together to obtain a set of 528 ($= 11 \cdot 8 \cdot 6$) observations of $Z_i$.

The $\chi^2$ test of the 528 $Z_i$ observations and the underlying histogram is shown on the left-hand side of Figure 10.5. The p-value of 0.0607 exceeds the confidence limit of 0.05. The ESG forecasts are not rejected, but the low p-value does not instill wholehearted confidence. An autocorrelation analysis reveals a marginally significant first-lag autocorrelation between the $Z_i$. The semi-parametric evaluation has a high p-value and does not reject the ESG forecasts. The likelihood ratio test of the parametric evaluation, which is the most powerful test, significantly rejects the null hypothesis of i.i.d. $U(0, 1)$ with a p-value of only 0.00021, which is far below a confidence limit of 0.05.

We have to accept the fact that the ESG forecasting method was rejected by our most powerful test. Fortunately, the testing methods also inform us on what exactly is rejected, and why. A closer look at the investigated out-of-sample period shows that our out-of-sample period is characterized by a fundamental difference from the in-sample period. It covers an economic situation after a marked decline of equity markets. The worsening economic situation caused low demand, low inflation and low interest rates. Most importantly, the US Federal Reserve chose a distinct policy which kept short-term interest rates low and the US yield curve artificially steep. This policy is specific to the years 2001–2004 and distinctly different from the policies of the in-sample period and the 1980s. It led to low values of low and medium term interest rates, much lower than the market forecasts based on forward interest rates indicated\(^{16}\). The example of Figure 10.4 can be seen as an illustration of the unexpectedly low short-term interest rates caused by this policy. In the first histogram of Figure 10.5, the low rates materialize in the significantly high frequency of $Z_i$ values in the leftmost bar.

Our hypothesis is that the unusual low-interest policy is the reason for the rejection of the forecasts. We test this hypothesis by excluding the three-month, two-year and ten-year interest rates, so the 30-year rate is the only interest rate in the test. In an analysis called study B, we do this only for the currencies USD (directly affected by the US Federal Reserve policy) and

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\(^{15}\)For the currency USD, choosing the FX rate against itself makes no sense. Instead, we added a US hedge-fund index to the US-based variables to be tested.

\(^{16}\)Some traders made a successful bet on the persistence of this anomaly and made profitable carry trades. They financed long-term deposits by rolling short-term debts forward.
EUR and CHF, where the central banks followed similar, if less pronounced policies. Thus the currencies JPY, GBP and AUD still have a full coverage of interest rates. Study B has a sample of 429 \( Z_i \) observations. In study C, we exclude short and medium term interest rates for all currencies and arrive at a sample of 330 observations. In both studies, B and C, the ESG forecasts are no longer rejected by any test. The \( \chi^2 \) tests have p-values of 0.1235 (B) and 0.2875 (C), both on the good side of the confidence limit of 0.05, see the middle and right histograms of Figure 10.5. The strongest test, the parametric evaluation, confirms this with p-values of 0.2313 (B) and 0.6017 (C). We conclude that the ESG forecasts are rejected only in the case of low and medium term interest rates of USD, EUR and CHF. Thus we report a qualified success of our ESG forecasts.

Is there a way to improve the method in order to give optimal forecasts for all variables? This is only possible if factors such as the policy of the US Federal Reserve or, more generally, economic cycles can be predicted. Neither the bootstrapping method nor any of its algorithmic modifications are able to do this, to our knowledge. Long data samples covering many decades and many economic cycles would help, but we are restricted to shorter samples for most of the modeled economic variables. Shifts in policies, economic cycles and market structures make future developments less predictable. In our bootstrapping method, a way to accommodate this would be to augment the resampled innovations by a slowly growing factor. Technically, this can be done the same way as the tail correction of Section 10.2.10, using Equation (10.9) with an increased cycle uncertainty multiplier.

A new test planned for 2008 will produce more significant results because the size of its out-of-sample period will more than double in comparison to that of the summer 2004 study. The plan is to repeat the test regularly using a newly developed, automated testing environment.

Although the tests based on PIT are powerful, they cannot test all possible aspects of model quality. Several competing models or simulation methods might pass a PIT-based test at the same time, but one model might still be better than another\(^\text{17}\). Some properties stay untested in our PIT-based method, most notably the dependence between returns of different variables in the simulated scenarios. We have added a study comparing correlations of simulated returns to those of actual returns, with good results. This is expected for a bootstrapping method which preserves dependencies in the innovations by design.

\(^{17}\)Example: Some variables might follow a complex nonlinear process that is captured by model A, whereas model B sees the same behavior as random noise. While none of the models is rejected in a PIT-based test, the nonlinear model A is better as it predicts narrower distributions.
10.5 Conclusion

Refined bootstrapping is our method to generate realistic scenarios of the future behavior of the global economy as represented by a set of key variables. We have presented many details that need to be observed in order to arrive at a realistic behavior of many different economic variables such as interest rates, foreign exchange rates, equity indices, inflation and GDP for several currency zones. A careful treatment of these modeling details, which include some subordinated parametric elements, is vital for the success of the bootstrapping method.

The following advantages of the bootstrapping method have been found:

- Wide coverage of economic variables, modularity and flexibility when extending the set of covered economic variables
- Automatic preservation of distributions and simultaneous dependencies between the innovations of different economic variables
- Exact reproduction of initial conditions at simulation start (no fitting of a model needed for that)
- Feasibility of long-term simulations (over decades), due to mean-reversion elements in expectations of variables
- Natural transition from the short-term behavior at start to the long-term behavior
- Easy ways to introduce modifications based on special studies or expert opinion (e.g. assuming expected equity returns lower than the mean of the historical sample)
- Good coverage of extreme risks, relying on the tail correction of Section 10.2.10 and large numbers of simulations
- No large calibration problems because the method is essentially non-parametric

Out-of-sample tests have confirmed the validity of the approach. A certain problem arises from the behavior of short and medium term interest rates of some currencies, reflecting an unusual low-interest policy of central banks during the out-of-sample period. We have discussed this behavior and possible solutions.

The final goal of our project has always been the application of the method in practice. We have implemented the refined bootstrapping method in our economic scenario generator (ESG). The results are regularly applied to Asset-Liability Management (ALM) studies that are part of the strategic decision making of the analyzed companies. We plan to include corporate
yield spreads and possibly other economic quantities to the set of bootstrapped variables in order to add new asset classes such as corporate bonds to ESG-based asset allocation studies.

10.6 Appendix: Robust Calibration of a GARCH Process

In Equation (10.8), a GARCH(1,1) process is defined. In our application, we need an especially robust calibration procedure. Following Zumbach [2000], we do not directly calibrate the three parameters $\alpha_0$, $\alpha_1$ and $\beta_1$. We rather reformulate the equation for the conditional variance as follows:

$$
\sigma_1^2 = \sigma^2 + \mu_{\text{corr}} \left[ \mu_{\text{ema}} \sigma_{i-1}^2 + \left(1 - \mu_{\text{ema}} \nu_i^2 - \sigma^2\right) \right], \quad (10.41)
$$

$$
\mu_{\text{corr}} = \alpha_1 + \beta_1, \quad \mu_{\text{ema}} = \frac{\beta_1}{\mu_{\text{corr}}}, \quad \sigma^2 = \frac{\alpha_0}{1 - \mu_{\text{corr}}}
$$

The parameters $\mu_{\text{corr}}$ and $\mu_{\text{ema}}$ have values between 0 and (less than) 1. While $\mu_{\text{corr}}$ describes the decay of the memory in conditional volatility, $\mu_{\text{ema}}$ determines the depth of averaging in the formation of the volatility memory.

The unconditional variance $\sigma^2$ is no longer regarded as a model parameter to be optimized through maximum likelihood. Instead, we directly take the empirical variance of the raw innovations as the moment estimator for $\sigma^2$. Thus we make sure that the unconditional variance of the process equals the empirical variance even if the GARCH process is misspecified or finite-sample problems lead to difficult behavior.

The two parameters $\mu_{\text{corr}}$ and $\mu_{\text{ema}}$ remain to be calibrated. The resulting GARCH(1,1) embeds two other processes: ARCH(1) if $\mu_{\text{ema}} = 0$ and a Gaussian random walk (Brownian motion, white noise) if $\mu_{\text{corr}} = 0$. In the latter case, the value of $\mu_{\text{ema}}$ becomes irrelevant.

The GARCH equation is evaluated iteratively at each time series point with index $i$. Therefore all $\mu$ parameters correspond to an exponential decay with time constant $\tau$:

$$
\mu_{\text{corr}} = e^{-1/\tau_{\text{corr}}}, \quad \mu_{\text{ema}} = e^{-1/\tau_{\text{ema}}}
$$

(10.42)

$$
\tau_{\text{corr}} = -\frac{1}{\log \mu_{\text{corr}}}, \quad \tau_{\text{ema}} = -\frac{1}{\log \mu_{\text{ema}}}
$$

(10.43)

where the $\tau$ values are in units of the time step of the time series.

If the maximum-likelihood procedure leads to a $\mu$ very close to 1, the time constants $\tau$ may reach extremely high values. Reason demands that $\mu_{\text{ema}}$ becomes irrelevant.

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$^{18}$Zumbach [2000] transforms $\mu_{\text{corr}}$ and $\mu_{\text{ema}}$ to other fitting variables by further mapping. We do not follow that approach as it pushes ARCH(1) and white noise (which are perfectly acceptable solutions) far away to the asymptotic limits of the parameter space.
we do not choose a time constant exceeding the sample size. This is why
our robust method sets an upper limit for $\tau$:

$$\tau_{\text{max}} = f \ n$$

where $n$ is size of the sample used for fitting and $f$ a constant factor; we
usually take $f = 0.5$. If we use a 10-year sample, for example, we do not ac-
cept decay models with time constants longer than five years. At the limit,
there are only two five-year volatility clusters within the 10-year sample,
at maximum. Two observations are not a large amount in statistics. This
fact may lead to an intuitive understanding of why we are not willing to ac-
cept even longer clusters with even lower significance in our robust GARCH
fitting procedure. Our condition is

$$0 \leq \tau_{\text{corr}} \leq \tau_{\text{max}}, \quad 0 \leq \tau_{\text{ema}} \leq \tau_{\text{max}} \quad (10.44)$$

$$\mu_{\text{corr}} \leq e^{-1/\tau_{\text{max}}}, \quad \mu_{\text{ema}} \leq e^{-1/\tau_{\text{max}}} \quad (10.45)$$

where the conditions for $\mu$ are derived from (10.42), (10.43). The un-
constrained solution of most practical fitting cases anyway obeys (10.44),
(10.45). However, in some misspecified or small-sample cases, the maxi-

mum likelihood may lie outside those conditions, and we prefer the robust
solutions ensured by Equation (10.44), (10.45). Our solutions not only observe the
stationarity limit condition but also keep a safe distance from that limit.

The logarithm of the likelihood function$^{19}$ is

$$l(\mu_{\text{corr}}, \mu_{\text{ema}}) = -\frac{1}{2} \ln \sigma^2 \left[ n \ln 2\pi + \ln \sigma^2 + \frac{r^2}{\sigma^2} \right]$$

with a total number of $n + m$ observations in the sample. We reserve a
considerable number $m$ of initial observations for the build-up of $\sigma^2_i$. At
start ($i = 1$), we use the initial value

$$\sigma^2_0 = \sigma^2$$

which has an initial error that exponentially declines over the GARCH it-

erations, Equation (10.41), from $i = 1$ to $m$. The larger $m$, the smaller is
the remaining error of $\sigma^2_i$. However, the remaining sample of size $n$ also
becomes smaller, given a limited total size $n + m$. This is a trade-off. In

$^{19}$This formulation assumes that $\varepsilon_i$ is generated by a Gaussian random process, which
is an acceptable assumption for our low-frequency data and the limited role of GARCH
within the whole bootstrapping algorithm. Assuming another distribution of $\varepsilon_i$ leads to
a different log-likelihood function.
our low-frequency case with quarterly data, this trade-off is almost desper-
ate. A 10-year sample has 40 quarterly observations – a modest number.
We need these 40 observations for the likelihood function in order to pro-
duce meaningful results. Reserving 20 observations for build-up and using
the remaining, meager 20 observations for GARCH fitting does not seem
to be a reasonable approach. For some economic variables, we have past
data older than ten years that we can use for the build-up. For some other
time series, this is not available. As a numerical trick, we can recycle the
scarce available data to build up an initial $\sigma_i^2$ through a zig-zag method.
We create a synthetic past. The real data are $r_{2n+1} \ldots r_{3n}$, so $m = 2n$;
the synthetic past consists of $r_1 \ldots r_n$ with $r_i = r_{2n+i}$ and $r_{n+1} \ldots r_{2n}$ with
$r_i = r_{4n+1-i}$. This is justified as the innovations $r_i$ are already detrended
and their temporal coherence, which is important for GARCH, is respected,
though partially in reverse order. We claim that the thus obtained $\sigma_{2n}^2$ value
is a better approximation of the true value than a simple initialization $\sigma_{2n}^2 = \sigma^2$. Of course, this claim should be substantiated through a theoretical or
statistical study.

Now we have to determine the maximum of the log-likelihood of Equa-
tion (10.46) by varying the parameters $\mu_{corr}$ and $\mu_{ema}$ under the constraints
of Equations (10.44), (10.45). This cannot be done analytically. The solution
of this non-linear optimization problem can be done with the help of
any appropriate method. All numerical methods lead to a local optimum
depending on the initial guess of parameter values. In order to obtain the
global optimum, it is important to run the optimization from different initial
parameters and to take the best among the obtained solutions. For some
GARCH(1,1) processes with large values of both $\mu_{corr}$ and $\mu_{ema}$, the white
noise solution ($\mu_{corr} = 0$) appears as a local optimum that is dominated by
the true, global optimum. Therefore one should always start optimizations
from at least two initial points: (1) white noise and (2) close-to-maximum
values of $\mu_{corr}$ and $\mu_{ema}$.
11

Interest Rate Hedges

Under the definition of interest rate hedges we classify those financial instruments that react to interest rate increases with an increase in market value. Within an investment portfolio such negative duration instruments represent a hedge against the losses that the bond investments would incur in case interest rates increase.

The document to describe the interest rate hedges is Debora Iannuzzi, Modelling of interest rate hedges in Converium’s internal ALM model, (Iannuzzi [2007]). The model for interest rate hedges is primarily determined by pricing formulas in conjunction with economic scenario generation (ESG), the latter amply documented in Müller et al. [2004]. The document on the interest rate hedges (Iannuzzi [2007]) aims to specify the type of the replicating assets and their pricing methods. This documentation of a model for interest rate hedges follows Iannuzzi [2007].

SCOR Switzerland’s portfolio currently has two classes of negative duration instruments in the three main currencies, USD, EUR and GBP:

- Swaptions with 6-month or 1-year maturity
- Structured notes with 5- or 10-year maturity: the Digital Notes, issued by the Bayerische Hypo- und Vereinsbank AG (HVB)

The economic scenario generator (ESG) generates scenarios of yield curves with a quarterly time granularity. The scenarios are imported in Giulio\(^1\) and therein linearly interpolated to daily steps. In correspondence of each of these daily scenarios Giulio calculates the market value of the hedges, as described in the following sections.

\(^1\) Giulio is SCOR Switzerland’s asset and liability model written in Igloo. The stochastic simulations based on the market risk model are largely performed in Igloo which is a technology applied in the simulation of multivariate probability distributions. Monte Carlo simulations of assets and liabilities are written in Igloo. For an overview of IT systems we refer to the documentation of IT Systems (Part VIII).
11.1 Swaptions

The price of the swaptions is calculated via traditional methods of option pricing (see Chapter 18 of Neftci [2004]).

Each swaption is assumed to hedge a portfolio of bonds (there are usually more swaptions for the same portfolio, with different durations). The corresponding bond portfolio is defined simply as the portfolio of bonds that have the same currency as the swaption (independent of the duration). We assume that the ratio $R$ between the swaption notional and the bond portfolio is given. This ratio can be considered as an intrinsic property of the swaption itself.

The swaptions are assumed to follow a rollover strategy. At expiry, each swaption is sold at market value and a new swaption is bought with the same duration and currency as the previous. The maturity of the new swaption is assumed to be always one year and the strike rate 75bps out-of-the-money. To calculate the notional needed for the swaption the model calculates the market value of the corresponding bond portfolio at the time of the transaction and multiplies it by the aforementioned ratio $R$.

This rollover strategy mimics the behavior of the asset manager ensuring that the swaption notional follows the market value of the bond portfolio that is intended to be hedged. We do not need to redefine the duration of the new underlying swaps because we assume that the asset manager follows a strict allocation target, including duration and currency splits. The bond portfolio, as well as the whole asset portfolio, is reallocated once a year (in the middle of the year) in such a way that the asset allocation, duration and currency split at the beginning and at the end of the year are the same. The reallocation takes into account all cash in- and outflows, including the trade of the swaptions. For the details regarding bond and equity modeling see Chapters 12 and 13 in this documentation.

11.2 Structured Notes

The structured notes require an initial investment, the nominal value of the note, which is guaranteed to be returned at expiry (after five or ten years from the issue date, depending on the contract). Within the life-span of the product, the bank will pay a certain coupon for each day when the reference interest rate is above a trigger rate, which was determined at inception of the product. The coupon is defined as a function of the reference rate, which is fixed once a year. In formulae, if $N$ is the notional, $T$ the trigger rate, $L$ the reference interest rate (in our case the Libor in a given currency, variable on a daily basis) and $Y$ the reference rate (equal to the Libor, but fixed once
a year), then the coupon $C$ (annualized) is defined as:

$$C = \begin{cases} 0, & \text{if } L < T \\ 1.99NY & \text{if } L \geq T \end{cases}$$

(11.1)

It is clear that, when interest rates increase, the market value of the notes will increase as well. Therefore these products will be typically chosen by investors who want to protect their bond portfolio against losses deriving from extreme increases in interest rates.

The calculation of the market value of the notes as a function of the interest rate curve is not straightforward. For ease of implementation, we have chosen a valuation method discussed with the issuing bank, HVB. The resulting valuation formula can be seen as a polynomial approximation of what the correct valuation formula would be.

The method is based on the so called *delta buckets*. In short, the method provides, for each note, yield shift and maturity, the impact of a shift of 25bps on the market value (excluding accrued interest) expressed as a percentage of the current market value of the note. The *delta buckets* are provided to SCOR Switzerland by HVB on a regular basis. For consistency reasons, HVB delivers also the volatility of SCOR Switzerland’s swaptions as of the date of the analysis of the notes. Such volatility is the one currently used in the option pricing formula. No volatility modeling is currently implemented.

The *delta buckets* method allows us to estimate the market value of the notes at the end of the year excluding the accrued interests. The coupon payments and the accrued interests are then easily calculated as a function of the reference Libor rates simulated by the ESG model according to the formula expressed by Equation (11.1).

Given that the five- or ten-year maturity notes had inception at the end of 2006, no rolling strategy is needed yet. The notes interact with the rest of the portfolio just due to the cash flow they produce via the coupon payments. Also this cash flow is assumed to happen in the middle of the year and reinvested within the rolling strategy of the rest of the asset portfolio.

### 11.3 Final Goal

The final goal of these hedging instruments is to lower the risk contribution of the fixed income portfolio to the shareholder equity worst case scenarios. This, of course, can only be monitored when the full ALM model is updated, whereas on a more frequent basis we can monitor stand-alone risk figures of the fixed income portfolio with and without interest rate hedges.

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2The proportionality factor in Equation (11.1) is chosen consistently with the treaty information contained in the contracts between the company and its banking partners.
11.4 Sensitivity

The sensitivity analysis of the interest rate hedges is done using the same methodology described so far: option pricing for the swaptions and bucket delta method for the notes. The only difference is that the analysis is done in an Excel file instead of Giulio. The same file is also used to crosscheck the results of Giulio.

11.5 Valuation Formulae

11.5.1 Swaptions

Inputs:
- Expiry of the option $E_{op}$
- End of swap $E_{sw}$
- Strike rate $r$
- Fixed leg $L_{fix}$ (which is 1 for EUR, 0.5 for GBP and USD)
- Notional (million) $N$
- Yield $y_i$ at time $i$ (interpolated, if necessary)
- Volatility $\sigma$

Valuation Formula: The market value at time $t$ (given the assumptions regarding the rollover strategy, for us $t$ is always smaller than or equal to $E_{op}$) is

$$M_t = NA \left[ P \cdot \Phi(D_1) - r \cdot \Phi(D_2) \right]$$

where $\Phi(\cdot)$ denotes the cumulative of the standard normal distribution, and we used

$$A = L_{fix} \sum_i (1 + y_i)^{-(i-t)}$$

with the time intervals $i$ determined according to the time leg $L_{fix}$. $P$ is the par yield

$$P = \frac{1}{A} \left[ (1 + y_{E_{sw}})^{-(E_{sw} - t)} - 1 \right]$$

while $D_1$ and $D_2$ are defined as

$$D_1 = \frac{\ln \left( \frac{P}{r} \right) + \frac{1}{2} \sigma^2 (E_{op} - t)}{\sigma \sqrt{E_{op} - t}}$$

$$D_2 = D_1 - \sigma \sqrt{E_{op} - t}$$
11.5.2 Notes

Inputs:

- Notional (million): $N$
- Accrued interest up to the valuation time $t$ (actual - for the spring exercise – or estimated using the reference yield curve and Equation (11.1) – for the autumn exercise): $M_0$
- Bucket delta matrix with entries $A_{ij}$ (for each note and each projection date different) where $i$ is the delta shift with steps of 25bps (from $-300$ to $+300$bps) and $j$ is the maturity ($j = 0m, 3m, 6m, 9m, 1y, 2y, 3y, 4y, 5y, 7y, 8y, 9y, 10y, 11y$)
- Reference yield curve (provided by the bank) $(f_j)_{j=0m,\ldots,11y}$ where $j$ is the maturity

Valuation formula: From the matrix $A_{ij}$ we extract, by linear interpolation, the matrix $B_{kj}$ where $k$ is the delta shift with steps of 1bp. Then the market value $M$ is defined as a function of a generic yield curve $G = (g_j)_{j=0m,\ldots,11y}$:

$$M(G) = N \sum_{k=0}^{g_j-f_j} \sum_{j=0m}^{11y} (B_{kj} + M_0)$$
12

Bond Portfolio Management

12.1 Introduction: Bonds, Equities, Cash

Simulating stochastic time series of bond and equity portfolios implies matching the cash flows between assets and liabilities (Müller [2006c]). Cash flow management is an essential ingredient to the risk model. While cash flows from or to the insurance liabilities are determined by the liability components themselves, the cash flows from and to the assets are subject to allocation guidelines. Cash flow management acquires an adjustment function when imbalances arise between the various asset classes caused by the inhomogeneous performance of asset classes. In multi period simulations the cash flows assigned to an asset component are modeled on the following assumptions: For each of the simulated time steps information is received from the modeled asset classes about the market values and return factors. The desired market value share which shall be sustained at the end of a simulation step must be additionally specified for each asset portfolio. The exchange of cash flows takes place at mid period of each simulated time interval in a multi period simulation.

Accounting for equity portfolios has been explicitly modeled (Müller [2006a]). The equity portfolio can be of the most general type, including hedge funds and real estate investments. The model assumes a continuous reinvestment of dividend payments. For the purpose of the model the initial configuration of an equity component is defined by the currency, the initial market and book values, the inclination to realize unrealized currency gains in excess of the realizations due to necessary transactions and similar specifications. The economic scenario generator (ESG) delivers the foreign exchange (FX) rates, return rates and dividend yields. External cash flows from and to the portfolio are modeled by the cash flow management (Müller [2006c]).

The bond portfolio model mimics a bond portfolio manager (Müller [2007a]). The bonds considered are of the highest possible maturity, govern-
ment bonds. Portfolios of bonds are modeled by an algorithm which governs the roll over of the portfolio and is determined by target values to be kept constant, either *mean time to maturity* or *duration*. For the purpose of a stylized bond portfolio in multi period simulations the roll over happens at the midpoint of each simulated time interval. The bond with shortest time to maturity is then sold and a new bond with much longer maturity is bought. The entire scenario depends on the economic scenario generator for zero coupon yield curves and foreign exchange rates. The policies are set at initialization stage by defining the turnover intensities in excess of the necessary turnover, historical coupon rates, the inclination to realize currency gains, the choice of the target variable (*held to maturity* or *duration*), the mode of the rollover, e.g. *rolling at maturity*, the currency and others. In multi-period simulations the number of bonds and their maturities held in a stylized portfolio are algorithmically inferred at each simulated time step. External cash flows from or to the portfolio are modeled in the cash flow management (Müller [2006c]).

*Corporate bond* portfolios are affected by default risks and are considered under the credit risk model (Müller [2007b]). The credit risk model conceives of a corporate bond as of the sum of a credit derivative of initial value zero and a risk free bond (government bond); which illustrates the link between credit risk and market risk (Müller [2007b]).

The models for equity, bonds and cash flows are firmly placed within the framework of stochastic economic simulation (Müller et al. [2004]) and are documented in *Accounting of an equity portfolio in simulations*, *Bond portfolio management and accounting in simulations*, *Managing cash flows of assets and liabilities* by Ulrich A. Müller (Müller [2006a], Müller [2007a], Müller [2006c]), a slightly edited version of which we are going to present in this and the following two chapters.

### 12.2 Bond Portfolios in Simulations

In the framework of ALM components written in Igloo\(^1\), the SCOR Switzerland Financial Modeling (FinMod) team uses a bond portfolio component which is able to manage a portfolio of bonds with predefined properties such as duration over the course of a Monte Carlo simulation, including proper accounting. The economic input for simulations, most importantly simulated yield curves, comes from an economic scenario generator (ESG).

An intelligent scheme of rollover and rebalancing of the bonds is needed to maintain the required properties of the portfolio over time. The goal is to mimic the behavior of fund managers tracking a target such as a mean time to maturity or a bond index, using a simplified scheme of bonds in

\(^1\)For an overview of this IT system we refer to the SST Systems documentation in Part VIII of this document.
a regular sequence of maturities. Even a simple model requires a rather complex computation of market values. The accounting variables such as book value, realized and unrealized gains, coupon income, amortization and impairment loss require an even larger number of carefully developed calculation formulas. The main application will need pro-forma accounting in US GAAP style, but the results can also be used for other standards.

Our bond portfolio management encompasses a consequent observance of the mean-time-to-maturity target, straightforward initialization of all variables, full support of market values and all accounting variables in local currency and accounting currency and total transparency of all formulas and algorithms.

Extreme precision of accounting results is not required since the results are based on simulated scenarios, and the internal details of the bond portfolio (such as the coupons and maturities of thousands of individual bonds of the portfolio) are treated in an aggregate, simplified form by an efficient model. The goal is the simulation of realistic projected balance sheets and income statements.

12.3 Timing: The Simulation Period

The simulation tool (Igloo\textsuperscript{2} or any other tools such as ReMetrica) has a basic simulation period of fixed size $\Delta t$. The simplest choice is yearly simulation from year end to year end. If this temporal resolution is insufficient, we may choose a multi-step approach with many small time steps such as quarterly periods.

The method allows for unlimited multi-period simulation. We keep the treatment of individual time steps as simple as possible, so the only way to achieve higher precision or temporal resolution is to choose smaller time steps in a multi-step approach. The first simulation period starts at the last regular time point for which full information on the portfolio and the economic variables is available. In each simulation scenario, new periods can be added by using the final values of the previous period as new starting points.

Cash flows from and to the bond portfolio typically happen over a whole period, not just at its beginning or its end. In order to have an efficient cash flow management in an ALM model with dozens of asset-and-liability components, we assume just one time point of the period at which all the cash flows happen instantaneously. Whenever this appears to be a too coarse simplification, we can use the same remedy as already stated above: replacing the large period by a sequence of shorter ones.

There is one reasonable choice for the time of the cash flow: the middle of the period. This is the average of all cash flow times, assuming that

\textsuperscript{2}See also Part VIII, pp. 408.
these are uniformly distributed over the year. Choosing the cash flow time at mid-period is in line with accounting-based concepts such as average FX rates and average investment levels.

Another advantage of the chosen concept is that we have stable situations and variable values (including mean time to maturity or duration) at both the beginning and the end of the simulation period, where the accounting is made.

For an appropriate bond portfolio treatment, we are forced to model a simplified internal structure explicitly, whereas an equity portfolio can be modeled in aggregated form. This internal structure leads to internal cash flows (coupons, rollover, reestablishment of the desired portfolio duration) which are assumed to happen at the midpoint of the period. In that respect, the internal cash flows are treated like the external ones.

12.4 The Stylized Bond Portfolio

The bond portfolio is a bunch of bonds denominated and expressed in one local currency, managed as an entity with a consistent policy (e.g. mean-time-to-maturity or duration target) and with continuous reinvestment of coupon payments.

The portfolio composition may be complicated in reality, with thousands of bonds. Modeling every bond of a real-life portfolio and simulating the rollover to hundreds of new bonds is regarded as inefficient. On the other hand, treating all bonds in a portfolio as one unstructured object probably leads to inaccurate results, given a simulated behavior of the full yield curve. This motivates us to go for an intermediate approach. We model few stylized bonds in a regular sequence of maturities. In the further course of the model explanation, we usually talk about $n$ stylized bonds, but we stay aware of the fact that each of the $n$ bonds acts as a proxy for another sub-portfolio which may again consist of many individual securities.

Real-life bond managers usually have a mandate with guidelines. Before bonds expire, they are typically sold, and new bonds are bought from the proceedings of the sale and the paid coupons. Through such a rollover procedure, the managers can achieve the desired goal of keeping either the mean time to maturity or the duration constant. In our approach, there is exactly one such rollover per simulation period which stands as a proxy for the whole set of real-life rollover operations during the period.

The bond portfolio does not exchange any funds with the outside world, except for a one-time cash flow from or to it at the middle of the simulation period. This one-time cash flow may have different external causes: claim payments, rebalancing between asset classes, investing new income.

The bond portfolio also has some internal cash flows due to reinvested coupons and reshuffling of the portfolio. These are not part of that external
cash flow and are regarded as an internal affair of the portfolio, invisible to other components. True coupon payments may be annual or semiannual or quarterly, depending on the type of the bond. Regardless of that frequency, the issuing dates, maturity dates and payment dates of the many real-life bonds are irregularly scattered over the whole period. Thus it would be arbitrary if we artificially defined some fixed semiannual payment dates, with so-called par yields based on semiannual payments. Instead, we always operate with annualized coupon rates (which can be calculated to correspond to par yields or any other coupon payment scheme as closely as possible), and we use only one yield curve, the zero-coupon spot yield curve. Par yield curves play no role, but users may translate the model results back to such concepts if so desired. Moreover, we assume exactly one payment time for coupons in the middle of the simulation period (which serves as a proxy and an average time of many such payments scattered over the year). Every alternative convention would add a lot of complexity to the cash flow timing without clear gain in overall modeling accuracy.

A bond portfolio modeled as described may also stand for a bond index, if that is a total-return index. Bond indices are usually set up with similar rules as our model portfolio, keeping the mean time to maturity or duration roughly constant over time. Thus the algorithm described here can also be used to explore the future behavior of a bond index in simulations.

The ESG provides economic scenarios for the zero-coupon yield curves which determine the behavior of each bond of the portfolio. The market value of each bond is given by the present value (PV) of the future cash flows.

The bonds are assumed to have the highest possible rating (government bonds). Future cash flows are assumed to be certain. Impairments of bond investments are therefore excluded. The modeling of bonds with credit risk and default risk (corporate bonds) is discussed under the credit risk model (Chapter 18).

### 12.5 Input Variables

A modeled bond portfolio encompasses many real-life bonds, all in the same currency, and maps them into a series of few stylized bonds. At initialization of a bond portfolio component, we know a few characteristic properties:

- The legal entity to which the bond portfolio is assigned
- The currency of the bond portfolio (= local currency)
- The accounting mode, held to maturity (HTM) or available for sale (AFS), where the trading mode may be approximately treated like AFS
The initial market value of the whole portfolio, \( MV_{\text{init}} \), in local currency

The corresponding initial book value \( BV_{\text{init}} \) in local currency

The initial level of the currency translation adjustment, \( CTA_{\text{init}}^{\text{acc}} \) (the part of unrealized gains that is due to currency shifts prior to the simulation start, which is the sum of the two items \( \text{Group UR Ccy Gain} \) and \( \text{Group UR Ccy Loss} \) in Mellon’s reporting tool)

The choice of the target variable to be kept constant, either mean time to maturity (MTM) or duration \((D)\)

The target value \( \bar{T} \) (for MTM) or \( \bar{D} \) of the target variable, e.g. a desired duration of \( \bar{D} = 4 \) years, assuming that the portfolio is on target with respect to that variable already at simulation start

The maturity \( T_r \) at which the rollover of each stylized bond has to be executed (= minimum accepted time to maturity), must be 0 (= rolling at maturity, a typical choice for held-to-maturity securities) or a positive integer multiple of the basic time interval \( \Delta t \), e.g. one year; a reasonable choice is clearly less than \( \bar{T} \) or less than \( \bar{D} \), respectively

Historical coupon rate assumed for the stylized bonds at initialization, either individual for each bond or simply an average historical coupon rate \( \bar{C}_{\text{hist}} \), assuming that all stylized bonds initially have that one

The average, relative coupon level \( c \) of newly purchased bonds, as a fraction of the current interest rate level, where \( c = 0 \) indicates a policy based on zero-coupon bonds and \( c = 1 \) indicates par bonds (assuming that the coupons of newly bought bonds equal the simultaneously observed zero-coupon yield, on average)

The turnover intensity \( u \) of the bond portfolio in excess of the necessary turnover needed to reinvest coupons and re-establish allocation targets, with \( 0 \leq u \leq 1 \) (further discussed in Section 12.11)

The FX turnover intensity \( u_{\text{FX}} \) of the bond portfolio, the inclination to realize unrealized currency gains in excess of realizations due to necessary transactions, with \( 0 \leq u_{\text{FX}} \leq 1 \) (further discussed in Section 12.12)

These variables refer to particular bond portfolios. In addition to these, there are some economic variables which affect all bond portfolios. An economic scenario generator (ESG) provides the necessary information, for the start as well as the end of a simulation step.
The initial zero-coupon yield curve of the local currency, with interest rates $r_{\text{init}}(T)$, where arbitrary maturities $0 < T \leq 30$ years are supported (through interpolation between the standard maturities of the ESG);

- The final zero-coupon yield curve of the local currency, with interest rates $r_{\text{final}}(T)$, analogous to $r_{\text{init}}(T)$ but for the end time of the period;

- The initial foreign exchange (FX) rate $f_{\text{acc}}^{\text{init}}$ (value of one unit of local currency expressed in the accounting currency);

- The final FX rate $f_{\text{acc}}^{\text{final}}$ (analogous to $f_{\text{acc}}^{\text{init}}$ but for the end time of the period).

All variables with superscript acc are expressed in the accounting (or consolidation) currency. If that index is missing, the quantity is expressed in the local currency of the bond portfolio.

Some initial variable values directly follow from the input variables above, so we can compute them right away. The following formulas apply to any type of asset, not just bond portfolios. The initial level of unrealized gains (or losses, if $< 0$) in local currency is

$$U_{\text{init}} = MV_{\text{init}} - BV_{\text{init}}$$

The initial market value in accounting currency is

$$MV_{\text{acc}}^{\text{init}} = f_{\text{acc}}^{\text{init}} MV_{\text{init}}$$

The book value in accounting currency is listed in the Mellon reporting tool under the header Group Book. We do not extract that quantity from there, however, because it can be computed with the help of the initial CTA:

$$BV_{\text{acc}}^{\text{init}} = BV_{\text{init}} \left( f_{\text{acc}}^{\text{init}} - \frac{CTA_{\text{acc}}^{\text{init}}}{MV_{\text{init}}} \right)$$

This can be interpreted as the book value translated to the accounting currency based on an old FX rate $f_{\text{acc}}^{\text{init}}$ rather than the current one:

$$f_{\text{acc}}^{\text{init}} = f_{\text{acc}} - \frac{CTA_{\text{acc}}^{\text{init}}}{MV_{\text{init}}} = \frac{BV_{\text{acc}}^{\text{init}}}{BV_{\text{init}}}$$ \hspace{1cm} (12.1)

This is the FX rate that was used when the bond portfolio was bought (or a weighted mean of such rates for a complex portfolio that was gradually built up in the past). The level of unrealized gains in accounting currency is

$$U_{\text{acc}}^{\text{init}} = MV_{\text{acc}}^{\text{init}} - BV_{\text{acc}}^{\text{init}} = U_{\text{init}}^{\text{acc}} + CTA_{\text{acc}}^{\text{init}}$$ \hspace{1cm} (12.2)

The last form shows that the unrealized gains consist of two components: one due to changes in the asset market, $U_{\text{init}}^{\text{acc}}$, and one due to changes in the
FX rate, $\text{CTA}_{\text{acc}}^{\text{init}}$. In the Mellon reporting tool we find $U_{\text{acc}}^{\text{init}}$ as the sum of the two items *Group UR Gain* and *Group UR Loss*. In our application we do not take it from there but rather compute it through Equation (12.2). When simulating US GAAP balance sheets we want to know the the two components of the unrealized gain separately. We compute $U_{\text{acc}}^{\text{init}}$ as

$$U_{\text{acc}}^{\text{init}} = U_{\text{acc}}^{\text{init}} - \text{CTA}_{\text{acc}}^{\text{init}}$$

12.6 The Bond Portfolio: Overview of the Algorithm

The stylized model portfolio, which stands for a more complicated real-life portfolio, has a fixed set of $n$ bonds with regular times to maturities $T_i$ at the beginning of a simulation period:

$$T_i = T_r + (i - 0.5) \Delta t = (m + i - 0.5) \Delta t, \quad 1 \leq i \leq n \quad (12.3)$$

with

$$m = \frac{T_r}{\Delta t}$$

The shortest time to maturity is thus $T_r + 0.5 \Delta t$. At the midpoint of the period, this time interval shrinks to $T_r$, which is the rollover maturity. Now the rollover happens: The bond is sold and a new bond with a much longer time to maturity, $T_r + n \Delta t$, is bought. At that time we shift the indices of all the bonds. The second-shortest bond takes the role of the shortest one and takes the index $i = 1$, followed by an analogous index shift for all other bonds. The new bond obtains the index $n$. At the same time there may be some additional and modified transactions due to reinvested coupons, external cash flows and the re-establishing of the target MTM or duration. At the end of the period, there are still $n$ bonds which then have the same times to maturity as those of Equation (12.3).

The scheme is set up in a way that the mean time to maturity, averaged over a whole period, is indeed close to $\bar{T}$. At period start and end, it is exactly so. In the first half of the period, it naturally shrinks by half the period size, but this is compensated by the rollover, after which the mean time to maturity exceeds $\bar{T}$ by half the period size. In the second half, it moves back to the target value $\bar{T}$.

12.7 Initial Bond Portfolio Set-Up

At the beginning of the first simulation period, the stylized bond portfolio has to be defined in line with the input parameters. At the beginning of a subsequent simulation period in a multi-period simulation, the state of
the bond portfolio is already known from the previous period. Thus the following initialization of the portfolio only applies to the first period.

The choice of \( n \) depends on \( T_r \) and the target of MTM or duration. If the mean time to maturity (\( \bar{T} \)) is the target, a reasonable choice is

\[
n = \text{ceiling} \left[ 2 \frac{\bar{T} - T_r}{\Delta t} \right]
\]  

(12.4)

where ceiling \([.]\) denotes the lowest integer number equal to or exceeding the argument. In order to have at least one bond, \( n = 1 \), we need the condition

\[ T_r < \bar{T} \]

The case \( n = 1 \) is unusual as there is only one bond that is replaced by another one at mid-period. This property limits the flexibility of reaching any given target MTM value, so we normally exclude the case \( n = 1 \).

If the target is duration instead of MTM, we use a rule of thumb for a suitable choice of \( n \) which slightly deviates from Equation (12.4). Only the case of MTM is pulled through. A duration target can be treated analogously.

We denote the coupon rate of each bond by \( C_i \). The coupon rate of all initial bonds of the initial period is assumed to be \( C_{\text{hist}} \). Starting at the first rollover, other coupon rates come into play.

For all \( n \) bonds, we can now compute the initial market value per unit of principal, based on \( T_i \), the coupon \( C_i \) and the initial zero-coupon yield curve:

\[
MV_{\text{init},i}^* = C_i \sum_{j=1}^{m+i} \left\{ 1 + r[(j - 0.5) \Delta t] \right\}^{-(j-0.5) \frac{\Delta t}{1 \text{ year}}} + \left\{ 1 + r[(m + i - 0.5) \Delta t] \right\}^{-(m+i-0.5) \frac{\Delta t}{1 \text{ year}}}
\]  

(12.5)

This is the usual formula for the present value of discounted cash flows. The market values \( MV_i \) of the bonds also depend on the size of the principals or redemption values \( P_i \):

\[
MV_{\text{init},i} = P_i MV_{\text{init},i}^*
\]  

(12.6)

We do not know \( MV_{\text{init},i} \) or \( P_i \) at the beginning, but we allocate the investment in a way to conform to the given input parameters through a simple linear allocation scheme:

\[
P_i = a T_i + b
\]  

(12.7)

The parameters \( a \) and \( b \) are determined by two conditions, namely the total initial market value \( MV_{\text{init}} \),

\[
MV_{\text{init}} = \sum_{i=1}^{n} MV_{\text{init},i} = \sum_{i=1}^{n} P_i MV_{\text{init},i}^*
\]  

(12.8)
and the target MTM \((= \bar{T})\) which is exactly observed at initialization, leading to

\[
MV_{\text{init}} \bar{T} = \sum_{i=1}^{n} MV_{\text{init},i} T_i = \sum_{i=1}^{n} P_i MV^*_{\text{init},i} T_i \quad (12.9)
\]

These equations, combined with Equation (12.7), lead to two linear equations for \(a\) and \(b\) with the solution

\[
a = \frac{MV_{\text{init}} \sum_{i=1}^{n} MV^*_{\text{init},i} T_i - \bar{T} \sum_{i=1}^{n} MV^*_{\text{init},i}}{(\sum_{i=1}^{n} MV^*_{\text{init},i} T_i)^2 - (\sum_{i=1}^{n} MV^*_{\text{init},i})(\sum_{i=1}^{n} MV^*_{\text{init},i} T_i^2)}
\]

and

\[
b = \frac{MV_{\text{init}} - a \sum_{i=1}^{n} MV^*_{\text{init},i} T_i}{\sum_{i=1}^{n} MV^*_{\text{init},i}}
\]

For \(n > 1\), there is always a unique solution \(\{a, b\}\) which can be reinserted in Equation (12.7) and then (12.6). Now we have all the relevant information on the initial bonds and their market values. At the beginning of a subsequent simulation step, the initialization procedure is not repeated because the allocation then is simply given as the final allocation of the previous step.

### 12.8 The Bond Portfolio at the Midpoint of the Period

The midpoint of the period is the time of all internal and external cash flows in our model. Before these cash flows, the portfolio keeps the same allocation with the same principals as at the beginning, given by Equation (12.7). The market value however changes in the first half of the period, for two reasons. The bonds are closer to maturity, and the yield curve is different. The ESG does not provide yield curves at the midpoints of periods. Lacking that information, the best approach is interpolation, which just means averaging in our case:

\[
r_{\text{mid}}(T) = \frac{r_{\text{init}}(T) + r_{\text{final}}(T)}{2}
\]

Of course there might be alternative averaging schemes, e.g. geometric averaging of yield factors or an interpolation based on initial forward interest rates.

At the midpoint, each of the \(n\) stylized bonds pays a coupon:

\[
C_{\text{paid},i} = C_i P_i
\]

All coupons together have a value of

\[
C_{\text{paid}} = \sum_{i=1}^{n} C_{\text{paid},i} \quad (12.10)
\]
Immediately after the coupon payments and before the rollover and the external cash flow, we obtain the following market value for each bond, per unit of principal:

\[
MV_{\text{before}C,i}^* = C_i \sum_{j=1}^{m+i-1} \left[ 1 + r_{\text{mid}}(j \Delta t) \right]^{-j \frac{\Delta t}{\text{year}}} + \{1 + r_{\text{mid}}[(m + i - 1) \Delta t]\}^{-(m+i-1) \frac{\Delta t}{\text{year}}}
\]

(12.11)

This is like Equation (12.5), except that maturity periods are reduced by half a period length \((\Delta t/2)\), one more coupon payment has already happened, and the updated interest rates \(r_{\text{mid}}\) are used.

The bond with index 1 is now reaching the rollover point and is sold for its market value,

\[
\text{Proceedings} = P_1 MV_{\text{before}C,1}^*
\]

which can be computed by using Equation (12.11).

Neither the coupon payments nor the sale of a bond leads to a change in the total portfolio value including the coupons paid and the sale proceedings, which is

\[
MV_{\text{before}C} = \sum_{i=2}^{n} P_i MV_{\text{before}C,i}^* + \text{Proceedings} + C_{\text{paid}}
\]

(12.12)

where the first term denotes the value of the remaining bonds and the last two terms the available free cash.

After the sale of the bond with index 1, the portfolio still has \(n - 1\) bonds. We rename those bonds by diminishing the index of each bond by 1. The bond closest to maturity now has the index 1 instead of 2. The index \(n\) becomes free and will be used for a new bond. All the following equations are based on the new indices.

A new bond with index \(n\) and time to maturity \(T_r + n\) is going to be bought now in order to complete the rollover. This time to maturity is in line with all current times to maturity of the new portfolio:

\[
T_{\text{mid},i} = T_r + i \Delta t = (m + i) \Delta t, \quad 1 \leq i \leq n
\]

Notice the effect of the index shift and the fact that maturity periods are reduced by half a period length, \(\Delta t/2\), as compared to Equation (12.3) at period start. The times to maturity at the end of the period can also be computed:

\[
T_i' = T_r + (i - 0.5) \Delta t = (m + i - 0.5) \Delta t, \quad 1 \leq i \leq n
\]

(12.13)

where the maturity periods are again reduced by half a period length, \(\Delta t/2\). This equation is the same as Equation (12.3), except for the fact that the index \(i\) now denotes different bonds.
The coupon rate \( C_n \) of the new bond depends on the market and the investment policy and may deviate from \( C_{\text{hist}} \). We use
\[
C_n = c \cdot r_{\text{mid}} [(m + n) \Delta t]
\]
where the input variable \( c \) stands for the coupon policy as explained in Section 12.5.

The market value of each bond, per unit of principal, is still given by Equation (12.11). We reformulate that equation by using the new, shifted indices \( i \):
\[
\text{MV}^*_{\text{afterC},i} = \sum_{j=1}^{m+i} \left[ 1 + r_{\text{mid}} (j \Delta t) \right]^{-j \frac{\Delta t}{\text{year}}} + \left[ 1 + r_{\text{mid}} [(m+i) \Delta t] \right]^{-(m+i) \frac{\Delta t}{\text{year}}}
\]
This formula is also valid for the new bond with index \( i = n \). The corresponding market values per unit of principal at period end can be computed similarly:
\[
\text{MV}^*_\text{final,i} = \sum_{j=1}^{m+i} \left[ 1 + r_{\text{final}} [(j-0.5) \Delta t] \right]^{-(j-0.5) \frac{\Delta t}{\text{year}}} + \left[ 1 + r_{\text{final}} [(m+i-0.5) \Delta t] \right]^{-(m+i-0.5) \frac{\Delta t}{\text{year}}}
\]
Here we are using the interest rates \( r_{\text{final}} \) at period end, and times to maturity that are reduced by half a period size.

A simple rollover rule would be to use the total amount of free cash, \( C_{\text{paid}} \), Proceedings, for buying the new bond. We need a more sophisticated rule, for two reasons. First, the bond portfolio receives some cash from the outside world or has to deliver cash. Second, we have to observe a target for the MTM or the duration. We can reach the target through a full rebalancing exercise which may lead to some shifts of the bond allocation in addition to the purchase of the new bond.

In reality, the bond managers are maintaining a duration target in continuous time, within a certain tolerance range, so the MTM or the duration is on target at every moment, including the end of the period. In our model we have all cash flows virtually concentrated at mid-period, but the MTM or duration only matters at the end of the period, where results are reported to users. We solve this problem by choosing an allocation now that will exactly reach the target at period end, which is possible as we already know the economic variable values at period end. We are forced to consider the further development of the portfolio already now.

Immediately after the external cash flow, the total market value of the portfolio, including cash, is
\[
\text{MV}_{\text{afterC}} = \text{MV}_{\text{beforeC}} + \text{CF} \quad (12.15)
\]

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where MV\(_{\text{beforeC}}\) comes from Equation (12.12) and the cash flow CF is a piece of external information produced in the course of the simulation, as to be further discussed in Section 12.9.

The total market value MV\(_{\text{afterC}}\) is now allocated by using a similar procedure as in the initial allocation. Since we do not know MV\(_{\text{afterC}}\) yet, we can only determine relative allocation \(p_i\) of principals now. The \(p_i\) values will eventually be scaled by a yet unknown constant to arrive at the true principals. This allocation is again defined as following a simple linear scheme as in Equation (12.7):

\[
p_i = a' T'_i + b'
\]

(12.16)

where \(T'_i\) is defined by Equation (12.13). The \(p_i\) values are constant over the whole second half of the simulation period, including its end, where the market values MV\(_{\text{final,i}}^*\) per unit principal are given by Equation (12.7). We are free to define the \(p_i\) values as scaled in a way to reach a final market value of 1:

\[
1 = \sum_{i=1}^{n} p_i \text{MV}_{\text{final,i}}^*
\]

(12.17)

This is one condition to find the \(a'\) and \(b'\) values, analogous to Equation (12.8), but now per unit market value at the period end. The second condition states that the target MTM (= \(\bar{T}\)) will exactly be observed at period end, leading to

\[
\bar{T} = \sum_{i=1}^{n} p_i \text{MV}_{\text{final,i}}^* T'_i
\]

analogous to Equation (12.9). Now we have two linear equations for \(a'\) and \(b'\) with the solution

\[
a' = \frac{\sum_{i=1}^{n} \text{MV}_{\text{final,i}}^* T'_i - \bar{T} \sum_{i=1}^{n} \text{MV}_{\text{final,i}}^*}{(\sum_{i=1}^{n} \text{MV}_{\text{final,i}}^* T'_i) - (\sum_{i=1}^{n} \text{MV}_{\text{final,i}}^*)(\sum_{i=1}^{n} \text{MV}_{\text{final,i}}^* T'_i)}
\]

and

\[
b' = \frac{1 - a' \sum_{i=1}^{n} \text{MV}_{\text{final,i}}^* T'_i}{\sum_{i=1}^{n} \text{MV}_{\text{final,i}}^*}
\]

Since \(n > 1\), there is always a unique solution \(\{a', b'\}\) which can be reinserted in Equation (12.16). The value of the portfolio with an allocation of principals \(p_i\) is \(\sum_{i=1}^{n} p_i \text{MV}_{\text{afterC,i}}^*\) after the cash flow and grows to 1 over the second half of the simulation period, see Equation (12.17), so the growth factor expressed in the local currency is

\[
g = \frac{1}{\sum_{i=1}^{n} p_i \text{MV}_{\text{afterC,i}}^*}
\]

(12.18)

This growth factor applies to the whole amount of the investment, regardless if it originates from an old investment or from cash newly injected and invested at mid-period.
12.9 Cash Flow from or to the Portfolio

At the midpoint of the simulation period, a cash flow comes into play. The size of this cash flow is essentially determined by the outside world. Cash enters in case of general premium growth and profitable underwriting, cash may be demanded from the component if claim payments exceed premium income.

However, the bond portfolio component also exerts a limited influence on the determination of the cash flows between components. The reason is rebalancing. Typical investors try to keep the allocation to different asset classes constant, which may require some shifts of cash between asset portfolios from time to time. These rebalancing cash flows can only be computed by an external cash flow management component (the treasurer of the model) if some information on market values is known there. The bond portfolio component has to provide some information before it gets back the desired information on the cash flow.

In Igloo, such circular information loops are not allowed. This is why we have to technically divide the bond portfolio component into two components:

1. The initial bond portfolio component is configured by the initial values of Section 12.5 (in the first simulation period) or the final values of a previous period and passes some information on its market value development to the cash flow manager.

2. The final bond portfolio component receives some basic information from its initial sister component and the cash flow information from the cash flow manager. Then it computes all results of the simulation period.

Rebalancing may encompass assets denominated in different currencies, but it is defined in terms of the accounting currency. The cash flow management is also done in accounting currency. In order to support cash flow calculations, we need to know the foreign exchange rate \( f_{acc}^{mid} \) at cash flow time. In accounting, such an average rate is sometimes provided. In the framework of our period-based simulations, we have no such information and choose the following obvious assumption:

\[
 f_{acc}^{mid} = \frac{f_{acc}^{init} + f_{acc}^{final}}{2} \tag{12.19}
\]

The aim is to have a well balanced portfolio at the end of the simulation period, at the time and in the currency of the accounting. A perfect balance at the period midpoint is not necessary as this midpoint is just an auxiliary time point with no accounting results to be shown to the users.

The cash flow manager thus needs the following information to make an informed decision on asset rebalancing:
• the market value $f_{\text{mv beforeC}}^{\text{acc}}$ at mid-period, just before the cash flow, in accounting currency, where $\text{MV}_{\text{beforeC}}^{\text{acc}}$ is available from Equation (12.12);

• the growth factor $g^{\text{acc}}$ by which the portfolio and the injected cash flow will appreciate between the middle and the end of the period, expressed in accounting currency.

The growth factor $g^{\text{acc}}$ is

$$g^{\text{acc}} = g \frac{f_{\text{final}}^{\text{acc}}}{f_{\text{mid}}^{\text{acc}}}$$

and thus has a dependency on the FX market. The local growth factor $g$ results from Equation (12.18). If cash is withdrawn from the bond portfolio, $g^{\text{acc}}$ refers to the size of the reduction caused by the withdrawal.

The cash flow manager (see Chapter 14, pp. 287) then determines the size of the cash flow $\text{CF}^{\text{acc}}$ to the component. In order to obtain the value in local currency, the final bond portfolio component has to do a division by the FX rate,

$$\text{CF} = \frac{\text{CF}^{\text{acc}}}{f_{\text{mid}}^{\text{acc}}}$$

This can be inserted in Equation (12.15). The cash flows $\text{CF}^{\text{acc}}$ and $\text{CF}$ can be positive or negative. In the latter case, the cash drain may be as high as to make the whole market value $\text{MV}_{\text{afterC}}^{\text{acc}}$ negative. That extreme event, which can only happen in case of bankruptcy, will in fact never be seen if we use a smart cash flow manager component with realistic insolvency handling or at least a cash overdraft component for negative market values.

### 12.10 The Bond Portfolio at the End of the Period

After inserting the cash flow in Equation (12.15), we obtain the market value $\text{MV}_{\text{afterC}}^{\text{acc}}$, which allows the final bond portfolio component to compute all the relevant results at the end of the simulation period. The final market value in local currency is

$$\text{MV}_{\text{final}}^{\text{acc}} = g \text{MV}_{\text{afterC}}^{\text{acc}}$$

(12.20)

The principals $P_i$ in the second half of the simulation period can be computed as

$$P_i = \text{MV}_{\text{final}}^{\text{acc}} \cdot p_i$$

(12.21)

using the results of Equation (12.16).

The final market value in accounting currency is

$$\text{MV}_{\text{final}}^{\text{acc}} = \text{MV}_{\text{final}} f_{\text{final}}^{\text{acc}}$$

(12.22)
In a multi-period simulation, the end state of the portfolio is identical to the initial state of the next period. A second simulation period will start from here instead of initializing the portfolio from scratch.

At this point of the description, we know all necessary things about the development of market values, but we do not yet fully support accounting views.

### 12.11 Accounting Results in Local Currency

The book value of the bond portfolio and other accounting variables are affected by the cash injection or withdrawal at mid-period and by the turnover policy of the asset manager. The accounting is first dealt with in the local currency. Then we move to the accounting currency, where the CTA comes into play.

Each bond has a book value which is defined similar to the market value as the present value of the future cash flows, except for using a fixed, frozen interest rate instead of the current yield curve for discounting. At the start of the simulation, we have the relation

$$\text{BV}_i = C_i \sum_{j=1}^{m+i} (1 + y_i)^{-(j-0.5) \frac{\Delta t}{\text{year}}} + P_i (1 + y_i)^{-(m+i-0.5) \frac{\Delta t}{\text{year}}} \quad (12.23)$$

which is similar to Equation (12.5) combined with Equation (12.6). The interest rates of the yield curve are now replaced by the fixed interest rate $y_i$, the so-called yield to maturity of the bond. This quantity stays constant over the lifetime of a bond, once the bond has been purchased. It is annualized only if the simulation time period $\Delta t$ has the size of 1 year. Otherwise it refers to the period $\Delta t$ rather than 1 year.

The sum in Equation (12.23) is now a regular geometric series as $y_i$ is identical for all coupon payment dates. Using the sum formula for geometric series, we obtain

$$\text{BV}_i = P_i (1 + y_i)^{0.5} \left[ \frac{C_i}{y_i} + \frac{1 - \frac{C_i}{y_i}}{(1 + y_i)^{m+i}} \right] \quad (12.24)$$

In case of a zero yield, $y_i = 0$, this equation numerically diverges. We can replace 0 by a very small number then.

We have a problem as we often have to compute the yield $y_i$ of each stylized bond from the book value rather than the other way around. Equation (12.24) cannot be inverted analytically in a closed form for that purpose. We need an iterative procedure such as IRR() of Excel. In Igloo simulations, where we have to compute thousands of $y_i$ values, we prefer not using an
iterative algorithm. Instead, we replace Equation (12.24) by an approximation that is analytically invertible. We expand the expression up to the second order of $y_i$, but we write $y_i'$ instead of $y_i$ to emphasize the fact that the expansion is just an approximation:

$$
BV_i = P_i \frac{1 + (m + i) C_i + [1 + (M_i + m + i) C_i] y_i'}{1 + (m + i + 0.5) y_i' + [M_i + 0.5 (m + i) + 0.25] y_i'^2}
$$

(12.25)

with

$$
M_i = 1.3 \frac{(m + i)(m + i - 1)}{2}
$$

The original second-order expansion does not have the factor 1.3 in this equation. We prefer using the factor 1.3 because it approximates the effect of the omitted higher-order terms and leads to smaller average approximation errors.

When using a correct $BV_i$ and then solving Equation (12.25) for $y_i'$, this $y_i'$ will be a good approximation of the true $y_i$.

The expansion of Equation (12.25) is set up as a rational function of $y_i'$ rather than a simple polynomial. An equivalent polynomial expansion exists, but some terms are negative there and may lead to a negative book value if $y_i'$ has an extreme value. We avoid that numerical problem by using Equation (12.25) where all terms in the numerator and the denominator are positive, so the book value is always positive, even for extremely large $y_i'$.

Equation (12.25) can be solved for $y_i'$ through a quadratic equation:

$$
y_i' = \frac{B_i \pm \sqrt{B_i^2 - 4 \left[ M_i + 0.5 (m + i) + 0.25 \right] \frac{BV_i}{P_i} \left[ \frac{BV_i}{P_i} - 1 - (m + i) C_i \right]}}{2 \left[ M_i + 0.5 (m + i) + 0.25 \right] \frac{BV_i}{P_i} \left[ \frac{BV_i}{P_i} - 1 - (m + i) C_i \right]}
$$

(12.26)

with

$$
B_i = 1 + (M_i + m + i) C_i - (m + i + 0.5) \frac{BV_i}{P_i}
$$

The resulting yield $y_i'$ is a reasonable approximation of the true yield for a sufficiently wide range of different maturities, coupon rates and yields. The quality of the approximation is shown in Table 12.11 for different choices of those parameters. Some unusually high values of coupons (12%) and yields (15%) are also considered there because these values challenge the quality of the approximation more than moderate values.

Whatever the maturities, coupons, and yields from Equation (12.26), Equation (12.25) exactly reproduces the original book value $BV_i$, leading to a consistent behavior of the algorithm with no unwanted jumps in $BV_i$ due to approximation errors.
The sum of all book values equals the total initial book value,

\[ BV_{\text{init}} = \sum_{i=1}^{n} BV_i \]  \hspace{2cm} (12.27)

which is analogous to Equation (12.8).

The initial modeling problem is that we know \( BV_{\text{init}} \) from accounting, but the \( BV_i \) values of the stylized bonds are unknown. A sophisticated approach would be to derive a realistic model for \( y_i \) or \( y'_i \) based on the yield curve history and the different times to maturity. Whatever the model, the resulting \( y'_i \) values must exactly lead to the correct initial book value when inserted in Equations (12.25) and (12.27). We use a procedure that satisfies this condition with no prior knowledge of yields.

The first step of the initialization is based on a coarse approximation for the total book value:

\[ BV_{\text{init}} = \sum_{i=1}^{n} P_i \sum_{i=1}^{n} P_i + \sum_{i=1}^{n} P_i (m + i) \frac{C_i}{\sum_{i=1}^{n} P_i (m + i)} \]  \hspace{2cm} (12.28)

This is like Equation (12.25), using one order less in the expansion and aggregating the formula for the whole portfolio. Here we simply assume that the initial yields are equal to one value \( y'_{\text{init}} \), which is not too bad an assumption for our stylized bond portfolio.

Now we have all the necessary ingredients to determine the initial book values of the bonds. This algorithm is only applied once before the first simulation step. We first determine the approximate yield \( y'_{\text{init}} \) by a transformed version of Equation (12.28):

\[ y'_{\text{init}} = \sum_{i=1}^{n} P_i \sum_{i=1}^{n} P_i + \sum_{i=1}^{n} P_i (m + i) \frac{C_i - BV_{\text{init}}}{BV_{\text{init}} \sum_{i=1}^{n} P_i (m + i)} \]

For all bonds, we insert the resulting yield \( y'_{\text{init}} \) in Equation (12.25) and obtain approximate book values \( BV'_i \) for each bond. The sum of all those \( BV'_i \) is the total book value. Due to the coarse choice of \( y'_{\text{init}} \), we can expect that this sum is not exactly \( BV_{\text{init}} \), but not too far from it. In order to reach the correct total book value, we now define the book values of the bonds by using a normalization factor:

\[ BV_{\text{init},i} = \frac{BV_{\text{init}}}{\sum_{i=1}^{n} BV'_i} BV'_i \]  \hspace{2cm} (12.29)

These initial book values are actually used.

However, there may be some turnover during the whole simulation step which requires some modeling already at this point, as to be explained. The modeled rollover policy minimizes the turnover of the bond portfolio, which
more or less corresponds to the behavior of real asset managers. The rollover happens at mid-period and affects book values only then, not at the start of the period, so the initial book values \( BV_{\text{init},i} \) are not affected.

Some asset managers may however generate some additional turnover for whatever reason, for example index tracking or corporate bond picking. We only refer to that additional turnover when using the word turnover in the following derivations. This turnover may happen at any time during the simulation period, but a practical implementation requires that we formally concentrate all the modeled turnover activities at very few time points. We might model them to happen at mid-period only, like the cash flows, but such a model would not be able to include an extreme case which we want to cover, namely the case of continuous realization of all unrealized gains. That limit case of very high turnover can be included if we allow for turnover at the period end. We also introduce another moment when turnover can happen, namely the period start, because the investment size may strongly differ between the two halves of the period and we want to model turnover in both halves.

The turnover in the first half period is modeled as an immediate correction of the initial book value:

\[
BV_{\text{initAT},i} = MV_{\text{init},i} + \sqrt{1 - u} \left( BV_{\text{init},i} - MV_{\text{init},i} \right)
\]  

(12.30)

where the parameter \( u \) is the turnover intensity, with \( 0 \leq u \leq 1 \). If \( u = 0 \), there is no turnover, if \( u = 1 \), the turnover is so strong that the book value moves to the market value. The index initAT means initial after turnover. The book value \( BV_{\text{init},i} \) originates from Equation (12.29) (at start of the first simulation step) or is the final book value of the previous simulation step. The initial market value \( MV_{\text{init},i} \) can be taken from Equation (12.6). We assume the same \( u \) value for all the bonds in our simple turnover model. We always assume that the turnover does not affect the properties of the investment because the newly purchased securities have – in our model – the same properties as the old ones. Thus the turnover only leads to shifts in accounting results, not market values. The turnover causes the following realized gain in the first half of the period:

\[
G_{\text{init},i} = BV_{\text{initAT},i} - BV_{\text{init},i}
\]

For all bonds together, we obtain

\[
G_{\text{init}} = \sum_{i=1}^{n} G_{\text{init},i}
\]

The book value \( BV_{\text{initAT},i} \) is now inserted in Equation (12.26) to obtain an individual yield to maturity \( y'_{i} \) for each bond. The resulting \( y'_{i} \) values replace the coarse approximation \( y'_{\text{init}} \) in the further course of accounting.
calculations. The yield \( y'_i \) of a bond keeps its constant value until maturity is reached (or other securities with the same maturity and another yield are additionally purchased and then mixed with the initial bond investment).

<table>
<thead>
<tr>
<th>Coupon rate</th>
<th>Maturity (years)</th>
<th>True yield to maturity</th>
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<tbody>
<tr>
<td></td>
<td>2%</td>
<td>5%</td>
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<tr>
<td></td>
<td>15%</td>
<td>4.98%</td>
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<td>2.5%</td>
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<td>15.69%</td>
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</table>

Yields to maturity resulting from approximation formulas applied to exact book values. A comparison to the true yields shows the approximation quality in different cases. Unlike the exact formula based on discounted cash flows, the approximations (Equations [12.26] and [12.37]), are analytically invertible (Equations [12.25] and [12.35]), so exact book values are reproduced when inserting approximate yields to maturity.

Table 12.1: Approximation for yields to maturity of bonds.

After computing the book values and \( y'_i \), we also want to know the book values at the end of the period and at the mid-point of the period, where the book values serve as intermediary results for computing some further results. In the first half of the period, before coupon payments, rollover and cash flow, the book value is only affected by the progress of time, that is by amortization. At this point, we have the following book values, analogous to Equation (12.11):

\[
BV_{\text{beforeC,i}}^* = C_i \sum_{j=1}^{m+i} [1 + y_i]^{-(j-1) \frac{dt}{1 \text{year}}} \cdot \{1 + y_i\}^{-(m+i-1) \frac{dt}{1 \text{year}}} (12.31)
\]

where the same \( y_i \) as at period start applies. For consistency reasons, we take an approximation analogous to Equation (12.25):

\[
BV_{\text{beforeC,i}} = P_i \frac{1 + (m + i) C_i + [1 + (M_i + m + i) C_i] y'_i}{1 + (m + i) y'_i + M_i y'^2} (12.32)
\]

where the same approximate yield \( y'_i \) as at period start is taken. Equation (12.32) looks slightly different from Equation (12.25) because the first coupon of the cash flow projection starts immediately now instead of half a
period later. The change in book value is the amortization of the portfolio in the first half of the period:

\[ \text{Amort}_{\text{init},i} = \text{BV}_{\text{before},C,i} - \text{BV}_{\text{init},i} \text{AT} \]  

(12.33)

and, for all bonds together,

\[ \text{Amort}_{\text{init}} = \sum_{i=1}^{n} \text{Amort}_{\text{init},i} \]

The amortization can be positive or negative, depending on the relative sizes of the coupon rates \( C_i \) and the yields \( y_i' \).

At mid-period, coupons are paid and the oldest bond is sold, as explained in Section 12.8. The book value of that oldest bond with index \( n \) does not matter as the proceedings of the sale correspond to the market value. After the sale but before the injection or withdrawal of cash, the indices of the bonds are shifted, so the index \( i = n \) becomes free for a new bond to be purchased. The external cash flow also leads to allocation changes as already explained in Section 12.8. All these steps affect the book values.

For some or all bonds, an additional amount of an already existing bond type has to be purchased. The problem is that the new amount of a bond with index \( i < n \) may have another yield than the old portion as the yield curve may have moved in the meantime. We have already limited the model complexity by assuming that the newly purchased portion has the same maturity and coupon as the existing portion. Now we add the assumption that the new bond with index \( i \), which consists of two portions with different yields, can be treated as a new homogeneous block with one yield, which is a sort of mixed yield of the old and the new portion.

After all transactions and the index shift, the following equation for book values can be formulated, analogous to Equations (12.11) and (12.31):

\[ \text{BV}^*_{\text{mid},i} = C_i \sum_{j=1}^{m+i} [1 + y_i - j \frac{\Delta t}{\text{year}} + (1 + y_i)^{-(m+i)} j \frac{\Delta t}{\text{year}}] \]  

(12.34)

For consistency reasons, we again replace Equation (12.34) by an approximation analogous to Equations (12.25) and (12.32):

\[ \text{BV}_i = P_i \frac{1 + (m + i) C_i + M_i C_i y_i'}{1 + (m + i) y_i' + M_i y_i'^2} \]  

(12.35)

where the same approximate yield \( y_i' \) as for the first half of the period (after the treatment of the turnover) is taken, except for the index shift, of course. Equation (12.35) looks slightly different from Equations (12.25) and (12.32) because the first coupon of the cash flow projection starts after one period instead of half a period or immediately.

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After the external cash flow and the re-balancing of the portfolio, the principals $P_i$ have changed. We denote the principal before the cash flow by $P_{\text{beforeC},i}$; the value results from Equation (12.7) (at simulation start) or Equation (12.21) as computed in the previous period (if we are not in the first simulation period). The principal after the cash flow is called $P_{\text{new},i}$ and results from Equation (12.21) as computed in the current period. Both $P_{\text{beforeC},i}$ and $P_{\text{new},i}$ use the new, shifted indices. Now we can calculate the book values after the cash flow:

$$
BV_{\text{afterC},i} = \begin{cases} 
P_{\text{new},i} & \text{if } P_{\text{new},i} < P_{\text{beforeC},i} \\
BV_{\text{beforeC},i} + (P_{\text{new},i} - P_{\text{beforeC},i}) MV^*_{\text{afterC},i} & \text{otherwise}
\end{cases}
$$

where $BV_{\text{beforeC},i}$ results from Equation (12.35) by inserting $P_{\text{beforeC},i}$ and $MV^*_{\text{afterC},i}$ from Equation (12.14). The change in $P_i$ also leads to a new contribution to the realized gains:

$$
G_{\text{mid},i} = \begin{cases} 
(1 - \frac{P_{\text{new},i}}{P_{\text{beforeC},i}}) (MV_{\text{beforeC},i} - BV_{\text{beforeC},i}) & \text{if } P_{\text{new},i} < P_{\text{beforeC},i} \\
0 & \text{otherwise}
\end{cases}
$$

For all bonds together, we obtain

$$
G_{\text{mid}} = \sum_{i=1}^{n} G_{\text{mid},i} \quad (12.36)
$$

The new book value $BV_{\text{afterC},i}$ is needed to compute the new, mixed yield $y'_i$ of both the old portion and the newly purchased portion together. This is done by using the inverted form of Equation (12.35):

$$
y'_i = \frac{B_i + \sqrt{B_i^2 - 4 M_i \frac{BV_i}{P_i} [\frac{BV_i}{P_i} - 1 - (m + i) C_i]}}{2 M_i \frac{BV_i}{P_i}} \quad (12.37)
$$

where the auxiliary quantity

$$
B_i = M_i C_i - (m + i) \frac{BV_i}{P_i}
$$

is used and the new book and principal values are inserted, $BV_i = BV_{\text{afterC},i}$ and $P_i = P_{\text{new},i}$. Equation (12.37) corresponds to Equation (12.26) which was derived for the period start rather than the midpoint.

We use Equation (12.37) for all bonds after the cash flow, including the newly purchased bond with index $n$. The only special feature of the new bond is that $P_{\text{beforeC},n} = 0$ is inserted in Equation (12.35). This leads to an initial book value equal to the initial market value of the new bond.
Now we can compute the final book values $BV_{\text{finalBT},i}$ by inserting the new yields $y_{i}'$ in Equation (12.25). The index finalBT means final before taking the final turnover into account. We can now compute the amortization in the second half of the period:

$$\text{Amort}_{\text{final},i} = BV_{\text{finalBT},i} - BV_{\text{afterC},i}$$

which is analogous to Equation (12.33). For all bonds together, we obtain

$$\text{Amort}_{\text{final}} = \sum_{i=1}^{n} \text{Amort}_{\text{final},i}$$

The resulting $BV_{\text{finalBT},i}$ values are now corrected by additional turnover modeled to happen at period end, which stands as a proxy for turnover in the second half period in general:

$$BV_{\text{final},i} = MV_{\text{final},i} + \sqrt{1-u} \left( BV_{\text{finalBT},i} - MV_{\text{final},i} \right)$$

analogous to Equation (12.30), with the same turnover intensity $u$. The final market value $MV_{\text{final},i}$ can be taken from Equation (12.20). The turnover causes the following realized gain in the second half of the period:

$$G_{\text{final},i} = BV_{\text{final},i} - BV_{\text{finalBT},i}$$

In the case of no additional turnover, which means $u = 0$, the evaluation of Equation (12.38) is superfluous as $BV_{\text{final},i} = BV_{\text{finalBT},i}$ and $G_{\text{final},i}$ vanishes. As soon as we have a portfolio with $u > 0$, we need Equation (12.38) and arrive at a new final book value $BV_{\text{final},i}$, where the bond with index $i$ is a mixture of an old portion and a portion consisting of newly purchased securities (due to additional turnover). The two portions have different yields, but we again compute a common, mixed yield $y_{i}'$ for efficiency reasons. The previously computed $y_{i}'$ should then be replaced by a new $y_{i}'$ value which results from a repeated application of Equation (12.37) to $BV_{\text{final},i}$.

For all bonds of the portfolio together, we obtain

$$G_{\text{final}} = \sum_{i=1}^{n} G_{\text{final},i}$$

and the total, final book value is

$$BV_{\text{final}} = \sum_{i=1}^{n} BV_{\text{final},i}$$

Based on the final book value $BV_{\text{final}}$ we can compute more accounting variables. Notice that the accounting of bonds and other fixed income investments may depend on the accounting standard and the category of the
investment. The two main categories are available for sale (AFS) and held to maturity (HTM). The balance sheet is typically based on market values of AFS securities and book values of HTM securities. In this document we present useful variables to support different accounting standards and different categories, but we do not discuss the choice of standard to be used in a particular application.

A first useful variable is the total unrealized gain $U$ at the end of the period,

$$U_{\text{final}} = MV_{\text{final}} - BV_{\text{final}}$$

For performance calculations we are sometimes interested in the unrealized gain $\Delta U$ of the period, which is

$$\Delta U = U_{\text{final}} - U_{\text{init}} = MV_{\text{final}} - BV_{\text{final}} - (MV_{\text{init}} - BV_{\text{init}})$$

The income statement focuses on income which consists of three parts, namely the realized gain $G$, the coupon income $C_{\text{paid}}$ from Equation (12.10) and the amortization (which is counted as income although there is no immediate cash flow):

$$\text{Income} = G + C_{\text{paid}} + \text{Amort} = BV_{\text{final}} - BV_{\text{init}} - CF$$

The last form states that the income is the change in overall book value excluding the external cash flow. The cash flow is excluded because it is not generated by the component itself.

The realized gain $G$ of the component is

$$G = G_{\text{init}} + G_{\text{mid}} + G_{\text{final}}$$

where the middle term originates from a reduction of the investment size at mid-period (Equation [12.36]) and the other two terms reflect gains due to additional turnover in the bond portfolio management. Realized gains can of course be negative, in which case we call them realized losses.

The total amortization of bonds in the whole period is computed as the sum of the two period halves:

$$\text{Amort} = \text{Amort}_{\text{init}} + \text{Amort}_{\text{final}}$$

The performance of the bond investment can only be measured if both the realized income and the unrealized gains are considered:

$$\text{TotalGain} = \text{Income} + \Delta U = MV_{\text{final}} - MV_{\text{init}} - CF$$

This relation can be derived from Equations (12.39) and (12.40). An investment return of the period can then be calculated by dividing the total gain by the mean market value:

$$\text{Investment Return} = \frac{\text{Total Gain}}{0.5 \ (MV_{\text{init}} + MV_{\text{final}})}$$

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12.12 Final Accounting Results in Accounting Currency

The accounting currency is used to do the final accounting and reporting of the analyzed company. The currency translation depends on the FX rate used. For the market value we use the final FX rate $f_{\text{final}}^{\text{acc}}$, see Equation (12.22), and for income statement variables originating from many time points in the course of the period, we use the average rate $f_{\text{mid}}^{\text{acc}}$ from Equation (12.19).

For the book value and the computation of the currency translation adjustment (CTA), we need the booked FX rate $f_{\text{final}}^{\prime acc}$ which is typically the average of rates that prevailed when the assets were bought, similar to $f_{\text{init}}^{acc}$ which we computed in Equation (12.1). A detailed derivation of $f_{\text{final}}^{\prime acc}$ would be based on tracking the FX rates used in all simulated transactions, which would be rather complex without adding much value. We prefer a simpler but realistic approach. Two effects essentially affect $f_{\text{final}}^{\prime acc}$: additionally purchased assets (mainly if the cash flow was positive) and deliberate realizations of the CTA. The Mellon reporting tool indeed shows quite a few securities (often with low investment sizes) with $\text{CTA} = 0$. Initial CTAs have been realized in those cases (or there may be a booking error in the Mellon tool). It does not matter whether such a realization is due to real market transactions or just a virtual booking operation, as long as it is correctly reported. The inclination to perform such CTA realizations is an input parameter called the FX turnover, $u_{\text{FX}}$, where $u_{\text{FX}} = 0$ means no such inclination and $u_{\text{FX}} = 1$ means continuous realization of all FX gains and losses. In practice, $u_{\text{FX}}$ often exceeds 0 as reinvested coupons are booked at new FX rates, thus leading to some natural FX turnover. Our model for $f_{\text{final}}^{\prime acc}$ is

$$f_{\text{final}}^{\prime acc} = u_{\text{FX}} f_{\text{final}}^{\text{acc}} + (1 - u_{\text{FX}}) \frac{MV_{\text{beforeC}} f_{\text{init}}^{\text{acc}} + \max(CF, 0) f_{\text{mid}}^{\text{acc}}}{MV_{\text{beforeC}} + \max(CF, 0)}$$

where $f_{\text{mid}}^{\text{acc}}$ originates from Equation (12.19) and $MV_{\text{beforeC}}$ from Equation (12.12). The cash flow of the component, CF, plays an asymmetric role here. As long as it is negative or zero (no new investment), it has no influence on $f_{\text{final}}^{\prime acc}$.

The book value in accounting currency can now be computed:

$$BV_{\text{final}}^{\text{acc}} = BV_{\text{final}} f_{\text{final}}^{\prime acc}$$

The total unrealized gain (or loss) is

$$U_{\text{final}}^{\text{acc}} = MV_{\text{final}}^{\text{acc}} - BV_{\text{final}}^{\text{acc}} = U_{\text{final}}^{\text{acc}} + \text{CTA}_{\text{final}}^{\text{acc}}$$ (12.43)

The last form shows the two components of the total unrealized gain that we ultimately need in accounting. The unrealized gain excluding currency...
gains and losses is

$$U_{\text{final}}^{\text{acc}} = U_{\text{final}} f_{\text{final}}^{\text{acc}}$$  \hspace{1cm} (12.44)$$

The Currency Translation Adjustment (CTA) has a final value of

$$\text{CTA}^{\text{acc}} = MV_{\text{final}} (f_{\text{final}}^{\text{acc}} - f_{\text{final}}^{\prime \text{acc}})$$  \hspace{1cm} (12.45)$$

This directly shows the impact of the FX rate difference $f_{\text{final}}^{\text{acc}} - f_{\text{final}}^{\prime \text{acc}}$. Equations (12.44) and (12.45) are consistent with Equation (12.43).

The income to be reported in accounting currency is

$$\text{Income}^{\text{acc}} = G^{\text{acc}} + C^{\text{acc} \text{paid}} + \text{Amort}^{\text{acc}} = BV_{\text{final}}^{\text{acc}} - BV_{\text{init}}^{\text{acc}} - CF^{\text{acc}}$$  \hspace{1cm} (12.46)$$

where $BV_{\text{init}}^{\text{acc}}$ is the book value at the start of the period. The cash flow is again subtracted in the last expression because it is external rather than being generated by the component itself. The relation between Income and $\text{Income}^{\text{acc}}$ is not straightforward as $\text{Income}^{\text{acc}}$ additionally contains some realized currency gains or losses.

The coupon income in accounting currency is

$$C^{\text{acc} \text{paid}} = C^{\text{paid} \text{acc}} f_{\text{mid}}^{\text{acc}}$$

using the FX rate of the time when coupons are reinvested, where $C^{\text{paid} \text{acc}}$ originates from Equation (12.10). The amortization of bonds happens over the whole period, where the booked FX rate is shifting. In lack of a detailed study of that rate, we use an average rate for the amortization gain in accounting currency:

$$\text{Amort}^{\text{acc}} = \text{Amort} \frac{f_{\text{init}}^{\text{acc}} + f_{\text{final}}^{\text{acc}}}{2}$$

where Amort is the result of Equation (12.41). The realized gain $G^{\text{acc}}$ in accounting currency is not just $G_{\text{mid}}^{\text{acc}}$, which is the part due to gains in local currency expressed in accounting currency, but it also contains a part due to realized currency gains. A simple way to compute the total realized gains $G^{\text{acc}}$ follows from Equation (12.46):

$$G^{\text{acc}} = \text{Income}^{\text{acc}} - C^{\text{acc} \text{paid}} - \text{Amort}^{\text{acc}}$$

12.13 Income Statement and Simulation Period Size

Income related variables as computed above can directly be reported in a projected income statement if the simulation period is one year. If smaller periods are used, the contributions of the periods within a year have to be added in order to produce a yearly income statement. In that case, we might also reformulate some equations by using an average FX rate for the whole year instead of individual periods.
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Accounting of an Equity Portfolio

13.1 Introduction

In the framework of ALM components written in Igloo\textsuperscript{1}, SCOR Switzerland’s Financial Modeling team (FinMod) developed an equity portfolio component which is able to keep track of all relevant accounting variables in the course of a Monte Carlo simulation. The equity portfolio can be of the most general type (including hedge funds and other funds, real estate investments etc.). The economic input for simulations comes from an economic scenario generator (ESG).

While the evolution of the market value is a straightforward simulation result, the accounting variables such as book value, realized and unrealized gains, dividend income and impairment loss require some deeper discussion. The main application will need pro-forma accounting in US GAAP style, but the results can also be used for other standards.

Extreme precision of accounting results is not required since the results are based on simulated scenarios, and internal details of the equity portfolio (such as the properties of individual equities in the portfolio) are not known. The goal is the simulation of realistic projected balance sheets and income statements.

13.2 Timing: the Simulation Period

Each simulation software (Igloo or any other tool such as ReMetrica) has its basic simulation period. The simplest choice is yearly simulation from year end to year end. If this temporal resolution is insufficient, we may choose smaller time steps such as quarterly periods. We keep the treatment

\textsuperscript{1}See again Part VIII in this documentation.
of individual time steps as simple as possible, so the only way to achieve higher precision or temporal resolution is to choose smaller time steps.

Cash flows from and to the equity portfolio typically happen over the whole period, not just at its beginning or its end. In order to have an efficient cash flow management in an ALM model with dozens of asset-and-liability components, we assume just one time point of the period at which all the cash flows happen instantaneously. Whenever this appears to be a too coarse simplification, we can use the same remedy as already stated above: replacing the large period by a sequence of shorter ones.

There is one reasonable choice for the time of the cash flow: the middle of the period. This is the average of all cash flow times, assuming that these are uniformly distributed over the year. Choosing the cash flow time at mid-period is in line with accounting-based concepts such as average FX rates and average investment levels.

Another advantage of the chosen concept is that we have stable situations and variable values at both the beginning and the end of the simulation period, where the accounting is made.

### 13.3 The Equity Portfolio

The equity portfolio is a bunch of equities of any kind, expressed in one local currency, managed as an entity with a consistent policy and with continuous reinvestment of dividend payments. There is only a one-time cash flow from or to it at the middle of the simulation period. This one-time cash flow may have different external causes: claim payments, rebalancing between asset classes, investing new income. The cash flow due to the own dividends of the portfolio is not part of that external cash flow and is regarded as an internal affair of the portfolio, invisible to other components. It has no influence on any market value, but it affects some accounting variables such as the dividend income as reported in the projected income statement.

We have an economic scenario for the overall behavior of the market value of an equity portfolio. The relevant variable is the total return rate $r$ over the period, assuming that the market value of the portfolio, including reinvested dividends, is $1 + r$ times the initial market value. The dividend yield $d$ is also known from the ESG and will be used for accounting purposes only.

### 13.4 Input Variables

At initialization of an equity portfolio component, we know the values of a few characteristic properties:

- The legal entity to which the equity portfolio is assigned
- The currency of the equity portfolio (= local currency)
- The initial market value of the whole portfolio, \( \text{MV}_{\text{init}} \), in local currency
- The corresponding initial book value \( \text{BV}_{\text{init}} \) in local currency
- The initial level of the currency translation adjustment, \( \text{CTA}^{\text{acc}}_{\text{init}} \) (the part of unrealized gains that is due to currency shifts prior to the simulation start, which is the sum of the two items \( \text{Group UR Ccy Gain} \) and \( \text{Group UR Ccy Loss} \) in Mellon’s reporting tool)
- The turnover intensity \( u \) of the equity portfolio, with \( 0 \leq u \leq 1 \) (further discussed in Section 13.7)
- The FX turnover intensity \( u_{\text{FX}} \) of the bond portfolio, the inclination to realize unrealized currency gains in excess of realizations due to necessary transactions, with \( 0 \leq u_{\text{FX}} \leq 1 \) (further discussed in Section 13.8)

These variables refer to particular equity portfolios. In addition to these, there are some economic variables which affect the whole equity market. An economic scenario generator (ESG) provides the necessary information, for the start as well as the end of a simulation step:

- The initial foreign exchange (FX) rate \( f^{\text{acc}}_{\text{init}} \) (= value of one unit of local currency expressed in the accounting currency)
- The final FX rate \( f^{\text{acc}}_{\text{final}} \) (analogous to \( f^{\text{acc}}_{\text{init}} \) but for the end time of the period)
- The total return rate \( r \) of the period (annual only if the period is defined as one year, otherwise for the period)
- The dividend yield \( d \) of the period (annual only if the period is defined as one year, otherwise for the period)

All variables with superscript acc are expressed in the accounting (or consolidation) currency. If that index is missing, the quantity is expressed in the local currency of the equity index.

Some initial variable values directly follow from the input variables above, so we can compute them right away. The following formulas apply to any type of asset, not just bond portfolios. The initial level of unrealized gains (or losses, if \( < 0 \)) in local currency is

\[
U_{\text{init}} = \text{MV}_{\text{init}} - \text{BV}_{\text{init}}
\]

The initial market value in accounting currency is

\[
\text{MV}^{\text{acc}}_{\text{init}} = f^{\text{acc}}_{\text{init}} \text{MV}_{\text{init}}
\]
The book value in accounting currency is listed in the Mellon reporting tool under the header *Group Book*. We do not extract that quantity from there, however, because it can be computed with the help of the initial CTA:

$$\text{BV}_{\text{acc}}^{\text{init}} = \text{BV}_{\text{init}} \left( f_{\text{acc}}^{\text{init}} - \frac{\text{CTA}_{\text{acc}}^{\text{init}}}{\text{MV}_{\text{init}}} \right)$$

This can be interpreted as the book value translated to the accounting currency based on an old FX rate $f_{\text{acc}}^{\text{init}}$ rather than the current one:

$$f_{\text{init}}^{\text{acc}} = f_{\text{acc}}^{\text{init}} - \frac{\text{CTA}_{\text{acc}}^{\text{init}}}{\text{MV}_{\text{init}}} = \frac{\text{BV}_{\text{acc}}^{\text{init}}}{\text{BV}_{\text{init}}} \quad (13.1)$$

This is the FX rate that was used when the bond portfolio was bought (or a weighted mean of such rates for a complex portfolio that was gradually built up in the past). The level of unrealized gains in accounting currency is

$$U_{\text{acc}}^{\text{init}} = \text{MV}_{\text{acc}}^{\text{init}} - \text{BV}_{\text{acc}}^{\text{init}} = U_{\text{acc}}^{\text{init}} + \text{CTA}_{\text{acc}}^{\text{init}} \quad (13.2)$$

The last form shows that the unrealized gains consist of two components: one due to changes in the asset market, $U_{\text{acc}}^{\text{init}}$, and one due to changes in the FX rate, $\text{CTA}_{\text{acc}}^{\text{init}}$. In the Mellon reporting tool we find $U_{\text{acc}}^{\text{init}}$ as the sum of the two items *Group UR Gain* and *Group UR Loss*. In our application we do not take it from there but rather compute it through Equation (13.2). When simulating US GAAP balance sheets we want to know the two components of the unrealized gain separately. We compute $U_{\text{init}}^{\text{acc}}$ as

$$U_{\text{init}}^{\text{acc}} = U_{\text{acc}}^{\text{init}} - \text{CTA}_{\text{acc}}^{\text{init}}$$

### 13.5 Cash Flow from or to the Portfolio

The situation is essentially the same as for cash flows from or to bond portfolios (*see p. 264*). In analogy with the bond portfolio, we divide the equity portfolio component into two components:

1. The initial equity portfolio component is configured by the initial values of Section 13.4 and passes some information on its market value development to the cash flow manager.

2. The final equity portfolio component receives some basic information from its initial sister component and the cash flow information from the cash flow manager. Then it computes all results of the simulation period.

Rebalancing may encompass assets denominated in different currencies, but it is defined in terms of the accounting currency. The cash flow management is also done in accounting currency. In order to support cash flow
calculations, we need to know the foreign exchange rate $f_{\text{acc mid}}$ at cash flow time. In accounting, such an average rate is sometimes provided. In the framework of our period-based simulations, we have no such information and choose the following obvious assumption:

$$f_{\text{acc mid}} = \frac{f_{\text{acc init}} + f_{\text{acc final}}}{2}$$

(13.3)

The aim is to have a well balanced portfolio at the end of the simulation period, at the time and in the currency of the accounting. A perfect balance at the period midpoint is not necessary as this midpoint is just an auxiliary time point with no accounting results.

The cash flow manager thus needs some information to make an informed decision on asset rebalancing. The market value of the portfolio, immediately before the cash flow, is

$$MV_{\text{before C}} = MV_{\text{init}} \sqrt{1 + r}$$

(13.4)

approximately assuming that the return factor in the first half period equals that of the second half. Altogether the cash flow manager thus needs the following information:

- The market value $f_{\text{acc mid}}MV_{\text{before C}}$ at mid-period, just before the cash flow, in accounting currency
- The growth factor $\sqrt{1 + r}f_{\text{acc final}} / f_{\text{acc mid}}$ by which the portfolio and injected cash flow will appreciate between the middle and the end of the period, expressed in accounting currency

If the cash is withdrawn from the equity portfolio, the latter factor also refers to the loss caused by the withdrawal.

### 13.6 Market Value Results

The final equity portfolio component receives a cash amount $CF_{\text{acc}}$ from the cash flow manager, corresponding to

$$CF = \frac{CF_{\text{acc}}}{f_{\text{acc mid}}}$$

in local currency. Then the component is able to compute the final market value in local currency,

$$MV_{\text{final}} = MV_{\text{init}} (1 + r) + CF \sqrt{1 + r}$$

and in accounting currency,

$$MV_{\text{acc final}} = MV_{\text{final}} f_{\text{acc final}}$$

(13.5)

At this point, we know all necessary things about the development of market values, but we do not yet support accounting views.
13.7 Accounting Results in Local Currency

The book value of the equity portfolio and other accounting variables are affected by several effects. They are listed in the sequence of their treatment here:

- Injection or withdrawal of cash at mid-period
- Dividends (which are reinvested)
- Turnover due to internal rebalancing of the equities in the portfolio
- Impairment of some equities depending on the size of losses

The accounting is first dealt with in the local currency. Then we move to the accounting currency, where the CTA comes into play.

Injected cash is used to buy new equity, where the purchase price equals the market value and the book value. Withdrawn cash means the liquidation of some equities, assuming that the book value of the remaining portfolio is diminished by the same proportion as the market value. For a portfolio without dividends, internal turnover or impairments, we obtain the following book value at the end of the period:

$$BV_{\text{final}}''' = \begin{cases} BV_{\text{init}} + CF & \text{if } CF \geq 0 \\ BV_{\text{init}} + CF \frac{BV_{\text{init}}}{MV_{\text{init}}} \sqrt{1+r} & \text{otherwise} \end{cases}$$  \hspace{1cm} (13.6)

Dividend payments are reinvested and immediately added to the book value. The book value of the equity investments that generated the dividend stays constant. In the absence of internal turnover and impairments, we further develop Equation (13.6) and obtain a final book value of

$$BV_{\text{final}}'' = BV_{\text{final}}''' + D$$

where the reinvested dividend income $D$ is defined as

$$D = d MV_{\text{init}} + \frac{d}{2} CF$$  \hspace{1cm} (13.7)

Here the dividend is composed of an initial contribution and another term due to the additional cash flow.

Asset managers hardly keep their equity portfolio totally constant over the period. If they have a passive index tracking mandate, several transactions are necessary to keep the tracking error low. Active managers may change the composition even more frequently. Whenever some equity is sold during these turnover processes, unrealized gains or losses are realized, and the book value of the newly purchased equity equals the market value, thus...
moving the overall book value closer to the overall market value. In absence
of impairments, the book value at the end of the period is

\[ BV'_{\text{final}} = u \cdot MV_{\text{final}} + (1 - u) \cdot BV''_{\text{final}} \]

where the parameter \( u \) is the turnover intensity, with \( 0 \leq u \leq 1 \). An
absolutely stable portfolio with no internal turnover has \( u = 0 \). In the case
of a passive mandate, the turnover is rather low with a roughly estimated
\( u = 0.2 \) for a yearly period.

Impairments of individual equity titles are necessary if the market value
moves below a threshold, a certain fraction of the book value, typically \( y =
80\% \) under US GAAP. In that case, the book value has to be taken back
to the market value. The size of this shift is the impairment loss \( I \). A
simple assumption is that an impairment test is made only once, at the
end of the period. Another simple assumption states that all equity titles
behave as fixed proportions of the portfolio. Under these assumptions, a
final impairment test of the whole portfolio leads to the following final book
value:

\[ BV_{\text{simpleImpairment}} = \begin{cases} 
BV'_{\text{final}} & \text{if } \frac{MV_{\text{final}}}{BV'_{\text{final}}} > y \\
MV_{\text{final}} & \text{otherwise}
\end{cases} \]  

(13.8)

A better approximation models a fraction \( v \) of equity titles to be impaired
that can take any values between 0 and 1. If \( \frac{MV_{\text{final}}}{BV'_{\text{final}}} \) is around \( y \),
the true \( v \) is likely to be somewhere in between. A heuristic model can be
made as follows:

\[ v = \frac{1}{1 + \left( \frac{MV_{\text{final}}}{0.82 \cdot y \cdot BV'_{\text{final}}} \right)^{30}} \]

This formula (which might be discussed and refined) leads to the final for-
mula of the book value at the end of the period:

\[ BV_{\text{final}} = v \cdot MV_{\text{final}} + (1 - v) \cdot BV'_{\text{final}} \]

The corresponding impairment loss is defined as a positive number:

\[ I = BV'_{\text{final}} - BV_{\text{final}} = v \cdot (BV'_{\text{final}} - MV_{\text{final}}) \]  

(13.9)

Now we can compute more accounting variables such as the income of
the component:

\[ \text{Income} = BV_{\text{final}} - BV_{\text{init}} - CF \]  

(13.10)

The cash flow is subtracted here because it is external rather than being
generated by the component itself. The income is reported in the income
statement and consists of three parts: realized gains, dividend income and
impairment loss. The latter two quantities are defined by Equations (13.7)
and (13.9), so we obtain for the realized gain

\[ G = \text{Income} - D + I = BV_{\text{final}} - BV_{\text{init}} - CF - D + I \]
Another useful variable is the total unrealized gain $U$ at the end of the period,

\[ U_{\text{final}} = MV_{\text{final}} - BV_{\text{final}} \]

For performance calculations we are sometimes interested in the unrealized gain $\Delta U$ of the period, which is

\[ \Delta U = U_{\text{final}} - U_{\text{init}} = MV_{\text{final}} - BV_{\text{final}} - (MV_{\text{init}} - BV_{\text{init}}) \quad (13.11) \]

The performance of the equity investment can only be measured if both the realized income and the unrealized gains are considered:

\[ \text{TotalGain} = \text{Income} + \Delta U = MV_{\text{final}} - MV_{\text{init}} - CF \]

This relation can be derived from Equations (13.11) and (13.10). An investment return of the period can then be calculated by dividing the total gain by the mean market value:

\[ \text{InvestmentReturn} = \frac{\text{TotalGain}}{0.5 \ (MV_{\text{init}} + MV_{\text{final}})} \]

### 13.8 Final Accounting Results in Accounting Currency

The accounting currency is used to do the final accounting and reporting of the analyzed company. The currency translation depends on the FX rate used. For the market value we use the final FX rate $f_{\text{final}}^{\text{acc}}$, see Equation (13.5), and for income statement variables originating from many time points in the course of the period, we use the average rate $f_{\text{mid}}^{\text{acc}}$ from Equation (13.3).

For the book value and the computation of the currency translation adjustment (CTA), we need the booked FX rate $f_{\text{final}}^{\prime \text{acc}}$ which is typically the average of rates that prevailed when the assets were bought, similar to $f_{\text{init}}^{\text{acc}}$ which we computed in Equation (13.1). A detailed derivation of $f_{\text{final}}^{\prime \text{acc}}$ would be based on tracking the FX rates used in all simulated transactions, which would be rather complex without adding much value. We prefer a simpler but realistic approach. Two effects essentially affect $f_{\text{final}}^{\prime \text{acc}}$: additionally purchased assets (mainly if the cash flow was positive) and deliberate realizations of the CTA. The Mellon reporting tool indeed shows quite a few securities (often with low investment sizes) with $CTA = 0$. Initial CTAs have been realized in those cases (or there may be a booking error in the Mellon tool). It does not matter whether such a realization is due to real market transactions or just a virtual booking operation, as long as it is correctly reported. The inclination to perform such CTA realizations is an input parameter called the FX turnover, $u_{FX}$, where $u_{FX} = 0$ means no such
inclination and \( u_{FX} = 1 \) means continuous realization of all FX gains and losses. In practice, \( u_{FX} \) often exceeds 0 as reinvested coupons are booked at new FX rates, thus leading to some natural FX turnover. Our model for \( f_{\text{acc}}^{\text{final}} \) is

\[
    f_{\text{final}}^{\text{acc}} = u_{FX} f_{\text{final}}^{\text{acc}} + (1 - u_{FX}) \frac{MV_{\text{beforeC}} f_{\text{init}}^{\text{acc}} + \max(CF, 0) f_{\text{mid}}^{\text{acc}}}{MV_{\text{beforeC}} + \max(CF, 0)}
\]

where \( f_{\text{mid}}^{\text{acc}} \) originates from Equation (13.3) and \( MV_{\text{beforeC}} \) from Equation (13.4). The cash flow of the component, \( CF \), plays an asymmetric role here. As long as it is negative or zero (no new investment), it has no influence on \( f_{\text{acc}}^{\text{final}} \).

The book value in accounting currency can now be computed:

\[
    BV_{\text{acc}}^{\text{final}} = BV_{\text{final}} f_{\text{final}}^{\text{acc}}
\]

The total unrealized gain (or loss) is

\[
    U_{\text{acc}}^{\text{final}} = MV_{\text{acc}}^{\text{final}} - BV_{\text{acc}}^{\text{final}} = U_{\text{acc}}^{\text{final}} + \text{CTA}_{\text{acc}}^{\text{final}} \quad (13.12)
\]

The last form shows the two components of the total unrealized gain that we ultimately need in accounting. The unrealized gain excluding currency gains and losses is

\[
    U_{\text{acc}}^{\text{final}} = U_{\text{final}} f_{\text{final}}^{\text{acc}} \quad (13.13)
\]

The Currency Translation Adjustment (CTA) has a final value of

\[
    \text{CTA}_{\text{acc}}^{\text{final}} = MV_{\text{final}} (f_{\text{final}}^{\text{acc}} - f_{\text{final}}^{\text{acc}}) \quad (13.14)
\]

This directly shows the impact of the FX rate difference \( f_{\text{final}}^{\text{acc}} - f_{\text{final}}^{\text{acc}} \). Equations (13.13) and (13.14) are consistent with Equation (13.12).

The income to be reported in accounting currency is

\[
    \text{Income}_{\text{acc}}^{\text{final}} = G^{\text{acc}} + D^{\text{acc}} - I^{\text{acc}} = BV_{\text{acc}}^{\text{final}} - BV_{\text{init}}^{\text{acc}} - CF^{\text{acc}} \quad (13.15)
\]

where \( BV_{\text{init}}^{\text{acc}} \) is the book value at the start of the period. The cash flow is again subtracted in the last expression because it is external rather than being generated by the component itself. The relation between Income and \( \text{Income}_{\text{acc}}^{\text{final}} \) is not straightforward as \( \text{Income}_{\text{acc}}^{\text{final}} \) additionally contains some realized currency gains or losses.

The dividend income in accounting currency is

\[
    D^{\text{acc}} = D f_{\text{mid}}^{\text{acc}}
\]

using the FX rate of the time when dividends are reinvested, where \( D \) originates from Equation (13.7). An impairment of equities may happen over the whole period, where the booked FX rate is shifting. In lack of a detailed
study of that rate, we use an average rate for the impairment in accounting currency:

\[ I^{\text{acc}} = I \frac{f_{\text{init}}^{\text{acc}} + f_{\text{final}}^{\text{acc}}}{2} \]

where \( I \) is the result of Equation (13.9). The realized gain \( G^{\text{acc}} \) in accounting currency is not just \( G_{\text{mid}}^{\text{acc}} \), which is the part due to gains in local currency expressed in accounting currency, but it also contains a part due to realized currency gains. A simple way to compute the total realized gains \( G^{\text{acc}} \) follows from Equation (13.15):

\[ G^{\text{acc}} = \text{Income}^{\text{acc}} - D^{\text{acc}} + I^{\text{acc}} \]

### 13.9 Income Statement and Simulation Period Size

Income related variables as computed above can directly be reported in a projected income statement if the simulation period is one year. If smaller periods are used, the contributions of the periods within a year have to be added in order to produce a yearly income statement. In that case, we might also reformulate some equations by using an average FX rate for the whole year instead of individual periods.
14

Managing Cash Flows of Assets and Liabilities

14.1 Introduction

In the framework of ALM components written in Igloo\(^1\), SCOR Switzerland’s Financial Modeling (FinMod) team developed a component to manage cash flows of assets and liabilities in simulations. Liabilities generate cash due to premiums and consume cash for paying claims. The cash flow management component takes these liability cash flows as given and does not pass any information back to the liability components. Asset components serve as a repository for positive cash flow and as a source of cash in case of a negative cash flow.

The cash flow management component manages all the cash flows of a firm at a central place. The cash flow component has the possibility and the task of allocating cash flows to different asset components, thereby maintaining a balance according to a predefined allocation scheme. This implies an information exchange between asset components and the cash flow management component.

In presence of sub-firms (legal entities), cash flow management becomes a more complex operation. Cash flows between legal sub-entities follow other rules than cash flows between assets and liabilities within an entity. Assets may be assigned to entities or the global firm, liabilities may be subject to internal reinsurance or retrocession between entities. In the current document, this sub-entity problem is ignored.

\(^1\)See also Part VIII of this document.
14.2 Timing: The Simulation Period

The simulation tool (Igloo or any other tools such as ReMetrica) has a basic simulation period of fixed size $\Delta t$. The simplest choice is yearly simulation from year end to year end. If this temporal resolution is insufficient, we may choose smaller time steps such as quarterly periods. We keep the treatment of individual time steps as simple as possible, so the only way to achieve higher precision or temporal resolution is to choose smaller time steps.

In reality, cash flows between the dozens of asset and liability components may happen on every day within a period. This property needs to be modeled in a simplified way for simulations in order to avoid an exceeding complexity within a single simulation period. Our concept is to have just one virtual time point for all the cash flows. Whenever this appears to be a too coarse simplification, we can use the same remedy as already stated above: replacing the large simulation period by a sequence of shorter ones.

There is one reasonable and symmetric choice for the virtual time of the cash flow: the middle of the period. This is the average of all cash flow times, assuming that these are uniformly distributed over the year. Choosing the cash flow time at mid-period is in line with accounting-based concepts such as average FX rates and average investment levels.

Another advantage of the chosen concept is that we have stable situations and variable values (including asset allocation) at both the beginning and the end of the simulation period, where the accounting is made.

14.3 Cash Flow from or to Liabilities

Cash from or to (re)insurance liabilities is determined by the liability components, which may refer to new business as well as the run-off of reserves. The payment times are the midpoints of the simulation periods. Example: Simulation start on Jan 1, 2006, with yearly simulation periods. Then we have payment dates on July 1 of the years 2006, 2007, 2008 and so on. This may not correspond to the dates of the original payment patterns of the liabilities. In that case, the patterns should be optimally mapped to the simulation cash flow dates. Original patterns often start at treaty inception rather than the beginning of the underwriting year. This fact should be accounted for when doing the appropriate pattern mapping.

The cash flow management component has no influence on the liability cash flows. It simply takes them as input values. Only the net sum of all liability cash flows matters. Cash may sometimes have to be transformed from one currency to another (within the liability and asset components), but the overall cash management is described in one accounting currency, denoted by the superscript “acc.” In fact, the cash flow manager does not bother about currencies (but the asset and liability components do). The
total liability cash flow is called $\text{CF}^{\text{acc}}$. A profitable, stable or expanding (re)insurance company is likely to have a positive value in a typical scenario, a shrinking company dominated by run-off normally has a negative value.

### 14.4 Cash Flow from or to the Asset Components

The cash flows from and to the $n$ asset components need to observe the Strategic Asset Allocation (SAA) guidelines\(^2\). When cash from the liabilities is distributed to the invested asset portfolios or cash is withdrawn from them, the cash flow management component has to make sure that the initial allocation split stays constant and conforms to the Strategic Asset Allocation guidelines. As a secondary effect, the different performances of different asset classes within a simulation scenario may lead to a disequilibrium in asset allocation which the cash flow manager will rebalance at the same time as it distributes the liability cash.

The virtual cash flow time at mid-period is important for the calculation but of no interest for final results. Market and accounting values are computed at the beginning and at the end of a simulation period. The asset allocation should aim at those time points rather than the time of the cash flow.

The following information is needed to perform the cash management:

- The desired asset allocation of all $n$ asset components at the end of the period: the market value share $\alpha_i$ of each asset portfolio (which may or may not be equal to the initial allocation at start of the period)

- The market value $\text{MV}^{\text{acc}}_{\text{beforeCF},i}$ of each asset component at mid-period, immediately before the cash flow, expressed in the accounting currency

- The total return factor $F^{\text{acc}}_i$ (not annualized) of the asset component, between mid-period and period end, expressed in the accounting currency and thus fully accounting for foreign exchange gains or losses

The last two variables originate from the asset components themselves. On the other hand, these components should also be the recipients of the resulting cash flow information. In Igloo, such circular information loops are not allowed. This is why we have to divide each asset component into two components:

1. The initial asset component which passes information on $\text{MV}^{\text{acc}}_{\text{beforeCF},i}$ and $F^{\text{acc}}_i$ to the cash flow manager

\(^2\)The SAA guidelines are described in *ALM report final version*, Converium, 28 June 2007.
2. The final asset component which receives some basic information from its initial sister component and the cash flow information from the cash flow manager as a basis of its final result computations

Between mid-period and period end, the market value of an asset component behaves as follows:

\[ MV_{\text{acc,final},i} = F_{\text{acc},i} \times MV_{\text{acc,afterCF},i} \]  \hspace{1cm} (14.1)

where \( MV_{\text{acc,afterCF},i} \) is the market value of the asset component, expressed in the accounting currency, at mid-period, immediately after being modified by cash injection or withdrawal as determined by the cash flow manager. All those market values together exceed the total market value before the cash flow exactly by the amount \( CF_{\text{acc}} \) of the total cash flow:

\[ \sum_{i=1}^{n} MV_{\text{acc,afterCF},i} = \sum_{i=1}^{n} MV_{\text{acc,beforeCF},i} + CF_{\text{acc}} \]  \hspace{1cm} (14.2)

The final asset allocation is defined by the \( \alpha_i \) values,

\[ MV_{\text{acc,final},i} = c \times \alpha_i \]

with an unknown constant \( c \). With the help of the performance factors, we can also determine the asset allocation at mid-period:

\[ MV_{\text{acc,afterCF},i} = c \times \frac{\alpha_i}{F_{\text{acc},i}} \]  \hspace{1cm} (14.3)

Both sides of this equation can be added over all asset components, leading to an equation for \( c \). By reinserting the resulting \( c \) in Equation (14.3) and using Equation (14.2), we arrive at a computation formula for the market values after the cash flow:

\[ MV_{\text{acc,afterCF},i} = \frac{\alpha_i}{F_{\text{acc},i}} \times \frac{\sum_{j=1}^{n} MV_{\text{acc,beforeCF},j} + CF_{\text{acc}}}{\sum_{j=1}^{n} \frac{\alpha_j}{F_{\text{acc},j}}} \]

After computing this, we immediately obtain the desired amounts of cash flow assigned to each asset component:

\[ CF_{\text{acc},i} = MV_{\text{acc,afterCF},i} - MV_{\text{acc,beforeCF},i} \]

The resulting cash flows \( CF_{\text{acc},i} \) are passed to all final asset components.

After doing this, the cash flow component has no further task. The final market values might be computed by Equation (14.1), but there is no need to do this within the cash flow component. This is done by the (final) asset components. The cash flows lead to shifts in book values, too, but those accounting-related calculations will also be done in the asset components.
14.5 Cash Flow with a Hedging Component

Some asset components present a special source of risk, e.g. US interest rate risk, that we want to hedge using derivatives. The values of these derivatives are also a part of the invested asset value. Some parts are static over the whole simulated period. They may produce some cash income which has to be accounted for. The cash manager treats those cash flows as an input, just like the cash flow from or to liabilities. This is the only interaction between static hedge and cash manager.

Another part of the hedge is not static but adapts to the development of the other invested assets. In the remainder of this section, we refer to this non-static part of the hedge. Our example is a set of swaptions bought to hedge the interest risk posed by the bond investments. The market values of the swaptions varies in a way to compensate adverse movements of bond values. Therefore it makes no sense to integrate these swaptions into the market value balancing algorithm we use for normal investments. We use another rule to adapt the size of the hedge to the rest of the portfolio. The total notional $N$ of all the swaptions has to be chosen in fixed proportion to the market value of the fixed income portfolio to be hedged:

$$N = p_{init} \sum_{i=1}^{m} MV_i^{acc}$$

The sum goes from $i = 1$ to $m$ rather than $n$. We assume that the first $m$ asset components represent the fixed income portfolio to be hedged, whereas the other $m - n$ components stay unhedged. The factor $p_{init}$ can be interpreted as the initial price of the swaption per notional. By using this framework, the hedging cash flows can be modeled by the same methods as other assets.
Foreign Exchange Risk

15.1 FX Risk and the Liability Model

Foreign exchange (FX) risk would be grossly overestimated if calculations were merely based on invested assets. A reinsurer will naturally invest in assets that match the currency in which the liabilities are expected to be paid. The ideal liability model will then reflect this currency spread. SCOR Switzerland’s internal liability model currently does not allow to distinguish the currencies in which the various lines of business are written. All liabilities are expressed in USD while the model of assets retains the full FX exposure. An improved way of calculating the FX risk takes into account the ratio of claim supporting capital versus invested assets (Dacorogna [2007a]). The adverse effect of a potential catastrophic event which would exceed the capital held in the respective currency is explicitly taken into account.

Foreign exchange (FX) risk is discussed in Michel M. Dacorogna, Computation of foreign exchange (FX) risk in our ALM model (Dacorogna [2007a]), a version of which is presented in the following section. It gives an estimate of the FX risk exposure in view of the current liability model used at SCOR Switzerland.

15.2 Computation of Foreign Exchange Risk in our ALM Model

Foreign exchange risk is central for a multi-line reinsurer who writes business internationally. SCOR Switzerland Ltd. is subject to this risk and needs to manage its assets to minimize it. Our general practice is to invest in assets that match the currency in which we expect related liabilities to be paid. We tend thus to invest our assets with the same currency allocation as our technical liabilities. This results in the same currency split for the assets backing our shareholders’ equity. In principle the FX value of our assets
varies in the same direction as the one of our liabilities. This means that they are hedged except for the part that represents our shareholders equity.

If the assets are kept in the currency of the liabilities, it is only the capital part that is affected by FX-risk. This is of course only true in expectation. A big catastrophe event can go much above the surplus kept in one currency. Nevertheless, we will assume in this document that most of the FX-risk related to the liabilities is covered by the assets held in the particular currency.

Our current liability model does not distinguish the currency in which a particular line of business is written. We express all liabilities in USD, while the model for the assets take into account the full FX-risk. Thus when we run the model with FX-risk we overestimate it since a good portion of it is hedged by the liabilities that move in the same direction. We need thus to develop a method to estimate the real FX-risk of our portfolio. To do this we run our ALM model once with the full FX-risk and once fixing the FX-rate, assuming that there is no FX-rate fluctuation. We thus obtain two sets of results for the risk-based capital (RBC): $RBC_{FX}$ and $RBC_0$. The true RBC will be somewhere in the middle. Assuming that we have similar portfolio in all currencies\(^1\), we can write an equation for RBC as follows:

$$RBC = (RBC_{FX} - RBC_0) K + RBC_0$$

Given the fact that our risk measure is coherent we can apply this equation to each component of our capital allocation. We need to determine the factor $K$, which represents the portion of the FX-risk that is not hedged by the liabilities. To do this, we shall assume that the FX-risk is proportional to the amount of total assets on the balance sheet. In our ALM model we apply the FX-rate to the invested assets, while we should apply it to the total assets; this leads to a second correction factor. Thus we can write

$$K = \frac{\text{Claim Supporting Capital}}{\text{Total Assets}} \cdot \frac{\text{Total Assets}}{\text{Invested Assets}}$$

which simplifies to

$$K = \frac{\text{Claim Supporting Capital}}{\text{Invested Assets}} \quad (15.1)$$

The capital computation is based on extreme risks: expected shortfall at a 99% threshold. Thus, we suggest correcting the formula above to account for a possible catastrophe event that would not be covered by the capital held in this particular currency. We choose for this the 1/250 years catastrophe loss. Equation (15.1) becomes then

$$K' = \frac{\text{Claim Supporting Capital} + \text{CAT Loss(99.6))}}{\text{Invested Assets}}.$$
This is the factor we apply to estimate the portion of FX-risk left in the capital at risk.

We need to apply a similar correction to the risk drivers, which are computed the same way but with a lower shortfall threshold of 5% or 10%. In this case, the catastrophe event will not play as big a role, so we can use the factor $K$ instead of $K'$, assuming that the catastrophe loss at this probability would be covered by the part of the surplus.

We realize that this method is at best a first approximation of the full FX-risk, but we believe it gives already some reasonable quantitative assessment of it. A better solution would be to split all the losses into their currency components and thus model simultaneously the FX-risk in assets and liabilities. Given our current resources, we need to postpone such a project to a future ALM exercise.
Limitations of the Market Risk Model

The economic scenarios (Müller et al. [2004]) expand into the future the historical experience of ten years. The knowledge about extreme trends is therefore limited and conclusions about dependencies among economic variables are conjectural.

The modeled bond and equity portfolios (Müller [2006a], Müller [2007a]) are stylized in the sense that they satisfy a parsimonious set of conditions which make them convenient to be investigated under the modeled economic scenarios. Within the model each single bond in a portfolio may actually be a proxy for an entire class of bonds. Each portfolio may equally stand for a category of sub portfolios. Alternatively the modeled portfolios may be completely theoretical when they serve to replicate the cash flows of insurance liabilities as is the case for GMDB liabilities (Erixon and Kalberer [2005]). The use of stylized portfolios therefore offers a viable opportunity for a unified approach. Yet by their theoretical nature stylized portfolios remain an approximation of real world portfolios.

Under the general framework of the cash flows manager (Müller [2006c]) practically any cash flows can be modeled. Assets components like hedges are implemented in a natural way and their cash flows are managed. The cash flow manager by itself seems therefore not limited.

Interest risk hedges include swaptions (Iannuzzi [2007]). In the pricing formulas for these swaptions the volatility of the underlying assets is kept constant over the course of one modeled year.

Foreign exchange (FX) risk (Dacorogna [2007a]) is modeled as an estimate of the portion of liabilities which are already hedged against the risk of exchange rate fluctuations by corresponding insurance assets. The main limitation is that the liability model displays all liabilities in one and the same currency (USD).
III

Credit Risk
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17

Introduction

17.1 Credit Risk in Reinsurance

A reinsurance company is affected by credit risk through several balance sheet items\(^1\). On the side of the *invested assets*, there are securities issued by companies with credit risk, mainly corporate bonds. The value of *retrocession assets* (retro claim reserves) depends on the solvency of retrocessionaires at the time when the payments are due. *Cedents* also hold some funds or payables that appear as assets on the reinsurer’s balance sheet. We will discuss how the *liability* side of the balance sheet is affected by some credit risk.

Credit risk materializes in two forms. First, *obligors* may not be able to fully meet their obligations once payments are due. Second, credit risk affects the pricing of securities such as *corporate bonds*, in the form of *credit spreads*. In the following sections we are going to specify a simple model to quantify all essential aspects of credit risk affecting a reinsurance company. This model was set up in the context of the asset-liability model and fulfills the requirements of the *Swiss Solvency Test* (SST). Since there is a market for credit risk with some dependency on *financial market* variables, we integrate a part of the simulation of credit risk in the economic scenario generator (Müller et al. [2004]).

17.2 Corporate Bonds

The credit risk model is a stochastic model. It quantifies the essential aspects of *credit risk* affecting the company\(^2\). The formulation of the model is with

\(^{1}\)In this section we extensively quote from the *abstract* of Müller [2007b].

\(^{2}\)The unified, simple credit risk model is consistently used for all the credit risks of SCOR Switzerland. The major credit risks arise from investments in corporate bonds, retrocession reserves and funds held by the cedents and by Zurich Financial Services.
an emphasis on numerical simulations\textsuperscript{3}.

The model makes assumptions on corporate bond yields translating into default probabilities and vice versa (Müller [2007b]). A set of mathematical conversion formulas relates default probabilities (KMV, Moody) with credit spreads\textsuperscript{4,5}. The ratings for the investigated counterparty companies are also used and occasionally replaced by a more precise data source, namely default probabilities from Moody’s KMV (FMCI, Bloomberg). Credit worthiness equivalently describes credit risk. The model establishes a concrete notion of creditworthiness which combines the dynamics of a bond issuer’s migration among credit classes with the fluctuating credit risk level of the entire currency zone\textsuperscript{6}.

Credit migration makes assumptions on the behavior of individual firms. The model assumptions are that any one simulation step over a discrete time interval will change a firm’s creditworthiness by a stochastic amount which is Pareto distributed.

The credit risk level of a currency zone is regarded as an economic variable. As such it is simulated in the economic scenario generator (ESG)\textsuperscript{7}. The ESG is the primary tool\textsuperscript{8} for modeling financial market risks. A dependence between credit risk and market risk is therefore correctly predicted.

For distinct firms, especially the retrocessionaires which are of primary importance to the company, input from Moody’s KMV is used to set the values of credit worthiness at simulation start.

Any portfolio of corporate bonds in one currency zone is simulated in the form of two stylized portfolios, one a credit derivative of net initial value zero, the other a risk free government bond portfolio. The portfolio is thus split into a component carrying the credit risk plus a stylized portfolio which is separately handled by the bond portfolio manager set up under the market risk model.

An extensive formulation of the credit risk model is Ulrich A. Müller,\textsuperscript{3}The stochastic simulations based on the credit risk model are built with Igloo. Igloo is a software for simulating multivariate probability distributions. Monte Carlo simulations of assets and liabilities are run in Igloo. We refer to the SST systems documentation, Part VIII at pp. 408.

\textsuperscript{4}The model relies on economic data input, including corporate yield curves, from the financial and economic database (FED) whose data have mainly been collected from Bloomberg

\textsuperscript{5}For details about FED we refer to Part VIII.

\textsuperscript{6}A portion of the credit risk depends on the overall credit markets, the credit cycle, rather than the peculiarities of individual companies. That part is modeled in the framework of the economic scenario generator (ESG).

\textsuperscript{7}The ESG simulates future development of key economic indices. Historical time series of economic indicators are analyzed, adjusted and processed. The historical time series of economic innovations is rearranged into future time series of economic innovations by means of statistical resampling. Input to the ESG most notably originates from the economic database FED which collects the data provided by external sources (Bloomberg).

\textsuperscript{8}We refer to Part VIII.
Modeling credit spreads and defaults: A simple credit risk model for ALM and SST (Müller [2007b]). The document puts emphasis on rating migration and the credit risk by individual firms. It deals at length with the connection between default risk and corporate yield spreads. The credit risk dynamics of bond portfolios is modeled. The model elucidates dependencies between market and credit risk. In the following we present a slightly adapted version of Müller [2007b]. The entire document, with its rigorous set of mathematical formulae, spans the range from the conceptual level down to almost the implementation level.

17.3 Funds Held by Reinsureds

A considerable component of net investment income arises from business written on a funds withheld basis\(^9\) which are reported under funds held by reinsureds. The credit risk due to the funds held by reinsured is considered non material and is therefore not modeled.

\(^9\)Converium annual report 2006, p. 38. According to the numbers presented in the consolidated balance sheet in the annual report 2006, funds held by reinsureds are roughly 20% of the asset value.
18

Modeling Credit Spreads and Defaults

18.1 A Simple Credit Risk Model for ALM and SST

Many invested or non-invested assets are, in one way or another, subject to credit risk: the uncertainty surrounding a counterparty’s ability to meet its financial obligations. A reinsurance company such as SCOR Switzerland is affected by credit risk through several balance sheet items:

- Invested assets: securities issued by companies with credit risk, mainly corporate bonds.

- Retrocession assets (retro claim reserves), depending on the future solvency of retrocessionaires at the time when the payments are due.

- Assets under control of cedents: payables and funds held by reinsureds. This type of credit risk is mitigated by the fact that obligations exist in both directions. If cedents cannot meet their obligations, reinsurers might offset these against their own obligations.

- Credit risks indirectly arise on the liability side through dependencies. Some liabilities (e.g. in the line of business Credit & Surety) are affected by developments in the credit market. Other lines such as Professional Indemnity or Marine also behave differently if the underlying companies are under financial stress. Some types of life reinsurance may be subject to substantial credit risk because of the long time span covered by contracts. We need a renewed effort to capture all these dependencies, but that is outside the scope of this document.

The proposed credit risk model is general enough to support the modeling of all credit risk types with a unified approach. The first two types of credit
risks are the most important ones, so we shall implement them with highest priority.

The model runs in the framework of ALM and fulfills the requirements of the Swiss Solvency Test (SST). The credit risk of a firm certainly depends on the individual properties of that firm. Another part of the risk, however, depends on the general market conditions and the general attitude of the financial markets towards credit risk. This part can be described by a generic credit risk variable for a whole market (of a currency zone) and may have some dependencies with other market variables, so we preferably integrate its simulation in the economic scenario generator (ESG). The ESG has no information on individual companies. Whenever we need to compute the company-specific part of credit risk, we have to do this outside the ESG, in the ALM model implemented in Igloo\(^1\).

At a given time, the credit risk of a company can be described by these variables:

1. The credit spread for zero-coupon bonds issued by the company. Usual bonds pay coupons and have a certain maturity date, and we know their price and their yield. The yield exceeds the simultaneous risk-free yield by the amount of the credit spread. If we know credit spreads, we can infer likely credit spread values for different coupons (including zero-coupon), different times to maturity or even different rating classes through formulas. We rarely obtain credit spread data for individual bonds or issuing companies, but we can collect them per main currency and rating class from economic and financial data vendors such as Bloomberg. For some currencies and rating classes, quotes are missing, or the reliability or updating frequency is limited. We additionally need a model to estimate some credit risk data through extrapolation from other classes or currencies.

2. Default probability: Moody’s KMV provides default probabilities or expected default frequencies (EDF) as results of their own model, for each firm. The EDFs refer to one full year, but a formula given by Denzler et al. [2006] allows for computing default frequencies for other time horizons (including the effect of credit rating migration over the years). KMV is a good data source to obtain the initial creditworthiness of a particular important firm (e.g. a retrocessionaire), but we avoid importing large sets of KMV data, which are model results rather than market data, in the ESG.

3. The rating: AAA, AA, A, BBB, BB, ... in the terminology of Standard & Poors, sometimes with narrower classifications such as A- or A+. For quantitative treatment, we shall map these codes to a numerical variable. We distinguish between official ratings by agencies

\(^{1}\)See the IT systems documentation, Part VIII.
(which are rather slowly reacting variables) and the hypothetical ratings which result from a calculation that incorporates new developments more quickly than the agencies. For the corporate bonds of our invested assets, we only have easy access to the official ratings.

For some calculations it is useful to introduce more variables such as the risk-neutral default probability and the creditworthiness or distance to default. The industry sometimes prefers another measure of credit risk levels: Indices based on credit default swaps (CDS).

The main modeling idea is to quantify credit risks by a parsimonious set of variables. There will be just one model variable per currency zone to describe the behavior of the general component of credit risk that is common to all firms. That component can be treated similar to other economic variables in the ESG. When describing the credit risk of one specific company or one group of companies, we just need one additional primary model variable. From few model variables, we can compute all the other quantities needed to describe different aspects of credit risk, in particular credit spread and probability of default, by a unified, simple model. Denzler et al. [2006] have established the groundwork that allows for modeling these two variables and assessing the value of one of them on the basis of the other one through a scaling law with respect to the time to maturity. The formula has a closed form and is invertible. We use it as the basis of our credit risk model.

The model offers a unified approach for all aspects of credit risk with a limited effort. The model would not be accurate enough for a bank or another company for which detailed credit expertise belongs to the core business. There the required credit risk models should be much more complex and rely on detailed information on companies stored in huge databases. Models of that complexity are the basis of commercial services such as CreditRisk+.

For a reinsurer measuring credit risk as one among other, more dominating risk factors, the model has an appropriate depth. It covers the main features and drivers of credit risk by applying closed-form equations to easily available market data. The model parameters can be chosen in a rather conservative way in order not to underestimate certain credit risks.

The organization of the document is as follows. First the key credit variables are presented and discussed. Then a set of formulas allows us to approximately compute different credit risk variables from the other variables, based on Denzler et al. [2006]. In Section 18.3, the dynamic behavior of credit risk is modeled: first the computation of common, market-dependent credit risk variables integrated in the ESG, then the risk modeling of individual firms and portfolios of such firms, which requires some stochastic simulation outside the ESG. Then the dependencies between credit risk and other risk factors are discussed. Section 18.5 offers some practical guidance on the set-up of the credit risk model.
18.2 Variables to Describe Credit Risks

18.2.1 Bond Yields and Credit Spreads

The bond markets provide yield curves for zero-coupon bonds issued by companies belonging to different industries and rating classes. Aggregated and averaged yield curves at an industry and rating class level are provided by the Financial Market Curve Indices (FMCI) database of Bloomberg. Yield curves consist of yield values $Y_j$ for different times to maturity $T_j$ where $j = 1, \ldots, m$. For each yield $Y_j$ there is a risk-free counterpart $\bar{Y}_j$ with the same maturity. Our data source is the fair-market zero-coupon yield curves of Bloomberg which we collect in the FED and use in the ESG. The credit spread of a firm can be defined as

$$\Delta Y_j = Y_j - \bar{Y}_j \geq 0$$

Credit spreads of bonds issued by firms under credit risk are positive. If negative values of $\Delta Y_j$ empirically occur although they violate theory, the reason may be related to errors or asynchronicity in the data. In such very rare cases, we should correct original $Y_j$ values to the risk-free values $\bar{Y}_j$. The spreads $\Delta Y_j$ play no further role as we directly use yields $Y_j$ in our data collection and the model.

Our data sources will not always provide yields for all firms, sectors, rating classes, currency zones and maturities in the desired data frequency and quality. Formulas to infer missing values from other available data in good approximation are needed and presented in this document.

18.2.2 CDS Indices

The industry often prefers CDS indices as a measure of credit risk levels. Credit default swaps (CDS) are instruments for the direct trading of credit risks, so their indices may reflect the market perception of credit risk more precisely and rapidly than other credit risk data. Some research is currently conducted to explore the connection between CDS indices and the other measures. Depending on the outcome, we may or may not add CDS indices as variables of our credit risk model.

18.2.3 Default Probabilities

Default probability: Moody’s KMV provides default probabilities or expected default frequencies (EDF) as results of their own model, for each firm. The EDFs refer to a time horizon of one full year. KMV is a good data source to obtain the initial creditworthiness of a particular important firm (e.g. a retrocessionaire), but we avoid importing masses of KMV data, which are model results rather than market data, in the ESG. We shall use EDF values to approximately compute yields of bonds issued by the firm.
and the other way around, using formulas derived from Denzler et al. [2006]. Although the EDF values from Moody’s KMV have a good reputation, they result from a model computation, whereas yields directly originate from markets. Yields reflect market reality better.

In addition to the EDF, an auxiliary risk-neutral default probability will be introduced.

### 18.2.4 Ratings

There are many corporate bonds in our investment portfolio. For most of them we have information on the credit rating of the issuing firm: AAA, AA, A, BBB, BB, ... in the terminology of Standard & Poors (S&P), sometimes with narrower classifications such as A– or A+. For some bonds we have ratings by other agencies aside from or instead of S&P ratings, but these can be translated to S&P’s system in order to arrive to a kind of average rating per bond in S&P terminology. For quantitative treatment, we shall map these rating codes to a numerical variable $Z \geq 0$ as shown in Table 18.1.

The choice of definition of the quantitative rating scale is arbitrary and of secondary importance as long as the model based on this definition is properly calibrated.

For the lower end around the $C$ rating there are some other codes describing different types of insolvency, but these are not relevant for the risk management of a reinsurer. The companies to be considered mostly belong to the investment grade category, which means a $Z$ value of 15 or higher.

<table>
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<th>Rating (S&amp;P code)</th>
<th>Numerical rating $Z$</th>
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<tbody>
<tr>
<td>AAA</td>
<td>25</td>
</tr>
<tr>
<td>AA</td>
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</tbody>
</table>

Numerical rating variable $Z$. If finer rating notches are given, they imply a shift of $Z$ by 1. Examples: AA– means $Z = 21$, BBB+ means $Z = 17$.

Table 18.1: The rating variable $Z$.

We distinguish between official ratings by agencies (which is a rather
slowly reacting variable subject to many problems of the rating process) and a synthetic rating derived from other variables that incorporate new developments more quickly than the agency ratings. For the corporate bonds of our invested assets, we only have access to the official ratings, so we have to use these.

Our data source has some industry-specific yield data aside from a classification according to ratings, leading to classes such as *Utility A*, *Utility BBB*, *Media BBB*, *Bank A*, *Bank BBB*, *Broker & Dealer A*, *Finance AA*, *Finance A* and *Telephone A*. Industry sectors are discussed later. If we are not interested in them, we prefer using *Composite* rating classes.

There is no constant, fixed relation between a company’s rating and its default probability over time, even if ratings are defined with high granularity and precision and no delay. This fact is demonstrated by the credit risk literature. In Denzler et al. [2006], Figure 1 shows that for a standard company of constant rating, modeled as the median of several groups of companies, the credit spread fluctuated substantially over the years. Depending on market conditions, high values of about 150 basis points and low values of about 50 basis points were reached.

We shall model this market-dependent component separately. Once this is done and we know the market-dependent credit risk level of that component at a certain time point, we shall be able to approximately compute the credit risk level for all rating classes.

### 18.2.5 Conversion Formulas: From Yield to EDF

The core of the credit risk model is a set of conversion formulas between all the introduced risk measures. Denzler et al. [2006] have shown that this is possible in quite good approximation. Here all these formulas are presented. The general scheme is

<table>
<thead>
<tr>
<th>Yield (credit spread)</th>
<th>Risk-neutral default probability</th>
<th>EDF</th>
<th>Creditworthiness</th>
</tr>
</thead>
</table>

Equation 3 of Denzler et al. [2006] contains the computation formula for the risk-neutral default probability $q$. This is defined as the default probability value that leads to a net present value of the expected future cash flows of a bond exactly corresponding to the observed yield $Y(T)$. The formula is

$$q(T) = \min \left\{ \frac{1}{1 - R^T} \left[ 1 - \left( \frac{1 + Y(T)}{1 + \bar{Y}(T)} \right)^{-\frac{T}{\text{year}}} \right], \; q_{\text{max}} \right\} \quad (18.1)$$

where $\bar{Y}(T)$ is the risk-free yield with the same maturity as the simultaneous, risk-prone yield $Y(T)$.

The time to maturity of the bond, $T$, is an important parameter. The minimum function (which has been added to the original form of the equation found in Denzler et al. [2006]) makes sure that $q$, which is a probability,
never exceeds 1. The obvious choice for $q_{\text{max}}$ is thus 1 (or a slightly lower value). In normal cases, $q$ is far below $q_{\text{max}}$. The recovery rate $R'$ is the NPV of the expected recovered funds in case of a default, expressed as a fraction of the NPV in the non-default case. In Denzler et al. [2006], a recovery rate of $R' = 40\%$ is generally assumed, based on some literature cited there. We adopt this general assumption because it makes little sense to pursue detailed recovery rate guesses for hundreds of bond issuers in the framework of the risk management of a reinsurer. However, for the unlikely case of $q = q_{\text{max}}$, the model implicitly assumes a lower recovery rate $R$ than $R'$. We define the true recovery rate $R$ in order to arrive at full consistency:

$$R = 1 - \frac{1}{q} \left[ 1 - \left( \frac{1 + Y(T)}{1 + Y(T)} \right)^{-\frac{T}{1\text{ year}}} \right]$$

This will result in $R = R'$ in normal cases and $R < R'$ in some rare cases where the bond market assumes that a default is near.

Equation 4 of Denzler et al. [2006] defines the annualized risk-neutral default probability:

$$\tilde{q}(T) = 1 - [1 - q(T)]^{\frac{1}{T}}$$  \hspace{1cm} (18.2)

The parameter $T$ of $\tilde{q}(T)$ is a reminder that $\tilde{q}$ still refers to a time to maturity $T$ although it has been annualized. If a constant annual default probability $\tilde{q}$ exists over a period $T$, the total default probability over that period is exactly $q(T)$.

Real markets have a risk aversion, so the true annual expected default probability (EDF) differs from $q$. A central empirical finding of Denzler et al. [2006] (in its Equation 11) is that annual EDFs, which we denominate by $p$, have an approximate relation to $\tilde{q}$. Here this relation is written in inverted form:

$$p = 2 \Phi \left[ \frac{1}{c} \left( \frac{T}{1\text{ year}} \right)^\alpha \Phi^{-1} \left( \frac{\tilde{q}(T)}{2} \right) \right]$$  \hspace{1cm} (18.3)

where $\Phi(\cdot)$ denotes the cumulative of the standard normal distribution and $\Phi^{-1}(\cdot)$ is its inverse function, i.e. $\Phi^{-1}[\Phi(x)] = x, \forall x \in \mathbb{R}$. Equation (18.3) is based on a power law with two empirical parameters $c$ and $\alpha$. Denzler et al. [2006] devote a large part of their paper to estimating $c$ and $\alpha$ from available data and measuring the quality of the results.

For our task at hand, we take a pragmatic approach. When we have direct values of $p$, we can take these. Whenever we do not have direct $p$ values but some raw data for similar companies, we can estimate $c$ and $\alpha$. Equation 12 of Denzler et al. [2006] describes a way to do this for one point in time and data for many times to maturity $T$. If our database is less well stocked, we may do the estimate for just one $T$ but different points in time. If we have no data except for $\tilde{q}$, we can still estimate $p$ through Equation
by using some standard values, \( c = 0.83 \) and \( \alpha = 0.03 \). These values correspond to the average behavior investigated in Figure 2 of Denzler et al. [2006], where the good fit of the power law is also demonstrated. Computations based on these values may not always be very accurate but are still useful for meaningful risk assessment.

Another auxiliary quantity is also useful, the creditworthiness (or distance to default). We use it for two purposes: (1) as a convenient variable for the mapping of numerical ratings, (2) for modeling the movements of credit risk over time. The latter modeling problem was already investigated by Denzler et al. [2006]. A Brownian diffusion model (Equation 8 of Denzler et al. [2006]) led to unrealistic results, but another dynamic model based on asymmetric credit risk shifts (drawn from a log-gamma distribution) succeeded in reproducing the empirical behavior in a Monte Carlo simulation study. The details of dynamic diffusion modeling will be presented later.

Our definition of the creditworthiness \( X \) is inspired by the successful model rather than the failing Brownian model. For a firm at a fixed time point, we define

\[
X = p^{-\gamma} - 1
\]

A defaulting company (with \( p = 1 \)) has \( X = 0 \), a permanent state from which it cannot recover. The parameter \( \gamma \) is related to the movements of the creditworthiness \( X \) over time and will be further discussed in Section 18.3. A good choice is about \( \gamma = 2 \) according to Denzler et al. [2006], but the choice of \( \gamma \) is subject to a new calibration for our model, using long data sample.

### 18.2.6 Conversion Formulas: Results for Different Rating Classes

The creditworthiness \( X \) is a convenient variable to model the relation between rating and other credit risk variables. It varies rather regularly as a function of the rating class as introduced in Section 18.2.4, for a given time point. In the literature, the relation between EDF and credit rating under certain market conditions has been studied. Based on Heitfield [2004], we quantify the following approximate relation between the rating \( Z \) and the creditworthiness \( X \) for a certain time point:

\[
X(Z) = C \cdot Z^\beta
\]

where the parameter values \( \beta = 3.3 \) and \( C = 0.001407 \) can be calibrated from the data presented in Figure 3 of Heitfield [2004], under the assumption \( \gamma = 2.1 \), for an average unstressed standard situation. A new calibration in Section 18.3.4 shows that the relation of Equation (18.5) is appropriate, but a slightly different choice of \( \beta \) and \( \gamma \) leads to a better reproduction of empirical behaviors. The relation depends on the market situation. Our
model assumption is that only the factor $C$ varies over time and describes the fluctuating market perception of credit risk. The parameter $\beta$, which defines the relative behavior of different rating classes in comparison to each other, stays constant.

We shall not use Equation (18.5) stand-alone, but rather as a kind of filter for extrapolating or translating $X$ values to a different rating class if all other conditions (the time point and the industry sector) are the same. The sensitivity against model parameter uncertainty is not as high when translating known $X$ values as in the case of computing unknown $X$ values from scratch.

Suppose we know $X_1$ for a company rated as $Z_1$, and we want to estimate the unknown creditworthiness $X_2$ of another company rated as $Z_2$ at the same time. If several candidates for reference companies or classes with known $X_1$ values exist, we choose one (or an average of companies) with a rating $Z_1$ as close as possible to $Z_2$, thus minimizing the error of the approximation. We can solve Equation (18.5) for $C$,

$$ C = \frac{X_1}{Z_1^{\beta}} $$

and insert the resulting $C$ and the new rating $Z_2$ in Equation (18.5). The result is the desired creditworthiness $X_2$, which can be further transformed to the other credit risk variables as explained in the next section.

18.2.7 Conversion Formulas: From EDF to Yield

The conversion formulas of Section 18.2.5 are often needed inversely. The whole sequence of conversions is now presented in reverse direction.

We start with a certain modeled creditworthiness $X$, which may be translated from another rating class as explained in Section 18.2.6. For any creditworthiness $X \geq 0$ we compute the annual EDF through the inverse form of Equation (18.4):

$$ p = (X + 1)^{-\gamma} $$

From $p$ we obtain the annualized risk-neutral default probability $\tilde{q}$ through the inverse form of Equation (18.3):

$$ \tilde{q} = 2 \Phi \left[ c \left( \frac{1 \text{ year}}{T} \right)^{\alpha} \Phi^{-1} \left( \frac{p}{2} \right) \right] $$

This corresponds to Equation 11 of Denzler et al. [2006].

The non-annualized risk-neutral default probability $\tilde{q}$ for a certain time to maturity $T$ can be computed by applying the inverse form of Equation (18.2):

$$ q(T) = 1 - (1 - \tilde{q})^{\frac{T}{1 \text{ year}}} $$

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The likely yield $Y(T)$ of a bond issued by the firm results from the inverse form of Equation (18.1):

$$Y(T) = [1 + \overline{Y}(T)] [1 - q(T) (1 - R)]^{-\frac{1\text{ year}}{T}} - 1$$

where the approximation $R = 40\%$, as discussed after Equation (18.1), can generally be used.

We stay aware of the fact that most conversion formulas contain an element of approximation rather than being exact. We only use them if market data are missing. The formulas will then bridge the data gaps by approximate values.

18.3 Dynamic Behavior of Credit Risks

18.3.1 Asymmetric Diffusion of Creditworthiness

In Denzler et al. [2006], a Monte Carlo simulation study of creditworthiness supports the model of Equations (18.3) and (18.8). The simulation succeeds, unlike the also studied Brownian model, because of its more realistic diffusion model for $X(t)$. The temporal changes of a firm’s $X(t)$ value are modeled as independent, discrete-time draws from a log-gamma distribution. This distribution has a fat lower tail with a tail index around $\gamma = 2.1$ that allows for downward shocks in the creditworthiness, whereas upward moves are much slower. The success of that model is in line with the observation that rapid, substantial downgrades and defaults of companies happen sometimes, whereas upgrades tend to be gradual.

In this document we set up a model for the dynamic behavior of the creditworthiness $X(t)$, inspired by Denzler et al. [2006], but with a clearer distinction between market factors and the idiosyncratic behavior of companies. This distinction is widespread in the credit risk literature. We consider

- The diffusion process $X_{class}$ for rating classes covering many firms, with no default (because downgraded and upgraded companies no longer belong to the class by definition) and mean reversion in the long run
- The diffusion process $X$ modeling an individual firm (or a group of firms); this differs from $X_{class}$ by an additional, idiosyncratic, trendless diffusion term

The model is explained in the following sections.

18.3.2 The Credit Risk of a Rating Class

We first consider the average creditworthiness of a large, representative set of companies belonging to one rating class $Z_{class}$. The rating $Z$ of all companies

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is in the following range:

\[ 0 < Z_{\text{class}} - 0.5 \Delta Z < Z < Z_{\text{class}} + 0.5 \Delta Z \]

The set of these companies is continuously changing as soon as companies migrate into the range or out of it. We reasonably assume that the definition of rating classes stays quite stable in the very long run. Therefore the average creditworthiness \( X_{\text{class}} \) of the set of companies has no long-term trend, just some fluctuations reflecting the credit market cycle. Moreover, there must be a mean reversion that keeps \( X_{\text{class}}(t) \) away from zero (bankruptcy) and avoids limitless upward trends of \( X_{\text{class}}(t) \) in the long run.

Equation (18.5) already relates \( X \) and rating \( Z \). We reformulate it for the creditworthiness as a function of time:

\[ X(Z_{\text{class}}, t) = C(t) Z_{\text{class}}^{\beta} \]

where \( C(t) \) describes the time variation of creditworthiness.

The historical behavior of the credit risk level \( C(t) \) can be computed from historical yield data for the same rating class using the formulas of Section 18.2.5 and then Equation (18.6). Since the dependency of \( X(Z_{\text{class}}, t) \) is captured by the factor \( Z_{\text{class}}^{\beta} \), we can expect the simultaneous values of \( C(t) \) for different rating classes to be quite close to each other. We therefore suggest to consider just one function \( C(t) \) per currency zone, defined as the average of all relevant rating classes and times to maturity for which sufficient historical yield data are available. This common \( C(t) \) offers a parsimonious way to model the general credit risk level.

The resulting credit risk level \( C(t) \) is known as a historical time series and is positive definite, as the creditworthiness of non-defaulting companies is positive. These are properties identical to those of many economic variables of the economic scenario generator (ESG) of Müller et al. [2004]. The obvious concept is to integrate \( C(t) \) as a new variable in the ESG. The ESG automatically models historical dependencies between economic variables. This will include the interdependence of the credit cycle and other economic variables, as soon as we integrate the variable \( C(t) \) in the ESG.

The treatment of variables in the ESG is explained by Müller et al. [2004]. As already mentioned, we assume that the credit level is confined in the long run rather than drifting away to arbitrary levels. Therefore the ESG will add a mean reversion of \( \overline{C}(t) \) to \( C(t) \). This is analogous to the mean reversion of some other economic variables, which is a method described by Müller et al. [2004].

In each simulation scenario, the ESG will produce a time series of simulated future \( C(t) \) values as one basic input of the credit risk model.

Instead of modeling just one \( C(t) \) per currency zone, we might consider several such credit levels for different industry sectors such as those mentioned in Section 18.2.4. Credit specialists have observed the phenomenon
of sector rotation. The market sometimes regards the credit levels of industry sectors such as banks or utilities favorably as compared to those of other sectors, even if the compared companies belong to the same rating class. This preferences tend to be temporary and limited to few years, after which other sectors enjoy the favor of the credit market. These temporary fluctuations of \( C(t) \) values per sector, compared to the general \( C(t) \) level, are well characterized by the term sector rotation.

The only sector of interest in our context might be the insurance sector, for modeling more accurately the insurers and retrocessionaires that are our potential obligors. The available data may not suffice for setting up a \( C(t) \) model for insurers only. At this stage we rely on general \( C(t) \) values.

18.3.3 Credit Risk Dynamics of Individual Firms, Rating Migration

Individual companies cannot be expected to have a constant credit rating forever. Fluctuations in the individual rating \( Z \) affect their creditworthiness in addition to the shifts in the general credit level \( C(t) \). In Denzler et al. [2006], the successful diffusion model was directly made for the creditworthiness \( X(t) \) rather than the rating \( Z(t) \). We follow that approach and consider \( X(t) \) before discussing \( C(t) \) (which we assume to be constant for now) and \( Z(t) \).

We follow the approach in a slightly simplified way. The log-gamma distributed shifts of \( X(t) \) are replaced by Pareto-distributed shifts with a similar lower tail. The resulting behavior of the modeled downside risk is almost identical, but the Pareto assumption allows for better tractability, using closed-form expressions.

In one discrete simulation step over a time interval of size \( \Delta t \), the creditworthiness changes by a stochastic amount of \( \Delta X \) which is Pareto-distributed with a cumulative distribution

\[
F(\Delta X) = \max \left[ \left( \frac{\gamma}{\gamma - 1} - \Delta X \right)^{-\gamma}, 1 \right]
\]  

(18.9)

The expectation of \( \Delta X \) is zero, so we do not assume a systematic drift, just a stochastic change. The upward potential is limited, but the heavy lower Pareto tail allows for large downward moves. It is straightforward to draw random \( \Delta X \) values from this distribution: Uniformly distributed random numbers between 0 and 1 are mapped to \( \Delta X \) through the inverse form of Equation (18.9).

At which value of \( \Delta X = \Delta X_{\text{default}} \) do we reach a default? We compute this by inserting the default frequency \( p \) from Equation (18.7). In order to keep the model consistent, we demand that it exactly reproduces the right EDF, \( p = F(\Delta X_{\text{default}}) \), and solve the resulting equation for \( \Delta X_{\text{default}} \).
We obtain

\[ \Delta X_{\text{default}} = \frac{1}{\gamma - 1} - X \]  \hspace{1cm} (18.10)

The simplicity of this result is due to using the same exponent \(-\gamma\) in both Equations (18.7) and (18.9). This explains why \(\gamma\) was used in Equations (18.4) and (18.7).

At a first glance, \(\Delta X_{\text{default}} = -X\) seems to be a more intuitive default condition than Equation (18.10) because it leads to the new value \(X = 0\) which characterizes a default. However, we should not forget that our discrete-time model only looks at the end of the simulation interval and neglects the possible occurrence of defaults in the middle of that interval. Equation (18.10) compensates for this by triggering defaults slightly more easily. In the Monte Carlo simulation for an individual firm we trigger a default as soon as the simulated \(\Delta X\) reaches a value of \(\Delta X_{\text{default}}\) or lower. In case of a default we set \(X = Z = 0\) for the end of the simulation step and for all times afterwards, and there is no need to consider the remaining equations of this section.

The usual time interval for simulations for SST and ALM is \(\Delta t = 1\) year. The EDFs are defined for one year. The ESG has shorter simulation intervals, but the resulting \(C(t)\) values are also available in the form of yearly simulation values. The formulas of this section refer to yearly simulation steps, but an adaptation to shorter intervals will be possible if needed.

If no default occurs, we can add \(\Delta X\) to the initial \(X\) value to obtain the new creditworthiness at the end of the simulation step: \(X(t_0 + \Delta t) = X(t_0) + \Delta X\), where \(t_0\) is the time at start of the simulation step. This result is appropriate if the general credit risk level \(C(t)\) stays constant over the simulated step. In reality, the dynamic behavior of the credit market leads to a change from \(C(t)\) to \(C(t_0 + \Delta t)\) which we take as given from our economic scenario generator. A tighter credit market, for example, reflects an increase in the default probability as perceived by the market, even if the rating does not change. Thus the creditworthiness \(X\) changes not only by an amount \(\Delta X\) drawn from Equation (18.9) but also by an additional amount due to the shift in \(C\).

Our model for the dynamic behavior now focuses on the change of \(X\) scaled by the simultaneous credit level \(C\). We model the change of \(X/C\) instead of \(X\) alone but we keep Equation (18.9) as our basis. The change of the variable \(X/C\) is thus

\[ \Delta(X/C) = \frac{\Delta X}{C} \]  \hspace{1cm} (18.11)

where \(\Delta X\) is still a stochastic variable drawn from Equation (18.9). The quantity \(\Delta(X/C)\) characterizes the whole simulation interval, so \(\bar{C}\) should also represent the whole interval. We model it as an average:

\[ \bar{C} = \frac{C(t_0) + C(t_0 + \Delta t)}{2} \]
Now the dynamic model for $X/C$ can be written:

$$\frac{X(t_0 + \Delta t)}{C(t_0 + \Delta t)} = \frac{X(t_0)}{C(t_0)} + \Delta \left( \frac{X}{C} \right)$$

where $\Delta(X/C)$ originates from Equation (18.11).

This model also determines the dynamics of the rating. We re-formulate Equation (18.5) for an individual firm:

$$X(t) = C(t) \beta(t)$$  \hspace{1cm} (18.12)

where both factors on the right-hand side vary with time. This is a separation model insofar as the whole dependency on the general credit market is entirely captured by the first factor $C(t)$ and the whole dependency on the individual firm and its rating $Z(t)$ by the second factor.

Equation (18.12) shows that the modeled quantity $X/C$ corresponds to $Z^\beta$. The dynamic model for $X/C$ turns out to be a rating migration model. By inserting Equation (18.5) we obtain

$$Z(t_0 + \Delta t) = \left\{ \max \left[ Z^\beta(t_0) + \Delta \left( \frac{X}{C} \right), 0 \right] \right\}^{1/\beta}$$  \hspace{1cm} (18.13)

We must not forget that Equation (18.10) may already have triggered a default which automatically leads to $Z(t_0 + \Delta t) = 0$. Equation (18.13) may lead to another default in the rare coincidence of a low random number with a distinct credit level decline. The maximum function in Equation (18.13) makes sure that such rare additional defaults are properly handled.

Equation (18.13) describes rating migrations in terms of numerical ratings. In the credit risk literature and in commercial software tools, rating migration is often modeled through migration matrices with transition probabilities between non-numerical rating classes. These models rely on the statistical analysis of historically observed rating migrations. As all statistical results, such matrices are subject to modifications and updates over time. By comparing results of Equation (18.13) to statistical migration tables we can calibrate the model in Section (18.3.4).

Now we are ready to run our complete quantitative credit rating migration model which is suitable to Monte Carlo simulations. The rating of a firm at $t_0$ is known to be $Z(t_0)$. If we do not yet know it in a simulation, we can compute it through solving Equation (18.12). The final rating is computed by Equation (18.13).

Eventually we calculate the final creditworthiness $X(t_0 + \Delta t)$ of a firm for the general case of a varying $C(t)$. This is straightforward now. We apply Equation (18.12) to time $t_0 + \Delta t$ and obtain

$$X(t_0 + \Delta t) = C(t_0 + \Delta t) Z^\beta(t_0 + \Delta t)$$
where $C(t_0 + \Delta t)$ is a result from the ESG and $Z(t_0 + \Delta t)$ results from Equation (18.13). Once $X(t_0 + \Delta t)$ is known, we can calculate all the other required credit risk variables at time $t_0 + \Delta t$ using the formulas of Section 18.2.7.

### 18.3.4 Calibrating the Dynamic Credit Risk Model

![Table 18.2: One-year rating transition and default probabilities.](image)

Some indications on the choice of the credit model parameters $\gamma$ and $\beta$ can be derived from Denzler et al. [2006] and Heitfield [2004], as already mentioned. That is not enough. We calibrate $\gamma$ and $\beta$ by using statistical results on empirically observed defaults and rating migrations. Standard & Poor’s have published such statistical results which are widely quoted.
and used by practitioners and which we use as a basis for our parameter calibration. Table 18.2 shows rating migration and default probabilities obtained from Poors [2005]. The original Table 21 of Poors [2005] shows the migrations to a further class named NR, i.e. not rated, with a comment that in such cases no default has been reported. Our model conservatively assumes that those NR cases are proportionally split between all other rating classes, including a share of defaults. Thus the horizontal sums of all the rows in Table 18.2 amount to 100%.

All the model parameters are calibrated in a way to minimize the weighted sum of squared deviations of modeled migration probabilities from historical ones as presented in Table 18.2. The squared deviation of a default probability has an arbitrary special weight of ten times that of another squared deviation, reflecting the need for a particularly good fit of default probabilities. The first calibration has led to the following choice of parameter values: $\gamma = 2.5$ and $\beta = 3.3$. The calibration exercise will be repeated based on new input data and other model improvements.

18.3.5 Credit Risk Dynamics of Portfolios and Dependent Companies

Investors typically hold well-diversified portfolios of corporate bonds rather than betting on few single companies. Similarly, a reinsurer prefers several retrocessionaires to just one. Portfolio diversification mitigates credit risks to a certain extent. Losses due to downgrades or even defaults become more likely for some firms, but these are averaged out by upgrades of many other firms of the portfolio.

However, we should not overestimate the diversification effects of portfolios. The benefits are limited for a number of reasons. The general credit risk level as explained in Section 18.3.2 affects all companies together, so there is always a common element in credit risk that cannot be diversified away. Our model captures this common element.

For discussing the other reasons, let us assume a portfolio of, say, 500 corporate bonds. A closer look may show that some of them originate from the same company or group, so only 400 are independent. The market values of the bonds typically vary in size. Some of them are so much smaller than the dominating ones that they do not count (or only if they are lumped together with other small bonds). We may be left with only 150 effective bonds that really matter. Some of the firms have strong ties or they share the industry sector, which leads to dependencies that are not (yet) captured by Section 18.3.2. The number of independent credit risk factors may be reduced to 80. The credit risk literature has found strong tail dependencies between the risks of different firms even if these behave independently under normal circumstances. Their creditworthiness tends to have strong co-movements in the extreme case of a crisis, as discussed by Li [2000].

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Extreme cases matter in risk management. In an extreme situation, the effective number of independent credit risk factors may shrink from 80 to 20 or 10. We should also consider that our credit risk model is quite simple. We know we have certain inaccuracies in our results, so we want to use the model with rather conservative assumptions on credit risk diversification.

A simple way of modeling a portfolio of companies, especially when modeling corporate bond portfolios, is to model the portfolio as a set of $N$ stylized, independent companies with the methods of Section 18.3.3. The number $N$ is chosen much smaller than the number of corporate bonds contained in the portfolio. We have seen that $N$ could be 10 even if the portfolio comprises 500 bonds. A fixed rule to determine $N$ for a given portfolio has yet to be worked out and may contain an element of judgment.

An alternative to modeling $N$ separate companies would be to derive a methodology for describing all of them together. That does not seem easy, and there may be no closed-form algorithm for the aggregated portfolio. At the moment, such an approach cannot be recommended.

For smaller portfolios such as the portfolio of retrocessionaires, we model each company individually, starting with the current EDF from Moody’s KMV. That is a more precise and up-to-date measure of credit risk than just using the company’s rating class provided by a rating agency. Then we model each company by using the methods of Section 18.3.3. Different retrocessionaires belong to the same industry sector, so we have to explicitly model some dependencies in their dynamic behavior. The most appropriate way is to link the random numbers used to determine the individual risk according to Equation (18.9) through a copula, as explored by Li [2000]. The Igloo software supports the generation of random numbers linked by a copula. The multivariate Clayton copula (see Section 8.2.4) is our choice for modeling the strong dependence in the lower tails of the distributions for all retrocessionaires.

### 18.4 Dependencies between Credit Risk and other Risk Factors

#### 18.4.1 Dependencies between Credit Risk and Market Risk

The SST demands that credit and market risks are modeled wherever they show up on the balance sheet. This includes the interdependence between these two risk factors. There is quite some academic research in this field, for instance in the long paper by Pesaran et al. [2003] where econometric models are explored. We rely on the ESG which includes a credit risk variable which exactly models the part of credit risk that depends on the market rather than on the individual fates of companies or industry sectors. Thus we rely on the bootstrapping technique which projects the historically
observed dependence between credit risks and other market variables into the simulated future. We do not need any additional element to introduce that dependence as the ESG already does the job of modeling it.

Empirical data show that there is a dependency between credit risk and market variables such as equity prices. Credit risks rose when the equity boom of the 1990s was over and gave way to negative equity returns. The ESG scenarios will reflect such empirically found behaviors.

18.4.2 Dependencies between Credit Risk and Liabilities

Liabilities may depend on market as well as credit risks. We may understand this as a triangle of dependencies: credit, market and liability risks. In earlier ALM models, we have already modeled the dependence of liabilities on market variables, for lines of business such as Credit & Surety. We argued that Credit & Surety claims are less likely and lower in times of a booming economy. We know that this often coincides with low credit risks.

Since we explicitly support credit risk levels in the ESG, we can formulate the dependence of credit risks and Credit & Surety losses more directly, without using market indicators such as equity index prices or GDP as intermediaries. For other lines of business such as Aviation, however, the main dependency will stay with the economy, and credit risk hardly has a direct influence.

18.5 Modeling Credit Risks of a Reinsurer

18.5.1 Modeling Bond Portfolios with Credit Risks

A corporate bond portfolio comprises many bonds of different ratings. For modeling them, we collect them in a few sub-portfolios, each with its characteristic range of rating classes. Each one has its average numerical rating $Z$ at simulation start. When averaging the $Z$ values of the bonds, we should not directly take the mean of $Z$ but rather the mean of the default probabilities $p$ computed from the $Z$ values through Equations (18.5) and (18.7). The mean of $p$ can then be transformed back to a mean rating $Z$ by using the inverse forms of these equations. Means are always meant to be weighted means, where the initial market values are the weights. By using such a procedure we make sure that the correct average default probability is preserved.

The ALM model of SCOR Switzerland already contains an elaborated model for simulating the behavior of risk-free (government) bonds, as described by Müller [2006b]. That model infers the market value development of bond portfolios from scenarios of the underlying risk-free zero-coupon yield curves.
Corporate bonds also depend on the yield curve dynamics, so we have a good reason to use the existing bond portfolio model also for corporate bonds. On the other hand, the credit risk model of this document cannot be easily integrated in that existing algorithm. We solve this dilemma by modeling a corporate bond portfolio as consisting of two components:

1. a risk-free portfolio modeled by the methods of Müller [2006b], corresponding to the corporate portfolio in every aspect except for its being risk-free;
2. a simplified portfolio consisting of a long position in a set of bonds with credit risk and a short position in the corresponding risk-free bonds.

This set-up has some advantages. The first component deals with the dependence on interest rate markets whereas the second component, which could be interpreted as a sort of credit derivative, captures the credit risk with the methods developed by this document. By keeping the components separate, we avoid the problem of merging two very different methods in one complicated algorithm. The portfolio composition of the second component is designed to support the credit risk modeling as outlined in Section 18.3.5, but there is no need to model the elaborated rollover scheme of Müller [2006b] there.

Moreover, the separation of the two components is beneficial for producing the desired results. While the first component reflects market risk and return, the second component reflects credit risk and return. In the SST and the risk driver analysis of ALM, we want to keep the two risk factors separate, so we can profit from a method where this separation is already built in.

The second component is affected by defaults, downgrades and upgrades. It may also be affected by an asset management guideline. Some asset managers have the mandate of keeping the rating of the managed bonds within a range. They will sell bonds as soon they move out of a rating range due to a downgrade or upgrade and buy new bonds that are well in the range. Such a rule may be followed very strictly or in a more relaxed way. Some substantial downgrades may be so sudden that the rule cannot be fully implemented. The algorithm has to mimic this behavior in acceptable approximation.

18.5.2 Modeling the Credit Risk of Retrocessionaires and other Obligors

Modeling the credit risk of individual firms such as retrocessionaires is much simpler than modeling corporate bond portfolios. The method has already been explained in Section 18.3.
Limitations of the Credit Risk Model

The exposure to individual retrocessionaire companies is indeed modeled (Müller [2007b]). However the model provides for no explicit dependency between the default probabilities of retrocessionaires and large natural catastrophe (nat cat) losses.

The credit risk of two balance sheet assets was treated as second priority so far, funds held by reinsureds and the Funds Withheld by ZFS. Some of these funds are hedged: The company would refuse claim payments corresponding to such funds in a default case. The funds held by reinsureds have not yet been modeled in the framework of the credit risk model.

The creditworthiness of a single counterparty is modeled $X = CZ^\beta$, where $C$ denotes the general credit cycle and $Z$ reflects the credit rating (Müller [2007b]). In a cycle of high perceived credit risks, the rating agencies may be inclined to more downgrades than upgrades. This sort of additional dependence has not been taken into account.
IV

Operational and Emerging Risks
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Operational Risk

The Operational Risk Management process consists of the following four steps:

1. **Risk Identification**, including collection of a list of operational risk schemes/scenarios and description of each operational risk scheme/scenario. The way we identified risks is twofold:
   
   (a) Annual conduction of *interviews* with process owners supplemented by further key people in the organization. The focus of the interviews is:
       i. Understanding general process flows and key process challenges.
       ii. Identifying new operational risk schemes.
       iii. Assessing the risk inherent in business processes.
       iv. Understanding current situation of and gaps in mitigation measures.

   (b) Annual conduction of a *survey* which distributed to all employees electronically and filled in online by the participants (answers are anonymous). The multiple choice questions are intended to receive feedback from the participants regarding the importance and current quality of mitigation on 33 predefined operational risks.

2. **Risk Assessment**, i.e. assessment of frequency and severity of every operational risk scheme/scenario and prioritization.

3. Risk Mitigation, i.e. definition of measures to minimize risk from high-priority operational risk schemes (e.g., educating people, training employees, setting guidelines).

---

1. This section is based on the company internal document (Langer [2008]).
4. *Incident Management*, i.e. detection of incidents (e.g., monitoring systems, red flags, whistleblower), set-up of incident management plans (e.g., emergency plans) and collection of incidents in incident database.
Emerging Risks

Emerging risks\(^1\) are those which may develop or which already exist, but are difficult to quantify and may have a high loss potential. These risks are marked by a high degree of uncertainty. Climate change, genetic engineering, nanotechnology, legal and regulatory changes are examples for emerging risks. Asbestos would have been an emerging risk before it materialized.

For emerging risks, we perform regular brainstorming sessions on potentially new arising risks. They are assessed in terms of relevance and impact on our portfolio. In case our portfolio is affected, a process is started to inform all necessary stakeholders; e.g. management, pricing and reserving department, underwriting etc. Policies and guidelines as well as trainings are reviewed and – if necessary – adjusted to account for the treatment of new risks.

\(^1\)This section is based on the company internal document (Langer [2008]).
V

The Economic Balance Sheet
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Aggregating Risk Inputs to Balance Sheet Outputs

This Part focuses on the economic valuation of assets and liabilities in order to generate the economic balance sheet (the “output”) from which the risk-bearing capital figures at different points in time can be derived as market value of assets minus best estimate of liabilities. The balance sheet values are valued at the beginning of the period and after one year (as contribution to the expected shortfall of the economic value of the company).

It should be said that risk input factors like market, credit and insurance risk potentially affect the economic balance sheet items at different places (e.g. interest rates influencing bond valuations and discounting of liabilities, foreign-exchange rates affecting assets and liabilities, reserve volatility influencing reserves and reinsurance assets), i.e. there is a many-to-one mapping from the risk factors to the economic balance sheet items after one year.

In general, the aggregation of the stochastic development of the insurance, credit and also market (e.g. interest rate) risk-related insurance business quantities (premiums, new losses, reserve developments, projected cash flows of life business) is done via Monte Carlo Simulation, i.e. for each simulation scenario the resulting liability cash flows and their pattern-dependent NPV valuations are added up to assess the overall value (change) of the insurance liabilities by balance sheet item. Simultaneously, also induced by the cash flows from the liabilities to be invested, the same is done for the (mainly) market-risk-dependent economic value development of the assets. (I.e. the stylized asset class portfolios of SCOR are affected by corresponding projected returns stemming from the economic scenario generator.) It has to be said that the impacts of the simulated insurance, credit and market risks and their modeled dependencies on the business quantities are assumed to be “complete” in the sense that no additional “extreme” risk scenarios (like the scenarios sent to the FOPI once a year) need to be added.
as this would result in a double counting of risks. Of course, the yearly analysis of extreme scenarios is used to crosscheck the outcome of extreme simulation results of the internal model.
23

Legal Entities and Parental Guarantees

23.1 SCOR Holding (Switzerland) AG

SCOR Switzerland AG (formerly Converium AG) is operating as a hub company within the SCOR Group. The Group’s operating segments are SCOR Global Property & Casualty Reinsurance as well as SCOR Global Life Reinsurance. The operating segment Property & Casualty aggregates results for Treaty Underwriting as well as Reinsurance of Specialty Lines & Business Solutions. Treaty Underwriting includes the following lines of business: General Third Party Liability, Motor, Personal Accident (assumed from non-life insurers) and Property. The lines of business in the Specialty Lines segment include Agribusiness, Aviation & Space, Credit & Surety, Engineering, Marine & Energy, Professional Liability, other Special Liability and Workers’ Compensation. In Life & Health Reinsurance, our lines of business include Life and Disability reinsurance, including quota share, surplus coverage and financing contracts and Accident & Health.

The SCOR Switzerland organizational structure is displayed in Figure 23.1.

23.2 Description of Parental Guarantees

1. In agreement with the rating agencies Converium AG issued a guarantee on 21 October 2004 for the policyholder obligations of Converium Rückversicherung (Deutschland) AG and Converium Insurance (UK) Ltd. (the Companies). This guarantee is a guarantee of payment, not of collection, and is unconditional. Converium AG undertakes to pay, upon receipt of a notice or demand in writing from the reinsureds, the respective amount if the Companies do not pay such amount due and payable under a reinsurance policy to a reinsured.
A diagrammatic overview of the SCOR Switzerland legal structure, as of 31.12.2007

Figure 23.1: SCOR (Switzerland) AG legal structure
2. In August of 2004, in order to retain certain US business, Converium AG endorsed for a number of selected cedents of Converium Reinsurance (North America) Inc. a parental guarantee with an option to novate business written for the 2003 and 2004 underwriting years. Some of these options to novate the business to Converium AG’s balance sheet were executed in the fourth quarter 2004. The remaining cedents did not execute the option and the business remained on Converium Reinsurance (North America) Inc.’s balance sheet. Due to the disposal of Converium’s North American operations to National Indemnity Company, Converium AG as the guarantor received from National Indemnity Company full indemnification of the potential outstanding liabilities. As of December 31, 2006, 2005 and 2004 these liabilities were USD 146.1 million, USD 95.7 million and USD 121.4 million, respectively.

3. Under the Quota Share Retrocession Agreement with Zurich Insurance Company, Converium AG, as at June 30, 2007, bears the credit risk for potential non-payment of retrocessionaires in the amount of CHF 319.6 million. As at December 31, 2006, the potential credit risks totalled CHF 324.2 million. Exceptions to this to this Quota Share Retrocession Agreement consist of liabilities that occurred as a result of the terrorist attacks in the USA on September 11, 2001.
Fungibility of Capital

The model of capital fungibility is outlined in the internal paper *A Simple Model of Capital Fungibility for Converium ALM* by Michel M. Dacorogna and the presentation *Fungibility of Capital* by Michael Moller presented to the FOPI. Our documentation on capital fungibility is based on these materials (Dacorogna [2007b]).

24.1 Introduction

The purpose of this paper is to give the framework for modeling capital fungibility in the ALM. We use the modeling framework suggested by Filipovic and Kupper [2006]. An insurance group with $m$ business units (BU) is considered. Values at the beginning of the period are known and denoted by small letters, while values at the end of the year are random variables and denoted by capital letters.

We use a coherent measure of risk, $\rho$, on the probability space $E$. We define the available capital of BU $i$ as the value of its asset-liability portfolio. We denote by $c_i \in R$ the capital at the beginning of the year and by $C_i \in E$ at the end of the year. This definition implies the assumption of a valuation principle $V: E \rightarrow R$ such that

$$c_i = V(C_i)$$

The available capital depends on the selection of liabilities to be covered and the assets backing these liabilities. We assume here that for each BU the relevant portfolio is determined as it is the case in our new ALM model.

The required capital $k_i$ is thus determined by the equation

$$k_i = c_i + \rho_i(C_i)$$

---

1This part of the documentation deals with the theoretical aspects of internal risk models. For the details about the ALM and SST processes and the ALM and SST modeling modules we refer to Part VII.
The asset-liability portfolio is considered acceptable if \( c_i > k_i \).

The objective of the group is to account for \textit{diversification} effects across the BU’s, which results in an aggregate group required capital, \( k_g \), being less than the sum of the stand-alone BU’s required capitals

\[
k_g \leq \sum_{i=1}^{m} k_i
\]

It is understood that the group required capital \( k_g \) is allocated to the BU’s according to their (marginal) risk contributions \( \hat{k}_i \)

\[
k_g = \sum_{i=1}^{m} \hat{k}_i
\]

### 24.2 Proposed Framework: Model Standardized C&R Transfers

We assume that there exists a well-specified finite set of legally enforceable capital & risk (C&R) transfer instruments with future contingent values modeled by some linearly independent random variables \( Z_0, Z_1, \ldots, Z_n \) in \( E \). Cash is fungible between BU’s as long as the payments at the end are determined at the beginning of the year. This is expressed by letting \( Z_0 \equiv 1 \).

The modified risk profile of BU \( i \) becomes

\[
C_i + \sum_{j=0}^{n} x_i^j Z_j
\]

for some feasible C&R transfer

\[
x_i = (x_i^0, \ldots, x_i^n) \in W_i, \quad i = 1, \ldots, m, \quad (24.1)
\]

such that

\[
\sum_{i=1}^{m} \sum_{j=0}^{n} x_i^j Z_j \leq 0 \quad (24.2)
\]

where \( W_i \) is some closed convex subset in \( R^{n+1} \) with

\[
0 \in W_i \text{ and } W_i + (r, 0, \ldots, 0) = W_i \text{ for } r \in R, \quad i = 1, \ldots, m.
\]

This means that there is no exogenous value added, because internal risk and capital transfers do not generate positive cash in total. The modified risk profile is solely due to a feasible redistribution of capital and risk by means of the instruments \( Z_0, Z_1, \ldots, Z_n \).

The aggregate required capital of SCOR Switzerland is

\[
k = c + \sum_{i=1}^{m} \rho_i \left( C_i + \sum_{j=0}^{n} x_i^j Z_j \right)
\]

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Lowercase indices $i$ count the legal entities, i.e. $i = 1, 2, 3$ stand for the business units Zurich, Cologne, UK, in which case we have $m = 3$.

Lowercase indices $j$ running from 0 to $n$ index the transfer instruments: Cash transfers, internal loans, retrocession programs, parental guarantees, sale of participations.

$x_i^j$ is the instrument exposure of instrument $j$ of Business Unit $i$.

$Z_j$ is the stochastic portion of instrument exposure $j$, it equals 1 for deterministic cash transfers.

$C_i$ is the stochastic available capital of Business Unit $i$ at the end of the year without risk transfer.

One task yet to be achieved in the future is to minimize the difference between required and available capital. One will have to find an optimal C&R transfer which solves the optimization problem

$$\min_{(x_1, \ldots, x_m)} \rho_i (C_i + \sum_{j=0}^{n} x_i^j Z_j)$$

subject to the feasibility and clearing conditions 24.1 and 24.2, respectively. The default risk of guarantors has to be taken into account.

### 24.3 Application to SCOR (Switzerland) ALM

In our ALM model, we do not directly model the retro but use gross and net distributions. This, of course, does not allow us to optimize the set of $x_i$ to obtain the best diversification. On the long run, we would like to do this and would then have to fully model the C&R transfers between the different legal entities. Meanwhile, we can still use the framework without optimization.

For each legal entity, we model the risk gross and net. Because the outward retrocession of our company is bought centrally by SCOR Switzerland Ltd., the difference between gross and net of the other legal entities become liabilities of Zurich. For those liabilities that are covered by an outward retrocession contract, we can assume that they will increase the gross liabilities but not the net. There are, however, some internal retrocession contracts that are not retroceded outside the company, particularly quota shares with London or Cologne. Those will be then direct contributions to the net liabilities of Zurich and would add to the diversification of the book.

Conversely the economic value of the various legal entities becomes an asset for SCOR Switzerland Ltd. In principle, a Ltd. can sell those legal entities if need be as we have done with Converium North America and
realize the economic value of this entity. The economic value of each legal entity owned by SCOR Switzerland Ltd. will thus be added to its available capital. In the model implementation, we thus have to add an economic valuation of each legal entity along the lines of the model we use for the impairment tests of our yearly balance sheet. Since we have to look at the economic view, we cannot simply take the values of the legal entities that are written on our balance sheet.

In practice, we need to model for each legal entity its own liability gross and net. The difference is then transferred to Zurich. The parental guarantee offered by Zurich to Cologne is not modeled by this approach. We need for this to establish few rules when such a guarantee can be exercised. We propose the following two rules:

- The parental guarantee is exercised when the internal retrocession does not suffice to assure the solvency of the legal entity.

- The parental guarantee is only exercised when it does not “put in danger” the solvability of the parent. If, for instance, the economic capital is extinguished or very low so that the likelihood of not fulfilling other obligations becomes high, the seniority of this guarantee might be subject of juristical debates between regulators and/or other stakeholders.

With these two rules we set the limit of the capital fungibility vis-a-vis the legal entities. While the selling of the legal entity establishes the fungibility in the other direction. We can also add another rule for this direction:

- The legal entity is allowed to transfer capital in favor of the parent only and only if the amount does not affect the solvability of the legal entity.

With this set of rules, we can have a first approximation of the capital fungibility. We will be able to measure the efficiency of the fungibility by aggregating the lines of business on the full company and looking at the difference between the capital needed in aggregate with the capital needed for the legal entity Zurich modeled as we just described. Of course, we cannot yet optimize the risk transfer between the legal entities as it would be possible if we would fully use the approach proposed in Filipovic and Kupper [2006]. We propose to do this in a second step, once we would have gained some experience with the application of this concept. Due to the big amount of highly liquid invested assets and the ability of transfer of capital between legal entities, it is at the moment assumed that liquidity risk is of minor importance for SCOR Switzerland.
24.4 Limitations of Modeling of Capital Fungibility

In case of financial distress of both the parent and a potential subsidiary in another country it is not clear to what extent the different regulators would limit the allowance of capital transfers.
The Valuation of Assets

The SST regulations propose a standard which evaluates assets generally at market conditions (economic valuation). The statement of the company’s financial condition and the report on the value of the company’s assets at a given point in time rely on the determination of the fair value of the company’s assets where the notion of fair value refers to the price, expressed in cash equivalents, at which property is exchanged between the seller and the buyer in an open and unrestricted market\(^1\). The company’s banking partners provide sufficient information to directly determine, for example, the amount of cash, equities and bonds and also the market value of derivative financial markets instruments, like swaptions (Iannuzzi [2007]) at \(t_0\). These market values constitute an essential part of the initial economic balance sheet and the valuation is typed mark to market. There are classes of assets, however, which equally influence the company’s financial position yet will not be read off the company’s bank accounts or are not traded in a deep and liquid market from where reliable and comparable market prices are available. An income approach may then be the most logical and most practical technique to estimate the fair value, by assessing the net present value\(^2\) (NPV) of the benefit stream of cash flows generated by the business. The NPV based valuation illustrates the mark to model approach with its contingency on the realism of financial model assumptions. As an illustration, the intrinsic value of SCOR Switzerland’s funds withheld assets is NPV based. With

\(^1\)In the absence of an actual exchange the fair value is a hypothetical price contingent on the assumptions about an idealized market. It is an estimate of the price at which assets or liabilities could be exchanged in transaction between knowledgeable, unrelated, willing parties.

\(^2\)The NPV, defined as the discounted excess of cash flows, i.e. discounted benefit cash flows minus the initial capital outlay, specifies one distinct approach towards the determination of a fair value. We emphasize that various classes of our company’s assets are valued by SCOR Switzerland’s banking partners, based on their own specialized models and the specific knowledge by these institutions. At this point we do not indicate anything about these methods which may of course deviate from an idealized NPV valuation or may be based on adequate principles other than an NPV valuation in the narrow sense.
complex financial instruments, marking to model is the only applicable valuation method. SCOR Switzerland holds structured notes (Iannuzzi [2007]) specifically designed to protect the company’s balance sheet against interest rate (IR) risk. There is no established market for these hedges as they are issued by banking partners (Hypo Vereinsbank, Germany) exactly for that purpose. The issuer combines the available market data into a market valuation which is finally reported to the global custodian (Mellon Bank, New York) from where, in turn, it indirectly enters the company’s balance sheet.

The issuers of financial instruments as well as the banking institutions which are responsible for safeguarding the company’s financial assets regularly provide relevant, reliable and comparable values on the company’s assets. They report the asset classes and the actual value of assets which serve as the starting point for stochastic simulation and for risk evaluation (see, for instance, economic scenario generation in Chapter 10 and credit risk, Part III).

In the real estate market an intricate aggregation of estimates is used by real estate fund managers for real estate appraisal, where different values (comparable sales, replacement costs, income approach: value in use) must be read against each other and are aggregated over a wide variety of individual assets. Fair values, as obtained by fund managers and also through a third party are once more registered by the global custodian.

SCOR Switzerland is holding shares in real estate investment trusts which are quoted on a stock exchange. These values are therefore market values.

Table 25.1 lists the classes of assets held by SCOR Switzerland. It specifies the valuation methods by which corresponding fair or mark-to-model values are assigned to them.

The required valuation at the end of the year, i.e. the contribution to the 1% expected shortfall of the economic value of the company, of most invested assets is based on the mapping of the returns of the economic scenario generator to the investment classes mentioned above. The handling of the accounting of the invested assets and the cash flows stemming from the insurance business is discussed in Part II.

Real Estate

SCOR Switzerland holds shares of the Morgan Stanley Euro Zone Office Fund, a private equity fund for institutional investors. Audited funds re-

---

3Most notably invested fixed maturity assets: Bank of New York Mellon, Asset Services, Global Custody.
4Again, the simplified nature of this example shall merely serve the purpose of illustration and shall not be applicable to infer the actual methods and assumptions used for valuation by the real estate fund managers.
5For the details of economic scenario generation see Chapter 10 of this documentation.
6SCOR Switzerland keeps its shares below 20%.
<table>
<thead>
<tr>
<th>Assets</th>
<th>Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Invested Assets</strong></td>
<td></td>
</tr>
<tr>
<td>Fixed maturities</td>
<td>Marked to market</td>
</tr>
<tr>
<td>Funds withheld</td>
<td>Marked to model (NPV based)</td>
</tr>
<tr>
<td>Investments in subsidiaries</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Equity securities</td>
<td>Marked to market</td>
</tr>
<tr>
<td>Derivatives (interest rate hedges)</td>
<td>Marked to model (by banking partners)</td>
</tr>
<tr>
<td>Real estate</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Short term investments and other investments</td>
<td>Marked to market</td>
</tr>
<tr>
<td><strong>Other assets</strong></td>
<td></td>
</tr>
<tr>
<td>Cash and cash equivalents</td>
<td>Marked to market</td>
</tr>
<tr>
<td>Premiums receivable</td>
<td>Marked to model (NPV based for best estimate)</td>
</tr>
<tr>
<td>Underwriting reserves, retrocession</td>
<td>Marked to model (NPV based for best estimate)</td>
</tr>
<tr>
<td>Funds held by reinsureds</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Deposit assets</td>
<td>Not material, treated as funds held by reinsureds</td>
</tr>
<tr>
<td>Insurance and reinsurance balances payable and other reinsurance receivables</td>
<td>Not yet fully modeled</td>
</tr>
<tr>
<td>Deferred policy acquisition costs and deferred income taxes</td>
<td>Set to zero</td>
</tr>
<tr>
<td>Reserves for unearned premiums, retrocession</td>
<td>Marked to model.</td>
</tr>
<tr>
<td>Derivatives</td>
<td>Marked to market (by banking partners)</td>
</tr>
<tr>
<td>Other assets and accrued income</td>
<td>Not yet modeled</td>
</tr>
</tbody>
</table>

Table 25.1: The classes of assets and their mode of valuation.
Porting is done as of end of year. It’s based on a third party assessment of the value. Quarterly evaluation and reporting is performed by the fund itself. The fair value, based on the quarterly numbers, is reported by to the global custodian (Mellon Bank, N.Y.). The entire process is accompanied by adequate SarbOx checks. The valuation at the end of the year is based on the simulation of real estate returns of the economic scenario generator.

**Derivative Instruments, Hedge Funds, Registered Bonds**

Market values of *structured notes* and *registered bonds* held by SCOR Switzerland are reported by the deposit banks to the custodian (Mellon Bank, N.Y.). *Hedge funds* report to the custodian (Mellon Bank, N.Y.) on a monthly basis.

- *Digital Notes* are non listed assets, with variable returns and a fixed notional (Iannuzzi [2007], see Chapter 11). The issuing banking institution (Hypo Vereinsbank, Germany) reports the fair values to Mellon Bank.

- *Registered bonds* are in the books of SCOR Switzerland (Cologne), these are non listed securities the fair value of which is calculated by the custodian banking institution (Commerzbank, Germany). SCOR Switzerland (Cologne) reports these values to it’s global custodian (Mellon Bank).

- The performance of *hedge funds* is closely monitored and the investment department as well as the custodian, Mellon Bank, receive monthly reports from the global hedge fund managers.

The analyzes and reports of Mellon Bank on these securities are regularly monitored by SCOR Switzerland. The end of year valuation of these assets is modeled in SCOR Switzerland’s simulation model. On the derivative side, the Black Scholes formula using ESG-input is used for valuing the swaptions. Structured notes are valued by building a linear interpolation of a function of the ESG-generated interest rate curve. ESG-returns from Hedge Fund Indices are used to simulate the value of the Hedge Funds at year-end.

**Participation**

The value of a participation as invested asset at $t_0$ equals the economic value of the subsidiary valued by SST principles, i.e. computed as market value of the assets less best estimate of the liabilities of this subsidiary. As we fully model our major subsidiaries (Cologne and UK) we compute the impact of the simulated values of the participation given the worst 1% scenarios of the parent at the end of the year. The average of these values is taken as the expected value of the participation at the end of the year. Other subsidiaries
with non-material value are taken as constants with their financial balance sheet value.

**Other Assets**

**Deferred policy acquisition costs.** These are acquisition costs principally representing commissions and brokerage expenses, premium taxes and other underwriting expenses, net of allowances from retrocessionaires, which vary with and are directly related to the production of new business, are deferred and amortized over the period in which the related premiums are earned\(^7\). The DAC-position is generally set to zero as the – economically relevant – cash outflows happened in the past. (The calculation of future income would contain the related deferred costs but this is only of theoretical interest as the SST-relevant future cash flows belonging to the business are just not lowered by the acquisition costs paid in the past.)

**Reinsurance Assets.** These are valued like reserves (see Chapter 3) except that they are also associated with a credit risk discount (see Part III). The credit risk due to the potential default of the retrocessionaire is modeled like the credit risk of a bond with similar rating and duration.

**Funds Held by Reinsureds.** The valuation of funds held by reinsureds is based on the NPV of the associated cash flows and a small credit risk (as these funds are partially hedged by related reserve positions). The credit risk related to the non-hedged part of the funds is modeled in the same way as the credit risk of the reinsurance assets.

\(^7\)Converium annual report 2006, p. 71.
In the following, the valuation of the major liability items are outlined. This relates mainly to reserves, funds held under reinsurance contracts, payables and debt which are valued at best estimate (MVM is treated separately) at the beginning of the year and as the contribution to the 1% expected shortfall of the economic value of the company at the end of the year. (The modeling of the stochastic behavior of the liability risk is outlined in chapter 2.) Generally, the valuation of liabilities contains the market value margin which are the costs of capital associated with the necessary capital for keeping the run-off portfolio after one year. Table 26.1 lists the classes of liabilities and their valuation without the market value margin. The calculation of the MVM is also documented in detail in chapter 2.

Reserves

The valuation of the reserves at $t_0$ is done at their best estimate. At the end of the year, based on the stochastic simulation result of the reserve development, the contribution to the expected shortfall of the value of the company is considered, see Chapter 3 for a comprehensive analysis of the valuation of reserves, their volatility and development over time. Clearly, the MVM is mainly related to the cost of capital associated with the capital needed to cover the future volatility of the run-off reserves after one year.

Unearned Premium Reserves

The risk associated with unearned premium reserves is modeled on the basis of the new business loss distribution per line of business. Consequently, the valuation of this position is relying on the NPV of the expected value (for $t_0$) and the contribution to the expected shortfall to the economic value of the legal entity at the end of the year.
### Table 26.1: The liabilities and their mode of valuation.

<table>
<thead>
<tr>
<th>Liability</th>
<th>Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underwriting reserves (gross)</td>
<td>Marked to model (best estimate: discounted reserves)</td>
</tr>
<tr>
<td>Insurance and reinsurance balances payable</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Reinsurance liabilities (retro)</td>
<td>Marked to model (NPV based for best estimate)</td>
</tr>
<tr>
<td>Life</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Reserves for unearned premium (gross)</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Other reinsurance liabilities</td>
<td>Not yet modeled</td>
</tr>
<tr>
<td>Ceded premiums payable</td>
<td>Marked to model (NPV based for best estimate)</td>
</tr>
<tr>
<td>Funds held under reinsurance contracts</td>
<td>Marked to model (NPV based for best estimate)</td>
</tr>
<tr>
<td>Deposit liabilities</td>
<td>Not yet modeled</td>
</tr>
<tr>
<td>Deferred income taxes</td>
<td>Excluded from market consistent balance sheet (not material anyway)</td>
</tr>
<tr>
<td>Other liabilities and accrued expenses</td>
<td>Marked to model</td>
</tr>
<tr>
<td>Debt/loans</td>
<td>Face value</td>
</tr>
<tr>
<td>Options and guarantees</td>
<td>Off balance sheet, modeled in liability cash flows</td>
</tr>
</tbody>
</table>


Funds Held under Reinsurance Contracts

These funds are valued on an NPV cash flow basis using related cash flow patterns.

Debt

Ordinary issued debt is in general valued like a corporate bond, see Parts II, III and especially Chapter 12 for market risk, credit risk and bond valuation.

To the extent these debts are subordinated they are considered as part of the capital basis. This means that their face value must be usable in case of financial distress and can not trigger the liquidation of the company (which is tested during the simulations). It is especially important that these debts are subordinated relative to the payments to policyholders.

Payables

This position is valued on an NPV cash flow basis using (short-term) cash flow patterns.

Other reinsurance liabilities, Pension liabilities

These liabilities currently comprise roughly 1.2% of SCOR Switzerland's liabilities and are not yet fully modeled in detail. A smaller fraction (30%) of these liabilities are currently related to pension liabilities to be paid to SCOR Switzerland's employees (a small part of the pension liabilities are reported as loss in “Other comprehensive income”) by SCOR Switzerland. This is basically the gap between the projected benefit obligations and the fair value of the plan assets of the pension fund, adjusted by the tax impact.

Off-Balance Sheet items

Embedded value of life insurance The valuation of the embedded value of life insurance business equals the NPV of the expected economic life-insurance related cash flows for \( t_0 \) and the contribution to the expected shortfall to the economic value of the legal entity at the end of the year. Options and guarantees are explicitly simulated in related models, see chapter 4.

CRTI, credit lines, contingent capital The impact of capital and risk transfer instruments on the valuation of the legal entity is directly simulated using information on parental guarantees or the retro structure. Contingent capital is negligible for SCOR Switzerland. Non collateralized Letters of credit are assumed to positively impact the policyholders’ interest but is not supposed to alter the economic value of the legal entity.
VI

Solvency Capital Requirements
This part summarizes the general technical terms and definitions used to analyze and compute the solvency capital requirements as documented by the FOPI in various documents and presentations. Basically, risk-bearing capital is defined as the market value of the assets less the best estimate of the liabilities using the values coming out of the economic balance sheet, see Part V.

The target capital (TC) is then the 1% expected shortfall ($ES_{1\%}$) of the one-year change in risk-bearing capital and the market value margin (MVM) that accounts for the cost of capital associated with the capital needs to keep the run-off portfolio over the years as expressed by Equation (26.1). Figure 26.1 shows the relationships.

\[ TC = -ES_{\alpha} \left( \frac{C_R(t_1)}{1 + r_1^0} - C_R(t_0) \right) + \frac{MVM}{1 + r_1^0} \] (26.1)

where the risk tolerance $\alpha$ is set at $\alpha = 1\%$ and $C_R$ denotes the risk-bearing capital. The one-year risk-free interest rate is $r_1^0$. The points in time $t_0$ and $t_1$ denote the start respectively the end of the calendar year. The market value margin is defined as

\[ MVM(1) = \eta \sum_{i \geq 1} SCR(i), \]

i.e. as the sum of the solvency capital requirements over all future years after the first year multiplied with the cost of capital of 6 %, given by the FOPI. (For the detailed outline of the computation of the market value margin see Chapter 2.)

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VII

Process Landscape
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# Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALM</td>
<td>Asset and Liability Management</td>
</tr>
<tr>
<td>BoD</td>
<td>Board of Directors</td>
</tr>
<tr>
<td>BS</td>
<td>Balance Sheet</td>
</tr>
<tr>
<td>CEO</td>
<td>Chief Executive Officer</td>
</tr>
<tr>
<td>CFO</td>
<td>Chief Financial Officer</td>
</tr>
<tr>
<td>CIO</td>
<td>Chief Investment Officer</td>
</tr>
<tr>
<td>COO</td>
<td>Chief Operating Officer</td>
</tr>
<tr>
<td>CPI</td>
<td>Consumer Price Index</td>
</tr>
<tr>
<td>CRO</td>
<td>Chief Risk Officer</td>
</tr>
<tr>
<td>dvlpmt.</td>
<td>development</td>
</tr>
<tr>
<td>ECS</td>
<td>Executive Committee Switzerland</td>
</tr>
<tr>
<td>ESG</td>
<td>Economic Scenario Generation</td>
</tr>
<tr>
<td>EUR</td>
<td>Euro</td>
</tr>
<tr>
<td>FED</td>
<td>Financial and Economic Database</td>
</tr>
<tr>
<td>FinMod</td>
<td>Financial Modeling</td>
</tr>
<tr>
<td>FOPI</td>
<td>Federal Office of Private Insurance</td>
</tr>
<tr>
<td>FX</td>
<td>Effects</td>
</tr>
<tr>
<td>GAUM</td>
<td>Global Aerospace Underwriting Managers</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
</tr>
<tr>
<td>GMDB</td>
<td>Guaranteed Minimum Death Benefit</td>
</tr>
<tr>
<td>LoB</td>
<td>Lines of Business</td>
</tr>
<tr>
<td>MDU</td>
<td>Medical Defense Union</td>
</tr>
<tr>
<td>Mgmt.</td>
<td>Management</td>
</tr>
<tr>
<td>MVM</td>
<td>Market Value Margin</td>
</tr>
<tr>
<td>Nat Cat</td>
<td>Natural Catastrophes</td>
</tr>
<tr>
<td>Non-prop.</td>
<td>Non-proportional</td>
</tr>
<tr>
<td>Prop.</td>
<td>Proportional</td>
</tr>
<tr>
<td>Q1</td>
<td>First quarter</td>
</tr>
<tr>
<td>Q3</td>
<td>Third quarter</td>
</tr>
<tr>
<td>RBC</td>
<td>Risk Based Capital</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>Standard &amp; Poors</td>
</tr>
<tr>
<td>SST</td>
<td>Swiss Solvency Test</td>
</tr>
<tr>
<td>strat.</td>
<td>strategic</td>
</tr>
<tr>
<td>USD</td>
<td>US dollar</td>
</tr>
<tr>
<td>UWY</td>
<td>Underwriting Year</td>
</tr>
</tbody>
</table>
Summary

The ultimate objective of the Swiss Solvency Test (SST) is to determine a company’s solvency status, using an all encompassing analysis of the respective company. The main ideas of the SST are described in the SST White Paper (FOPI [2004]) issued by the FOPI. In the past, SCOR Switzerland used its internal stochastic model to manage assets and liabilities. This model has now been enhanced and expanded to include all SST relevant aspects and elements. Going forward, the internal model will be used for both asset and liability and solvency calculations for the purposes of the SST.

The SST is of great relevance for the management decision processes at SCOR Switzerland. With our internal stochastic models we calculate results on an ultimate basis for the Asset and Liability Management (ALM) and on a one year basis for the SST. These results define such business relevant issues as the group’s strategic asset allocation, the exposure of the group to main risk classes or the capital requirements needed. Additionally, a forecast of the asset performance is provided. These results serve as guidance for the group’s risk appetite and risk mitigation, the asset management strategy and the capital management. The internal model therefore passes the requirements of a use test.

From the viewpoint of the process documentation we understand the SST as a use test. The main purpose is to show the processes of how we get to the SST relevant results. Our internal stochastic models serve as the starting point for the ALM. Preparing for the SST, we enhanced our ALM process to be fully inline with the SST requirements, set up by the Federal Office of Private Insurance (FOPI). The SST is therefore closely linked to the ALM process.

The ALM process at SCOR Switzerland is specified by five process
**modules**: “economic scenario generation,” “reserving,” “asset management,” “liability management” and “aggregated results calculation.” This document provides a detailed description of input-to-output processes within the single modules. **Insurance risk**, **market risk**, and **credit risk** modeling are carried out with specific focuses in these modules.

The SST process documentation is an integral part of the supporting documentation produced and submitted to the FOPI:

- The design of the ALM modules and the process steps in particular are based on the ideas as highlighted in the SST White Paper (FOPI [2004]) issued by the FOPI.

- The high-level dataflows and process steps as specified in the modules of the ALM process form the basis for the IT Systems documentation (*Part VIII, pp. 408*).

- The methodology used for the calculation is described in this documentation (*see, above all, the Parts I, II, III in this document*).
Structure of the Process Landscape Documentation

The documentation of the process landscape at SCOR Switzerland Ltd. is based on *Swiss Solvency Test (SST) at SCOR Switzerland: Process Landscape* (Skalsky and Bürgi [2008]) and is structured as follows:

We first give an overview of the embedment of the ALM process within SCOR Switzerland. The SST is performed based on the results from the ALM process. The overview includes a comprehensive description of roles and responsibilities and the decision processes related to the results of the SST and the ALM Report. This first part shows that the ALM process is an integral part of the overall company processes.

We subsequently describe the ALM process in detail. It is structured in modules which are combined to workflows. Each module consists of a number of input and output data entities as well as a number of process components. The roles and responsibilities are defined by module, and for the output data entities, the corresponding sign-off procedures are described.

We then provide an overview of the process of creating the scenarios used for the SST only. The main purpose of this section is to show the general process steps that are performed every year to decide upon the relevance of the used scenarios and, if necessary, the production of new scenarios needed. This part mainly refers to the detailed scenario documentation (Gencyilmaz [2007]), provided to FOPI on a yearly basis as an integral part of the SST Report.

In the Appendix, a general overview of the SST project setup, the timeline of implementation, as well as the project roles and responsibilities and
the sign-off matrix of the overall project is presented. The SST project has the goal to achieve SST compliance of SCOR Switzerland and to implement methods and processes in order to satisfy the FOPI requirements.
29

Overview

29.1 The Swiss Solvency Test / Asset and Liability Management Process

This section gives an overview of the embedment of the ALM process within SCOR Switzerland. As required by the FOPI, the production of the SST report is fully integrated in the ALM process and is produced from results in the ALM process. The main difference between the SST and the ALM Report is to be found in the granularity and level of detail of reported results. The integration of the ALM process and the production of the SST report in the overall corporate process landscape is shown in Figure 29.1.

![Figure 29.1: Embedment of the ALM process at SCOR Switzerland – organizational overview.](image-url)
### 29.1.1 Process Description

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset management strategy</td>
<td>The asset management strategy covers strategic assumptions on</td>
</tr>
<tr>
<td></td>
<td>- The proportion of asset classes (e.g. fixed income vs. equity vs. other financial products)</td>
</tr>
<tr>
<td></td>
<td>- The target duration of bonds</td>
</tr>
<tr>
<td></td>
<td>The Chief Investment Officer is responsible to define the asset management strategy. The asset management strategy is approved by the ECS and the BoD.</td>
</tr>
<tr>
<td>ALM process</td>
<td>The ALM process takes the risk tolerance, the financial plan, and the asset management strategy as an input. The main products of the process are the ALM report and the SST report. The results listed in these reports cover</td>
</tr>
<tr>
<td></td>
<td>- The exposure of the group to the main risk classes</td>
</tr>
<tr>
<td></td>
<td>- The overall capital requirements needed given the plan and the asset management strategy</td>
</tr>
<tr>
<td></td>
<td>- A forecast of the asset performance</td>
</tr>
<tr>
<td></td>
<td>- A proposal for the strategic asset allocation</td>
</tr>
<tr>
<td></td>
<td>The overall responsibility of the ALM process lies with the Financial Modeling team. The results of the SST and the ALM Report are approved by the ALM Committee. Furthermore, the ALM Committee decides on proposals for risk mitigation and asset management strategy.</td>
</tr>
<tr>
<td>Proposal for risk mitigation</td>
<td>If the overall capital requirements or the risk components as calculated by the ALM process violate the risk tolerance of the group, the risks have to be mitigated or the risk tolerance has to be redefined. The ALM process comes up with a suggestion for risk mitigation, which is proposed to the BoD/ECS. Depending on the outcome, the risk tolerance, the financial plan or the asset management strategy may have to be revised.</td>
</tr>
</tbody>
</table>
### Proposal for asset management strategy

Within the ALM process, the efficient frontier of the group is calculated. From this and from other results of the ALM process, a strategic asset allocation is proposed, which feeds back into the asset management strategy process.

### Capital management/rating agency communication

Once the overall capital requirements are calculated and approved, the capital can be actively managed. Strategies can be defined to deal with surplus capital (e.g., share buy-back or dividend payments). Capital shortages can be detected at an early stage and solid mitigation strategies can be defined. Furthermore, the rating agency capital requirements can be calculated and the communication strategy with the rating agencies can be defined.

#### 29.1.2 Roles and Responsibilities

The following overall roles and responsibilities are defined for the ALM process within SCOR Switzerland:

**Chief Risk Officer (CRO)**

- Define the risk tolerance
- Participate in the ALM Committee

**Chief Financial Officer (CFO)**

- Provide the financial plan of the company
- Chair the ALM Committee
- Manage the capital according to the results as produced in the ALM process
- Ensure consistency of rating agency communication with the ALM results
- Define the investment strategy in line with the investment guidelines and the ALM results
**Chief Investment Officer (CIO)**

- Define the asset management strategy based on ALM results
- Sign-off on the investment forecasts as produced in the ALM process
- Participate in the ALM Committee

**Financial Modeling (FinMod)**

- Manage the ALM process overall
- Create and validate appropriate models
- Produce the various calculation results (for the ALM as well as for the SST report)
- Achieve a sign-off from the relevant parties
- Ensure that the ALM results are appropriately dealt with
- Secretary of the ALM Committee
- Reporting of the ALM results to ECS and BoD

**ALM Business Owner**

- Ensures that the end product is fit for its purpose and that the solution will meet the user/business needs within the constraints of the business case,
- Provides business resources and must be of sufficient authority to take decisions on business matters
- Ultimate responsibility for sign-off on the projects products

**Local Chief Risk Officer**

- Produce the SST report
- Obtain sign-off of the SST report
- Publish the SST report and submit it to the FOPI
ALM Committee

- Sign-off on adequacy of models and results
- Propose the strategic asset allocation to ECS and BoD
- Propose an investment strategy based on the ALM results
- Propose risk mitigation strategies as needed based on the ALM results
- Ensure that the ALM results are reflected in related decisions of the company
- Publication of the ALM report – sign-off and publication of the SST report

Executive Committee Switzerland and Board of Directors
(ECS/ BoD)

- Decide on the risk tolerance
- Agree on the strategic asset allocation
- Decide on the investment strategy,
  decide on risk mitigation if necessary
- Decide on capital management

29.2 General Timeline of the SST and the ALM Process at SCOR Switzerland

The ALM process is carried out twice a year within SCOR Switzerland. The SST is performed once a year, based on the ALM calculations in Q1. A high level overview is given in Figure 29.2.

29.3 Q3 ALM process

The Q3 ALM process starts in September and ends in December. Its purpose is to forecast results of the next financial year based on the financial plan of the company, which is based on the risk tolerance defined. The ALM process is carried out as described below.

The capital requirements are calculated, and the risk limits are monitored. If the financial plan violates the risk tolerance, it has to be revised.
29.4 Q1 ALM process

The Q1 ALM process starts in March and ends in July. Based on the results of the renewals to date and based on the investment results to date, the forecast of the Q3 ALM process are refined to reflect the actual developments.

The Q1 ALM results are parsed to the SST Report. Additionally, the sections specific to the SST report only are produced. The SST Report is submitted to the FOPI.
30

The ALM Process

30.1 General Overview

The ALM process at SCOR Switzerland is specified by process modules, lined up in Figure 30.1. Detailed descriptions of input-to-output processes within the single modules will be discussed in the following subchapters. Insurance risk, market risk, and credit risk modeling is carried out with specific focuses in these modules. This will be specified in the respective detailed module descriptions.

The ALM results are formatted into the ALM Report, which serves as the basis for the final SST Report. The main difference between the ALM and the SST Report lies partly in the level of granularity and partly in the choice of provided data. The SST focuses on SST relevant quantities like “target capital” or “market value margin” with a time horizon of one year for compliance with the FOPI-criteria/requirements. The whole process is audited by the Group Internal Audit (GIA) team.

Figure 30.1: ALM process – functional overview.
30.2 The Execution of the ALM Model

This chapter describes the process of executing the ALM model. This process encompasses:

- The collection of all relevant data entities
- The re-modeling of some of these data entities in order to match the desired level of granularity and aggregation
- The calculation of the overall result distribution as well as risk measures, strategic asset allocation etc. as desired by the various stakeholders (management, regulators, rating agencies etc)

The ALM model execution process is structured in five modules which are represented as a workflow in Figure 30.2.

![Figure 30.2: ALM process – modules.](image)

In summary, the scope of the modules can be described as follows:

**Economic scenario generation:** A forecast of the world economy is generated in the form of various economic scenarios and their respective probabilities. Historic time series serve as input and are projected to the future.

**Reserve modeling:** The starting point of the reserve model are the figures contained in the balance sheet. Based on reserving triangles (claims incurred, claims paid, premiums received etc.), patterns are determined and the reserve volatility is calculated.

**Liability modeling:** The loss models of single acceptances or line of business fragments are gathered and aggregated. The dependency structure is modeled. The loss models are projected to the planning figures. The gross liability cash flows are calculated by year, legal entity and line of business. Internal and external retro is deducted to generate net cash flows.
**Asset modeling:** The asset portfolio is stylized in order to reduce the complexity of the calculations. Similar assets are gathered to asset classes. The asset management – module delivers results for the invested assets performance, stand-alone risk-based capital, sensitivity of invested assets and the efficient frontier analysis, which is a core result of the ALM process. The efficient frontier graph sets the expected shortfall in relation to shareholders equity and compares SCOR Switzerland’s asset portfolio with the most efficient strategic asset allocation.

**Aggregated results calculation:** The aggregated results calculation module is the last module in the ALM process. It calculates the main results needed for capital management and the Swiss Solvency Test.

**30.2.1 Roles and Responsibilities**

Two roles with an overall coordinative function are specified in general terms for the ALM and SST processes. Roles and responsibilities of the ALM and SST process managers are defined:

**ALM process manager**
- Plan the process and manage changes
- Nominate persons to the defined roles and responsibilities, including the nomination of the respective deputy functions
- Progress monitoring and reporting
- Manage process definitions

**SST process manager**
- Produce the SST report from results of the ALM report
- Obtain a sign-off of the SST report from the ALM Committee
- Submit the SST report to the FOPI
30.3 Detailed Module Description

30.3.1 The Economic Scenario Generator

Economic scenario generation forms the basis for many of the other modeling steps in the ALM process. The module can be described as shown in Figure 30.3.

![Figure 30.3: ALM process – economic scenario generator.](image)

**Data entities**

Several time series are involved in the calculation of the economic scenarios. In reality, many time series are correlated with each other. This correlation is also modeled in the economic scenario generator. The data entities associated with the economic scenario generator are displayed below:

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rates</td>
<td>Interest rates are a basic quantity modeling the economy. For each currency zone to be modeled, the corresponding government yield curve serves as a basis to model risk-free interest rates. The risk-free interest rate model is coupled to the inflation of a currency zone through a mean reversion, and it serves as a basis to simulate other time series, such as FX, bond indices etc.</td>
</tr>
</tbody>
</table>
Inflation index

The inflation in a currency zone is another important quantity modeling basic economic behavior. The consumer price index (CPI) is taken as a measure for inflation. Inflation and interest rates are coupled through a mean reversion. Furthermore, the inflation has an impact on FX rates and equity indices modeled.

FX rates

The FX rates are defined with respect to the home currency (USD for SCOR Switzerland, EUR for SCOR). The FX rate model depends on the risk free interest rates as well as on the inflation of the corresponding currency zone and of the home currency zone.

GDP

The modeling of the gross domestic product is independent of other variables of the economy. However, the modeled GDP is used to model investment indices.

Investment index

Investment indices are typically equity indices, bond indices, real estate indices, hedge fund indices or similar. The modeling of investment indices depends on other basic quantities such as the inflation and the GDP or interest rates of a currency zone. The investment indices are used to valuate the asset portfolio in the risk simulation.

Credit spread

Credit spreads are input in the form of corporate yield curves. The difference between the corporate yield and the government yield serves as a basis to calculate credit cycles. Two types of credit spreads currently serve as input for two different ratings. It is thus possible to model cycle levels which depend on the rating of a company.

Credit cycle

The credit cycles are calculated from the credit spreads. They serve as a basis to calculate the credit risk in the ALM model.

GMDB cycle

The GMDB (Guaranteed Minimum Death Benefit) cycle is modeled in order to valuate the GMDB business. The GMDB model depends on up to date information on the replicating GMDB portfolio delivered by the life pricing team, as well as US equity and yield curve scenarios.
The market cycle is not yet calculated in the current process. This is why its borders are dashed in the graph above. It would serve as a basis to scale some reinsurance losses.

### Process Steps

A number of process steps are carried out in order to generate the simulated economic scenarios:

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collect historical time series</td>
<td>All historical time series which serve as a basis for the risk simulation are collected. The data can be imported from external data sources, such as Bloomberg or from the Financial and Economic Database (FED) of SCOR Switzerland. Some of the economic variables are pre-processed and transformed internally to calculate other variables. For instance, the credit levels are calculated from the corporate yields and the government yields. For a detailed description of the ESG input, please refer to the Systems Documentation (<em>Part VIII in this document</em>).</td>
</tr>
<tr>
<td>Analyze and adjust historical time series</td>
<td>It is important to analyze the historical data first. Even the automatically imported data can be inconsistent. This happens, for instance, when an index is rebased. In this case, the rebasing has to be undone in the input for the ESG. Typically, this process step encompasses the calculation of mean and standard deviation of the time series, which leads to a first overview. Furthermore, the data is verified by eye in order to spot inconsistencies. Additionally, statistical tests help detecting suspicious entries. For instance, it can be checked whether a value deviates from the forecast as given by an exponential moving average by more than a number of standard deviations.</td>
</tr>
</tbody>
</table>
Simulate future time series

Once the historical time series are collected and adjusted, the simulation of the future scenarios are generated by using bootstrap simulation. Several parameters control the simulation, e.g. the number of historical periods to be considered is chosen, parameters steering the methodology are set etc.

Post process and analyze scenarios

The simulated scenarios are post processed: In order to speed up the calculations and reduce the number of simulation runs needed for the risk simulation, the scenarios are post-processed. Extreme scenarios are identified and kept as they are. The number of average scenarios is reduced by keeping typical average scenarios and increasing their weight. Sometimes series are scaled with respect to performance expectations. The post-processed scenarios are used as an input for the other modules. Forecast tables and graphs are produced for the communication with Top Management and for inclusion into the ALM and the SST Report.

Roles and Responsibilities

The main role involved in the economic scenario generation process is the economic scenario modeler. The main responsibility is to update all economic scenarios needed for the ALM process. All other risk modelers have the responsibility to order additional time series needed for their parts. Roles and responsibilities are as follows:

**ESG modeler**

- Ensure correctness of the methods and algorithms used in the simulation and their implementation
- Calibrate internal model parameters

**ESG data manager**

- Collect the historical time series necessary for simulation
- Ensure correctness and consistency of data
- Choose the appropriate set of parameters to be used for the scenario generation
Obtain a sign-off of the generated scenarios
Ensure correct import of the economic scenarios in the other modules
Manage orders from other risk modelers and order new scenarios if appropriate

Other risk modelers
Order scenarios for the time series needed for their parts from the ESG data manager
Ensure correct application of the time series in their corresponding modules

Chief Investment Officer (CIO)
Discuss performance targets of economic variables with ESG data manager
Provide sign-off of economic scenarios

Sign-off Procedures
The sign-off of economic scenarios is closely linked with the result data entities of other modules. Yield Curve, Investment Index and FX Rates have a strong impact on the asset performance. The correctness of the data is approved together with the asset modeling module. Market cycle and inflation index have an impact on the liability modeling and are signed-off together with the liability modeling module. The final sign-off to the ESG scenarios is given by the CIO. An overview of the given sign-offs in the ALM process is provided in the Appendix.
30.3.2 Reserve Modeling

The purpose of the reserve modeling is to calculate the patterns needed to calculate the liability cash flows in the liability-modeling module, as well as to analyze the reserve run-off and the reserve duration. The outline of the reserve-modeling module is given in Figure 30.4.

Data Entities

We display the data entities which occur in the Reserve-Modeling module:

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield curve</td>
<td>The historic yield curve is used to discount cash flows.</td>
</tr>
<tr>
<td>Incurred, paid, case reserve and premium triangle</td>
<td>The reserving triangles by line of business and by legal entity are the basis for the reserve-modeling module. They serve as a basis to calculate the patterns, as well as to calculate the corresponding reserve volatility. The triangles are typically given by underwriting year and development year.</td>
</tr>
</tbody>
</table>
The patterns describe yearly developments of claims and premium payments, as well as earnings. These patterns are needed in the liability-modeling module in order to calculate the yearly liability cash flows. The claims paid pattern refers to the payment of claims, whereas the premium pattern refers to the premiums received, and the earning pattern to premiums earned.

<table>
<thead>
<tr>
<th>Claims paid, premium and earning pattern</th>
<th>The BS reserve is calculated from the reserving triangle and the initial BS reserve. The reserving triangles are used to break down the initial BS reserve to line of business and legal entity granularity.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance sheet (BS) reserve</td>
<td>The reserve volatility is one of the main output data entities of the reserve-modeling module. It is used in conjunction with a lognormal distribution to calculate the reserve developments in the liability-modeling module. Note that, for this purpose, the reserve volatilities need to be calculated on a calendar year basis rather than an underwriting-year basis.</td>
</tr>
<tr>
<td>Reserve volatility</td>
<td>The reserve duration denotes the average duration of reserves. The reserve duration is calculated in order to propose the strategic asset allocation.</td>
</tr>
<tr>
<td>Reserve run-off</td>
<td>Discounted historic run-off of the reserves (difference between the most recent balance sheet and the prior balance sheet) including claims payments, reserve increases, etc.</td>
</tr>
</tbody>
</table>
## Process Steps

The following process steps have to be carried out in reserve modeling:

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate patterns</td>
<td>The basis of this analysis is the reserving triangles. They are automatically imported by line of business and by legal entity from the reserving system. Chain ladder patterns are calculated for further processing in the liability-modeling module. Disclaimer: It would be preferable to import the reserving patterns from the reserving system directly. However, the granularity of the patterns calculated by reserving currently does not match the granularity of lines of business as needed in the ALM model. Discussions with reserving are ongoing to implement the calculation of the patterns at the right granularity for input in the ALM model.</td>
</tr>
<tr>
<td>Calculate balance-sheet reserv-</td>
<td>The reserve ultimates are calculated by line of business and legal entity using the chain ladder method. Disclaimer: As for the patterns, it would be desirable to obtain the right granularity of the ultimates from the reserving system directly. Discussions concerning this are ongoing.</td>
</tr>
<tr>
<td>ing</td>
<td></td>
</tr>
<tr>
<td>Calculate reserve volatili-</td>
<td>The reserve volatilities are calculated from the reserving triangles and the patterns. Disclaimer: As for patterns and ultimates, it would be desirable to obtain the reserve volatility directly from the reserving system. Discussions concerning this are ongoing.</td>
</tr>
<tr>
<td>ties</td>
<td></td>
</tr>
<tr>
<td>Calculate duration and statis-</td>
<td>The duration and statistics analysis is carried out on reserving data directly, as the data granularity can be different from the liability modeling baskets. The claims paid patterns are imported from the reserving system, the historical yield curve serves for discounting. An automated script imports the basic data and calculates the reserve run-off and the reserve duration.</td>
</tr>
<tr>
<td>tics</td>
<td></td>
</tr>
</tbody>
</table>
Roles and Responsibilities

The following roles and responsibilities are part of the reserve-modeling module:

Reserve modeler
• Import and check the raw reserving triangles
• Ensure that the methods and implementations thereof are appropriate and correct
• Ensure consistency of the patterns, reserve duration, and reserve run-off with other reserving analyzes
• Obtain a sign-off of the reserve volatility, patterns, reserve duration, and reserve run-off from the reserving actuary

Reserving actuary
• Provide explanations on the reserving triangles imported
• Support the reserve modeler during the process as required
• Provide a sign-off of the reserve volatility, the patterns, the reserve duration, and the reserve run-off

Sign-off Procedures
Patterns, reserve duration, and reserve run-off are signed-off by the reserving actuary. The reserve modeler is responsible to coordinate and achieve the sign-off process. Please see the Appendix for an overview of the sign-offs given in the ALM process.
30.3.3 Liability Modeling

The main scope of the liability-modeling module is to calculate the risk and performance inherent in the reinsurance business. The overall process is illustrated in Figure 30.5.

Figure 30.5: ALM process – liability modeling.
Data Entities

The following data entities occur in the liability modeling module:

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss model</td>
<td>There are various types of loss models involved in the liability modeling process:</td>
</tr>
<tr>
<td></td>
<td>• Wherever available, the single loss models of each contract belonging to a modeling basket are imported from the pricing systems and aggregated.</td>
</tr>
<tr>
<td></td>
<td>• In some cases (e.g., for the nat cat perils or the joint ventures) the loss models are aggregated by the corresponding pricing teams and delivered in form of aggregate distributions.</td>
</tr>
<tr>
<td></td>
<td>• For certain types of life business, the loss modeled is delivered in form of cash flows.</td>
</tr>
<tr>
<td></td>
<td>All the non-life lines of business are eventually modeled as aggregate distributions by modeling basket (i.e., by Lines of Business (LoB) and legal entity). The life models are implemented as a cash flow model (e.g., Financial Reinsurance business) or as a scenario based model (e.g., GMDB business).</td>
</tr>
<tr>
<td>Claims paid pattern, earning pattern</td>
<td>These data entities are taken from the reserve module. Please refer to the corresponding definitions. They are used to model the yearly loss cash flows as well as the portion of unearned business.</td>
</tr>
<tr>
<td>GMDB cycle</td>
<td>This entity is taken from the economic scenario generation module. Please refer to the corresponding definition. The GMDB cycle is used to model the GMDB line of business.</td>
</tr>
<tr>
<td>Credit cycle</td>
<td>This entity is taken from the economic scenario generation module. Please refer to the corresponding definition. The credit cycle is used to calculate the credit risk of liabilities (credit and surety business) and retrocessionnaires.</td>
</tr>
<tr>
<td>Retrocession</td>
<td>Information on internal and external retrocession is used to model the reinsurance recoverables. This data is provided by the retrocession department.</td>
</tr>
<tr>
<td>BS reserve</td>
<td>The initial reserve state is taken from the start balance sheet and split by line of business/legal entity in the reserve modeling module. Within the ALM model, the reserves of the future balance sheet are forecast.</td>
</tr>
<tr>
<td>Reserve volatility</td>
<td>This entity is taken from the reserve modeling module. Please refer to the corresponding definition. The reserve volatility is used to calculate the balance sheet forecast of the reserves.</td>
</tr>
<tr>
<td>Yield curve</td>
<td>This entity is taken from the economic scenario generation module. Please refer to the corresponding definition. The yield curve is used in order to discount results.</td>
</tr>
<tr>
<td>Credit risk</td>
<td>On the liability side, the credit risk is currently calculated for the credit and surety business, as well as the credit risk of retrocessionnaires.</td>
</tr>
<tr>
<td>New business results</td>
<td>Reinsurance results, such as loss ratios, combined ratios, premium volumes and cost ratios by line of business and legal entity. Loss statistics are calculated both on a gross and on a retro basis.</td>
</tr>
<tr>
<td>UWY RBC</td>
<td>The liability stand-alone risk based capital consumption is calculated by line of business and legal entity. The diversified risk-based capital is calculated in the aggregated results calculation module.</td>
</tr>
<tr>
<td>Return on UWY RBC</td>
<td>The return on the liability stand-alone RBC is calculated in order to monitor the profitability of the new business.</td>
</tr>
<tr>
<td>Liability cash flow</td>
<td>The liability cash flow is one of the main outputs of the liability modeling module. They are used in the asset-modeling module in order to calculate the rollover of the asset portfolio.</td>
</tr>
</tbody>
</table>
Market cycle, inflation index

These data entities are taken from the economic scenario generation module. Please refer to the corresponding definitions. Market cycle and inflation index will be used to model a stochastic dependency of claim sizes with the economic scenarios. This feature is not yet implemented in the ALM model. This is why these entities are dashed in the graph overview above.

Process Steps

The following process steps have to be carried out in liability modeling:

<table>
<thead>
<tr>
<th>Process steps</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model baskets and dependencies</td>
<td>Depending on the plan, the granularity of model baskets has to be determined. The main decision criteria are to generate homogeneous modeling units with a hierarchical structure. Furthermore, all business segments in the plan have to be modeled. Among all modeling baskets within a node of the hierarchical tree, a copula dependency is modeled. This dependency structure is reviewed at the beginning of each ALM process together with underwriters and actuaries.</td>
</tr>
<tr>
<td>Aggregate in-force business</td>
<td>The basis for modeling the new business is the in-force business. In many cases, the individual loss distributions by contract are loaded from the pricing system and aggregated by modeling basket. Whenever aggregate loss distributions are input (e.g. for nat cat perils), the aggregation step is trivial.</td>
</tr>
<tr>
<td>Adjust to planned volume</td>
<td>The distributions obtained from the in-force business are scaled in order to reflect the planned volume. Proportional and non-proportional business is scaled separately by legal entity and by LoB. Furthermore, a distinction is made between growth in number of contracts versus growth in share (per-risk versus per-event scaling).</td>
</tr>
</tbody>
</table>
Calculate gross reinsurance loss including unearned business

For the new non-life business, the losses are modeled by aggregate distributions and their dependencies. An analogous computation is done for the unearned business of previous UW years. Losses are simulated from the corresponding loss models, i.e. the loss distributions, the GMDB cycle etc. The payout patterns are taken from the reserve model and are used to calculate the loss cash flow per year. Inflation index and market cycle reflect a correlation or anti-correlation of reinsurance losses with the economic scenarios; this feature, however, is not yet implemented. The credit cycle is used to calculate the credit risk of the reinsurance business (credit and surety business).

Calculate recoverables

Recoverables from internal and external retrocession are calculated. This yields the net cash flows from the gross cash flows.

Calculate reserve developments

The reserves are taken from the initial balance sheet. Together with the reserve volatilities, log normal distributions are modeled to yield the reserve developments of the calendar year. The reserve of the projected balance sheet is calculated.

Calculate cash flow and statistics

The liability cash flows are calculated gross and net reflecting the new business loss, the retrocession recoverables, as well as the reserve developments. Statistics are calculated by line of business and legal entity based on the stand-alone liability risk.

Roles and Responsibilities

The roles and responsibilities involved in the liability-modeling process are described in the following subsections.

**Liability modeler:** The liability modeler is responsible for the overall process of the liability module. The responsibilities are to

- Ensure overall correctness and consistency of all methods and implementations used in the liability module
- Ensure overall consistency of data within the liability modeling module
• Coordinate the overall liability-modeling process

• Coordinate the overall sign-off procedures for the liability-modeling module

**Planning coordinator:** The planning coordinator has the responsibilities to

• Provide the planned volumes by line of business, legal entity, and type of business (prop. vs. non-prop)

• Provide the planned loss ratios by line of business, legal entity, and type of business (prop. vs. non-prop),

• Provide the planned cost ratios by line of business, legal entity, and type of business (prop. vs. non-prop)

• Provide information on the growth assumptions (new business vs. growth of existing business)

• Assist in the refining of the granularity of planning data to modeling baskets

• Provide a sign-off on the final planning data by modeling basket and legal entity

• Help ensuring consistency between pricing and planning

• Participate in the sign-off of the ALM liability model

**Pricing actuary:** There are several pricing actuaries involved in the ALM process, depending on the organizational structure of the company. Currently, separate pricing actuaries are responsible for the standard non-life business, the nat cat business, life business, special contracts (Lloyd’s, Global Aerospace Underwriting Managers (GAUM), Medical Defense Union (MDU)) etc. The responsibilities of the pricing actuary in the ALM process are to

• Timely deliver loss models on the corresponding business if not automatically available in the pricing systems

• Provide explanations on the loss models, especially also regarding the interaction with loss models of the other business segments

• Participate in the sign-off of the ALM liability model
**Unearned business modeler:** The unearned business modeler has the responsibilities to

- Derive and implement the loss distributions of unearned business by modeling basket and legal entity from the new business model based on earning patterns
- Ensure correctness of the methods and implementations used to create these loss models and ensure their consistency with the rest of the ALM model
- Participate in the sign-off of the ALM liability model

**New business modeler:** The new business modeler has the responsibilities to

- Create the modeling baskets according to the planned business
- Discuss and review the dependency tree with underwriters and pricing actuaries
- Refine the granularity of planning data if necessary
- Collect the loss models from the pricing system or from the corresponding pricing actuaries
- Generate the aggregated loss models by modeling basket and legal entity
- Model cost ratios per modeling basket and legal entity
- Coordinate the feedback with the planning coordinator and pricing actuaries for explanations regarding the loss models and consistency between planning and pricing
- Ensure correctness of the methods and implementations used to create the loss models and ensure their consistency with the rest of the ALM model
- Achieve the sign-off on the gross underwriting year loss statistics

**ESG manager:** The responsibilities of the economic scenario modeler in the liability modeling module are to

- Timely deliver the economic scenarios for yield curves, market cycle, inflation index, GMDB cycle and credit cycle
- Provide explanations on the economic scenarios if required
**Retrocession coordinator**: The retrocession coordinator is the contact in the retrocession department for the ALM process. The responsibilities are to

- Provide up-to-date information on internal and external retrocession
- Provide explanations on the retrocession information as needed
- Participate in the sign-off of the net underwriting year loss statistics

**Retrocession modeler**  The responsibilities of the retrocession modeler are to

- Model internal and external retrocession in the ALM model
- Ensure correctness of the retrocession methods and implementations used in the ALM model
- Collect the up-to-date information on internal and external retrocession from the retrocession department
- Achieve the sign-off of the net underwriting-year loss statistics

**Reserve modeler**  The responsibilities of the reserve modeler in the liability modeling module are to:

- Timely deliver balance sheet reserve states and calendar year reserve volatilities
- Provide a mapping of reserving lines of business to the new business modeling baskets
- Ensure consistency of the reserve model with the new business model
Sign-off procedures

Overall, the liability modeler is responsible that the data entities of the liability modeling module achieve the appropriate sign-off. Please see the Appendix for an overview of the sign-offs given in the ALM process. Within the liability-modeling module, sign-off has to be achieved for the following data entities:

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross UWY loss</td>
<td>The new business modeler is responsible to coordinate and achieve the sign-off of gross underwriting-year loss statistics. The sign-off is given</td>
</tr>
<tr>
<td>statistics</td>
<td>by the planning coordinator and the pricing actuary. As there is an impact of the economic scenarios on the gross underwriting-year loss statistics, the economic</td>
</tr>
<tr>
<td></td>
<td>scenario modeler is also involved in the sign-off procedure.</td>
</tr>
<tr>
<td>Net UWY loss</td>
<td>The sign-off of net underwriting-year loss statistics is based on a sign-off of the gross underwriting-year loss statistics. The retrocession</td>
</tr>
<tr>
<td>statistics</td>
<td>modeler is responsible to achieve the sign-off from the planning coordinator and the retrocession coordinator.</td>
</tr>
</tbody>
</table>
30.3.4 Asset Modeling

The main purpose of the asset modeling module is to calculate the performance and risk inherent in the asset management. The asset portfolio is stylized, valuated, and rolled-over taking into account the cash flows on the liability side. The asset modeling module is illustrated in Figure 30.6.

![Asset Modeling Diagram](image)

**Data entities**

The following data entities are involved in the asset-modeling module.

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Asset portfolio</strong></td>
<td>The asset portfolio is the collection of assets held by the company. The initial asset portfolio is delivered by the asset management department. The asset portfolio consists of various asset classes such as government bonds, corporate bonds, equity, real estate, interest rate hedges etc. Within the asset-modeling module, it is stylized and rolled forward according to the development of the economy and the liability cash flows. The final asset portfolio is output from the module.</td>
</tr>
<tr>
<td><strong>Yield curve</strong></td>
<td>This data entity is taken from the economic scenario generation module. Please refer to the corresponding definition. The yield curve is used for calculating present values of assets.</td>
</tr>
<tr>
<td>Entity</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>FX rates</td>
<td>This data entity is taken from the economic scenario generation module. Please refer to the corresponding definition. FX rates are used to convert assets to the home currency.</td>
</tr>
<tr>
<td>Investment index</td>
<td>This entity is taken from the economic scenario generation module. Please refer to the corresponding definition. Several investment indices are used to calculate the development of asset positions. These are, for instance, equity indices and real estate indices.</td>
</tr>
<tr>
<td>Credit cycle</td>
<td>This entity is taken from the economic scenario generation module. Please refer to the corresponding definition. The credit cycle is used to calculate the credit risk of corporate bonds.</td>
</tr>
<tr>
<td>Strategic asset allocation</td>
<td>The strategic asset allocation is an entity describing the strategic target portfolio of investments such as the proportion of equity, bonds and other investments, as well as the maturity of newly acquired bonds. The strategic asset allocation is used to roll over the asset portfolio.</td>
</tr>
<tr>
<td>External parameters</td>
<td>For some asset classes, external parameters are used for the valuation. For instance, for interest rate hedges, bucket deltas are used which are delivered by the HypoVereinsbank.</td>
</tr>
<tr>
<td>Liability cash flow</td>
<td>The liability cash flows are taken from the liability-modeling module. They are used to calculate the rollover of the asset portfolio.</td>
</tr>
<tr>
<td>Asset performance</td>
<td>The asset performance is the return generated by asset class as calculated from the forwarded asset portfolio with respect to the initial asset portfolio.</td>
</tr>
<tr>
<td>Risk-based capital</td>
<td>Within the asset-modeling module, the entity risk-based capital denotes the stand-alone capital consumption needed for the asset management.</td>
</tr>
<tr>
<td>Credit risk</td>
<td>Within the asset-modeling module, the credit risk of corporate bonds is calculated.</td>
</tr>
</tbody>
</table>
Asset sensitivity

The asset sensitivity consists of a calculation of the impact that certain economic scenarios (such as interest rates shifts or equity crises) would have on the market value of the invested assets portfolio.

**Process Steps**

The following process steps have to be carried out:

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model stylized portfolio</td>
<td>In order to simplify the calculations, the asset portfolio is stylized at the beginning of the asset-modeling module. This means that, instead of considering each and every single asset position for calculation, asset classes are generated reflecting the main properties of the asset portfolio.</td>
</tr>
<tr>
<td>Model development of asset portfolio</td>
<td>Based on the stylized portfolio, the development of the asset portfolio is calculated. For this purpose, all assets are valued, the asset cash flows are calculated and the liability cash flows are subtracted. Given the overall net cash flows, assets are sold or bought in a way that the strategic asset allocation is achieved.</td>
</tr>
<tr>
<td>Calculate asset performance and risk</td>
<td>The asset performance is calculated from the final asset portfolio with respect to the initial asset portfolio. The standalone risk-based capital inherent in the asset management is calculated. Furthermore, the credit risk of corporate bonds is calculated.</td>
</tr>
<tr>
<td>Calculate asset sensitivities</td>
<td>For different compositions of the asset portfolio, e.g. for different proportions of equity, the risk-based capital as well as the S&amp;P capital consumption is calculated. This is an important step regarding the proposal of the strategic asset allocation.</td>
</tr>
</tbody>
</table>
Roles and Responsibilities

The roles and responsibilities are defined for the asset-modeling module as follows:

**Asset modeler:** The asset modeler is the main role of the asset-modeling module. The responsibilities are to

- Collect the information needed on the current asset portfolio from the asset management coordinator
- Ensure correctness of the input data
- Ensure that all methods and implementations in the asset-modeling module are appropriate and correct
- Model the stylized portfolio
- Calculate the final asset portfolio, the asset performance, risk-based capital, the credit risk of corporate bonds, as well as the asset sensitivity
- Achieve a sign-off of the final results from the chief investment officer

**Asset manager:** The asset management coordinator is responsible to

- Timely deliver the information needed on the current asset portfolio
- Provide explanations on the asset portfolio as needed
- Participate in the sign-off of the final results

**Chief Investment Officer (CIO):**

- Provide the sign-off on performance target of the asset portfolio

Sign-off Procedures

A sign-off is needed for the asset performance. This sign-off is closely related to the economic scenarios and is therefore to be coordinated with the economic scenario modeler. The asset performance is signed-off by the chief investment officer. Please see the Appendix for an overview of the sign-offs given in the ALM process.
30.3.5 Aggregated Results Calculation

The aggregated results calculation module is the last module in the ALM process. It calculates the main results needed for capital management and the Swiss Solvency Test. The module is illustrated in Figure 30.7.

Figure 30.7: ALM process – aggregated results calculation.

Data Entities

The following data entities are involved in the aggregated results calculation module:

<table>
<thead>
<tr>
<th>Data entity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset portfolio</td>
<td>This is the final asset portfolio as calculated within the asset modeling module.</td>
</tr>
<tr>
<td>Balance sheet reserve</td>
<td>This is the final balance sheet reserve as calculated within the reserve modeling module.</td>
</tr>
<tr>
<td>New business results</td>
<td>These are the gross and net loss statistics as calculated within the liability-modeling module.</td>
</tr>
<tr>
<td>Finance instruments</td>
<td>Non-reinsurance liabilities, such as senior debts, hybrid equity</td>
</tr>
</tbody>
</table>
Non-invested assets | Non-invested assets, such as retro assets, funds held by reinsured, deferred tax assets, goodwill.
---|---
Balance sheet | The main items of the nominal balance sheet are calculated. Furthermore, the Swiss Solvency Test and Solvency II require calculation of the main items of the economic balance sheet.
Income statement | The main items of the nominal income statement are calculated. Furthermore, the Swiss Solvency Test and Solvency II require calculation of the main items of the economic income statement.
Risk-based capital (RBC) | The overall diversified risk-based capital is calculated on a calendar year basis. Statistics are provided on details of the business segmentation such as the net underwriting risk, the net reserving risk and the asset risk. Furthermore, the diversity gain is calculated by comparing the stand-alone RBC to the diversified RBC.
Return on RBC | The return on RBC is calculated as a measure of the performance.
MVM market value margin | As defined by FOPI, see SST white paper (FOPI [2004])
SST solvency capital requirements | SST solvency capital requirements: As defined by FOPI, see SST white paper (FOPI [2004])
Risk drivers | The main risk drivers are calculated for both the assets and the liabilities.
Efficient frontier | Efficient frontiers are calculated by varying asset allocations for the portfolio rollover.
Strategic asset allocation | A new strategic asset allocation is derived from the efficient frontier.
Process Steps

The following process steps have to be carried out:

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate financials</td>
<td>The nominal as well as the economic balance sheet and income statement are calculated by mapping the final asset portfolio, the final loss statistics, as well as the final balance sheet reserves to the corresponding positions of the balance sheet and the income statement.</td>
</tr>
<tr>
<td>Calculate SST solvency requirements</td>
<td>The calendar year risk based capital is calculated by calculating the risk measure, i.e. the 1% shortfall, of the overall result distribution. The RBC is split to the various sub-segments, and the return on RBC is calculated. The MVM and the SST solvency capital requirements are calculated.</td>
</tr>
<tr>
<td>Calculate risk drivers</td>
<td>The main contributions of single risk drivers to the calendar year RBC are calculated and listed as the risk drivers.</td>
</tr>
<tr>
<td>Calculate asset management strategy</td>
<td>The ALM model is evaluated multiple times using different allocations for the asset portfolio rollover. Each calculation yields a point for the efficient frontier. From the efficient frontier, the strategic asset allocation is determined.</td>
</tr>
</tbody>
</table>

Roles and Responsibilities

We display the roles and responsibilities involved in aggregated results calculation:

**Aggregated results calculator**

- Ensure consistency and correctness of methods and implementations used in the module
- Ensure consistency of the input data as collected from the other modules
- Calculate the output data entities of the module
- Achieve sign-off from the ALM business owner
Financial controller

- Provide data and explanations on finance instruments and non-invested assets

ALM business owner

- Provide sign-off on aggregated results

ALM Committee

- Provide sign-off on projected balance sheet and income statement results
- Provide sign-off on efficient frontier
- Provide sign-off on the ALM and SST Reports

Sign-off Procedures

As the results of the aggregated results calculation module are the main results of the ALM process, and as there are no modeling steps involved apart from the ones carried out in the previous modules and signed-off there, the output data entities of the aggregated results calculation module are signed off by the ALM Committee directly.

As mentioned in the overview section, the results are assembled in two reports: The ALM report and the SST report. The ALM Committee signs off on both reports. Eventually, the SST report is submitted to the FOPI. Please see the Appendix for an overview of the sign-offs given in the ALM process.
Scenarios

31.1 Overview

Creating a scenario is a result of collaboration between many departments within SCOR Switzerland: actuarial, reserving, financial-risk modeling, underwriting and finance. Besides, a number of departments are involved in terms of providing data, partial information and impact assessment from their areas. The scenarios are updated, maintained and checked for relevance on a yearly basis. For detailed information, see the specific SST Scenario Report for FSAP 2007 update (Gencyilmaz [2007]) created as a substantial part of the SST Reporting. Figure 31.1 shows an overview of the scenario production process at SCOR Switzerland.

Figure 31.1: Scenario production process.
### 31.2 Process Steps

The following steps are taken during the scenario production process:

<table>
<thead>
<tr>
<th>Process step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback to scenario owners</td>
<td>The yearly feedback by the FOPI is communicated by the risk management team to all scenario teams in separate meetings.</td>
</tr>
<tr>
<td>Internal workshop</td>
<td>In a workshop the specific teams define which scenarios are the largest threat to SCOR Switzerland’s portfolio, respectively which scenarios need to be accomplished continuously and which can be held back.</td>
</tr>
<tr>
<td>Calculation of yearly figures</td>
<td>The data providers hand over the data for the scenario production. For further details, see the specific scenario documentation.</td>
</tr>
<tr>
<td>Update of scenario documentation</td>
<td>The scenario documentation is maintained and reviewed for up-to-dateness. The major goal of this step is to focus on improvements in terms of transparency and ability to replicate the analysis in the future.</td>
</tr>
<tr>
<td>Peer review</td>
<td>Each scenario is peer-reviewed by SCOR Switzerland’s risk management team and the feedback is incorporated into the document.</td>
</tr>
<tr>
<td>Production of combined scenario document</td>
<td>SCOR Switzerland’s risk management team writes a draft of the combined scenario document, including an executive summary. The document is reviewed by the ARMS (actuarial and risk management services) leadership group. It is also signed off for submission to senior management.</td>
</tr>
<tr>
<td>Senior management and external review</td>
<td>The document is reviewed by the ECS.</td>
</tr>
</tbody>
</table>
Finalization of the combined scenario document/ sign-off

ECS feedback is incorporated and the final document gets signed-off by ECS for delivery to FOPI.

31.3 Roles and Responsibilities

The following roles and responsibilities are defined for the scenario production process:

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SST process manager</td>
<td>• Overall responsibility for the production of all scenarios</td>
</tr>
<tr>
<td></td>
<td>• Perform peer review</td>
</tr>
<tr>
<td></td>
<td>• Manage flow of information to update scenario documentation</td>
</tr>
<tr>
<td></td>
<td>• Performing the role of the scenario documentation owner in collaboration with the risk management team</td>
</tr>
<tr>
<td>Scenario teams</td>
<td>• Calculate and document respective scenarios</td>
</tr>
<tr>
<td></td>
<td>• Define, acquire and check all data needed for calculation</td>
</tr>
</tbody>
</table>
Risk management
team

- Manage flow of feedback from internal senior management and external review to the scenario owners
- Performing the role of the scenario documentation owner in collaboration with the SST project leader

ECS

- Challenge whether scenarios include all possible losses
- Challenge whether there are other more severe scenarios
- Review the scenarios for delivery to FOPI
- Sign-off by CEO, CFO and COO

Scenario documentation owner

- Main point of contact for FOPI regarding scenarios
- Define scenarios in cooperation with senior management

Scenario documentation editor

- Edit and structure the scenario document
- Communicate requirements to individual scenario teams
- Ensure timely delivery of scenarios by scenario teams
31.4 Sign-off Procedures

Sign-off (Table 31.1) is given by local CEO, CFO and COO for delivery to FOPI. In addition there is peer review and senior management review of scenarios.

<table>
<thead>
<tr>
<th></th>
<th>senior mgmt.</th>
<th>ECS</th>
<th>peer review</th>
</tr>
</thead>
<tbody>
<tr>
<td>methodology</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>parameters</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>Data</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>processes</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Results</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 31.1: Scenario production process – sign-off matrix.
Appendix

32.1 Sign-off Matrix

<table>
<thead>
<tr>
<th>Entity</th>
<th>Sign-off</th>
<th>Legal</th>
<th>Internal</th>
<th>Project</th>
<th>Roles</th>
<th>SLT</th>
<th>STG</th>
<th>ALM</th>
<th>SSTG</th>
<th>Management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finance</td>
<td></td>
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<td>IT</td>
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<tr>
<td>HR</td>
<td>1</td>
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<tr>
<td>IT</td>
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<tr>
<td>ALM</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSTG</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Management</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

32.1.1 Timeline of Project

SCOR Switzerland has set up the SST implementation project in February 2006. All internal models for the ALM process have been reviewed, adjusted where needed and validated considering the guidelines provided by the FOPI for the SST. The ALM process within SCOR Switzerland is considered industry state-of-the-art over years (reference: Report from Oliver Wymann, 2003). Based on this ALM process, with the necessary adjustments to be fully SST compliant, SCOR Switzerland will create the yearly SST Report from Q1/2008 onwards. Figure 32.1 shows SCOR Switzerland’s timeline to implement the SST within its organization.
32.1.2 Project – Sign-off Matrix

To ensure consistency of model methodology, implementation, data and processes, SCOR Switzerland has developed a sign-off matrix for the SST implementation process (Table 32.1). As can be seen from the sign-off matrix, the documentation of all steps is subject not only to internal review but also to external review, especially, a full disclosure to the FOPI.

<table>
<thead>
<tr>
<th>public disclosure</th>
<th>internal review</th>
<th>external review</th>
<th>FOPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>methodology</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>implementation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>processes</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 32.1: SST project – sign-off matrix.

32.1.3 Project Roles and Responsibilities

Establishing an effective organization structure for a project is crucial to its success. Every project has a need for management, direction, control and communication, so does the SST implementation process at SCOR Switzerland. The following roles bring together the various interests, skills, management and support:
Steering committee:
The role of the steering committee is to confirm the project objectives, provide guidance and direction to the project and agree the project approach. The steering committee approves deviations to plans and signs off on the completion of project stages.

The steering committee commits the necessary resources to the project, arbitrates conflicts and assists in the management of issues as escalated by the project manager. The group is responsible for the acceptance of the project’s risk management strategy and oversees the execution of that strategy.

Project sponsor:
The project sponsor is the executive ultimately accountable for the project within SCOR Switzerland. He/she ensures that the project gives value for money and as such owns and ensures the validity of the business case.

Business owner:
Ensures that the end product is fit for its purpose and that the solution will meet the user/business needs within the constraints of the business case. The business owner is responsible for provision of business resources and must be of sufficient authority to take decisions on project matters. The business owner is ultimately responsible for the sign-off on the project’s products. The new organization aids in the implementation of this model as a result of global responsibilities.

Resource provider:
The resource providers are accountable for the quality of the products delivered. They commit significant resources to the project and are of sufficient authority to take decisions on project matters. They deliver the results desired by the business owner within the time and cost parameters of the business case.

Solvency II liaison:
Ensures the alignment of SCOR Switzerland Cologne regarding the preparatory activities for Solvency II in the EU with the SST project, capitalizing on synergies.
FOPI liaison: Takes the lead in FOPI liaison and is ultimately responsible for ensuring that the regulatory requirements are met by the solutions proposed.

Project manager: The project manager is responsible for the day-to-day management of the project. Delivers the project within the agreed time, scope and cost. Due to the complexity, strong project management will be required.

Project core team: The core team may be supplemented with additional resources and may change as a result of detailed planning and progressing work. The above team reflects the envisaged goals and work during 2006 (Phase 1). The core team will be in charge of ensuring the timely delivery of the results. The team will naturally include the project manager.

32.1.4 Project Organizational Overview of Roles and Responsibilities

The above mentioned roles are organized in three different segments: steering / structuring and validating / executing. This separation of segments is crucial to the success of the project as it strengthens internal audit and objectiveness of results.
Figure 32.2: Organizational overview of roles and responsibilities.
VIII

SST Systems
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Summary

This Part of the documentation, based on *SST Systems Documentation* (Gut and Bürgi [2008]), gives an overview of the software systems which are relevant for the calculation of SST results at SCOR Switzerland. The high level architecture of existing systems and new developments is discussed. The IT systems are mapped to the ALM process modules\(^1\).

\(^1\)The documentation of the ALM and SST process together with the relevant process modules is Skalsky and Bürgi [2008] and is incorporated in Part VII of this documentation, see pp. 353.
Introduction

In the past decades, the importance of information technology in the financial industry has increased significantly. With the growing data density grew the need to develop appropriate techniques to extract information from the data. The systems were interlinked, the system landscape was integrated, initially with a strong design focus on accounting and administration. The complexity of handling data increased, especially in business areas which were not the main design focus for the IT system.

Figure 34.1: Overview of the ALM systems: functional needs.

Figure 34.1 shows schematically the functional needs of ALM systems. The ALM process is at the very end of the corporate process landscape: It takes information from many different departments. The main data providers are planning, asset management, pricing, reserving, accounting,
and Bloomberg. One of the purposes of ALM is to generate a holistic picture of all cash flow relevant activities in the company. This step of matching code worlds, of resolving definition conflicts, of refining data to match the required content definition and granularity, is an extremely demanding and time-consuming step when calculating capital requirements of a reinsurance company.

It is not surprising that a large part of the SCOR Switzerland ALM system landscape consists of tools which were developed to handle these interfaces. Data have to be entered into the system. In some cases, data can be transferred through automatic interfaces, in other cases data have to be reformatted and entered manually into the system. This depends mostly on the availability of appropriate systems on the counterparty end. Data often have to be enhanced (e.g. mapping codes or adjusting data to the required granularity of data).

An important input to the internal model is a statistically balanced set of economic scenarios. SCOR Switzerland has developed a bootstrapping algorithm to generate economic scenarios. The ESG tool has been developed in house.

The means of computer simulation has proven to be efficient to implement the models. The ability to efficiently implement complex calculation algorithms is fundamental to building a sound and robust internal model. The risk simulation imports all the required data from the data entry and enhancement and the ESG parts and simulates the result-distribution of the company and all the other quantities relevant for the ALM and SST reports.

Eventually, the quantities generated in the risk simulation have to be analyzed and formatted for use in the ALM and SST reports.

### 34.1 Scope of the IT Systems Documentation

The system landscape involved in the calculation of the ALM and SST reports is rich. The ownerships of the interfacing systems reside in the corresponding departments. The ownership of ALM specific systems resides in the Financial Modeling department. For a detailed overview of ownerships, refer to Appendix (p. 448).

Since a large part of the data generated in the company are cash flow relevant in one or the other way, most of the systems used in the company are somehow involved in the ALM process. There is a core of ALM systems with the risk simulation part in its centre and the other modules around it as described above. This documentation focuses mainly on the ALM specific systems. Furthermore, it gives an outline of the interfacing systems. The further away a system is from the core risk simulation, the less it will be described in this documentation.
34.2 ALM Systems and ALM Process

A sound integrated management should consist of both, a solid methodology and calculation process, and a robust ALM process. The main purposes of the ALM system landscape are to:

- Provide tools and methods to retrieve and refine input data
- Calibrate the models used in the ALM methodology
- Calculate all results necessary for the ALM and SST reports
- Support the ALM process

In this sense, the SST Systems Documentation is strongly interlinked with the SST methodology\(^1\) and the SST process landscape\(^2\). The modules of the SST Process Landscape were designed to give a functional overview of the SST Process. In this document, the implementation in the ALM system landscape will be shown. This is a different perspective, and the charts will therefore not be fully congruent. The link between the ALM systems and the functional modules described in the ALM Process Landscape will be described in detail.

\(^1\)See parts I, II, III, IV, V, VI of this document.
\(^2\)Swiss Solvency Test (SST) at SCOR Switzerland: Process Landscape, Skalsky and Bürgi [2008], incorporated into this document as Part VII.
This chapter is divided in two major sections: one section describing the architecture of the existing systems and the other section highlighting the architectural principles used for new developments. The Economic Scenario Generator (ESG) has already been built along these principles. A project is in preparation to implement these principles for all ALM specific components.

A vast variety of technologies is involved in the calculation of the ALM and SST reports, especially due to the high number of systems involved. The core risk simulation is implemented in Igloo, a generic DFA software developed by EMB\(^1\).

The majority of the system landscape of former Converium relevant to the SST was built in the .NET framework with Microsoft SQL databases. The new ESG has also been built in this technology. The SCOR systems consist mostly of Java and PowerBuilder applications with Sybase databases. The decisions on the future technologies to be used are pending at the time of writing this documentation.

Most of the data input to the ALM model are usually exchanged in Excel or text file format. Exceptions: ESG data is read from the MS SQL database directly into Igloo; the reserving data is transferred from MS SQL via an OLAP layer to Excel. The data enhancement steps (mapping of codes, refining data to match the granularity needed, etc.) are usually implemented in Excel. Final inputs to the risk simulation in Igloo are stored in Excel or text format and read via Igloo links into the risk simulation. Simulation outputs are stored in Excel or text file format.

\(^1\)http://www.emb.co.uk/
35.1 Software Development Process

35.1.1 Software Development Life Cycle (SDLC)

The Purpose of SDLC

The Software Development Life Cycle (SDLC) guidelines define the principles to ensure delivery or acquisition of software according to business needs, with business involvement and testing, using standards and methods and providing appropriate documentation. The result will enable SCOR Switzerland to make proper use of the application and the technological solution.

Process ownership

The main responsibility lies with the Head of IT Development at SCOR Switzerland who is the process owner and thus responsible for designing, implementing, changing, documenting and executing the process.

The processes covered by SDLC

The SDLC covers six processes:

1. It is ensured that the business side is appropriately involved in the design of applications, selection of packaged software.

2. It is ensured that an adequate development process is in place. The process covers security, availability, and processing integrity aspects.

3. When new systems are implemented or existing systems are modified, controls are added, modified or redesigned to ensure that applicable control objectives are achieved (including change management).

4. Controls exist to ensure there is adequate testing for the development or acquisition of systems and applications. Testing is signed off by both the business side and IT.

5. Controls are in place to ensure that an appropriate system, user, and control documentation is developed for new systems and applications, as well as for modifications of existing systems and applications.

6. Controls are in place to ensure that users are trained on new systems/applications used during financial reporting processes in accordance with an appropriately defined training plan.
35.1.2 Application Change Management (ACM)

The Purpose of ACM

The Application Change Management (ACM) process is a business-driven and cost efficient method of handling changes to SCOR Switzerland’s applications. It ensures minimum negative impact and risks for SCOR Switzerland’s business by requiring planning and initial impact analysis of all change requested. Changes cannot be built or implemented outside the ACM process. The ACM process will increase substantially the quality of service SCOR IT delivers to the business side.

Process ownership

The ACM process is owned by the Head of IT Operations department.

Process Objectives

Business side objectives for the ACM process are:

- Ensure the business side is involved in the process of specifying, authorizing and setting priorities.
- Ensure that the Sarbanes-Oxley control environment for business controls are maintained.
- Ensure delivered solution meets requirements and specifications of change request.
- Ensure that the change has no negative impact on business.
- Ensure coordination within the business community.
- Ensure that the correct sign-offs are collected for changes.
- Ensure all audit and compliance requirements are met.
- Ensure efficient, prompt and documented handling of all changes.

Functional objectives for the ACM process are:

- Ensure Sarbanes-Oxley IT general controls have been performed.
- Minimize impact of changes to production environments.
- Ensure a clear segregation between development and production environments.
- Ensure change has no negative impact on IT Service Delivery.
- Ensure IT service quality is maintained as agreed.
• Ensure the use of standardized methods and procedures.
• Ensure configuration data and documentation are maintained.
• Ensure the IT Security of systems and applications is maintained.
• Save Cost by distinguishing changes by impact and priority.
• Save Cost by specifying solutions to problems before any development or implementation work is done.

Process Scope

The scope of the ACM process includes all software referenced in the scope section of the policy for Configuration Management. Basically the scope includes all changes to financially relevant application software developed in-house or acquired for specific use within SCOR Switzerland. Also system software supporting to these applications are in scope.

The scope includes the Test/QA and Production environments and the following application component changes:

• Program code
• Data structures
• Back-end data corrections
• Configuration files and Automated Jobs/Agents/Tasks
• Documentation and procedures associated with running, supporting and maintaining the application

The customizing of third part software is out of scope.

35.2 Security

The .NET / MS SQL systems are secured by role-based access management (RBAM). The RBAM is maintained according to the corporate guidelines and can be found in the corresponding documentation.

All Excel and text files as well as the Igloo model are stored on NFS drives with Windows access management. Access to the spread sheets, text files and the Igloo model is restricted to the members of the Financial Modeling department and single interfacing roles.
35.3 Revision Control

The key to gaining overview and transparency of data, methods and documentation is a sound versioning and historization concept. All data entities, methods and documents underlie a clear release concept. Releases are documented and labelled. Changes are documented. In order to ensure reproducibility and auditability of data, a clear historization concept has been elaborated and implemented.

For all the systems which are maintained by the IT department, i.e. all the .NET / MS SQL systems, the revision control system used is IBM Clear Case. For all the Excel and text files, as well as the Igloo model, the versioning system used is Microsoft Visual SourceSafe. All features for check-in, check-out, roll-back are readily provided by the corresponding revision control systems. Changes can be documented at the point of check-in, releases can be labelled.
36

Detailed Description of Systems

This part of the SST Systems Documentation is strongly interlinked with the SST Process Landscape (which is Part VII in this document, see pp. 353). Whereas the functional modules described in the latter document are designed to reflect the content in its context, the present documentation aims at describing the systems in their context. This leads to a slightly different structure; however, the link to the functional modules is given explicitly in all sections.

36.1 Language Definition

Throughout the next sections of this document the IT systems as well as the relevant data processing steps will be graphically displayed. The graphics serve to illustrate the tools used and the data flows as specified in the text. Unidirectional or bidirectional arrows indicate the flow of data between IT systems. Due to the complexity of the data processing steps, spread sheet calculations are often involved. In order to differentiate between systems or other data processing entities, database storages, data files and documents, numerical operations, and manual input, we introduce the basic shapes shown in the following table.
Whenever we use color coding the colors shall help group data processing steps or data entities according to the scope of application.

### 36.2 Economic Scenario Generation

Economic scenario generation is a fundamental step to calculating the results of the ALM and SST reports. SCOR Switzerland has chosen its own approach of generating economic scenarios by bootstrapping historic time series\(^1\).

#### 36.2.1 Functional Overview

Figure 36.1 shows the economic scenario generation module.

\(^1\)For a detailed description of this bootstrapping methodology, refer to Part II in this document.
The input data to the module are the economic variables to be simulated. Table 36.1 gives an overview of the input variables. All variables are available on Bloomberg.

Note, however, that the bootstrapping algorithm needs a sufficient history in order to work reliably. In some cases, it is necessary to extrapolate a history. A few examples of this are:

- The Euro has existed only since the end of the 1990s. Historic FX rates have to be calculated out of the other FX rates.

- For the Euro risk-free yield beyond the existence of the Euro, the risk-free yield of the Deutsche Mark was chosen, as this is the most dominant currency in the Euro.

- The same considerations also apply for the corporate yield curves.

- Certain investment indices do not date back to a sufficiently large history. E.g., a hedge fund or real estate index might only have been created later. In these cases, a history has to be extrapolated. To do this, an index is taken which is believed to behave similarly. From this basis, and adding a stochastic noise, a history for the corresponding index is extrapolated.

### 36.2.2 System Overview

The functional module as displayed in Figure 30.3, pp. 371, needs to be translated to a systems perspective. This is shown in Figure 36.2. Time series are synchronized from Bloomberg to the Financial and Economic Database (FED). From there, the time series are imported into the ESG Historic Time Series module of the ESG. The time series are analyzed and completed.
Government bonds are not quite risk-free – there is still a small credit risk of the corresponding country priced in. Bloomberg has defined a set of fair-value risk-free yield curves which are calculated out of several liquid market instruments for each currency. This set is taken as input for the risk-free interest rates.

The consumer price index (CPI) of the corresponding currency zone. For the Euro zone, an overall CPI is used. All CPIs are available in Bloomberg.

The real GDP, i.e. the GDP net of inflation, is needed. In Bloomberg, there is a mixture of nominal and real GDPs available. Wherever the real GDP is available, it is used as input. Otherwise, the nominal GDP is imported and converted to a real GDP within the ESG Historic Time Series module (see Part II, pp. 193).

The FX spot rates are chosen at the time of each simulation interval (i.e. end of a quarter or month).

The investment indices matching the asset portfolio are chosen for simulation. These are the gross equity indices (MSCI daily), and for some currencies hedge fund or real estate indices.

Corporate yields are imported for a number of ratings (currently for A and BBB). From the corporate yields and the risk-free yields, the credit spreads are calculated.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Time series used</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Risk-free yield</strong></td>
<td>Government bonds are not quite risk-free – there is still a small credit risk of the corresponding country priced in. Bloomberg has defined a set of fair-value risk-free yield curves which are calculated out of several liquid market instruments for each currency. This set is taken as input for the risk-free interest rates.</td>
</tr>
<tr>
<td><strong>Inflation Index</strong></td>
<td>The consumer price index (CPI) of the corresponding currency zone. For the Euro zone, an overall CPI is used. All CPIs are available in Bloomberg.</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>The real GDP, i.e. the GDP net of inflation, is needed. In Bloomberg, there is a mixture of nominal and real GDPs available. Wherever the real GDP is available, it is used as input. Otherwise, the nominal GDP is imported and converted to a real GDP within the ESG Historic Time Series module (see Part II, pp. 193).</td>
</tr>
<tr>
<td><strong>FX Rates</strong></td>
<td>The FX spot rates are chosen at the time of each simulation interval (i.e. end of a quarter or month).</td>
</tr>
<tr>
<td><strong>Investment Index</strong></td>
<td>The investment indices matching the asset portfolio are chosen for simulation. These are the gross equity indices (MSCI daily), and for some currencies hedge fund or real estate indices.</td>
</tr>
<tr>
<td><strong>Credit Spread</strong></td>
<td>Corporate yields are imported for a number of ratings (currently for A and BBB). From the corporate yields and the risk-free yields, the credit spreads are calculated.</td>
</tr>
</tbody>
</table>

Table 36.1: Economic variables used for economic scenario generation.
The ESG Simulation module provides the bootstrapping algorithm, which results in the Economic Scenarios. From there, the data can be exported in the Igloo Interface and then to Igloo. For ALM, there is no manual input of time series required. This facility is only provided for other consumers of economic scenarios.

36.2.3 Financial and Economic Database (FED)

The Financial and Economic Database (FED) is the central data provider of time series. It is an in-house development. FED is implemented in Microsoft SQL Server and .NET. The main data provider of FED is Bloomberg. All subscribed time series are updated nightly.

The scope of FED is to be an encapsulation layer between data providers such as Bloomberg and the company internal systems. This encapsulation layer has the advantage that several data sources, usually with different code worlds (e.g. their tickers) and database schemas, can be merged into one common data source, from where it can be served to the other applications. This situation is shown in Figure 36.3.

FED also has a user interface which allows to retrieve financial and economic data through a web browser or exported to an Excel sheet. FED is owned by the Financial and Risk Modeling department and maintained by IT Development.

The Economic Scenario Generator (ESG) is one of the main consumers of FED data. The data are imported directly from the FED database to the ESG Historic Time Series module. There, they can be adjusted and completed for simulation purposes.
36.2.4 ESG Application

The ESG application consists of two distinct modules: the ESG Historic Time Series, and the ESG Simulation modules as described in the next subsections. The ESG application is developed in the .NET framework with an MS SQL database.

The ESG application is owned by the Financial and Risk Modeling department and maintained by IT Development.

ESG Historic Time Series

The ESG Historic Time Series module has the purpose to collect all time series required for simulation from FED. There is the concept of an ESG Ticker, which contains the basic definition of a time series, such as the

- Code to be used in Igloo
- Type of ticker (GDP, CPI, equity etc.)
- Value definition (gross, price, dividend, total return)
- Inflation correction (nominal, real)
- Link to the FED source

The management of the ESG Tickers is enabled in the ESG application. The role of the ESG Ticker Manager is responsible to administer the ESG Tickers such that they relate to the desired data source and to the corresponding part in the Igloo model.

The role of the ESG Modeler is responsible to assemble the time series history. Once the desired ESG Tickers are chosen for the ALM model, the
data can be automatically imported from FED. Note that, typically, the ESG Historic Time Series module is updated by copying a previous one and by importing the most recent FED data. The ESG application supports this use case strongly by providing a color coding for which values correspond to the FED and which ones not, and by enabling a partial update of figures for a selection of individual items.

The ESG Historic Time Series module also provides facilities for inter- and extrapolating time series (vertically in time) and yield curves (horizontally in maturity). The vertical inter- and extrapolation is needed if the simulated time series are not up-to-date, i.e. if a time series such as the GDP has a certain delay of being reported. The horizontal inter- and extrapolation of yield curves is needed if maturities other then the ones supported by Bloomberg are to be simulated.

Since it is difficult to validate a large amount of data, the ESG Historic Time Series module also contains statistical tests which highlight deviations of time series by more than the expected deviation. This feature allows to get a quick overview of suspicious values which have to be checked within Bloomberg. This would, for instance, be the case if an index was re-based. The application would highlight the place of re-basing, and the user could manually convert the index to one base.

For all deviations from the FED values, comments must be added to the dedicated section. Once the time series history is completed and validated, the ESG Historic Time Series module is locked by the ESG Modeler. Locking will disable the update of the module. This is the procedure how a version of the data is administered. In order to create a new version, the ESG Historic Time Series module can be copied, modified and locked.

**ESG Simulation**

The ESG Simulation module has the purpose to assemble the set of time series to be simulated, to select the simulation parameters, and to run the simulation.

A selection of time series is necessary, as there is not a one-to-one correspondence between historic time series and simulated ones: For instance the corporate yield curves are evaluated, together with the risk free yields, to the credit level, which is output from the simulation.

There are only few simulation parameters exposed on the user interface. The main ones are:

- The time interval denoting the historic period to be used for bootstrapping
- The time interval to be simulated
- The number of scenarios to be simulated
For investment indices, there is also an option to enter expert opinions: The main purpose of the ESG is not to calculate accurate forecasts but to calculate an accurate spread around the forecasts. As the expert opinions are based on a broader range of observations than pure stochastic bootstrapping from the history, they are to be preferred. It is thus possible to use (a stochastic variant of) the expert opinions to adjust the expected net return.

All other parameters, i.e. the parameters underlying the economic models, are hidden from the user. These parameters are the result of calibration calculations. Typically, they are only modified when the analysis of historical data is updated or enhanced. This is a lengthy procedure which can, for instance, be done in the framework of a master’s thesis. When adjusting the low-level parameters, and after extensive testing, the application would be re-deployed with the new models.

A post-processing step allows for a further set of operations:

- The scenarios can be stochastically interpolated to other time intervals (e.g. from quarters to months). This is usually not needed in the ALM context.

- The scenario set can be concentrated, i.e. the number of scenarios can be reduced. This is done along an extremeness metrics, which defines extreme scenarios. The most extreme scenarios are all kept. The number of remaining (‘normal’) scenarios is reduced along the metrics, increasing the weights of the normal scenarios correspondingly. Scenario Concentration makes sense if the data transfer (e.g. import to Igloo) does not perform well enough.

- Time series can be scaled to a target over a given simulation interval. Studies show that, over a long time horizon such as 10 years, the return of an investment index is less volatile than the bootstrapping algorithm would predict. Therefore, it is possible to scale scenarios to yield a certain long-term return. This feature is used for the scenarios submitted to the life department, which cover a time horizon of several decades.

Another feature of the ESG Simulation module is that it can produce forecast tables over a specified forecast horizon. This forecast table is an integral part of the ALM report.

Once the ESG Simulation is completed and appropriately commented, it is locked by the ESG Modeler. Only if a simulation is locked, it can be exported to Igloo.

### 36.2.5 Data Export

Within the ALM process, there are two main consumers of the economic scenarios: The actuarial life department who model the GMDB and financial
reinsurance life business using these scenarios, and the main risk simulation in Igloo. For transferring the data to the life department, they are exported to an Access database on an NFS drive.

For exporting the data to the main risk simulation in Igloo, the data are transferred to the Igloo interface (Figure 36.4). Only locked simulation can be exposed to Igloo. In Igloo, the economic scenarios can be read from the MS SQL Server, and are parsed to a proprietary Igloo file format.

![Figure 36.4: The ESG – Igloo Interface.](image)

### 36.3 Reserve Modeling

The purpose of reserve modeling\(^2\) is to calculate the reserve volatilities as well as to calculate the incurred (reported), paid and earnings patterns by line of business and by legal entity. Furthermore, the reserve duration and the reserve run-off are calculated for inclusion in the ALM Report.

#### 36.3.1 Functional Overview

Figure 36.5 shows the functional overview of the reserve modeling module.

![Figure 36.5: ALM process – reserve modeling](image)

\(^2\)The detailed aspects of the reserve model are considered in Part I of this documentation.

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The initial data to the module are triangles (UW year / development year) of incurred and paid claims, case reserves, and premiums by line of business (LoB) and legal entity (LE). From the triangles, the chain ladder patterns are determined which are later imported in the risk simulation. The total reserves are taken from the initial balance sheet, and are broken down by LoB and LE. The reserve volatilities are calculated from the triangles as described in Part I.

The reserve duration is calculated as it has an impact on the strategic asset allocation as proposed in the ALM Report. The reserve run-off is calculated as a historic figure for comparison with the reserve risk in the ALM Report.

### 36.3.2 Systems Overview

The translation of the functional overview to the systems overview is shown in Figure 36.6. The main reserving database is CORE. Reserves are calculated using ResQ, an EMB reserving tool. The results are stored back to CORE. ReCube provides an aggregated hypercube of the CORE database. From ReCube, triangles can be imported directly in Excel. The patterns, as well as the reserve volatilities and the initial balance sheet reserve split are calculated in Excel. The results are stored in Excel format and are then imported in the risk simulation.

The reserve duration and run-off is calculated directly from the CORE database by means of an SQL script.

![Figure 36.6: Data processing steps in reserve modeling.](image-url)
36.3.3  CORE

CORE – Converium Reserving Estimator – is a reserve analysis tool and database developed in-house. The analysis part of the application has been replaced by ResQ in 2007. The CORE database continues to exist as the data provider to ResQ and main storage of all reserving data and for bookings and accruals. The CORE database is implemented in Microsoft SQL Server, the front end is in PowerBuilder.

The application and data owner is the Reserving team (*Actuarial – Non-Life*). CORE is maintained by IT Development.

36.3.4  ReCube

ReCube was internally developed by the company as a reporting tool to analyze reserving data and to produce reserve triangles. ReCube is a hypercube on CORE (*OLAP* layer). Triangles can be imported directly from ReCube in Excel.

The applications and data owner is within the Financial Planning & Analysis team. ReCube is maintained by IT Development.

36.4  Modeling New and Unearned Business

Next to the reserving risk, the risk inherent in the new business\(^3\) is the second major part of the insurance risk. Currently, life and non-life business is completely separated systemwise. Since SCOR Switzerland is mainly writing non-life business, its system landscape is richer on the non-life part than on the life part.

36.4.1  Functional Overview

Figure 36.7 shows the functional overview for modeling new and unearned business.

The planning data is analyzed carefully and based on it, and based on the existing business, the modeling baskets are created. A dependency structure between the baskets is defined. The basket structure and dependency tree are hierarchical.

Loss models are imported from the pricing systems. This can either be done by contract, upon which the contracts would be aggregated using the dependency within the basket. In some cases, the contracts are aggregated in a different manner; for nat cat perils, aggregate distributions are calculated using an event-set based approach. Large contracts are modeled in separate baskets with the aggregate distributions entered directly into Phobos. For life business, there is a separate process in place. Generally, the loss scenarios

\(^3\)The detailed aspects of the new business model are considered in Part I.
Figure 36.7: ALM process - liability modeling
depend on the economic scenarios as delivered by the ESG. Therefore, the actual loss distributions are generated only during the risk simulation. They are merged at this stage to the basket structure according to the dependency tree.

For the majority of the in-force business, loss models are available from pricing. However, the plan includes also business which is not yet written or which has not been priced. The assumption is taken that the non-priced business will behave similarly as the priced part of the portfolio. Therefore, the loss models as obtained from pricing are scaled to match the planned volume.

The underwriting-year statistics of the new business on a stand-alone basis (i.e. disregarding developments in reserves, assets etc.) can be generated at this stage. It is an important consolidation step to compare the planned volumes, loss ratios, profits and stand-alone RBC required in their aggregated form among the departments (pricing, nat cat and planning). This helps to enhance the overall quality of data in the company significantly.

The underwriting-year-based loss models are then processed with the payment-and-earnings-patterns in order to yield the gross cash flow models for new and unearned business. Information on the retrocession covers is provided by the retrocession department. The loss models are imported in the risk simulation, together with the data from the reserve modeling and the information on the retrocession covers. In the risk simulation, the models are evaluated in conjunction to yield the net liability cash flow model.

36.4.2 Systems Overview

The translation of the functional overview as given above to the non-life systems overview is shown in Figure 36.8.

The Global Planning Tool (GPT) is used to plan the business by contract. From GPT, an Excel Pivot table is extracted, which is the entry point in the ALM model. The Pivot table is analyzed, the modeling basket structure is prepared by segmenting the data further. Eventually, the planned premiums and losses as well as the planned costs are available for each modeling basket.

The basket structure is entered in Phobos, the main aggregation engine. The dependencies are added to the basket structure. Phobos has a direct interface to MARS, the main non-life treaty pricing system. The MARS treaties can be assigned to the modeling baskets, and the NPV, premium, loss, cost distributions and also development patterns can be imported automatically for aggregation. Other data sources can be added by copying and pasting aggregate discrete distributions.

Once the reference volumes of the in-force part of the business is known, separate scaling factors for growth in the number of contracts and growth in share can be entered in Phobos. These scaling factors are calculated in a
spread sheet which is based on the pre-processed planning data.

For the distributions of nat cat perils, a slightly different procedure applies. During the pricing process, MARS stores information about the event loss tables to GCDP. For the ALM process, the nat cat department aggregates the event loss sets and projects them to the planned portfolio. The resulting aggregate distributions by peril and region are stored in MARS and enter the ALM model from there.

The final premium, loss, and cost models are commented, stored, and locked on the Phobos database. From there, the loss models are imported in the risk simulation.

Another output of Phobos are the diversification functions between the modeling baskets. They are exported from Phobos to MARS, so that MARS can allocate the capital to the single contracts during the pricing process, given the planned portfolio.

36.4.3 Phobos

Phobos is an in-house development which allows the manipulation of aggregated risk variables. The application and data owner is the Actuarial Risk Modeling department. Phobos is developed in the .NET framework with a Microsoft SQL database. It is maintained by IT Development.

The user interface allows to
- Administer the basket structure
- Administer the dependencies within and among the baskets
- Assign (in-force) contracts from MARS to the modeling baskets
- Copy and paste aggregate distributions of baskets
- Set scaling factors,
- Aggregate premium, loss, and cost distributions and view results
- Calculate premium, payment, and cost patterns

Once the loss distributions are calculated, the corresponding Phobos tree is appropriately documented and locked. From there, the data are imported in Igloo. The Phobos to Igloo interface is currently under reconstruction, with the aim to come up with a similar solution as for the ESG Igloo Interface. Documentation should be amended as soon as it is specified and implemented.

The diversification functions are updated in MARS for the main January renewal period.

36.4.4 MARS

MARS (Model for Analyzing Reinsurance Submissions) is a pricing tool for non-life treaty business. It is used by actuaries and underwriters to determine if a given business opportunity will generate an appropriate return. This platform was developed in-house in the .NET framework and with a Microsoft SQL database. Application and data owner is the non-life pricing department. MARS is maintained by IT Development.

The calculation of Net Present Value distributions is calculated in MARS. The excess of the actual performance versus the target performance is calculated in MARS based on the parameters as calculated in Phobos.

36.4.5 Global Cat Data Platform

The global cat data platform (GCDP) is a central system for the aggregation of natural catastrophe risk and for risk accumulation control. The application and data owner is the Natural Hazards department. The GCDP is built in Java and Microsoft SQL Server. The GCDP is maintained by IT Development.

The GCDP stores information on event loss set tables, i.e. the contribution of each contract to a given event loss. From GCDP, all in-force nat cat business is aggregated using an event-set based methodology, and the resulting loss and NPV distributions are calculated in MARS. From MARS, the loss and NPV distributions are entered in Phobos.
36.4.6 Planning and Business Projections: GPT

The GPT – Global Planning Tool – creates business projections during the planning cycle of the company. It is a forecasting, reporting and planning tool for future business based on historical data. The application and data owner is Financial Planning and Analysis department. The GPT is built in Java and Microsoft SQL Server. GPT is maintained by IT Development.

GPT is used globally by Underwriters to create business projections during the company’s planning cycle. With GPT underwriters can create multi-year projections based on historical data and on projections of future business. A forecast module calculates results based on projections of future business and historical data. Results are converted to calendar year results and displayed as n-year projections. The reports module generates reports including Input, profit & loss and summary reports.

The GPT data is exported to a Pivot table in Excel and enters the ALM model in this form.

36.4.7 Retrocession

This section considers retrocession modeling. An estimate of the recoveries through retrocession (reinsurance assets) is implemented in Igloo in form of a linear regression which models the recoveries as a fraction of losses. Data input for a calibration of the model is received from the department Retrocession & Portfolio Management and Financial Planning & Analysis in the form of Excel files. These Excel files contain the

- Structure of the retrocession acceptances
- Premiums paid for the retrocession covers
- Information on the unearned premiums and losses

Reinsurance assets cover the balances from retrocessionaires for losses and ceded unearned premiums. We have to bear in mind that the reinsurance assets, though by nature another class of assets, are completely distinct from the asset portfolio. The model of recoveries is integrated in the liability risk models, as, from the viewpoint insurance risk, the recoveries constitute a contribution to the liability cash flows in stochastic simulations.

For the business segments which cover large events by retrocession contracts, further information has to be collected in order to model the frequency and the severity separately. Furthermore, information on the run-off of retrocession contracts of previous years allows for the modeling of the development in the reinsurance assets.

\textsuperscript{4}The detailed aspects of retrocession modeling are considered in Part I.
The Finance department and the team Retrocession & Portfolio Management provide the data, in form of Excel excerpts, which are then preprocessed in Excel and finally imported in the Igloo model.

### 36.4.8 Life & Health Liabilities

Figure 36.9 shows an overview of the systems involved for the Life and Health models\(^5\). There are two distinct processes and system components implemented for GMDB and financial reinsurance business. They are thus described separately.

**GMDB**

GMDB business is modeled by a replicating portfolio. The entry point to the ALM model is the submission of the replicating portfolio by the Life actuarial department. The data is delivered in form of an Excel sheet. A full pricing model for the GMDB replicating portfolio is implemented in Excel. This Excel book also imports the economic scenarios as provided by the ESG. The output of this pricing workbook are the GMDB total returns by ESG scenario. The GMDB total returns are imported in the risk simulation

\(^5\)The detailed aspects of these models are considered in Chapter 4 in Part I of this documentation.
in Igloo and are used in conjunction with the same ESG scenarios for the other parts of the risk simulation.

Financial Reinsurance

For financial reinsurance business, the life department prepares cash flow distributions directly. They are submitted as the entry point in the ALM model as an Excel file. This file is imported in Igloo, and the financial reinsurance cash flows are added to the other liability components.

36.5 Assets

36.5.1 Functional Overview

The functional asset module is shown in Figure 36.10. The individual asset items of the company are allocated to a stylized asset portfolio. This is the main process step described in this section. The further functional steps are carried out in the risk simulation and will be discussed there in detail.

36.5.2 Systems Overview

Figure 36.11 shows an overview of the systems involved in receiving data and creating the stylized portfolio. As mentioned in the functional overview, the functional steps 2–4 are carried out in the risk simulation in Igloo or in form of a post-processing of Igloo simulation results and will be discussed there.

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The details of the risk model involving stylized asset portfolios are considered in Part II.
The detailed information on the composition of the company’s asset portfolio is from the Finance department in form of an Excel excerpt. An aggregation and re-allocation step is performed in Excel, resulting in the stylized asset portfolio which in turn constitute the Excel input into the Igloo model.

For the calculation of the interest rate hedges, there are external parameters for the hedging bucket delta method as described in Chapter 11. These parameters are delivered by an external provider (a banking institution, currently HypoVereinsbank). They are delivered in form of Excel files by e-mail. The parameters are linked to the risk simulation in Igloo.

### 36.6 Risk Simulation

The main purpose of the risk simulation is to assemble and evaluate all models as imported and enhanced in the previous process steps. The risk simulation is a Monte Carlo simulation: At simulation start, an economic scenario together with a new business scenario and a reserve scenario are chosen. Each simulation step yields one point in the distributions to be monitored (e.g. the results distribution of the company).

#### 36.6.1 Functional Overview

The risk simulation covers three functional modules:

1. Assembly of the new and unearned business and the reserve models, including the calculation of retrocession recoverables (see liability modeling Figure 30.5, Steps 4–7).

2. Development, performance, and sensitivity calculations of assets (see Asset Modeling: Figure 30.6, Steps 2–4).

3. Calculation of the Aggregate Results (Figure 30.7).

The loss models for the new and unearned business are imported in the risk simulation, the reinsurance losses for a given simulation step are generated out of the loss model. The retrocession recoverables are calculated, and the reserve development is added in order to yield the full liability cash.
flows for this simulation step. The results as obtained for the liability part of the model are merged with the asset models: The (stylized) asset portfolio, the non-invested assets, as well as the other financial instruments such as hybrid debts are imported. The asset portfolio is rolled over, i.e. the development of the asset portfolio during a simulation year is calculated. The asset portfolio is then evaluated at year end in order to yield the market value of the assets. A sensitivity analysis is carried out, i.e. the results of the asset simulation are post-processed, in order to determine the impact of certain economic scenarios, such as interest rate shifts or equity crises, on the market value of the invested asset portfolio.

Eventually, the aggregated results are calculated: The simulation results are assembled to the (nominal and market value) balance sheet and income statement. The capital requirements are calculated according to the guidelines issued in the SST White Paper (FOPI [2004]). The main risk drivers are determined, and the efficient frontier is calculated leading to a new proposal of the strategic asset allocation.

36.6.2 Systems Overview

The risk simulation is programmed in Igloo, an external tool developed by EMB\(^7\). Figure 36.12 shows the Igloo model in its context.

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\(^7\)http://www.emb.co.uk/

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Figure 36.12: Aggregate results calculation in Igloo.

As described in the previous sections, with the exception of the ESG data the modeling outputs are stored in Excel or text file format. Addi-
tional input to the risk simulation, such as the non-invested assets and the finance instruments, are also stored in Excel or text file format. All data are imported in the risk simulation using Igloo links: Igloo is capable of reading simple text or Excel files in tabular format and storing them in its marginals. Furthermore, Igloo provides an ODBC link via which SQL queries can be submitted to the Microsoft SQL Server. This is the method which is used to import ESG data in Igloo.

### 36.7 Presentation of Results

Igloo results comprise the CY and UWY risk based capital (RBC), target capital (TC), as well as asset returns and volatilities. Igloo exports results in the form of Igloo Reports. Igloo model output is post-processed and formatted with Excel. From there, the result tables are included in the ALM and SST reports.
Architecture of New Developments

In January 2007, a project was launched to re-implement the Economic Scenario Generator (ESG) as a state of the art software product. In this context, a new strategy for developing the ALM systems landscape was elaborated, see *ALM Information Architecture: Strategic Long-Term Vision* (Bürgi [2007]). For our description of the architectural strategy we extensively draw on the ideas, concepts and formulations originally presented in that paper.

In the context of the general information disclosure policy being introduced in the reinsurance industry, with a special focus on the Swiss Solvency Test (SST), new requirements on the ALM Information Architecture have been established, with an emphasis on the documentation of methods and data together with the verification of a correct implementation of the related processes. Data and methods have to be associated with a particular version, correctness of implementation has to be guaranteed, and basic IT security aspects have to be fulfilled. All these aspects are addressed by the ALM Information Architecture.

37.1 Basic Architecture of Applications

The basic architecture of the applications (so far the ESG) is reduced to a few powerful components: the user interface, the object model including the communication layer, the mathematical kernel, and the ALM information backbone. Figure 37.1 graphically displays the basic application architecture.

The object model is at the centre of the architecture. It is used to transfer data from the user interface to the mathematical kernel and to the data base. It consists of simple data objects which can get and set properties. The communication layer enables to transfer, load, and save data from and
to the data base, as well as xml, csv, or other files.

The object model defines a clear interface between the user interface, the mathematical kernel and the data base. It is ideal to allow for simultaneous programming of the parts in separate teams.

### 37.2 ALM Information Backbone

The ALM information backbone is the core part of the ALM information architecture (see also Figures 34.1 and 37.1). It is a data warehouse storing all data needed for the risk simulation in various stages of refinement, as well as the results of the risk simulation. The data model is comprehensive and all entities are properly interlinked. Data is properly historised. Role-based access management restricts access of data to the relevant roles.

Currently, only the parts needed for the ESG are implemented in the ALM information backbone. The data model, however, is set up to accommodate the other parts of the ALM model. New developments will be integrated in the ALM information backbones.

### 37.3 Full Data Historization

As pointed out, MS Visual Source Safe is currently used to version data and methods. For methods, versioning is adequate. For data, however, versioning merely provides snapshots of data at different points in time. It is desirable to fully historise data, i.e. to make data comparable over time.

The key to enable full data historization is to solve the problem of slowly
changing dimensions: Codes such as line of business, business unit etc. vary over time. Codes including structural elements (e.g. hierarchical grouping), validity of codes (code versioning), migration of codes (management of succeeding codes) etc. have to be administered with utmost care in order to guarantee full data historization. It will thus be possible to link or compare data sets provided at different points in time. A clever code administration system is the foundation of the ALM information backbone.

37.4 Separating Data from Methods

Technologies such as Excel have the disadvantage that they couple data tightly to methods. It is only possible with significant efforts to get Excel to perform the same calculation with the same code for multiple data sets.

The downside of linking data and methods is that tools are difficult to test; one of the preconditions to a high testability is the ability to build a test suite, which requires full decoupling of data from methods.

In the ALM information architecture, data is only foreseen to be stored on the ALM information backbone. The methods are provided by the applications and are thus fully separated from the data.

37.5 Design Principles

Four design principles of overarching importance shape the long term strategy and the formulation of the information architecture:

1. Extensibility: Allowing for an adaption to new methods as the methodology progresses with time, easily adapting to changes in the data model when new or higher quality data become available. The data model, the modules, as well as the user interfaces evolve with time.

2. Maintainability: Low maintenance and the ability of keeping up with the changes for example in data formats. Flexibility, in terms of a swift implementation, with respect to a varying combination of data and methods.

3. Testability: The ability to test the IT components for errors and malfunctions at various hierarchy levels, using an exhaustive set of predefined test cases.

4. Re-usability: The ability to recombine programming code and system parts. Each code part should be implemented only once if reasonable. Thus, the consistency of code segments is increased significantly.
37.5.1 Extensibility

The methodologies developed in financial modeling constantly evolve. More and higher quality data become available over time, and methods become more sophisticated. Changes can be significant, and they have impacts on the ALM Information Architecture, such as the data model, the methods modules, and the user interfaces.

It is not possible to design a universally extendable model and system. Nevertheless, good design techniques can provide a minimum standard in universal extensibility. A consistent separation of data and methods, which has so far been achieved only for the ESG, will deliver a substantial gain in this respect.

In order to develop models and systems it must be anticipated which extensions will likely be needed later on, and the design has to be chosen appropriately. In the case of unanticipated changes it is important to ensure that the extension is cleanly implemented, and that code segments are refactored appropriately.

Currently, the modules provide limited support for extensions. This implies that a change in the modules potentially leads to major rebuilds. The current model implementation is strongly linked to technology.

Special care has to be taken regarding extensions which will be developed by external partners. It is important to guarantee a stable and clean API which will underlie a clean change management process.

37.5.2 Maintainability

In the current ALM infrastructure, substantial efforts have to be invested in the maintenance of data and systems. Due to the representation in Excel, the data modeling is restricted to two-dimensional tables. This leads to substantial efforts maintaining them.

Maintainability of data can be improved significantly by using appropriate technology. The example of ReCube (which is a hypercube on the reserving database CORE) shows an intelligent way of maintaining data from the reserving process. Data inputs can be conveniently linked in Excel, and the update of the sheets is much better supported in this process.

An important aspect to maintainability is a clear data and methods versioning. The ALM process security can be significantly enhanced by combining different data and methods versions. For instance, the new methods can be developed using the old data in order to assess the quantitative impact of the changes.

37.5.3 Testability

To test an IT component for its functionality in a reliable manner, several layers of test cases are needed. The lowest level is the unit test layer. Ideally,
Each method has a corresponding set of test cases which is as exhaustive as possible. Most of the defects are then detected on the unit test level. This is where they can be fixed at very low cost.

Above the unit test level is the component test layer. The component tests typically cover a whole module. They can be created synthetically or can be chosen from productive data sets. The component test suite should contain both a simplified and a real complexity set of test cases which are well known. They can ideally be used for developing new methods and should thus be well studied.

The last test level is the integration test layer. It covers the integration of modules with each other and focuses on the interfaces, i.e. the transfer of data, evolution of data after application of multiple modules etc.

If all these test layers are created and maintained, the error potential during development and refactoring of code can be reduced significantly. Therefore, testability enables to fulfill the design criteria extensibility and maintainability.

The issue of testability is also addressed by the corporate software development and application management processes as described in Section 35.1.

### 37.5.4 Re-usability

Smart reuse of code and system parts leads to a clear design and supports all other design criteria. Several aspects of re-usability are important:

1. Obtain full consistency: If components or modules are implemented multiple times, experience shows that the implementations will diverge sooner or later. It is therefore essential to elect and label one current version of each concept, method, data set etc., and to keep the implementation unique. This implies that the design of modules supports usage in multiple contexts.

2. Keep specification, implementation and documentation close: In many cases, it is possible to combine specification, implementation and documentation in one place. For instance, a conceptual data model is created and kept as the unique place for change. From the conceptual data model, a physical model can be generated automatically. Furthermore, an object model can also be generated automatically from the conceptual data model (object oriented communication layer to be used, e.g., between user interface and mathematical kernel). A documentation of the data model can be compiled from the conceptual data model. This allows for keeping specification, implementation and documentation in sync at all times without additional maintenance effort. The concept has been fully realized for the economic scenario generator.
3. Provide basic components for new modules: If code is designed for re-use, a large collection of tools will be gathered over time. It will be easy to generate new modules making use of this collection. Thus, calculation requirements to Financial Modeling can be met at best.

4. Run models from the past: It should be possible to combine different model and data versions. This allows for assessing the effect of method and data changes and optimized auditability and reproducibility of results.

Even though re-usability may appear as an obvious design criterion, it is not always desired. Modules and data need to be well encapsulated so that they can be changed independently. In some of these cases, re-usability may not be the right criterion to optimise, and code duplication may not always be a bad choice. It is important to find the right balance between reuse and encapsulation.

37.6 Security

All newly built parts of the ALM information architecture will incorporate role-based access management. The ability to read, create, update, and delete data entities will be given according to the roles in the ALM Process.

Entities which are used for production of the ALM or SST reports, or which are communicated otherwise internally or externally, can be locked. Locks are cascading, i.e. when locking a data entity, all entities which feed data into this entity must be locked as well. When an entity is locked, the data becomes read-only, and it is not possible to delete it. An audit trail can thus be guaranteed.
Appendix

We list the IT systems and their owners. For commercial software tools, we also list (in braces) the company from which the licence or the application was purchased.

<table>
<thead>
<tr>
<th>IT System</th>
<th>Ownership</th>
</tr>
</thead>
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<tr>
<td><strong>GPT – Global planning tool</strong></td>
<td><strong>Financial Planning &amp; Analysis</strong></td>
</tr>
<tr>
<td>Business projections during the planning cycle of the company</td>
<td></td>
</tr>
<tr>
<td><strong>MARS – Model for analyzing reinsurance submissions</strong></td>
<td><strong>Actuarial – Non-Life</strong></td>
</tr>
<tr>
<td>Pricing tool for non-life treaty business</td>
<td></td>
</tr>
<tr>
<td><strong>Phobos</strong></td>
<td><strong>Actuarial – Non-Life</strong></td>
</tr>
<tr>
<td>Manipulation of aggregated risk variables and modeling-basket dependencies in non-life treaty business</td>
<td></td>
</tr>
<tr>
<td><strong>GCDP – Global cat data platform</strong></td>
<td><strong>Natural Hazards</strong></td>
</tr>
<tr>
<td>Aggregation of natural catastrophe risk and risk accumulation control</td>
<td></td>
</tr>
<tr>
<td><strong>ResQ</strong></td>
<td><strong>Actuarial – Non-Life</strong></td>
</tr>
<tr>
<td>Reserving tool, commercial software (EMB Consultancy)</td>
<td></td>
</tr>
<tr>
<td>IT System</td>
<td>Ownership</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>CORE – Converium reserve estimator</td>
<td>Actuarial – Non-Life</td>
</tr>
<tr>
<td>Reserve analysis tool and database</td>
<td></td>
</tr>
<tr>
<td>ReCube</td>
<td>Financial Planning &amp; Analysis</td>
</tr>
<tr>
<td>Analyzing reserving data, producing reserve triangles</td>
<td></td>
</tr>
<tr>
<td>ESG – Economic scenario generator</td>
<td>Financial and Rating Modeling</td>
</tr>
<tr>
<td>Generating stochastic economic scenarios</td>
<td></td>
</tr>
<tr>
<td>FED – Financial and economic database</td>
<td>Financial and Rating Modeling</td>
</tr>
<tr>
<td>Providing historic financial and economic time series</td>
<td></td>
</tr>
<tr>
<td>Igloo</td>
<td>Financial and Rating Modeling</td>
</tr>
<tr>
<td>Stochastic simulation of assets and liabilities. Commercial DFA software (EMB Consultancy)</td>
<td></td>
</tr>
</tbody>
</table>
IX

Final Remarks
In July 2008, the first SST report of SCOR Switzerland will be delivered to the FOPI. It is a reflection of the outcome of SCOR Switzerland’s internal model that was to a good extent already in use at Converium for a couple of years and constituted the basis of various ALM reports of Converium. However, to fulfill the further requirements of the SST, substantial enhancements of the internal model have been recently built in. This relates to a stricter “pure” legal entity view including internal retrocession and parental guarantees, a much more comprehensive documentation regarding systems, processes and the various models. However, several limitations are still associated with various parts of the internal model (as mentioned in the sections on model limitations in this document). This also refers, last but not least, to the modeling of management rules concerning dividends, capital management, strategic asset allocation, business mix and volume during the business cycle and global risk mitigation. We aim to continuously improve the internal model regarding those limitations and fully understand the potential need for additional models that will be always a consequence of writing new lines of business, investing in new types of investments etc. This document will therefore be updated to the extent those enhancements materialize in the internal model.

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## Acronyms

**A**
- **AAD**  Annual aggregate deductibles.
- **AAL**  Annual aggregate limits.
- **ACM**  Application change management.
- **ALM**  Asset liability management.
- **ATP**  Application testing process.

**B**
- **BoD**  Board of Directors.
- **BS**  Balance sheet.

**C**
- **CDS**  Credit default swaps.
- **CEO**  Chief Executive Officer.
- **CFO**  Chief Financial Officer.
- **CIO**  Chief Investment Officer.
- **CoCM**  Cost of capital margin.
- **COO**  Chief Operating Officer.
- **CORE**  Converium reserve estimator.
- **CPI**  Consumer price index.
- **CRO**  Chief Risk Officer.
- **CTA**  Currency translation adjustment.
- **CY**  Calender year.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>DFA</td>
<td>Dynamic financial analysis.</td>
</tr>
<tr>
<td>dvlpmt</td>
<td>Development.</td>
</tr>
<tr>
<td>ECS</td>
<td>Executive Committee Switzerland.</td>
</tr>
<tr>
<td>EDF</td>
<td>Expected default frequency.</td>
</tr>
<tr>
<td>ESG</td>
<td>Economic scenario generator.</td>
</tr>
<tr>
<td>EUR</td>
<td>Euro.</td>
</tr>
<tr>
<td>FED</td>
<td>Financial and economic database.</td>
</tr>
<tr>
<td>Finance Re</td>
<td>Financial reinsurance.</td>
</tr>
<tr>
<td>FinMod</td>
<td>Financial Modelling.</td>
</tr>
<tr>
<td>FOPI</td>
<td>Federal Office of Private Insurance.</td>
</tr>
<tr>
<td>FX</td>
<td>Foreign exchange.</td>
</tr>
<tr>
<td>GAUM</td>
<td>Global Aerospace Underwriting Managers.</td>
</tr>
<tr>
<td>GCDP</td>
<td>Global cat data platform.</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross domestic product.</td>
</tr>
<tr>
<td>GMDB</td>
<td>Guaranteed minimum death benefit.</td>
</tr>
<tr>
<td>GPT</td>
<td>Global planning tool.</td>
</tr>
<tr>
<td>ILF</td>
<td>Increased limit factor.</td>
</tr>
<tr>
<td>KMV</td>
<td>Moody’s KMV.</td>
</tr>
</tbody>
</table>
LE  Legal entity.
LGD  Loss given default.
LoB  Line(s) of business.
MARS  Model for analyzing reinsurance submissions.
MDU  Medical Defense Union.
MKMV  Moody’s MKMV.
MSCI  Morgan Stanley Capital International.
MVM  Market value margin.
Nat cat  Natural catastrophe.
NCB  No claims bonus.
NFS  Network file system.
Non-prop  Non-proportional.
NPV  Net present value.
ODBC  Open database connectivity.
OLAP  Online analytical processing.
ORP  Optimal replicating portfolio.
Prop  Proportional.
Q1  First quarter.
Q3  Third quarter.
<table>
<thead>
<tr>
<th>Abbr</th>
<th>Term</th>
</tr>
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<tbody>
<tr>
<td>R</td>
<td>Risk adjusted capital.</td>
</tr>
<tr>
<td>RAC</td>
<td>Risk-adjusted capital.</td>
</tr>
<tr>
<td>RBAM</td>
<td>Role-based access management.</td>
</tr>
<tr>
<td>RBC</td>
<td>Risk-based capital.</td>
</tr>
<tr>
<td>ROP</td>
<td>Roll-up policies.</td>
</tr>
<tr>
<td>S</td>
<td></td>
</tr>
<tr>
<td>SAA</td>
<td>Strategic asset allocation.</td>
</tr>
<tr>
<td>SDLC</td>
<td>Software development life cycle.</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>Standard &amp; Poors.</td>
</tr>
<tr>
<td>SQL</td>
<td>Structured query language.</td>
</tr>
<tr>
<td>SST</td>
<td>Swiss solvency test.</td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
<tr>
<td>TRAC</td>
<td>Time and risk adjusted capital.</td>
</tr>
<tr>
<td>U</td>
<td></td>
</tr>
<tr>
<td>USD</td>
<td>US Dollar.</td>
</tr>
<tr>
<td>UW</td>
<td>Underwriting.</td>
</tr>
<tr>
<td>UWY</td>
<td>Underwriting year.</td>
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The documentation of the internal risk models at SCOR Holding (Switzerland) Ltd. is supposed to be understandable by readers who have a reasonable knowledge of the business and who are willing to study the information diligently. The sources from which we extensively excerpted in order to document the company’s internal risk models are Möhr [2008], Hummel [2007a], Hummel [2007b], Müller et al. [2004], Muller [2006a], Müller [2007a], Müller [2006c], Müller [2007a], Iannuzzi [2007], Dacorogna [2007a], Dacorogna [2007b], Briçonnet and Kalberer [2005], Guettanner et al. [2007], Pals and Matter [2008], Gallati [2007a], Gallati [2007b], Wollenmann [2007], Müller [2007], Langen [2008], Trachsler and Küttel [2008], Gut and Bürgi [2008], Skalsky and Bürgi [2008], not all of which are presented in full length though the original wording is preserved if possible. These sources, largely consisting of company internal papers and documentation, are included in an adapted form in the designated sections of this document. The level of authenticity and accuracy remains unaltered while at the same time we added explaining sections so as to reduce the technical character and to enhance the readability of the entire documentation. As we avoided a dual approach we were bound to cut the sources, leaving out title pages, abstracts, we abbreviated introductory parts or concluding remarks, the relevant contents of which we sought to absorb into various parts of our documentation where they are cited.