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SCOR Papers

Market Value Margin: Practical calculations under the Solvency II Cost of Capital approach

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Abstract

In the Solvency II framework, Market Value Margin (MVM) comes in addition to the Best Estimate valuation of liabilities as an attempt to provide a market-consistent value of technical provisions.
In the "Cost of Capital" approach that has now been adopted, the MVM is defined as the present value of the current and future costs of capital required to support the liabilities until full run-off. The capital itself is defined as the amount of funds needed over a one-year time horizon to ensure solvency within a 99.5th confidence level, and this is computed by the Value at Risk (VaR) of the Available Capital, which in turn depends on the MVM. Hence, a mutual dependency between Capital and MVM arises. Several simplifications and approximations, most of them ignoring the circularity, have been suggested in order to project future capital requirements in the MVM calculations. However, little research has been done to quantify these approximations. The subject of this note is to propose a set of analytical methods to derive a "closed-form solution" for MVM for a Non-Life insurance company, from a theoretical model first, and then to fit this model to a given set of simulations of future cash-flows, obtained from real claims history data. The constraints are those imposed to date by CEIOPS in its interpretation of the texts of the European Directive.

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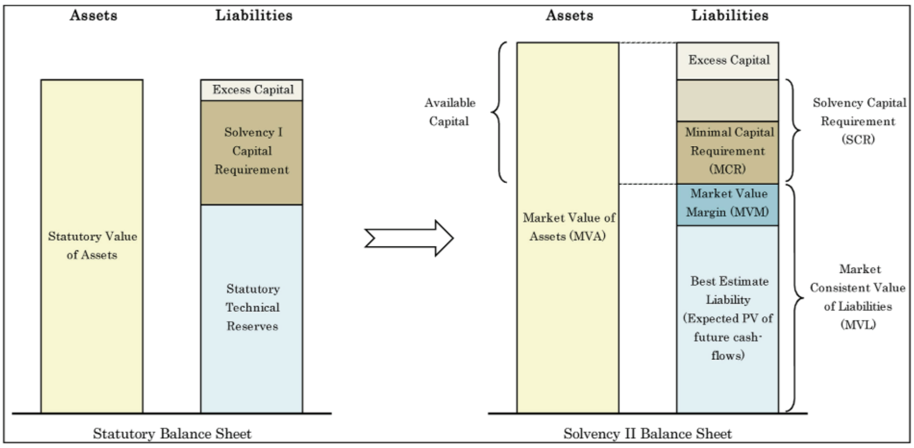
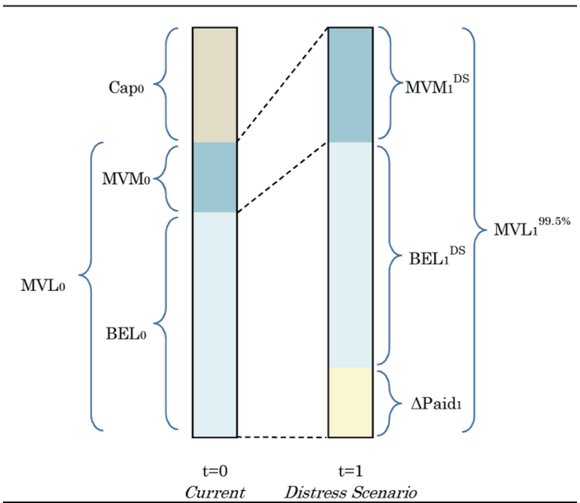
1. Background

Fair Valuation of risks

Under Solvency II, the Market Value Margin (MVM) is meant to bring technical provisions to a fair value, and is to be computed using the Cost of Capital approach. In the background lies the Market Consistent economic balance sheet which reflects what Solvency II seeks to achieve: a fair valuation of risks.



Limiting ourselves to the reserve risk only – as will be done in the rest of this note – the following graph shows that the Capital should be sufficient to restore the balance sheet to a fair value of liabilities after a 1 in 200 event:



For Solvency II, the Solvency Capital Requirement (SCR) is meant to cover one year of deterioration, meaning that only "shocks" applied to the following year are considered. The graph depicts, on the liability side of the economic balance sheet, how the capital funded at time $t=0$ is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year, where both the Best Estimate of Liabilities (BEL) and the MVM are subject to a distressed scenario.

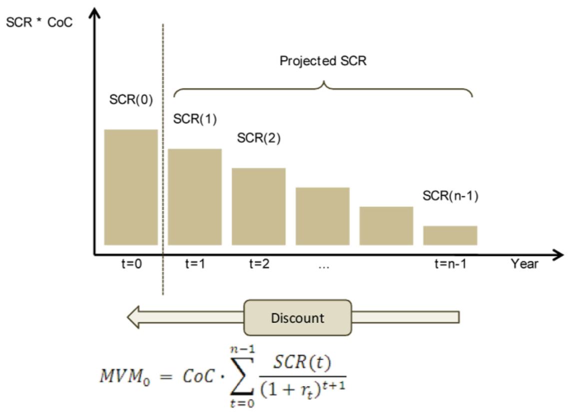
Cost of Capital approach

The CoC approach takes the perspective that sufficient capital is needed to be able to run-off the business. Here, the risk margin is estimated by the present value of the expected cost of current and future SCRs for non-hedgeable risks to support the complete run-off of all liabilities.

Schematically, the MVM calculation can be carried out in 4 steps:

- First, project the expected SCR until all liabilities run-off. This puts into the equations the fact that an undertaking taking over the portfolio has to put up future regulatory capital $SCR(1)$, $SCR(2)$, … , $SCR(n-1)$ until the portfolio has run-off completely at time $t=n$;
- Second, multiply all current and future SCR by the Cost of Capital rate (c or CoC). This captures the fact that the insurer selling the portfolio has to compensate the insurer taking over the portfolio for immobilizing future capital requirements;
- Third, discount everything to time 0;
- The sum then gives the CoC risk margin.

These steps result in the derivation of the MVM at time 0. They are illustrated in the chart below.



The circularity issue

The SCR, in its extended formulation as implicitly given in the European Directive, is computed as the Value at Risk (VaR) of the Available Capital. This Available Capital depends on the SCR via the Risk Margin, as can be seen from the formula

$$MVM_0 = CoC \cdot \sum_{t=0}^{n-1} \frac{SCR(t)}{(1+r_t)^{t+1}}$$

expressed in the chart above. Put differently, the Risk Margin is described in terms of the SCR and the SCR depends on the potential movement in the Risk Margin which makes up part of the Market Value of Liabilities (MVL), hence a mutual dependency.

In QIS5, several proxies have been suggested to bypass this issue, assuming that the Risk Margin within the Standard Formula does not change under a stress scenario.

This note intends to provide an estimate of what an exact solution to this issue would be as well as to provide a measure of the simplifications suggested in practise.

2. Theoretical MVM – a closed-form solution

The current and future Capital and MVM come under two general simultaneous equations:

$$\boldsymbol{Cap_t = 99.5\%[MVM_{t+1} + E(C_n|\mathcal{F}_{t+1}) - MVM_t - E(C_n|\mathcal{F}_t)|\mathcal{F}_t]}$$

$$\boldsymbol{MVM_t = c\sum_{i=t}^{n-1} E[Cap_i|\mathcal{F}_t]}$$

where:

- the notation 99.5%[…] indicates the percentile amount for the risk measure, which is the VaR at the 99.5th level.

- the notation $|F_t$ denotes that we are conditioning on information about claims development available at time t.

- Cap_t and MVM_t are the distributions of the Capital and MVM respectively required at time t, all in time-equal-t money

- C_t is the discounted cumulative claims payments as at time t

n is the time of complete maturity (i.e. $C_n = C_{n+1} = C_{n+2} = \ldots$)

- c is the Cost-of-Capital rate

It can be seen that in order to overcome the underlying circularity, we need to solve this backwards, using a recursive approach from time n when all the liabilities are run-off. The capital requirement is then null and so is the MVM, as there is no requirement to hold future solvency capital.

Solving using an analytical model

A simple claims process is studied, with a number of explicit assumptions.

Namely, we will restrict ourselves to the reserve risk only, gross of reinsurance, for a single line of business. The CoC rate is set constant at 6% and a deterministic discount yield curve is used.

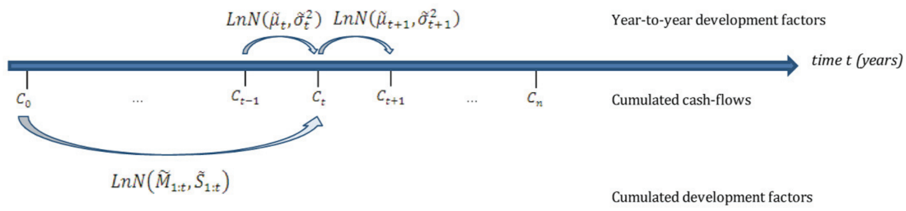
The claim structure assumes that the cumulative paid claims follow the following regime:

$$\boxed{C_t = C_{t-1}.LnN(\tilde{\mu}_t, \tilde{\sigma}_t^2)}$$

for $t=1,\ldots,n$, with n being the time where all liabilities run-off.

Here we assume that the development factors are log-normally distributed with a dependency structure assumption imposing independent Gaussian Copulas between different time periods.

The following graph shows the model's run-off of liabilities as well as its dependency structure between time periods:



A closed-form solution is then obtained, providing a theoretical value both for the MVM and the Capital at each point in time – if the claims were to follow this simple analytical structure.

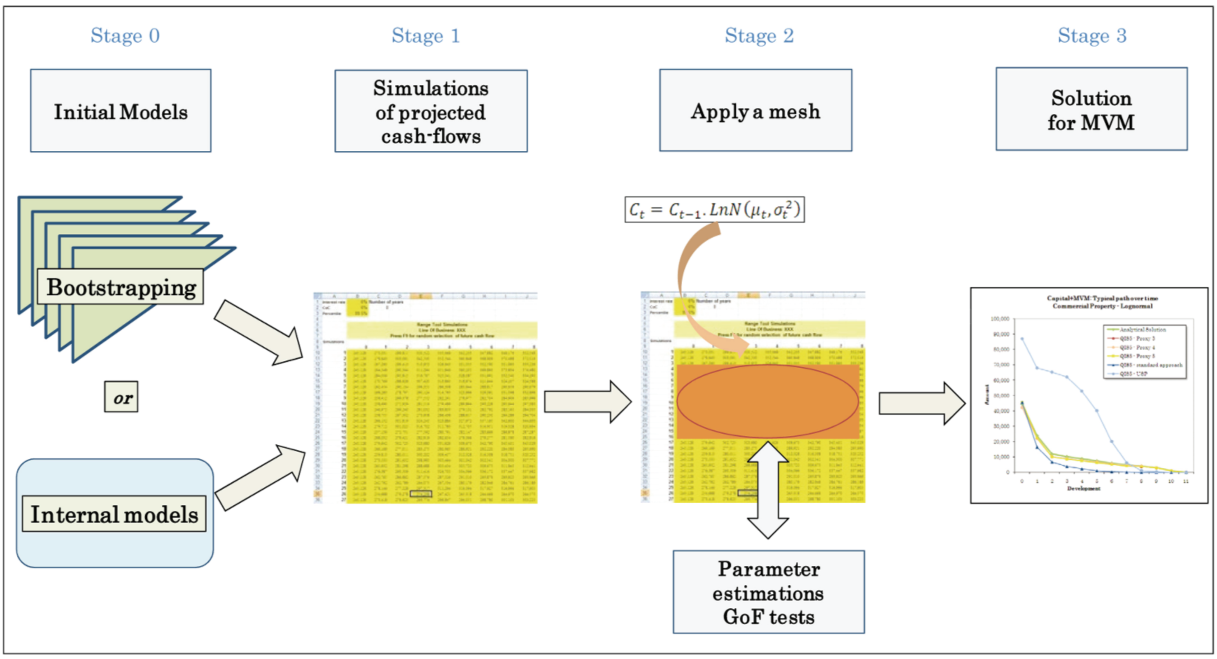
3. Application: MVM Model

Building on this, we will now transpose those theoretical results into finding an estimate for more general claims models, provided the outputs of these can be extracted in the form of a set of simulations of future cash-flows.

The approach is to apply a "mesh" on the sets of simulations – obtained either by some Bootstrapping or by an internal model – by fitting those outputs to either of the two analytical models that will act as "supra" models to describe the initial models more concisely for the sole purpose of the "closed-form" computation of the MVM.

An interesting advantage of this approach is that in general no additional simulations are required for the sole purpose of computing the MVM. A set of projected cash-flows is in practice usually at hand from work carried out elsewhere, as many companies for example derive ranges in their reserving exercises or, if using an internal model for Solvency II, its outputs could be used. This then avoids having to perform simulations on simulations to capture conditional predictive distributions of claims.

The following graph gives an overview of how the different processes used in the case studies piece together:



– Stage 1 is the inputs data required as a basis of the mesh-fitting of Stage 2. They are obtained through the initial losses development triangles from which some initial modeling is carried out, as depicted in Stage 0, in order to derive projected simulations of the claims distributions until run-off.

– Stage 2 is where a mesh – i.e. our analytical model – is fitted to the projected simulations of cash-flows from Stage 1. It is supposed to capture the whole projected underlying distributions of cash-flows into one analytical claims development structure. In order to fit the cash-flows to the mesh, parameters need to be estimated. The fit will be measured through Goodness Of Fit tests (GoF).

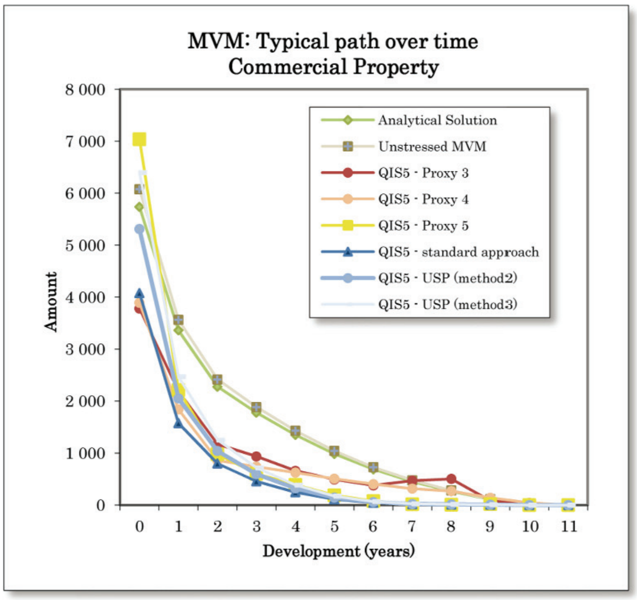
– Stage 3 gives the results of the MVM estimation.

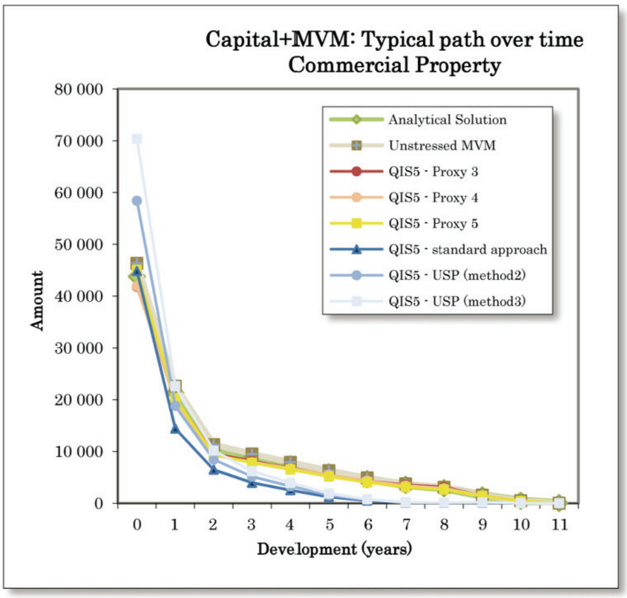
MVM results

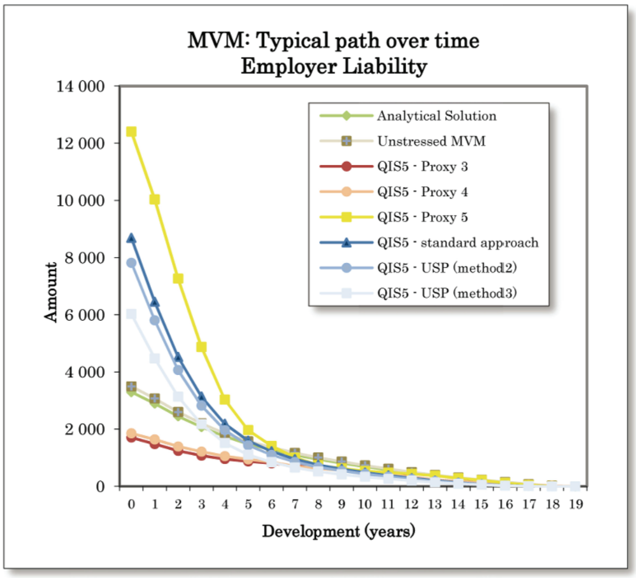
The theoretical solutions have then been applied to two real case studies along a typical selected path. The results for the two lines of business (Commercial Property and Employer's Liabilities) are presented below, in terms of MVM amounts first and then also in terms of MVM + Capital amounts. The latter element (MVM + Capital) is actually the most important and meaningful from an economic point of view. Indeed, the mutual relationship between MVM and Capital holds in the way each of the two components move against each other. If the MVM is larger, then the Capital should be smaller since the MVM can be used to fund future capital requirements, as MVM drops to zero over time and that release from the technical provisions is an offset to capital.

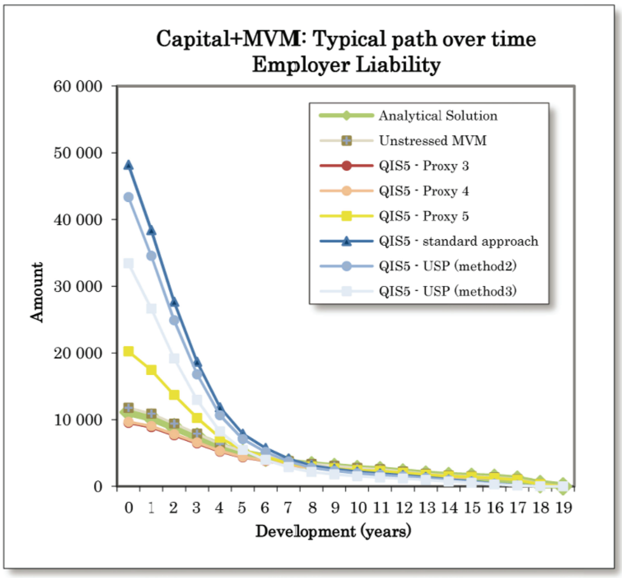
Then we present some high-level results of comparisons against classes of business. The underlying data comes from internal models outputs, the aim here being to get a wider picture of the QIS5 proxies materiality.

Results on a typical projection path

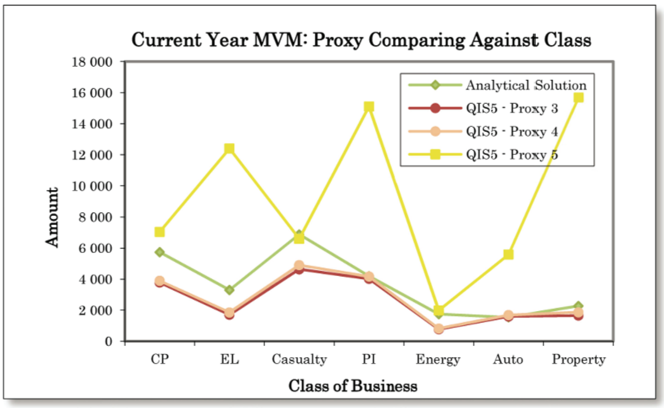


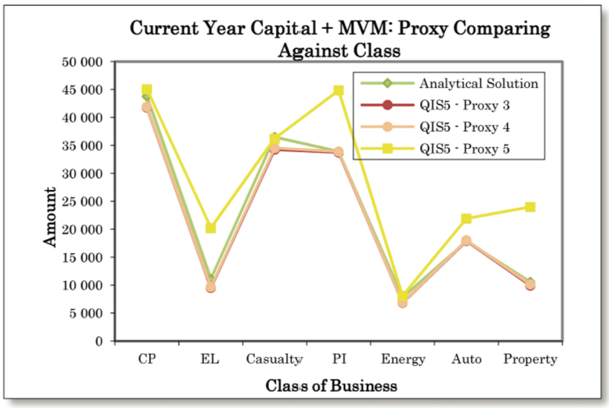






Comparing against class





Comments on results

First, a few explanations on what exactly lies behind the various results being compared here:

Analytical solution: it is the one solved for (Cap_t, MVM_t) and calibrated to the data.

Unstressed MVM: this gives the analytical result for (Cap_t, MVM_t) where the capital ignores the movement of the MVM under a stress scenario over a 1-year horizon. This will provide a quantification of the materiality of this currently used simplification.

QIS5 – proxy 3 (the "proportional approach" – where the expected payment pattern over the run-off of the liabilities is used as an estimate of how the capital requirement will run-off over the lifetime of all liabilities) / QIS5 – proxy 4 (the "duration approach") / QIS5 – proxy 5 (the "simplest approach" where the MVM is defined as a set percentage of BEL): these elements are meant to quantify the proxies suggested by QIS5, on the MVM approximation in isolation. This means that the capital base these are using is the one determined under the analytical solution, in order to strip out the effect of the approximations also applied when determining capital requirement under the Standard Approach in QIS5 (cf. below). As a result, the volatility taken into account (in the capital calculation, and hence, in the MVM number) is estimated by the mesh parameterisation, which in turn finds its roots in the input cash-flows.

QIS5 – standard approach: the Standard Formula provides a simplified way of assessing the capital amount at time t=0. Under the scope of this study, the capital amount is made of non-life reserve risk capital requirement for a single line of business only. The technical specifications[1] provide the following calculations steps:
where σ is the standard deviation for the reserve risk (per reserve unit) provided in the Technical Specifications[2] and R_t the reserves at time t. With the capital defined as such, the MVM is further approximated using Proxy 3.

$$NL_{res(t|t)} = Cap_t^{QIS5} = \left(\frac{e^{\Phi^{-1}[99.5\%].\sqrt{\ln(\sigma^2+1)}}}{\sqrt{\sigma^2+1}} - 1 \right).E(R_t|\mathcal{F}_t)$$

QIS5 – USP[3] : in an extension to the previous, the undertaking can use its own specific parameters (Undertaking Specific Parameters) as a measure of the standard deviation for the reserve risk and use these in the Standard Formula. At the undertaking level, the purpose is to contribute to a more risk-sensitive capital requirement and allow a better assessment of the underwriting risk that undertakings are exposed to. At the European level, encouraging companies to calculate these USP will help revising the calibration of the corresponding market parameters prescribed under the Standard Approach. A credibility element is attached to the final volatility to be used.

Then, bearing in mind that some of the conclusions do not claim to be generalized in their application to all lines of business and to all portfolios within each line of business, as the results might be due to the specific characteristics of the data chosen in the case studies, the following can be observed.
First, as a generic comment that is independent of the class of business, the "Unstressed MVM" analytical solution, defined as the Expected Cost of Capital Risk Margin, with capital capturing the expected BEL deterioration only (i.e this is the simplified analytical solution that by-passes the circularity issue) gives a higher result than the exact solution, with a spread roughly capped by the Cost of Capital rate.

(1) QIS5 Technical Specifications SCR.9.14 (cf. [3])
(2) QIS5 Technical Specifications SCR.9.29 (cf. [3])
(3) QIS5 Technical Specifications SCR.10.6 (cf. [3])

Working backwards, this is mainly due to the fact that the MVM replaces the "simplified" capital at time $t=n-1$ with a reducing effect at each previous projection year.
Then, overall on most classes, Proxies 3 and 4 seem to systematically underestimate the MVM amount throughout the whole run-off period, while Proxy 5 seems to overestimate it, especially over the first year(s) depending on the class of business, suggesting that the calibration factors were set to be quite punitive in this scenario. In the case of Proxy 3, the results confirm the intuition that Capital/BEL cannot be a constant ratio over time as we get less diversification within a book as claims are paid out and are settled. All these results are further mitigated when looking at the MVM + Capital amounts as we then have a diluting effect.

4. References

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[2] QIS4 Technical Specifications
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