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A risk-profitability analysis of multi-line reinsurance contracts in the Solvency II framework

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A chi c'è sempre stato...

Chi più in alto sale,

più lontano vede;

chi più lontano vede,

più a lungo sogna.

Walter Bonatti

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Introduction

The introduction of the Solvency II directive has changed completely the way according to which insurance companies are managed. A great role is played by the measurement of the insurer's risk exposure, through the computation of a capital requirement. Therefore, in order to improve its solvency position and meet the requirements, the undertaking can put in place some risk mitigation solutions, among which there are reinsurance treaties.

An exhaustive explanation about alternative reinsurance contracts is provided in Chapter 1. Great attention is paid to the main proportional and non-proportional treaties, highlighting their differences, advantages and drawbacks.

A great role in the choice of the optimal cover is played by the reinsurance premiums since they influence the net profitability of the direct insurer. Therefore, reinsurance pricing process has been described in Chapter 2, separately for proportional and non-proportional treaties.

In Chapter 3, the three-pillars structure characterizing the Solvency II framework has been briefly depicted. The main focus is on the quantitative requirements, in particular on the risk modules most affected by the introduction of a reinsurance treaty.

By carrying out the business in more than one segment, the insurance company can be interested in underwriting a multi-risk product. Therefore, in Chapter 4, multi-line contracts are introduced, as a more efficient alternative to simple per-peril contracts.

Chapter 5 describes the mathematical tools which can be applied inside an Internal model for properly fitting the dependence structure among different lines of business. Special attention is paid to copula functions.

In Chapter 6, it is defined the Internal Model which will be used for assessing the premium risk capital requirement and the profitability of the direct insurer.

Finally, in Chapter 7, a case study has been carried out providing a risk-profitability analysis of different reinsurance treaties. The typical trade-off between underwriting risk and return on equity for the traditional mono-line reinsurance treaties is firstly extended to a multi-line policy and secondly, by including the counterparty default risk.

1 Reinsurance

1.1 What is reinsurance and why reinsurance

Reinsurance can be easily defined as the insurance for the insurer. It is the transfer of an insurance risk from one party to another one through a contractual agreement under which one party, namely the reinsurer, agrees, in return for a reinsurance premium, to indemnify another party (the cedent, the primary insurer) for some or all the financial consequences of certain loss exposure covered by the cedent's policies. The duration of the treaty is fixed (typically one year). Notice that reinsurance is something different with respect to coinsurance. Indeed, in coinsurance, the policyholder underwrites a contract with a pool of insurance companies. The risk and the premium, paid by the policyholder, are split among the different companies according to percentages fixed in the agreement. Conversely, if an insurance companies underwrites a reinsurance treaty, there is not a direct link between the policyholder and the reinsurer. The insurer will remain responsible for the whole loss and it has the right to recover from the reinsurer the part of loss the reinsurer is responsible for. Before describing the main types and characteristics of reinsurance treaties, it is quite relevant to highlight why an insurance company should be interested in buying reinsurance:

- *Increase the insurer's underwriting capacity.*

  If an insurance company cedes part of its underwriting risk, it is able, given the same available capital, to underwrite more policies of the same kind which could be desirable for different reasons like market share targets.

- *Stabilize business result and loss experience.*

  The insurer's results fluctuate over time. A reinsurance contract enables the cedent to reduce the volatility of its financial result, since in good times the insurer's results get smaller due to the ceded premium, while in bad years the reinsurer pays part of the claims.

- *Provide catastrophe protection.*

  A catastrophe event usually involves several customers of the primary insurer for a relatively small area. The insurer could have to pay a lot of claims brought by a single event.

- *Increase the insurer's solvency through a reduction of the capital requirement.*

  Under the Solvency II framework, insurance companies must have an adequate capital in the relation to their risk exposure in order to protect beneficiaries and policyholder. Since reinsurance is a way to reduce the probability to suffer losses, with a reinsurance contract, the insurer is able to reduce the capital requirement, due to the lower aggregated risk.

- *Access benefits from larger diversification pools.*

  Typically, insurance company's business is restricted to a local area and extending it to other markets for getting a larger diversification benefit could be costly and inefficient, in particular for very risky contracts. On the other hand, reinsurers carry out their business at an international level, having theoretically more possibilities of diversification. As a result, the amount of capital needed by the reinsurer for some risks is lower than the insurer's one due to the larger diversification.

- *Access to reinsurer's expertise and services.*

  Often, reinsurer may share its expertise and data on the respective risks with the cedent. In some situations, an insurance company could not have enough data points or competences in order to analyse the risks and their tails. It is often cheaper to pass on those risk to an entity with much more experiences, like a reinsurer, than dealing with them through other means.

1.2 Distinction between obligatory treaties and facultative treaties

It is relevant to distinguish between obligatory treaties and facultative treaties. In obligatory treaties, a binding agreement is applied to all risks of a specified risk class. Once the Line of Business (LoB) or the portfolio covered by the reinsurance treaty has been established by the two parties, the insurance company is forced to cede all the risks included in the LoB or in the portfolio, whereas the reinsurance company is forced to accept all of them. Usually, an obligatory treaty aims at reducing the problem of asymmetric information. From the reinsurer's point of view, it avoids that the insurer makes an arbitrary selection of the contracts to be ceded, ceding only the worst risks. On the other side, it allows the insurer to cede very risky contracts. Typically, obligatory treaties are mainly for portfolios and for Lines of Business. Since the insurance contracts are not all written at the beginning of the year like a reinsurance treaty, it is necessary to determine a cession basis, which is the criterion defining the way in which risks are ceded by the insurer to the reinsurer.

The typical cession bases are:

- *Risk Attaching During.*

  The reinsurer agrees to indemnify the primary insurer for losses from policies issued within the reinsurance period, irrespective of the loss's occurrence date and of the time at which the loss is reported. It is called risk attachment since risks underwritten during a certain reinsurance period attach to that reinsurance period.

- *Losses Occurring During.*

  The reinsurer agrees to indemnify the primary insurer for losses occurring during the period of reinsurance regardless the issue date of the original policy and the time at which the loss is reported. Notice that the loss occurrence date refers to the loss occurrence date for the insurer and not for the policyholder.

- *Claims Made Basis/Loss Discovered Basis.*

  All the losses reported during the (reinsurance) policy period will be recovered, regardless the occurrence data and the policy inception date. The reinsurer could introduce a retroactive date in order to make more practical and reasonable the time period in which losses can be picked by the contract and to limit the applicability of the contract to losses occurred after a specific date.

The typical cession basis used in practice is the Loss Occurrence due to its simplicity. Some problems could arise if the reinsurance program on Loss Occurrence basis is not renewed because losses for policies in-force occurring after the termination date would not be covered. This shortfall can be overcome by introducing a run-off clause, according to which the reinsurer remains liable for claims occurring after the termination of the contract.

Conversely in facultative treaty, both parties have the option to decline or accept a particular risk. Being highly customized, facultative treaties are usually applied to big risks, to emerging and new risks or to risks excluded from the obligatory treaties. Both parties have to analyse specifically the risk prior to commitment, implying no ex-ante guarantee of cession or coverage. If the reinsurer believes that a particular exposure generated by the primary insurer is inconsistent with its own risk tolerance, it can decline to write the cover. On the other hand, if an insurer holds an attractive and profitable risk, it may choose to retain the entire exposure. In order to cede a risk under facultative treaty, the cedent has to submit to the reinsurer some information needed for assessing the quality of the ceded business. It typically regards the original policyholder, the original wording of the policy and the loss history, quite relevant for determining the performance of the ceded business in the past years and the expectation in the future. The obligatory treaty process is efficient and economical, being less expensive on a "per risk" basis than facultative cover, but it also reduces a certain amount of the reinsurer's underwriting power since it is accepting all the risks without investigating them one by one. As a result, it is possible to write also contracts which are facultative for one party and obligatory for the other one. The choice of the kind of the reinsurance treaty depends also on the bargaining power of the insurance company and of the reinsurer.

Before explaining the characteristics of the different reinsurance treaties, it is necessary to briefly specify the notation used in the following paragraphs. Denote with $\tilde{X}_t$, the total claim amount at time t, with $\tilde{Z}_i$ the severity of the i-th claim occurred in the year t. If the insurance company underwrites a reinsurance contract is possible to rewrite the aggregate claim amount as:

$$\tilde{X}_t = \tilde{D}_t + \tilde{R}_t$$

where $\tilde{D}_t$ represents the retained amount by the insurance company and $\tilde{R}_t$ is the amount paid by the reinsurer.

1.3 Proportional Reinsurance and Non-proportional Reinsurance

A typical distinction in reinsurance treaties is between proportional treaties and non proportional treaties. In proportional treaties, the primary insurer will cede a proportion, defined at the issue of the reinsurance treaty (ex-ante), of each risk to the reinsurer and it will pay a reinsurance premium equal to the same proportion of the overall premium charged to the policyholder. The methodology for assessing the proportion of risk to be ceded to the reinsurer depends on the type of contract, respectively Quota Share or Surplus. Conversely, in non-proportional treaties, it is not fixed in advance the ceded proportion of losses and there is not a direct link between the reinsurance premiums and the ceded proportion of losses. It is fixed a priority on the claim size in case of an Excess-of-Loss (XL) or on the aggregate claims amount in case of Stop-Loss (SL) over which the reinsurer is responsible for, capped at a value called limit. The premium to be paid to the reinsurer is no more a function of the premium charged by the insurance company to the policyholder, but it is computed separately by the actuarial department of the reinsurance company. In the following paragraphs, it will be shown the main characteristics of proportional treaties and of non-proportional treaties.

1.4 Proportional Reinsurance

1.4.1 Quota Share

The quota share treaty is the simplest reinsurance contract from a conceptual and administrative point of view. Premiums and aggregate claims amount are divided at a fixed ratio α between the insurance company and the reinsurer.

$$\tilde{R}_t = \alpha \tilde{X}_t, \ with\ 0 < \alpha < 1$$

$$\tilde{D}_t = (1 - \alpha)\tilde{X}_t$$

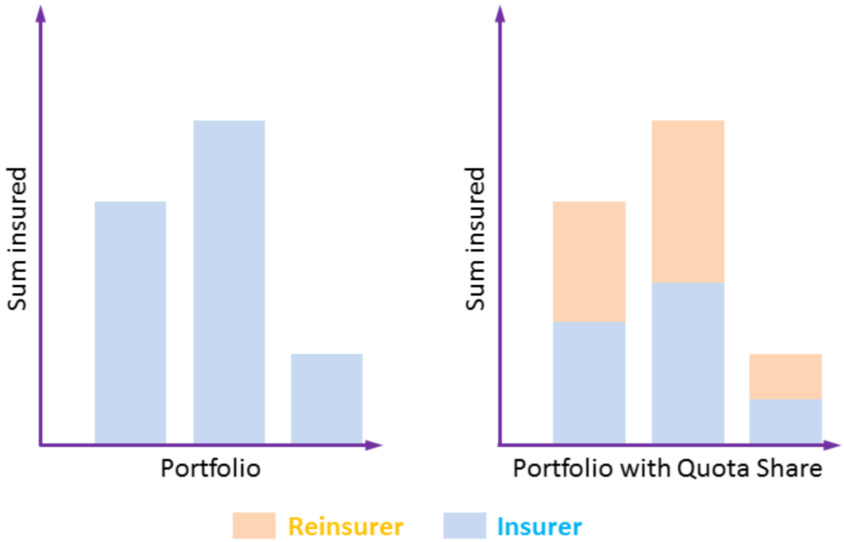

Figure 1: Optimization of profit and minimization of probability of ruin for insurer by choosing reinsurance treaty, Gulnaz Khishamova

As Figure 1 shows, it is worth to underline that all the risks are shared in the same proportion regardless the loss and the quality of the risk. The advantage of Quota Share treaty is to allow the insurer to diversify its risks and to improve its solvency ratio. The main disadvantage of the treaty regards the fixed proportion of risks ceded. On one side, the simple fixed proportion of losses retained by the insurer avoids some forms of moral hazard. On the other side, it limits the benefits which could arise from sharing the risk. Indeed, via a Quota Share, the insurer has to cede also small risks which could properly be retained, but it retains too much of very large risks.

For better catching the benefits the insurer obtains from a reinsurance treaty, it is necessary to compare the distribution and the moments of the aggregate claims amount gross of reinsurance $\tilde{X}_t$ and net of reinsurance $\tilde{D}_t$. Recalling its linearity property, the expected value of $\tilde{D}_t$ is:

$$E(\tilde{D}_t) = E((1-\alpha)\tilde{X}_t) = (1-\alpha)E(\tilde{X}_t)$$

Looking only at the expected value of the distribution of the aggregate claims amount net of reinsurance is not sufficient, since through a reinsurance treaty, the cedent tries to also reduce the risk (and not only the expected value). Therefore, it is necessary to also compute the variance, recalling that it is a quadratic operator:

$$\sigma^2(\tilde{D}_t) = \sigma^2((1-\alpha)\tilde{X}_t) = (1-\alpha)^2\sigma^2(\tilde{X}_t)$$

The standard deviation is easily derived as:

$$\sigma(\tilde{D}_t) = (1-\alpha)\sigma(\tilde{X}_t)$$

It is possible to notice that a Quota Share allows for an equal reduction of the standard deviation and of the expected value equal to α, the ceded proportion of risk. Indeed, the CV of the distribution of the aggregate claims amount net of reinsurance is equal to the CV of the gross distribution:

$$CV(\tilde{D}_t) = \frac{(1-\alpha)\sigma(\tilde{X}_t)}{(1-\alpha)E(\tilde{X}_t)} = \frac{\sigma(\tilde{X}_t)}{E(\tilde{X}_t)} = CV(\tilde{X}_t)$$

All the risk indexes are unchanged since through a Quota Share treaty, the distribution of the aggregate claims amount is only rescaled, as Figure 2 shows. A Quota Share is able to reduce the absolute volatility, but not the relative volatility. In particular, the reduction of the risk is equal to the reduction of the expected technical profit (ignoring the effect of the reinsurance commission). It implies a reduction of the required capital for that risk, but there is not a reduction of the capital absorption since the CV does not decrease. Indeed, typically, quota share contracts are used by small companies in order to broad chances for underwriting policies and to gain experience in new markets bearing a limited amount of risk. Conversely, an insurance company able to manage its portfolio in a profitable way has not so much need of a Quota Share, since it will transfer to the reinsurer not only the fixed proportion of claims, but also the same proportion of the premiums and, consequently, of the profits.

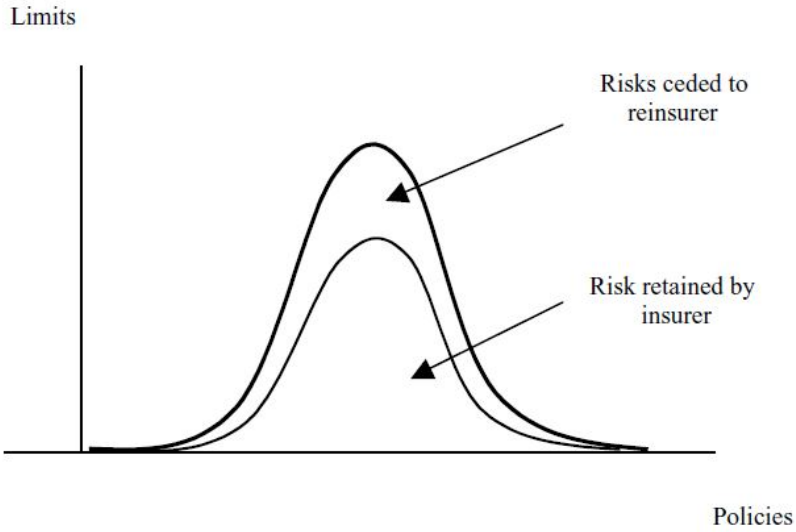

Figure 2: Distribution of the aggregate losses on gross and net basis. Alternative risk transfer, Erik Banks

1.4.2 Surplus Reinsurance

Surplus is a proportional treaty working as a Quota Share with the only difference that the proportion α is not the same for all risks in the portfolio, but it depends on the coverage limit of the underlying policy. As a result, it is able to overcome the drawbacks of a Quota Share, keeping its main advantages. Let Q_i be the sum insured of claim Z_i and M the fixed retention line. The amount paid by the reinsurer is defined as:

$$\tilde{R}_i = \left(1 - \frac{M}{Q_i}\right)\tilde{Z}_i \cdot 1_{\{Q_i > M\}}$$

where $1_{\{Q_i > M\}}$ is the indicator function. Conversely, the retained amount is defined as:

$$\tilde{D}_i = \tilde{Z}_i \cdot 1_{\{Q_i \leq M\}} + M\frac{\tilde{Z}_i}{Q_i} \cdot 1_{\{Q_i > M\}}$$

For each single risk, if the sum insured is lower than the deductible, the risk remains entirely to the direct insurer. Conversely if the sum insured is higher than M, premiums and claims are shared between the reinsurer and the direct insurer in the proportion $Q_i - M : M$. It is possible to write a closed formula for the ceded portion of claims to be applied to each risk under a Surplus treaty:

$$\alpha_i = \max\left(0; \frac{Q_i - M}{Q_i}\right)$$

As the sum insured increases Q_i, given the same deductible, the ceded proportion of claims increases. Indeed, the proportion ceded is higher for the largest risks and smaller or zero

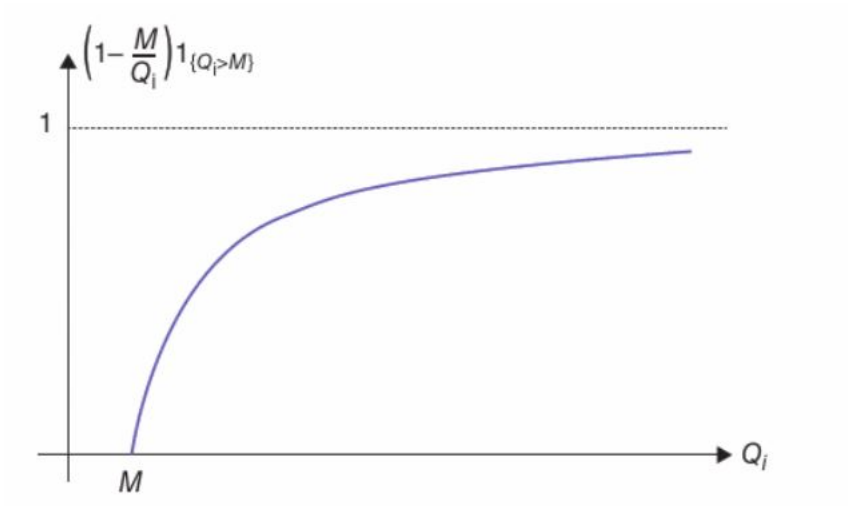

Figure 3: REINSURANCE, Hansjörg Albrecher, Jan Beirlant, Jozef L. Teugels, 2017

for smallest risks. The surplus holds the simplicity of ceding claims in a proportional way (as a Quota Share does) to the reinsurer which is responsible mainly (or only) of the largest policies. Once computed α_i, the surplus works as a Quota Share.

It is quite simple to obtain the amount retained by the insurance company and the amount paid by the reinsurer as:

$$\tilde{D}_t = \sum_{i=1}^{\tilde{N}_t} \tilde{D}_i$$

$$\tilde{R}_t = \sum_{i=1}^{\tilde{N}_t} \tilde{R}_i$$

It is possible to show that a Surplus is able to reduce the standard deviation of the net aggregate claims amount $\tilde{D}_t$ more than in terms of the expected value. It implies a reduction of the relative volatility (CV) and a better risk mitigation, also in relative terms, with respect to a Quota Share, since riskier is the contract, larger is the participation of the reinsurer. In addition, since the maximum retained size of each claim is M, the surplus reinsurance contract homogenizes the portfolio of the insurer.

Typically, a surplus reinsurance treaty is used for insurance contracts characterized by a sum insured like fire insurance, property, accident, and marine insurance. It is possible to write a surplus treaty also for reducing the volatility of the retained portfolio of the cedant company because there is a tendency to cede to the reinsurer the largest risks. It should be clear that the main difference between a Quota Share and a Surplus is in terms of the proportion ceded to the reinsurer (α). In the extreme case all the proportions α_i of a Surplus are equal, the Surplus degenerates into a Quota Share. More the proportions α_i

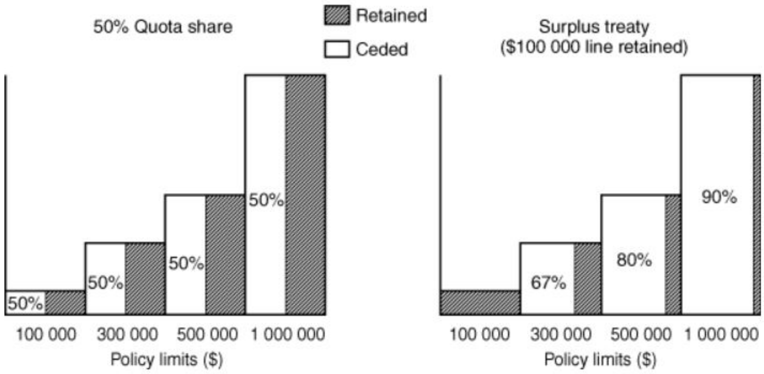

Figure 4: Surplus Treaty, Ana J. Mata

are different, more the (relative) risk is reducing. In choosing between a Quota Share and a Surplus, a lot of attention must be paid also the premium paid to the reinsurer and to the reinsurance commission.

Regarding the purposes for which an insurance company could be interested in underwriting a reinsurance cover, proportional treaties allow the insurer to increase the underwriting capacity, since the insurer cedes a portion of the risk. Surplus is more effective than quota share because it implies a better harmonization of the portfolio. Both proportional contracts provide catastrophe protection, even though non-proportional contracts are more effective. Regarding the stabilization of the loss experience, quota share is quite useless, because it reduces losses in the same proportion for all the risks in the portfolio, whereas surplus is very effective since the cession rate is different for each risk according to the sum insured, making the portfolio more homogeneous. Finally, proportional treaties are very effective in increasing the insurer's solvency, especially quota share since they significantly reduce the insurer's risk exposure.

1.5 Non-proportional Treaties

1.5.1 Excess of Loss (XL)

In a XL treaty L xs D, the reinsurer agrees to pay for any incurred claim which is greater than a certain amount D, called the retention, up to a limit L[1]. The amount ceded to the reinsurer, named layer function, is defined as:

$$\mathcal{L}_{(D,L)}(Z_i) = \min(\max(Z_i - D, 0), L)$$

[1] The retention is called also deductible or priority, while the limit can be called also cover.

Since the XL treaty acts on the single claim size and not on the aggregate claims amount, it does not affect the number of claims. Indeed, the insurer can reduce its exposure against extreme severity, but not against extreme frequency. In order to be covered also against the extreme frequency, a SL treaty, explained in the paragraph 1.5.2, is more suitable. The amount retained by the insurance company and the amount paid by the reinsurer can be defined as:

$$\tilde{D}_t = \sum_{i=1}^{\tilde{N}_t} \min(Z_i, D) \cdot 1_{\{Z_i \leq D+L\}} + (Z_i - L) \cdot 1_{\{Z_i > D+L\}}$$

$$\tilde{R}_t = \sum_{i=1}^{\tilde{N}_t} \mathcal{L}_{(D,L)}(Z_i) = \sum_{i=1}^{\tilde{N}_t} \min(\max(Z_i - D, 0), L)$$

The risk mitigation provided by an XL treaty does not assure the intervention by the reinsurer. Indeed, it will cover losses only if the single loss lies in the range $D - D + L$. If all the single claim amounts reported to the primary insurer lie below the deductible, there is no intervention by the reinsurer. The reduction of the insurer's risk exposure is still in place since it is protected from potential extreme single claims. In addition, it should be noticed that it is true that the reinsurer at most covers a single loss equal to L, but it could intervene infinite times (in case all the single losses are between D and $D + L$). Indeed, the reinsurer could introduce some clauses, respectively Annual Aggregate Deductible (AAD) and Annual Aggregate Limit (AAL) in order to limit its exposure in terms of number of claims. AAD and AAL will be presented in the following paragraph.

There are two types of excess of loss contracts:

- *Excess of loss per risk*, often referred to as Working excess of loss (WXL). The direct insurer retains a deductible of D for each risk affected by a loss. This type of treaty protects the direct insurer from individual major losses.

$$\tilde{R}_t = \sum_{i=1}^{r} \mathcal{L}_{(D,L)}(Z_i) = \sum_{i=1}^{r} \min(\max(Z_i - D, 0), L)$$

  Notice that the layer function is applied to each loss the insurer has to pay, $L_{(D,L)}(Z_i)$, and, then, all the results coming from the layer function are summed up considering all the r losses of the portfolio occurred in the year.

- *Excess of loss per event.* This is a per event cover, common in property insurance, where the direct insurer retains a deductible D per event. This type of treaty is used if many risks can be affected by a loss event at the same time.

$$\tilde{R}_t = \sum_e \mathcal{L}_{(D,L)}\big(\tilde{Z}_r^e\big)$$

All the l^e losses related to a specific event are considered as a single loss to which the layer function is applied. Later, the results of the layer function for the different events are summed.

Usually a per-risk contract has a lower deductible and not so huge limit with respect to a per-event XL. It must be paid great attention to the definition of "event" since it defines the set of claims to which the layer function must be applied.

In an XL treaty, the reinsurer suffers from adverse selection since the insurer seeks protection in particular from risks hard to be modelled and/or having heavy tails or a quite limited past claim experience. Some moral hazard problems can arise, since for a claim larger than the retention, in case of an infinite cover, the primary insurer has not incentives to carefully settle it. One possible solution is the introduction of a limit or clause imposing that the reinsurer pays only a pre-specifies fraction of the original reinsured amount in the XL treaty. The XL is typically used in casualty and fire insurance, since it reduces the exposure of the insurance company in an effective, but quite simple, form.

Annual Aggregate Deductible (AAD) and Annual Aggregate Limit (AAL). As mentioned in paragraph 1.5.1, in an XL treaty, the reinsurer can insert some clauses in order to limit its exposure. Introducing an AAD, the insurer will pay both the deductible D and the AAD, whereas the reinsurer starts to pay losses when the AAD is exhausted. The idea behind the AAD clause is that the direct insurer retains a large deductible for the first claim, (namely D+AAD), and a small deductible D in the case of future claims. In practice, the AAD clause is rarely introduced, conversely the AAL clause is quite typical. Without any AAL, there will be a limit L to be applied to each claim, but the total amount to be paid by the reinsurer could be unlimited since it depends on the number of losses. If the reinsurer wants to limit its exposure to the XL treaty (for instance for capital purposes), it can put an AAL, since it will be responsible for losses until the AAL is not exhausted. Often AAL is a given as a multiple of L, the limit per loss. In the case where $AAL = (1 + k)L$, the treaty is described as having k reinstatement. In presence of Annual Aggregate Deductible

AAD and Annual Aggregate Limit AAL, the amount paid by the reinsurer will be:

$$\tilde{R}_t = \min\Bigg(\max\Bigg(\sum_{i=1}^{\tilde{N}_t} \min(\max(Z_i - D, 0), L) - AAD, 0\Bigg), AAL\Bigg) =$$

$$= \mathcal{L}_{AAD,AAL}\Bigg(\sum_r \mathcal{L}_{D,C}\big(\tilde{Z}_r\big)\Bigg)$$

Notice that, in terms of expected losses, a stop loss contract with deductible D and cover L is equivalent to an excess of loss with deductible 0, unlimited cover and AAD=D and AAL=L.

Reinstatements. Referring always to an XL treaty $L\ xs\ D$ with a null Annual Aggregate Deductible, the contract with k reinstatement is a common variant in particular in property and casualty insurance. At the beginning, only an initial premium P_0 is paid for the coverage of a first layer defined as: $min(\tilde{R}_t, L)$. When a claim occurs, the layer could be totally used up, leaving the insurance company without any coverage for the remaining part of the reinsurance period. In reinsurance contracts with reinstatements, the layer can be filled again by paying a reinstatement premium. With a reinstatement clause, the premium payment is no more deterministic, but it depends on the loss history of the portfolio under reinsurance treaty during the coverage period. The clause is particularly attractive for the primary insurance since it has the possibility to pay premiums for purchasing more coverage only if needed, with a lower financial burden at the beginning.

Indexation Clause. In an XL treaty, a large role is played by claim inflation. Some claims, like large bodily injury claims, could be settled over a long period. Consequently, a claim occurred today might be paid and settled at a much higher value than the amount at which similar claims are settled now. As a result, reinsurance layers could be used much more frequently than intended at the time of designing them. In order to protect reinsurers from the consequences of claims inflation, it can be introduced an indexation clause, which links both the deductible and the limit of the layer to an index, normally a wage index, calculated at the time at which the claim is settled. From the point of view of the insurer that purchases excess of loss reinsurance, the effect of the indexation clause is to make reinsurance a bit cheaper and to reduce the amount of cover it gets.

1.5.2 Stop Loss (SL) Reinsurance

The purpose of Stop Loss reinsurance is to limit the aggregate claims amount of the insurer in a given year, across different kinds of causes, different events, or different classes of insurance. Indeed, the aggregate claims amount faced by the insurer over the time interval (typically one year) is capped at the priority D. The reinsurer will cover the excess up to a limit L:

$$\tilde{R}_t = \min\big(\max\big(\tilde{X}_t - D, 0\big), L\big)$$

Notice that in a XL treaty, the layer function is applied to the claim severity, while in a SL treaty, to the aggregate claims amount. Through a SL treaty without limit, the insurer is ceding the whole tail of the distribution of the aggregate claims amount to the reinsurer, being covered from the extreme severity (as in an XL treaty), but also from the extreme frequency. Therefore, for limiting its risk exposure, the reinsurer can put a limit L to its intervention not insuring the very large aggregate claim amounts for which the primary insurer is still responsible. For being covered from the tail (above L), the insurer has to write another reinsurance treaty (multi-layer contract). Typically, the priority and the limit are expressed in terms of Loss Ratio of the insurance portfolio under reinsurance agreement. It is very appealing for an insurance company since it reduces the volatility related to the performance of the insurance company. Indeed, in a proportional treaty, the insurer will retain the same proportion $1 - \alpha$ for all claims, but the cession has no impact on the Loss Ratio of the retained portfolio. Conversely, assuming an infinite cover and a deducible equal to $x\%$ of the LR, the SL allows a reduction of the volatility concerning the loss ratio because the LR of the retained portfolio will be more or less $x\%$ of the loss ratio of the original portfolio.

Being all-encompassing, typically a Stop Loss treaty is quite expensive and difficult to be obtain, also for the required, but complicated, modelling of the tail. Indeed, since the reinsurer is covering all the aggregate claims amount between D and $D + L$, there are a lot of ingredients required for setting a good Stop Loss treaty:

- Final loss burden established quickly and reliably.
- High and long-lasting trust relationship between the reinsurer and the insurance company.
- Good knowledge of the insurance portfolio.
- No peaks in the portfolio.

- Etc...

The SL is a typical coverage in agricultural insurance, especially crop insurance. Compared with proportional treaties, SL and XL are much more effective in providing catastrophe protection and in stabilizing the loss experience. In addition, the XL per risk is good in increasing the large line capacity, while the SL allows an increase in the insurer's solvency.

2 Pricing

Since reinsurance can be thought as a form of insurance, one solution to price a reinsurance treaty is to apply principles similar to the ones used in insurance pricing. Before explaining the typical premium principles, it is important to recall some important differences among insurance contract and reinsurance treaty. First of all, in a reinsurance treaty, the parties have to define how to share administrative costs for the acquisition of insurance policies and the settlement of the claims. In addition, the reinsurance market is typically characterized by a lower loss experience, a lower number of reinsurance companies and limited diversification possibilities. As a result, the reinsurance premiums are typically adapted faster to the loss experience than in insurance market. As for the insurance contracts' pricing principles, typically a reinsurance premium consists of the expected aggregate claims amount of the underlying risk increased of a safety loading and of a margin for taking into administrative costs, runoff expenses, taxes, and profit. Other features have an influence on the premium. For instance, it has not to be forgotten the default risk arising from a reinsurance treaty. The insurer will remain responsible for the entire losses towards the policyholder even if the reinsurer will be in default. According to the Solvency II directive, a capital requirement for the default risk related to the reinsurance contracts has to be computed. Actually, the size of the default risk arising from the reinsurance treaty and borne by the insurer depends on the rating of the reinsurance company. Worse is the reinsurer's rating, higher will be the default risk SCR. As a result, the cedent could ask for a reduction of the reinsurance premium in case of bad reinsurer's rating. Another factor influencing the demand and the supply of reinsurance treaty is the risk appetite of the top management of the two parties involved in the reinsurance treaty.

2.1 Some principles of premium calculation

Typically, an insurer, before selling an insurance contract, has to assess the riskiness of its overall position. According to the fair principle, the (fair) premium has to be equal to the expected value of the aggregate claims amount. If the insurance company asks a premium exactly equal to the fair premium, the expected profit will be null. As a result, the fair premium is without any economic interest since it does not provide any expected profit at the inception of the coverage. A safety loading needs to be added to the fair premium and, in the Non-Life business, it is typically computed explicitly. The presence of the safety loading can be justified in two way. First of all, the insurer is bearing some risks

by underwriting an insurance policy and it is asking a remuneration for them. Secondly, the safety loading represents a buffer covering the company against adverse loss realizations. Indeed, the fair premium covers the expected value of the aggregate claims amount, but it leaves the insurance company uncovered against any deviations from it. The first resource to be used for covering extreme losses is the safety loading.
Therefore it has to be defined a *premium calculation principle*, a rule which assigns the aggregate claims amount $\tilde{X}_t$ to a premium P_t through a function Ψ such that its position becomes acceptable in terms of profitability and safety:

$$P_t = \Psi(\tilde{X}_t)$$

Before issuing a reinsurance cover, the insurer and the reinsurer have to perform a similar analysis on the aggregate claims amount they expect to be responsible for (respectively $\tilde{D}_t$ and $\tilde{R}_t$). The premium calculation principle of the reinsurer has not to be the same of Ψ (the insurer's one), but it has to reflect its risk attitude and the diversification possibilities. Ψ could be whatever function the company prefers. Here some examples of premium principles are presented, by denoting with Y the generic risk with Cumulative Density Function F_Y:

- *Expected value principle.*

$$P(Y) = (1 + \theta)E(Y), \ \textit{with}\ \theta > 0$$

  According to this principle, the safety loading is equal to a fixed proportion θ of the expected aggregate claims amount. The strength of the expected value principle relies on its simplicity and its transparency. It is typically used in insurance and reinsurance practice due to the poor data availability and to the limited reliability of information on the risk beyond the first moment. Its drawback is that the expected value does not consider properly the riskiness of the portfolio. The principle can be slightly improved by defining the proportion θ as a function of the riskiness of the portfolio.

- *Variance Principle.*

$$P(Y) = E(Y) + \alpha Var(Y), \ \alpha > 0$$

  The shortfall of the expected value principle is overcome by the variance principle, by linking the safety loading to the variance of the risk. It is used in the insurance practice if more information about the risk is available.

- *Standard Deviation principle.*

$$P(Y) = E(Y) + \beta\sqrt{Var(Y)},\ \beta > 0$$

The advantage of using the standard deviation instead of the variance is given by the unit of measure. Indeed, the standard deviation is expressed in the same unit measure of Y, while the variance, being a quadratic risk measure, in the square of it. Considerations similar to the ones of the variance principle can be made.

- *Zero utility principle.*

Denoting with w the deterministic current surplus and with $u(x)$ the utility for the insurer of having capital x, it holds:

$$u(w) = E(u(w + P(Y) - Y))$$

$P(Y)$ is defined as the premium which makes indifferent the insurer to enter or not the contract in terms of expected utility. If the insurer asks a premium equal to $P(Y)$, its expected utility arising from the issue of the coverage will be equal to the utility in case of no policy. The utility function should be:

  - Non-decreasing: larger risks require larger premium.
  - Concave, denoting risk aversion.

If the variance of the risk is small, it is possible to approximate the premium as:

$$P(Y) \approx E(Y) + \frac{|u^{''}(w)|}{2u^{'}(w)} Var(Y)$$

The main complexity of the zero-utility principle concerns the definition of an appropriate utility function describing the risk attitude for all magnitudes.

- *Ruin probability principle.*

The principle assumes the knowledge of the probability distribution of Y. Since the fair premium is equal to the expected value of Y, an insurance company will be able only to cover the expectation of Y, but not any deviations from it. By charging only the fair premium, sooner or later the company will go probably in ruin. In order to define a safety loading according to the ruin probability principle, the company has to fix a level of ruin probability in absence of capital (ϵ), by accepting a probability equal to ϵ that the aggregate claims amount will be higher than the pure premium. Knowing the entire probability distribution of Y, it is possible to compute the quantile at level $1-\epsilon$, which is equal to the pure premium. As a result, the safety loading can be easily computed as the distance between the $1-\epsilon$ th quantile and the expected value. The choice of ϵ is playing a big role. Lower is ϵ, higher will be the safety loading and higher will be the part of risk borne by the policyholder. The ruin probability principle has the advantage of considering information coming from the entire probability distribution of Y, conversely to the expected value principle and the variance principle which consider only the first two moments. Indeed, it considers of skewness and kurtosis, which are important features in the aggregate claims amount distribution. The strength of this principle is also its drawback because it is quite difficult to precisely know the distribution of the risk.

Up to now, the main principles relevant in premium computation have been presented. In the following paragraphs, the pricing of reinsurance treaty will be deeply analysed distinguishing between proportional treaties and non-proportional treaties.

2.2 Pricing Proportional Reinsurance

Denoting with B_t the gross premium charged from the insurance company to the policyholder, the reinsurance premium can be easily computed as:

$$B_t^{RE} = \alpha B_t$$

Computing the premium paid from the insurer to the reinsurer in a proportional way implies that the reinsurer is not able to influence the pricing method of the direct insurer. If the reinsurer will receive entirely B_t^{RE}, it will obtain a too much favourable result. In general, the gross premium B_t charged by the insurance company is made up of a risk premium P_t, equal to the expected aggregate claims amount, a safety loading, and an expense loading. From a technical point of view, it is correct to cede to the reinsurer a proportion α of the risk premium, since, on expectation, the reinsurer has to cover $\alpha\%$ of the aggregate claims amount. In addition, also the reinsurer will charge a premium for taking the risk (safety loading), but it could make a different risk evaluation with respect to the direct insurer. As a result, ceding $\alpha\%$ of the safety loading is only partially correct. Regarding expenses, since the reinsurer does not afford to all the costs an insurer does (like acquisition costs of policies, costs for the estimation and settlement of the claims and other administration expenses), it is not correct to transfer $\alpha\%$ of the expense loading. Indeed, in order to truly "share the fortune", it is necessary that the reinsurer pays back a commission to the direct insurer. It is possible to notice that the negotiation process in proportional treaties takes place through the reinsurance commissions. Indeed, on one hand, it is true that the reinsurer has not power to intervene on B_t^{RE}, being computed as a percentage of B_t, but on the other one, it can introduce its own risk evaluation through the reinsurance commissions. If the reinsurer shares the same pricing with the direct insurer, the reinsurance commission will be exactly $\alpha\%$ of the expense loading. If it believes that the direct insurer under-prices the contracts, it will pay back an amount lower than $\alpha\%$ of the expense loading. To sum up, the amount of the reinsurance commissions depends on the quality of the portfolio ceded and also on the bargaining power of the insurer and of the reinsurer.

The reinsurance treaty can have fixed commissions or scaling commissions. In the first case, the amount of the reinsurance commission to be paid back by the reinsurer is fixed in advance at the cover issue. Lower is the reinsurance commission, higher is the price assessed by the reinsurer to the risk and higher will be the impact on the insurer's profitability. In practice, typically, the treaty involves scaling commission, implying that the concrete amount of the reinsurance commissions is not known in advance, but it depends on the actual loss experience. In the treaty, it is defined the computational methodology of the commission, which usual forms are:

- *Sliding scale commissions.*

  It is fixed a provisional commission, in terms of percentage, and a reference loss ratio, defined as the incurred claims, including settlement expenses, divided by the earned premium. For each percentage point the actual loss ratio deviates from the reference LR, the commission percentage is inversely adapted to it, according to the upper and lower limits fixed in the treaty. As a result, the effective amount of the reinsurance commission is defined ex-post, after having observed the real realizations, in terms of aggregate claims amount, of the portfolio under reinsurance agreement. Worse is the quality of the portfolio ceded, higher will be the LR, lower will be the commission rate. Since the amount of the reinsurance commission is unknown in advance, the scaling commission introduces an additional source of volatility in the insurer's technical result.

- *Profit sharing provisions.*

  If the participation of the reinsurer in a year in losses is very successful/low, it passes back part of the premiums according to some predefined terms.

- *Loss corridors.*

  In order to reduce its risk exposure, the reinsurer can define an agreement such that it covers only below $a\%$ and above $b\%$ of its LR, whereas, between $a\%$ and $b\%$, it is responsibility of the direct insurer.

It must be noticed that, once defined the computation method, the effective calculation of the reinsurance commissions is quite straightforward, and it depends mainly on the Loss Ratio. As a result, great attention has to be put on the distribution of the Loss Ratio, which has to be estimated from information on historical data. As explained before, the LR is defined as the ratio between claims and premiums. For understanding which premiums and claims must be considered, it is necessary to look at the cession basis of the reinsurance treaty. If it is on loss occurring basis, the earned premiums and the accident year losses are relevant. If the treaty is on risk attaching basis, losses on policies written during the treaty period are covered. As a result, written premiums, and the losses related to those policies should be considered. Usually, catastrophes losses and shock losses[2] are removed and the remaining historical losses are developed to the ultimate values. Historical premiums need to be adjusted to future level in order to include the rate's changes expected during the treaty period. Given the data points, the expected loss ratio is estimated by the arithmetic average of the historical loss ratios adjusted to the future level. In addition, it has to be adjusted by adding a loading for including catastrophe losses. At the end, the reinsurer can also modify the final estimate according to his own experience on those claims.

2.3 Pricing Non-proportional Reinsurance

The pricing of a non-proportional treaty is much more involved than for proportional treaties. The basic idea is to compute the pure premium as the sum of the expected aggregate claims amount the reinsurer will be responsible for and a loading which takes into account profit expectation, cost of capital and internal and external costs. Denoting with $Y : \Omega \to \mathbb{R}^d, d \geq 1$ the random vector representing all the risks of the company, it is possible to define the pure premium principle for the generic risk Y_i as:

$$P_i = E(Y_i) + h(F(\mathbf{Y}))$$

[2]It is important to distinguish between catastrophe losses which cause many claims from shock losses which cause single very large claims.

F is the distribution function of $\mathbf{Y}$ and h is a generic function suitable for the reinsurance company. The risk premium is equal to the expected aggregate claims amount related to the specific risk Y_i, whereas typically the safety loading is computed as a function h applied to a group of risks. It should be clear that more reliable information about the aggregate claims amount distribution the company has, more accurate and flexible will be the premium. Typically, the most applied principles are the expected value principle and the variance principle. Indeed, non-proportional treaty, in particular in presence of clauses like reinstatement, are much more complex than proportional treaties and the determination of the first two moments is not straightforward.

Two main approaches can be distinguished in the computation of the pure reinsurance premium:

- *Experience Rating.*

  The experience rating is the classical approach followed by reinsurance companies for pricing non-proportional treaties. The fair premium is estimated according to the experience observed in the last years, by taking the expectation of the losses the reinsurer is responsible for. It is adopted the same logic of the computation of the fair premium of an insurance contract, but in practical terms it is much more complicated. Indeed, typically, the reinsurer is responsible for very extreme claims which data availability is generally quite poor.

- *Exposure Rating.*

  The exposure rating aims at pricing a reinsurance policy when data for a specific risk is not sufficient to produce a reliable severity model. It is based on the use of the exposure curves, which are reengineered severity curves based on losses coming from a large number of risks (like at market level). The strength of the methodology concerns the curve calibration at a market level which involves a larger set of available data. This could be also a drawback since the assumptions underlying the construction of the curves could be not aligned with the specific portfolio of the company. Exposure curves are typically used in property reinsurance, like for a XL per risk.

2.3.1 Exposure Rating

Typically, exposure rating is applied when the experience rating is not reliable. As it will be explained in the paragraph 2.3.2, experience rating works properly if there is a sufficient data availability which allows to fit properly a severity model and a frequency model and to combine them through simulations. Some problems may arise with a property risk:

- First of all, in the severity model, it is implicitly assumed that no claim generated exceeds the value of the most expensive property in the portfolio, by capping the severity distribution. It should be more realistic to also consider the value of other properties in the portfolio.
- Secondly, fitting a realistic severity model is not always so simple.
- The assumption of stable risk profile is not always satisfied in practice and changes in exposure can affect both the severity and the frequency models.

It appears clear how another pricing method has to be applied, since property risk needs an individual analysis. The rating of non-proportional reinsurance treaties should not only rely to the past loss experience, but also on the actual exposure. Regarding per risk covers (here analysed for a property line), the exposure rating is based on risk profiles. Severity curves for an individual property risk can be interpreted as a curve providing the probability that the damage loss is lower than a given percentage of the sum insured or of the maximum possible loss[3], typically expressed as a fraction of the sum insured. In addition, it has to be considered that the probability of having a total loss (determining the complete destruction of the property) is finite. By defining the damage ratio as the ratio between the loss and the total loss expressed as the sum insured or as the Maximum probable loss, typically, the severity curve may look like in Figure 5.

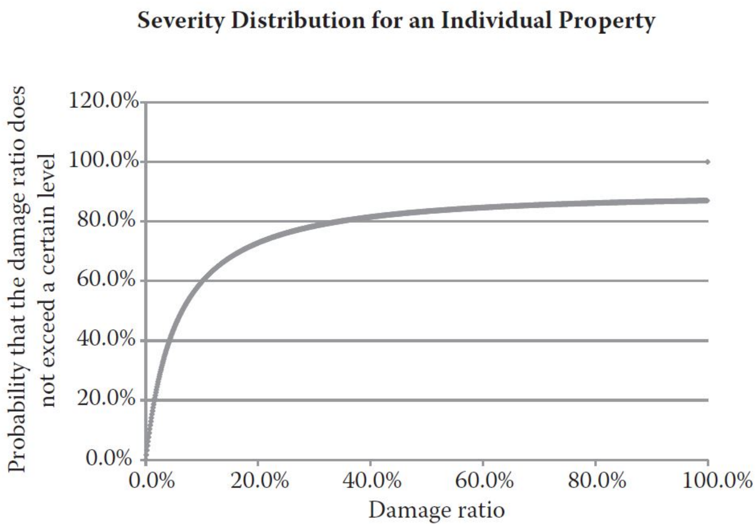

Figure 5: Severity curve for an individual property, Pricing in general insurance, Pietro Parodi, 2015

[3]The Maximum Possible Loss is typically expressed as a function of the sum insured. A benchmark value could be 70%, but it depends on the type of property and on the underwriting's experience.

It is an increasing and concave function with a jump for a Damage ratio equal to 100% (total loss). The height of the jump is the probability of having a total loss.

An empirical severity curve can be built by assuming that properties relatively similar will have a similar severity curve once rescaled in terms of Damage ratio. The first step involves the collection of individual losses. Secondly, they need to be divided with respect to the sum insured or to the MPL, in order to obtain relative losses between 0 and 1. Finally, they have to be sorted in ascending order. In addition, it is possible to fit to the empirical severity distribution a model, which can be used as relative severity model in the rating of the reinsurance treaty.
The problem arising from this methodology is that the client (the reinsurer) could not have enough losses in its portfolio to build a severity curve for a homogeneous type of property. Therefore, it is reasonable to aggregate the loss experience of different companies and produce market severity curves for different types of property which may be used to complement or to replace the client-specific severity curves.

The exposure curves. An exposure curve gives the percentage of risk which is retained by the reinsured (or primary insurer) if a given deductible is imposed. It has an increasing, continuous, and concave shape as a severity curve and it goes up to 100% (infinite deducible), regardless the probability that a total loss occurs. In this paragraph, the construction of the exposure curves for a property XL $L\ xs\ D$, without index clause, is provided. Given a loss Z, the loss to the layer $(D, D+L)$ the reinsurer is responsible for is:

$$Z(D, L) = \min(Z, D+L) - \min(Z, D)$$

Therefore, the expected loss to the layer is:

$$E(Z(D, L)) = E\big(\min E(Z, D+L)) - E(\min(Z, D)\big)$$

Denoting with N the number of losses, the expected total losses to the layer are:

$$E(R(D, L)) = \big(E(\min(Z, D+L)) - E(\min(Z, D))\big)E(N)$$

The previous relation holds by assuming that the claim size is i.i.d. and that the number of claims and the claim size are independent. Since a property line is considered, the loss Z cannot exceed the sum insured or the maximum probable loss, denoted with M. Therefore, it is reasonable to describe any loss occurring to that property as a percentage of its maximum value M, $z = \frac{Z}{M}$. Through this normalization, losses are no more affected by inflation and currency fluctuations.

The expected loss can be rewritten as:

$$E(Z) = E\left(\frac{Z}{M}M\right) = E\left(\frac{Z}{M}\right)M = E(z)M$$

and

$$E(\min(Z, D)) = E(\min(z, d))M$$

where $d = \frac{D}{M}$.

Dividing and multiplying by the expected severity, the expected total losses to the layer $(D, D + L)$ can be written as:

$$E(R_{D,L}) = \frac{E(\min(Z, D + L)) - E(\min(Z, D))}{E(Z)} E(N)E(Z)$$

Divide both the numerator and the denominator by M:

$$= \frac{E\big(\min(z, d + l)) - E(\min(z, d)\big)}{E(z)} E(X)$$

$$= \big(G(d + l) - G(d)\big)E(X)$$

where

$$G(u) = \frac{E(\min(z, u))}{E(z)}$$

$G(u)$ is a function from [0,1] to [0,1].

It should be clear that $G(d) = \frac{E(\min(z,d))}{E(z)}$ can be interpreted as the percentage of risk retained by the reinsured (primary insurer) after the imposition of a deductible $D = dM$. If a loss Z is experienced, the loss retained by the cedent is the minimum between the loss and the deductible:

$$Z_{ret} = \min(Z, D) = M \cdot \min(z, d)$$

The average expected retained loss is defined as:

$$E\big(Z_{ret}\big) = E\big(\min(Z, D)\big) = M \cdot E\big(\min(z, d)\big)$$

Conversely if there was not a reinsurance treaty, the average expected loss would be:

$$E(Z) = M \cdot E(z)$$

The percentage of risk retained by the reinsurer is their ratio:

$$\frac{E(Z_{ret})}{E(Z)} = \frac{M \cdot E\big(\min(z,d)\big)}{M \cdot E(z)} = \frac{E\big(\min(z,d)\big)}{E(z)} = G(d)$$

Plotting $G(d)$ as a function of d returns the exposure curve. Given the exposure curve and knowing the deductible, the computation of the expected losses to the layer given the overall expected losses is quite straight-forward.

Relationship between exposure curve and severity curve. Given the one-to-one correspondence between severity curve and exposure curve, it could be interesting to find a way to move from one to the to the other. The numerator of $G(u)$ can be written as:

$$E(\min(z,u)) = \int_0^1 \min(z,u) f(z)dz = \int_0^u zf(z)dz + \int_u^1 uf(z)dz =$$

$$= \int_0^u zf(z)dz + u \cdot \int_u^1 f(z)dz = \int_0^u zf(z)dz + u \cdot \big(1 - F(u)\big)$$

By integrating by parts the integral, it is possible to obtain:

$$\int_0^u zf(z)dz + u \cdot \big(1 - F(u)\big) = u \cdot F(u) - \int_0^u F(z)dz + u \cdot \big(1 - F(u)\big)$$

$$= \int_0^u \Big(1 - F(z)\Big)dz$$

Using this result, the function G(u) can be written as:

$$G(u) = \frac{\int_0^u \big(1 - F(z)\big)dz}{E(z)}$$

whereas its derivative is:

$$G^{'}(u) = \frac{1 - F(u)}{E(z)}$$

Since $F(0) = 0$, it follows that the first derivative of G computed in 0 is equal to the reciprocal of the expected loss:

$$G^{'}(0) = \frac{1}{E(z)}$$

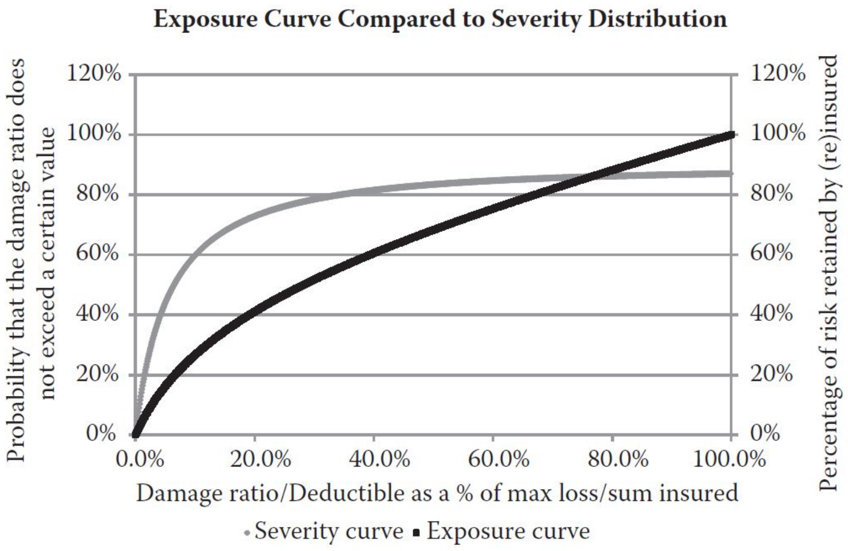

Figure 6: Comparison between exposure curve and severity curve. Pricing in general insurance, Pietro Parodi, 2015

Therefore, it is possible to write the cumulative distribution function $F(z)$ highlighting the relation between exposure curve and severity curve:

$$F(z) = \begin{cases} 1 - \frac{G^{'}(z)}{G^{'}(0)}, \ 0 \leq z < 1 \\ \\ 1, \ z = 1 \end{cases}$$

One of the properties of the function G(z) is the concavity. A strong concavity implies that even a small deductible determines a strong reduction in the risk ceded to the reinsurer and a negligible probability of total loss. On the other hand, if the exposure curve has not concavity at all ($G^{'}(u) = 1$ for all u), all losses are total losses and a deductible of $d\%$ achieves a reduction of the loss equal to $d\%$.

Use exposure curves in order to price an XL. The fitted exposure curve can be used in order to rate the XL reinsurance for a portfolio of properties. As shown in the previous paragraphs, the expected total losses to a layer $L \ xs \ D$ for an individual property is defined as $(G(d+l) - G(d)) \cdot E(X)$. Once fitted the exposure curve, the value of $G(d+l) - G(d)$ is known. Since typically the exposure rating pricing methodology is applied in case of poor data availability, the expected value of the aggregate claims amount can be computed using the premium originally charged by the direct insurer. Recalling the definition of the expected loss ratio:

$$E(LR) = \frac{E(X)}{Premium}$$

it is possible to estimate the expected aggregate claims amount as:

$$E(X) = E(LR) \cdot Premium \approx LR \cdot Premium$$

The relationship can be approximated by considering the actual Loss Ratio obtained by the insurer over a number of past years properly adjusted for the underwriting cycle and other biasing factors. Therefore, the expected total losses to the layer are computed as:

$$E(R_{D,L}) = (G(d+l) - G(d)) \cdot E(LR) \cdot Premium$$

Considering a number of homogeneous properties in the portfolio equal to K in the portfolio, it is possible to obtain:

$$E\big(R^{D,L}\big) = \sum_{k=1}^{K} E(R_{D,L})$$

$$= \sum_{k=1}^{K} \big(G_k(d_k + l_k) - G_k(d_k)\big) \cdot E(X_k) =$$

$$= \sum_{k=1}^{K} \big(G_k(d_k + l_k) - G_k(d_k)\big) \cdot E(LR_k) \cdot P_k =$$

where:

- $G_k(z)$ is the exposure curve for the k-th property
- d_k and l_k are the limits of the layer of the k-th property expressed in terms of M_k, its sum insured or MPL.
- P_k is the premium paid by the policyholder to the insurer related to the k-th property.
- $E(LR_k)$ is the expected Loss Ratio for the k-th property.

If the portfolio involves homogeneous properties such that it is possible to assume the same exposure curve and if the expected Loss Ratio does not depend on the value of the property, the previous relationship becomes:

$$E(R^{D,L}) = E(LR) \cdot \sum_{k=1}^{K} \big(G_k(d_k + l_k) - G_k(d_k)\big) \cdot P_k$$

The accuracy of the exposure rating applied by the reinsurer will depend on the accuracy of the rating process of the insurer. In practice, the reinsurer could be not able to get from the direct insurer the entire list of properties under reinsurance agreements. In such a case, properties with similar values of sum insured or MPL will be grouped in B bands and information about these bands is provided more likely to the reinsurer. The expected losses to the layer can be approximated as follows:

$$E\big(R^{D,L}\big)\approx E(LR)\cdot\sum_{\beta=1}^{B}\big(G(d_\beta+l_\beta)-G(d_\beta)\big)\cdot P_\beta$$

where $d_\beta=\frac{D}{M_\beta}$ and $l_\beta=\frac{L}{M_\beta}$, being M_β the average sum insured or Maximum Possible Loss in a band, defined as:

$$M_\beta=\frac{M_{\beta,min}+M_{\beta,max}}{2}$$

P_β is equal to the sum of all the premiums paid for properties in the band β.

Parametrization of Exposure curves. If a company decides to use exposure curves in order to price a reinsurance treaty, it has to calibrate them. There are typically two possibilities:

- *Use market exposure curves.*

  Among the last years, exposure curves were derived by institutions empirically from collections of historical losses and they are actually available in graphical or tabulated forms or implemented in computerized underwriting tools. The main shortfall is that they are provided only for a limited set of parameters which could be not suitable for the specific portfolio. Indeed, if it were possible to have a continuous set of parameters, the exposure curve would be much more smoothed. In addition, it could be difficult to find a set of parameters to be associated with the information available for a band of similar properties.

- *Build its own MBBEFD class exposure curve.*

  What is needed in order to build an exposure curve is a large number of claims for a portfolio of reasonably similar properties and their Maximum Possible Loss. The first step concerns the division of each claim by the MPL and sort them to form an empirical relative severity curve. Secondly it is needed to fit a severity curve in the form:

  $$F(z)=\begin{cases}\frac{b(g-1)(1-b^z)}{b(g-1)+(1-bg)b^z},\, z<1\\[2ex] 1,\ z=1\end{cases}$$

  to the empirical severity curve using numerical optimization method for estimating the parameters b and g. Finally, the exposure curve is defined as[4]:

$$G(z) = \frac{\ln\left(\frac{b(g-1)+(1-bg)b^z}{1-b}\right)}{\ln(bg)}$$

Aggregate Loss Distribution Using Exposure Rating. It could be also interesting to develop the Aggregate Loss distribution and the volatility around the point estimate just obtained from the exposure rating. It is only needed the exposure curve G, the properties scheduled by bands, the premium for each band and the expected Loss Ratio. Regarding the severity model, it has been highlighted in the previous paragraphs the one-to-one correspondence between the exposure curve and the severity curve. From the exposure curve, the severity curve and model are easily built for the losses in the band b and it can be estimated the average of the individual loss amount for each band, $E(Z_b)$. Regarding the frequency model, only the expected number of losses for each band has to be estimated. From the definition of the expected Loss Ratio, it is possible to compute the expected total losses for the band b as:

$$E(X_b) = E(LR_b) \cdot P_b$$

According to a collective risk model, the expected number of losses for the specific band can be estimated as:

$$E(N_b) = \frac{E(X_b)}{E(Z_b)}$$

Once estimated it, it is only matter of fitting a theoretical distribution of interest as a Poisson or a Negative Binomial. Once fitted the severity model and the frequency model, it is possible to obtain the aggregate loss model across all bands via Montecarlo Simulations, by assuming independence among different bands.

Sources of Uncertainty. The reliability of the results and the robustness of a model depends on its underlying assumptions and on the uncertainty introduced by them. Regarding exposure rating methodology, there are some critical issues to be taken into account:

- *Exposure curve.*

  More the exposure curve describes realistically the property considered, more correct are the results. Since each property in the portfolio has unique features, the exposure curve will always be an approximation of the true exposure.

[4]For further details, look at “The Swiss Re Exposure Curves and the MBBEFD Distribution class”, Stefan Bernegger.

- *Loss Ratio.*

  The Loss Ratio introduced in the exposure rating depends on the assumptions on the underwriting cycle and on the consistency of the insurer's pricing strategy. It could be also affected by some fluctuations with respect to the past values implying parameters uncertainty.

- *Property schedule* to be properly updated overtime in order to reflect the real risk.

To conclude the explanation of the exposure rating, it could be interesting to list situations where this pricing methodology works properly:

- The company resembles the industry.
- Experience is limited in volume to be relied on.
- The past will not predict properly the future due to some changes in the classes, states, and limits.
- The company uses market data to capture better the severity of loss.
- If used as another view to the company's experience.

Conversely, the exposure rating does not work properly if:

- The exposure data is incomplete or unreliable.
- The experience rating is very robust and stable.
- Exposure curves are not available.

2.3.2 Experience Rating

In experience rating, the reinsurer bases the calculations of the premium on the loss experience of the portfolio. It is a technique applying past year's loss experience to the today's reinsurance contracts to get an estimate of the expected losses. As a result, the pricing methodology works properly only if there is a sufficient, credible, and reliable claim experience. Only claims data above a certain level, named reporting threshold a, are disclosed by the insurer to the reinsurer for a certain past period (typically last 5-10 years). Notice that data collected are not only the ones laying in the interval $(D, D+L)$, but typically also above a percentage (usually 50%) of the deductible.

This approach shows different shortfalls regarding the data collected from the insurer:

- *It assumes stable risk profile.*

  Since the experience rating uses past data for predicting future ones, the underlying assumption is that the portfolio of risks which has generated claims in the past is very stable and similar to the portfolio which will generate losses in the future. If the assumption is not met in practice, it is necessary to correct data for the exposure.

- *It does not consider the inflation.*

- *It does not consider the unused capacity.*

  The unused capacity is the part of the cover which is not "attached" by the losses. If all the claims related to a portfolio do not exceed $D + L$, the experience rating will provide the same price to treaty with the same deductible and limits higher than the maximum loss observed in the past.

- *It ignores IBNR claims.*

For having comparable data points, it is necessary to make adjustments in order to consider some factors affecting the claim size and/or the frequency:

- *Inflation.*

  Data needs to be inflation-corrected by adjustment indices varying across lines of business (like consumer price index, construction cost index). Another point to be highlighted is that in a XL treaty a claim which did not touch the layer may nowadays be above the deductible due to the effect of the inflation. Indeed, the threshold established for reporting past claim size to the reinsurer is not at D, but below D. Without considering inflation, the severity burden inside the layer of an XL is defined as:

  $$E\big(Z^{D,L}\big)= \int_{D}^{D+L} \big(1 - F_Z(z)\big)dz$$

  By introducing inflation with a factor $\delta > 1$, it can be rewritten as:

  $$\int_{D}^{D+L} \big(1 - F_{\delta Z}(z)\big)dz = \delta \int_{\frac{D}{\delta}}^{\frac{D+L}{\delta}} \big(1 - F_Z(z)\big)dz = \delta \cdot \mathbb{E}\big(Z^{D/\delta, L/\delta}\big)$$

  It is equal to the scaled (by the factor δ) expected reinsured claim size in the layer $\big[\frac{D}{\delta}, \frac{D+L}{\delta}\big]$. It should be clear that inflation may affect both the claim size and the claim number in different ways. There could be situations in which the layer with deductible D/δ and limit L/δ has a smaller loss expectation that the original layer, but typically for all relevant cases the opposite holds meaning that the loss in the layer grows by a factor higher than δ.

- *Portfolio Size Changes (exposure).*

  Data points need to be adjusted to the portfolio volume at the current year. Different measures are suitable for the volume. A suitable volume measure are the original premiums charged by the insurer since they provide the best measure of exposure, if priced consistently. According to the line of business and to the type of cover, it could be assumed that:

  - The claim sizes increase proportionally or according to other functional relationships with the volume.
  - Portfolio size changes affect both number of claims and claim size. Typically, in per-risk XL, volume affects only the number of claims, whereas in cumulative XL, it will also influence the size of the aggregate claims per event.

- *Loss development.*

  For long-tail lines of business (like liability), due to the low settlement speed, it is possible that claims are not fully developed. The data reported to the reinsurer will consists of development patterns and current estimates of the final loss burden. In order to have comparable data point in terms of number of claims and of sizes of the claims, the reinsurer has to apply reserving techniques on an individual claim basis.

Once made all the adjustments, the burning cost rating is quite straightforward to be applied. Indeed, it is possible to build the empirical cumulative density function which can be used for pricing. It is quite useful the resulting expected claim size, named burning cost, but recall that the empirical Cumulative Density Function is typically not sufficient for modelling the entire risk. Indeed, it is implicitly assumed that the largest possible claim has already occurred. In addition, it is possible to have few data in the layer. In general, if the loss experience is not fully representative, an additional model is needed for pricing the treaty. A possibility could be fit an analytical loss model to the observed data. Therefore, once data points are adjusted, it is typically possible to fit a frequency-severity loss model which defines the aggregate claims amount to be paid by the reinsurer in a year t as:

$$\tilde{R}_t = \sum_{i=1}^{\tilde{N}_t} \tilde{Z}_i$$

$\tilde{N}_t : \Omega \rightarrow \mathbb{N}$ is a random variable modelling the frequency. $\tilde{Z}_i : \Omega \rightarrow \mathbb{R}^+$ are iid random variables modelling the severity. The frequency and the severity components are supposed to be independent. In symbols, $\tilde{N}_t \perp \{\tilde{Z}_i : i \in i, ..., N\}$. The Wald's identity simplifies the calculation of the expected value of the sum of a random number of random variables. Applied to a Collective Risk Model, if the variance of the severity and of the frequency are finite $\big(Var(\tilde{Z}) < \infty, Var(\tilde{N}_t) < \infty\big)$, the expected aggregate claims amount can be expressed as the product between the expected severity and the expected frequency:

$$E(\tilde{R}_t) = E(\tilde{N}_t) \cdot E(\tilde{Z})$$

It can be noticed the complementary role of the average claim size and the average claim number. The knowledge of the expected value of the aggregate claims amount can be helpful when pricing according to the expected value principle. In addition, it is possible to compute the variance of the total annual aggregate claims amount in a closed form:

$$Var(\tilde{R}_t) = Var(\tilde{Z}_t) \cdot E(\tilde{N}_t) + Var(\tilde{N}_t) \cdot E(\tilde{Z}_t)^2$$

Knowing the variance of the aggregate claims amount, it is possible to apply a premium calculation principles based on the first two moments of $\tilde{R}_t$. In the particular case the claim count follows a homogeneous Poisson process, $\tilde{N}_t \sim Poi(\lambda)$, it holds that:

$$E(\tilde{R}_t) = \lambda \cdot E(\tilde{Z})$$

$$E(\tilde{R}_t) = \lambda \cdot E(\tilde{Z}^2)$$

Before applying a specific model is always fundamental to check whether the underlying assumptions are met by the data. Important methodological results like the determination (or approximation) of the distribution of the aggregate claims amount and its moments would not be possible without them. In reality, there are situations where one or more of these assumptions do not hold (e.g. modelling attritional claims and large claims together or treating as one claims belonging to very different LoBs).

In order to proper evaluate the moments and, if possible, the distribution of the total aggregate claims amount, it is necessary to define a proper distribution for both the frequency component and the severity component.

2.4 Frequency Analysis

Regarding frequency analysis, the number of claims with claim size above the reporting thresholds should be disclosed to the reinsurer. The reinsurer has to fit a prospective model predicting the number of losses in the future, implying that past data have been adjusted in order to describe pattern of the future portfolio. For instance, the number of claims has to be adjusted for the exposure in a quite complicated way depending on the contract basis. For Risk Attaching During policies, the correct exposure to be used are the premiums written during the reinsurance policy period, which is matched to the claims attached to the policy year n (claims effectively covered by the reinsurance treaty in the year). Differently, for Loss Occurring During policies, the correct exposure are the premiums earned during the reinsurance period. They could be matched to the claims incurred during the period. If the earned premiums are not available, under the assumption of uniform writing of premiums and of occurring claims, it is possible to define the earned premium as the arithmetic average of the written premiums of two consecutive years.

After having adjusted data, the following step is to fit a frequency model which provides the probability that a given number of claims occurs during a given period. Even though in the literature there are a lot of possible distributions to be fitted, in practice the typically used distributions for the frequency component are the binomial distribution, the Poisson distribution and the Negative Binomial distribution. They belong to the Panjer class (denoted also as [a,b,0] class), which is characterized by the fact that exist constants a and b such that:

$$\frac{p_k}{p_{k-1}} = a + \frac{b}{k}, \;\; k = 1, 2, 3, ...$$

p_k is the probability of having exactly k losses and p_0 is determined by imposing the condition $\sum_{k=0}^{\infty} p_k = 1$. This characteristic is used in the calculation of the aggregate claims amount distribution by applying the Panjer recursive formula. The three distributions differ in terms of variance/mean ratio, named also index of dispersion, which can drive the choice of the model to be used. The variance/mean ratio is 1 for the Poisson distribution. If it is greater than 1, as for the Negative binomial, the claim number process is overdispersed. Conversely, if it is lower than 1, as for the binomial distribution, the process is underdispersed. Typically, the Binomial distribution is used in individual risk models, while the Poisson distribution and the negative Binomial distribution are typically used in collective risk model.

2.4.1 Binomial distribution

The binomial distribution models the probability of having k successes in an experiment repeated n times with the probability of any given experiment being successful equal to p. In insurance and reinsurance practice, it is commonly used when there are n independent and identically distributed risks. Each risk has a probability p of incurring in a claim and it is assumed that only one claim for risk is possible which implies a maximum number of claims per year equal to n. This feature is suitable in individual risk model. Conversely, if the probability of making a claim depends on the risk, the binomial distribution cannot be used. The probability of having k claims from the n risks is defined as:

$$P(\tilde{N} = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

The mean and the variance are respectively equal to:

$$E(\tilde{N}) = np$$

$$Var(\tilde{N}) = np(1-p)$$

By increasing the number of risks (n), the expected value and the variance increase. Conversely, the coefficient of variation decreases due to the mutuality:

$$CV(\tilde{N}) = \sqrt{\frac{1-p}{np}}$$

Finally, the skewness is defined as:

$$\gamma_1 = \frac{(1-2p}{\sqrt{np(1-p)}}$$

The sign of the skewness mainly depends on the value of the probability of getting a claim. In particular if $p > 0.5$, the skewness will be negative, implying a left tail. If $p < 0.5$, the skewness will be positive and the distribution presents a right tail. Finally, if $p = 0.5$, the distribution is perfectly symmetric.

To sum up, the binomial distribution is appropriate for portfolio with (finite) small number of risks that create small homogeneous events with equal probability p.

2.4.2 Poisson Distribution (Poi)

A stochastic process is supposed to be Poisson if it satisfies the following properties:

- Starting in 0.
- Stationarity. The distribution of the number of events in an interval depends only on the length but not on the starting point of the interval. In other words, the probability of having k simultaneous events is constant among disjoint time intervals.
- The number of events occurring in disjoint time interval are independent.
- Events are rare. The probability of obtaining two events in a small-time window is negligible and the probability of having one event in that time window is proportional to the length of the window by a constant λ denoted as Poisson rate.

The Poisson process is useful for large number of risks and small probability of claim occurrence, or if the expected number of events is much smaller than the theoretically possible maximum number of events. It is also applicable to situations in which the event arrivals can be assumed to be independent, with expected number λ per time unit (typically a year). The probability of having n events in a time interval of length t is given by:

$$Pr(\tilde{N} = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

Notice the key assumption of constant rate at which losses happen. Otherwise, it is needed to fit other distribution like the Negative Binomial. In the interval [0,1] (as one year), the mean and the variance are given by:

$$E(\tilde{N}) = Var(\tilde{N}) = \lambda$$

The coefficient of variation, measure of relative volatility, is:

$$CV(\tilde{N}) = \frac{1}{\sqrt{\lambda}}$$

In terms of skewness, it holds:

$$\gamma_1(\tilde{N}) = \frac{1}{\sqrt{\lambda}}$$

For any value of λ, the skewness is always positive, implying a right tail and a mass probability to have an extreme number of losses, quite dangerous situation for the insurer/reinsurer.

An increase of the Poisson rate λ leads to an increase in the expected value and the variance and to a reduction of the relative volatility (CV) and of the skewness. Indeed, the distribution will be more volatility in absolute terms, but less skewed. The Poisson distribution presents two limitations:

- It is not suitable for taking systemic risk into account.
- Parameter Uncertainty. The Poisson rate is always estimated basing on a small number of years and it has a significant parameter uncertainty. Since it makes the variance and the dispersion index increase, it should be more appropriate to fit a distribution with variance higher than the mean like a Negative Binomial.

2.4.3 Negative Binomial (NB)

Conversely to the Poisson, under the Negative Binomial distribution, the probability of occurrence of the event is not equal for all the elements. The Negative Binomial distribution can be defined also as the discrete probability distribution of getting k successes in a sequence of Bernoulli trials before $r > 0$ failures occur. Denoting the probability of failure by $\frac{1}{1+\beta}, \beta > 0$, the probability of getting k successes is given by:

$$P(\tilde{N} = k) = \binom{k+r-1}{k} \left(\frac{\beta}{1+\beta}\right)^k \left(\frac{1}{1+\beta}\right)^r$$

The mean and the variance of the negative Binomial are given by:

$$E(\tilde{N}) = r\beta$$

$$Var(\tilde{N}) = r\beta(1+\beta)$$

Since $\beta > 0$, it follows that $Var(\tilde{N}) > E(\tilde{N})$. Indeed, the Negative Binomial is suitable where the Poisson distribution does not have enough volatility, since the variance is always larger than the mean (dispersion index higher than 1). The skewness is always positive and defined as:

$$\gamma_1(\tilde{N}) = \frac{2+\beta}{\sqrt{r(1+\beta)}}$$

It decreases as r increases.

It should be highlighted that a Poisson with a parameter λ Gamma distributed corresponds to a Negative Binomial. If $N \sim Poi(\tilde{\lambda})$ where $\tilde{\lambda} \sim Gamma(\theta, \alpha)$, it follows that $\tilde{N} \sim NB(\beta = \theta, r = \alpha)$.

The Negative Binomial is able to overcome the shortfalls of the Poisson distribution. Indeed, the Negative Binomial is more suitable than the Poisson distribution to model real-world situations since the extra volatility enables to take into account of the systemic variations. In addition, a Poisson with parameter uncertainty can be also modelled as a Negative Binomial distribution, by approximating the variation around the central value of λ as a gamma distribution rather than a normal distribution.

2.4.4 Excess Frequency

Since the reinsurer has data regarding claims above the reporting threshold a, it is necessary to introduce the concept of excess frequency. The number of claims exceeding the reporting threshold a is defined as:

$$N_a = \sum_{i=1}^{N} 1_{\{Z_i > a\}}$$

Denoting with π, the probability that the single loss exceeds the threshold a ($\pi = P[Z > a]$), the distribution of the number of claims exceeding the threshold N_a is of the same kind of the distribution of N, the total number of claims, but with different parameters:

- If $N \sim Poisson(\lambda) \to N_a \sim Poisson(\lambda\pi)$
- If $N \sim Binomial(n, p) \to N_a \sim Poisson(n, p\pi)$
- If $N \; NegBin(a, p) \to N_a \sim NegBin(r, \frac{p}{p+\pi(1-p)})$, where $p = \frac{1}{1+\beta}$

In addition, the expected excess frequency is defined as:

$$E(N_a) = E(N) \cdot P[Z > a]$$

It is possible to rewrite the layer function applied in a XL treaty in terms of the excess frequency as:

$$\sum_{i=1}^{N} \mathcal{L}_{D,L}(Z_i) = \sum_{i=1}^{N_a} \mathcal{L}_{D,L}(Z_i^*)$$

where $Z_i^* = Z_i | Z_i > a$. The relevance of the (expected) excess frequency will be highlighted in the computation of the risk premium, in particular in the computation of the expected total losses to the layer $L \; xs \; D$.

2.4.5 Fitting Process

Theoretically, in order to choose the most suitable model, one can refer to the common characteristics of the Binomial, Poisson and Negative Binomial distribution which is their belongness to the Panjer class. Inside a class [a,b,0], it could be possible to fit the model $\frac{p_k}{p_{k-1}} = a + \frac{b}{k}$ on the empirical frequencies $\hat{p}_k$ and estimate the parameters a and b by standard linear regression models in order to identify the most proper distribution. The method is quite elegant, but in practice it is very improbable to have a sufficient number of empirical frequencies allowing for good and reliable estimate of the parameters. Therefore, in practice, it is better to rely on some general criteria. In particular, the use of the binomial distribution is reasonable only for individual risk model where each risk can have at most one loss. In a collective risk model, characterised by the absence of an overall limit to the number of claims in a given year, it is preferred to fit a distribution characterized by a variance/mean ratio at least equal to the one of a Poisson process. The Poisson distribution is justifiable only if the Poisson rate is quite constant and where there is a negligible effect of systemic risk. Since in reality systemic risk is quite common, through a Poisson distribution, it could be underestimated the volatility of the loss experience. As a result, in general it is better to fit a Negative Binomial, estimate its parameters $\hat{r}$ and $\hat{\beta}$ and check from the historical claim counts by method of moment or by maximum likelihood. If the constraints $\hat{r} > 0$ and $\hat{\beta} > 0$ are met, fit a Negative Binomial, otherwise fit a Poisson distribution, and estimate the Poisson rate λ.

2.5 Severity Analysis

Data transferred from the insurer to the reinsurer involves claims above the reporting threshold a. Before determining the conditional loss distribution for the single loss $(Z > a)$, it is necessary to properly adjust the statistical material. In paragraph 2.3.2, the adjustments regarding economic changes (like inflation) and changes in the portfolio have been presented. Another adjustment necessary for the severity component regards the IBNER (Incurred But Not Enough Reserved) which involves the difference between the final settled amount and the overall estimate (consisting of paid and outstanding). The possible solutions to IBNER are:

- Ignore IBNER, if there are not sufficient information on the development of the individual reserve or on the split between paid and outstanding. The shortfall of this solution is the bias introduced in case of under or over reserving.

- Use just Closed claims. The approach still introduces some bias since the claims still open are typically the largest ones.

- Identify trends in the estimated reserves for individual claims and use them in order to predict the ultimate value of each claim.

In addition, recall that the claims reported to the reinsurer are the ones above the reporting threshold which is typically lower than the deductible, whereas the aim of the loss model is to price a treaty with deductible D. Indeed, it has to be defined how many losses should be taken into account for fitting the chosen distribution. If from one hand, it is desirable to consider as many claims as possible in order to provide a reliable statistical basis, on the other hand the distribution of small losses is typically very different from the distribution of high losses. The Pareto distribution, which is typically used for modelling large losses, solves the issue, being closed to truncation as described in the following paragraph. Once historical data have been adjusted, it is possible to build a severity model by fitting a proper distribution. It should be highlighted that rather than fitting all the possible distributions from a distribution-fitting tool, it is preferred to restrict the number of admissible models, selecting them according to the experience and to theory. For instance, distributions with negative domain must be excluded. In addition, model complexity has to be penalized, since models more complex than needed make poor predictions. It is suggested, if there is sufficient data, to split the dataset int the training set (used for parameter estimation) and a test set (used for selection and validation), whereas if the data is insufficient, to use the AIC[5] criterion.

Some considerations about extreme value theory (EVT). Reinsurance pricing involves modelling claims above a certain threshold. According to the Extreme Value Theory, it is necessary to consider a threshold distribution, which is a distribution taking values above a certain positive threshold. In addition, the severity distribution density should decrease with size since a large loss is typically less likely than a smaller loss. The Normal Distribution has to be excluded since its domain involves real numbers. In actuarial pricing practice, typically a lognormal distribution is fitted. Since in a non-proportional treaty, claims are reported above the threshold, it is more proper to fit a shifted lognormal distribution, having a density function defined as:

$$f_X(x) = \frac{1}{\sigma(x-d)\sqrt{(2\pi)}} e^{\frac{(\ln(x-d)-\mu)^2}{2\sigma^2}}, x > d, d > 0$$

Moments are defined as:

$$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right) + d$$

[5] $AIC = -2loglik + 2d$, where d is the number of parameters. Due to the presence of the penalty term $2d$, the model with the smallest AIC achieves a compromise between fit and complexity.

$$Var(X) = \exp\big(2\mu + \sigma^2\big)\big(\exp\big(\sigma^2\big) - 1\big)$$

$$\gamma(X) = \big(\exp\big(\sigma^2\big) + 2\big)\sqrt{\exp(\sigma^2) - 1}$$

Notice that the variance and the skewness are the same of a Lognormal distribution starting in 0. Only the expected value is shifted by the parameter d.
With respect to a Normal distribution, the Lognormal distribution overcomes the problem of possible negative values in the domain. In case of a non-proportional treaty, the (shifted) Lognormal distribution has to be still avoided due to its behaviour not totally decreasing.
A distribution more suitable for describing reinsurance losses is the Pareto distribution or the Generalized Pareto Distribution (GPD). According to the Pickland-Balkema-de Haan theorem, the tail of any distribution can be modelled by a Generalized Pareto Distribution (GPD):

$$F(x) = \begin{cases} 1 - \big(1 + \xi\frac{x-\mu}{\sigma}\big)^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1 - \exp\big(-\frac{x-\mu}{\sigma}\big),\ \xi = 0 \end{cases}$$

The probability density function is 0 up to the value μ and then it declines to 0 more or less quickly according to the scale parameter ξ. In particular:

- If $\xi > 0$, the distribution goes to zero as power law (quite slowly).
- If $\xi = 0$, the distribution goes to zero as exponential (very fast).
- If $\xi < 0$, the distribution is capped and it is zero above a certain threshold.

σ is a scale parameter which stretches the distribution horizontally without impacting its behaviour towards 0. Parameters can be estimated via Maximum Likelihood.

Regarding Excess of Loss severity model, the fundamental and mostly used distribution is the Pareto distribution. Recalling that the Pareto distribution is a particular case of the GPD, its cumulative distribution function and its density function can be defined as:

$$F(x) = \begin{cases} 1 - \big(\frac{x}{\mu}\big)^{-\alpha}, x > \mu \\ 0,\ else \end{cases}$$

$$f(x) = \begin{cases} \alpha \cdot \mu^{\alpha} \cdot x^{-\alpha-1},\ x > \mu \\ 0,\ else \end{cases}$$

The parameters α and μ are strictly positive. Its density function is 0 up to μ and then it shows a decreasing J-shaped behaviour approaching the orthogonal axis asymptotically.

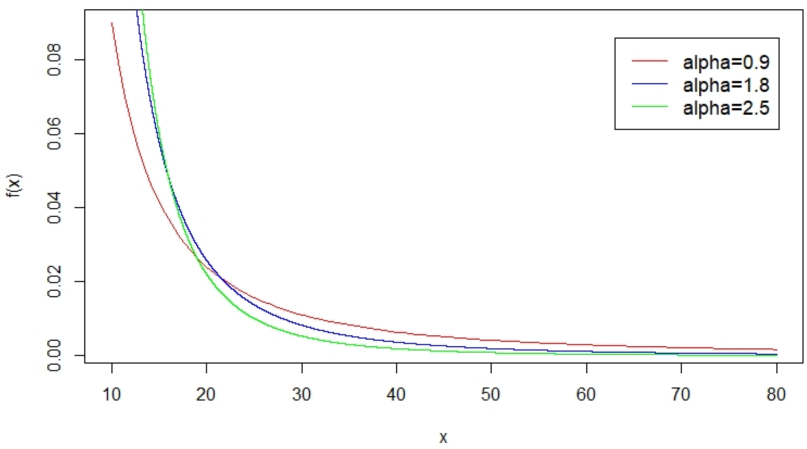

Figure 7: Density function of a Pareto distribution for different values of α

α is the shape parameter influencing the tail behaviour of the distribution, while μ is the scale parameter, defining the minimum value the Pareto distribution can take. Figure 7 shows the density function of Pareto distributions with the same minimum value equal to 10 and different values of α, to detect the effect of the shape parameter. Higher is the shape parameter α, lower is the relevance of the tail.

The moments of a Pareto distribution are given by:

$$E(X^k) = \begin{cases} \frac{\alpha \cdot \mu^k}{\alpha - k}, & k < \alpha \\ +\infty, & k \geq \alpha \end{cases}$$

The k-th order moment only exists for $k < \alpha$. Specifically, the expected value and the variance are:

$$E(X) = \mu \cdot \frac{\alpha}{\alpha - 1}, \ \alpha > 1$$

$$Var(X) = \mu^2 \cdot \frac{\alpha}{(\alpha - 1)^2 \cdot (\alpha - 2)}, \ \alpha > 2$$

In order to parametrize the Pareto distribution, it is possible to apply method of moments (MM) or the Maximum Likelihood (ML). Due to the poor data availability, it is better to use a Bayesian estimator, combining the ML/MM estimates with some typical values of α provided for certain lines of business. In particular:

- Earthquake and storm, $\alpha \approx 1$.
- Fire, $\alpha \sim 2$, fire in industry $\alpha \approx 1.5$.
- Motor Liability $\alpha \approx 2.5$.

- Motor Liability $\alpha \approx 2.5$.
- General Liability $\alpha \approx 1.8$.
- Occupational injury $\alpha \approx 2$.

It should be noticed the case of earthquake and storm, for which there could be some problems. In particular, if $\alpha < 1$, the expected value does not converge and it is not possible to run Montecarlo Simulations for pricing. Another characteristic is worth remembering regards its behaviour in forming certain conditional distributions. In particular, if X is Pareto(μ, α)-distributed and $T \geq \mu$, then:

$$X|X > T \sim Pareto(T, \alpha)$$

$$P(X > x|X > T) = \frac{P(X > \max(x, T))}{P(X > T)} =$$

$$= \begin{cases} \left(\frac{x}{T}\right)^{-\alpha}, x > T \\ 1, \ else \end{cases}$$

By truncating a Pareto distribution, it is obtained still a Pareto distribution, with the same shape parameter α. Indeed, this distribution is closed under truncation. The closure under truncation property is very crucial given the data at disposal of the reinsurer. Indeed, if it is possible to say that if losses above the reporting threshold a are Pareto distributed, also losses above the deductible D will be Pareto distributed with the same shape parameter α (but different starting point).

Finally, Figure 8 allows a comparison between the density function of a Pareto distribution and a Lognormal distribution having the same mean and variance in order to show how the Pareto distribution satisfies much more the properties of a threshold distribution with respect to a shifted Lognormal.

Expected Losses to the Layer. Once fitted a distribution for the individual reinsured claim size and for the number of claims, it is quite simple to compute the expected aggregate claims amount of the reinsurer. For an XL treaty, the loss to the layer $(D, D + L)$ can be defined as:

$$\mathcal{L}_{D,L}(Z) = \min(Z, D + L) - \min(Z, D)$$

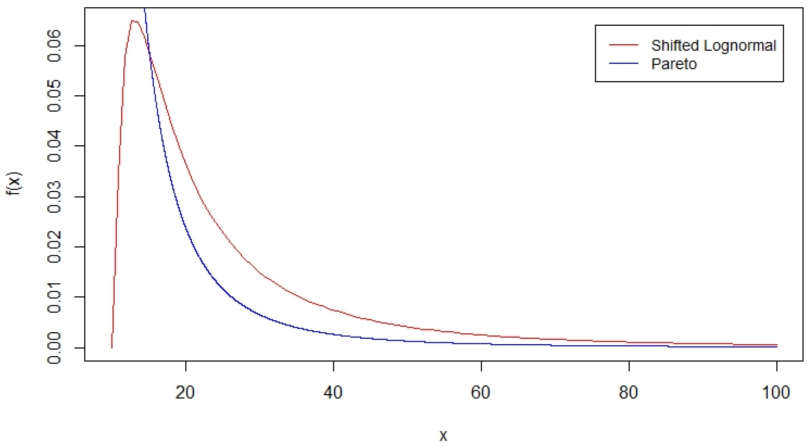

Figure 8: Comparison between the Pareto and the shifted Lognormal density functions.

The expected loss to the layer, if an individual loss has occurred before, is computed as:

$$E\big(\mathcal{L}_{D,L}(Z)\big) = E(\min(Z, D+L) - \min(Z, D)) = E(\min(Z, D+L)) - E(\min(Z, D))$$

In particular, if the assumptions of independence between claim size and frequency and of independence and of identical distribution of the claim sizes hold, the expected total losses of a $L\ xs\ D$ cover can be expressed as follows:

$$E(R(D,L)) = E\left(\sum_{i=1}^{\tilde{N}_t} \mathcal{L}_{D,L}(Z)\right) = E(\tilde{N}_t) \cdot E(\mathcal{L}_{D,L}(\tilde{Z})) =$$

$$= E(\tilde{N}_t) \cdot \big(E(\min(Z, D+L)) - E(\min(Z, D))\big)$$

Recall that $\tilde{N}_t$ is the number of losses from the ground up (meaning the number of losses reported to the insurer). Given the assumptions hold, the expected aggregate claims amount, of which the reinsurer is responsible for, can be written as the product between the expected frequency and the expected severity burden for the reinsurer. The expected severity burden $E(\mathcal{L}_{D,L}(\tilde{Z}))$ can be expressed in terms of the distribution function of the loss, F_Z, using the stop-loss transformation. The stop-loss transform of a random variable X is the function in real numbers:

$$slt_X : \mathbb{R}^+ \to \mathbb{R}^+$$

$$slt_X(u) = \int_u^{+\infty} \big(1 - F_X(t)\big) dt$$

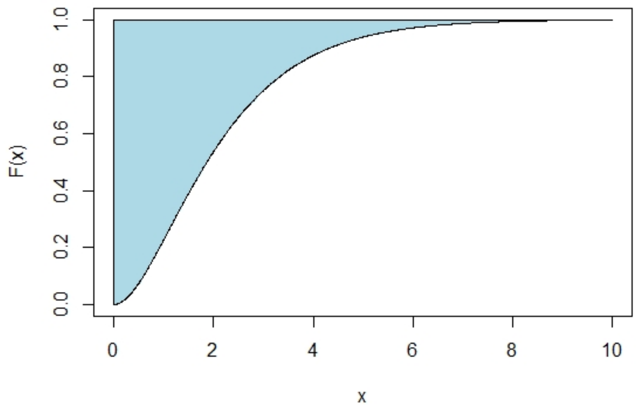

Figure 9: Stop-loss transformation computed in 0.

Its name comes from the fact that it provides exactly the expected loss in a stop-loss treaty with infinite cover as a function of the deductible. It is a decreasing convex function with extremes equal to[6]:

$$slt_X(0) = \int_0^{+\infty} \big(1 - F_X(t)\big)dt = E(X)$$

$$slt_X(\infty) = 0$$

Therefore, according to the Stop-Loss Transformation, the expected severity burden within a layer with cover equal to L and a deductible D can be computed as the integral of the survival function of Z between the extremes D and $D+L$:

$$E\big(\mathcal{L}_{D,L}(Z)\big) = \int_D^{D+L} \big(1 - F_X(t)\big)dt$$

Notice that if Z is Pareto distributed, $Z \sim Pareto(\alpha, \mu)$, the expected severity burden can

[6]Indeed, according to the Darth Vader rule, the expected value of a non-negative random variable X with finite expectation can be related with its distribution function as follows:

$$E(X) = \int_0^{+\infty} \big(1 - F_X(t)\big)dt$$

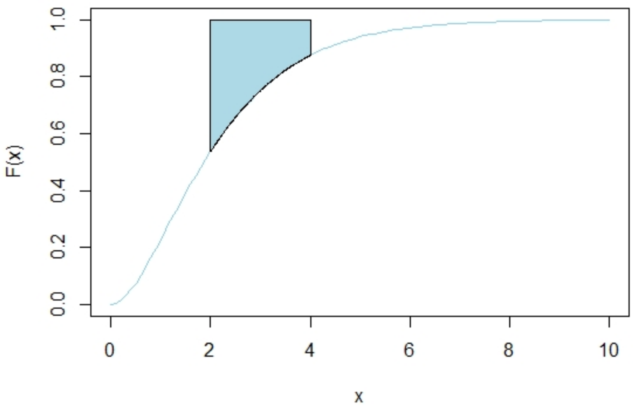

Figure 10: $E\big(\mathcal{L}_{2,2}(Z)\big)$

be expressed in a closed form[7]:

$$E\big(\mathcal{L}_{D,L}(Z)\big) = \mu^{\alpha} \cdot \frac{1}{1-\alpha}\Big((D+L)^{-\alpha+1} - D^{-\alpha+1}\Big)$$

This result is very powerful in particular in the case the Pareto distribution has a parameter $\alpha \leq 1$. Indeed, in that case, the distribution has an infinite expected value, but the expected severity burden will always exist thanks to a limit on the severity equal to L.

The shortfall of this formulation is that typically, the expected claim count from the ground up, $E(\tilde{N}_t)$, is typically unknown. Indeed, claims are reported to the reinsurer only up to the reporting threshold. Therefore, it is more useful to rewrite the previous formula in function of the expected number of claims exceeding the threshold a:

$$E(R(D,L)) = \frac{E(\min(Z, D+L)) - E(\min(Z, D))}{Pr(Z > a)} \cdot E(N_a)$$

where $E(N_a) = E(N) \cdot Pr(Z > a)$. The ratio $\frac{E(\min(Z,D+L))-E(\min(Z,D))}{Pr(Z>a)}$ is the average severity conditional on the severity being above the reporting threshold. It can be rewritten by knowing the probability of having a claim higher than the threshold a, based on the

[7]
$$E\big(\mathcal{L}_{D,L}(Z)\big) = \int_D^{D+L} \big(1 - F_Z(t)\big)dt = \int_D^{D+L} \left(\frac{t}{\mu}\right)^{-\alpha} dt =$$
$$= \left| \frac{\mu\left(\frac{t}{\mu}\right)^{-\alpha+1}}{-\alpha+1} + k \right|_D^{D+L} = \frac{\mu}{\mu^{-\alpha+1}} \cdot \frac{1}{-\alpha+1} \cdot \left| t^{-\alpha+1} \right|_D^{D+L} =$$
$$= \mu^{\alpha} \cdot \frac{1}{-\alpha+1}\big((D+L)^{-\alpha+1} - D^{-\alpha+1}\big)$$

severity model above a:

$$E(\min(Z, D+L)|Z > a) - E(\min(Z, D)|Z > a)$$

To sum up, the fair premium of the reinsurer can be computed as:

$$E(R(D, L)) = E(\min(Z, D+L)|Z > a) - E(\min(Z, D)|Z > a) \cdot E(N_a)$$

2.6 Distribution of the Aggregate Claims Amount

In order to make sensible risk-management decisions, it is necessary to know the aggregate claims amount distribution. Generally, its calculation is quite difficult to be solved analytically. The cumulative distribution function of the aggregate claims amount can be expressed as:

$$F_R(r) = Pr(Z_1 + ... + Z_N \leq r)$$

The complexity arises from the fact it is involved a sum of random variables with distribution $F_Z(z)$ and the distribution of the sum of random variables is a complicated function, named convolution function, of the individual distributions. In addition, the number of random variables is still a random variable. Since different values of N correspond to mutually exclusive events, it is possible to write $F_R(r)$ as a weighted sum of the probability of mutually exclusive events:

$$F_R(r) = Pr(Z_1 + ... + Z_N \leq r) = \sum_{n=0}^{\infty} Pr(N = n) \cdot Pr(Z_1 + ... + Z_n \leq r)$$

By conditioning with respect to N, the problem has been reduced to calculating the distribution of the sum of a fixed number of random variables. The previous equation can be rewritten, by calculating the convolutions of a fixed number of variables:

$$F_R(r) = \sum_{n=0}^{\infty} Pr(N = n) \cdot F_Z(r)^{*n}$$

where $F_Z(r)^{*n}$ is the n-th convolution power of $F_Z(r)$. If the severity distribution is continuous, a similar relationship holds for the probability density function:

$$f_R(r) = \sum_{n=0}^{\infty} Pr(N = n) \cdot f_Z(r)^{*n}$$

where $f_Z(r)^{*n}$ is the n-th convolution power of $f_Z(r)$. According to this formulation, the cumulative distribution function of the aggregate claims amount can be seen as an infinite sum of multi-dimensional integrals and typically it is not possible to write down its solution in a closed form. Therefore, numerical methods are needed. Typically, in non-proportional reinsurance pricing, an additional complexity can arise in presence of non-differentiable function operating possibly on single claims amounts, on aggregation of them, and even on the frequency component. Nevertheless, in order to calculate the distribution of the aggregate claims amount, there are three typical approaches:

- Parametric Approximations.
- Numerical quasi-exact methods.
- Montecarlo Simulation Methods.

2.6.1 Parametric Approximations

Unlucky, it is not possible to apply the Central Limit Theorem[8] since the number of claims is itself a random variable. As a result, the variance is not simply given by the sum of the variances of the severity distributions, but it must take into account also of the volatility of the number of losses. In addition, there is no guarantee that the resulting distribution is still Gaussian. It could be still possible to approximate the aggregate claims amount distribution by a Gaussian distribution (Normal Approximation) by modifying the variance for taking the extra volatility into account.

$$F_R(r) \sim \Phi\left(\frac{r - E(R)}{\sigma(R)}\right)$$

This approach has two main shortfalls:

- The error can be significantly high when the frequency is very low and its upper limit is quite difficult to be calculated.
- The aggregate loss distribution is typically not symmetric and the Gaussian does not capture the right-hand tail. Higher is the skewness, poorer is the accuracy.

[8] The Central Limit Theorem (CLT) states that, given a sequence of i.i.d random variables $X_1, \ldots, X_n$ with expected value μ and variance $\sigma^2 < \infty$, where $\bar{X}_n$ denotes its empirical average, it holds that:

$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$$

where $\xrightarrow{d}$ stands for convergence in distribution.

If the skewness of R is available, it is possible to apply a Normal Power Approximation:

$$F_R(r) \sim \Phi\left(-\frac{3}{\gamma(R)} + \sqrt{\frac{9}{\gamma^2(R)} + 1 + \frac{6}{\gamma(R)} \cdot \left(\frac{r - E(R)}{\sigma(R)}\right)}\right)$$

The relationship holds for the right tail and for skewness index lower than 1.
In addition, if the three first moments are available, it is possible to fit a translated gamma approximation which fits a shifted gamma distribution to the aggregate claims amount.

$$R \sim \Gamma\left(\frac{4}{\gamma^2(R)}, \frac{2}{\gamma(R) \cdot \sigma(R)}\right) + E(R) - \frac{2\sigma(R)}{\gamma(R)},\ with\ \gamma(R) > 0$$

It fits a skewed distribution, but the way in which modelling the tail and quantifying the error is still an unsolved issue. Regarding both parameters approximations methods, the accuracy is very poor, in particular when the number of claims is small and the severity tail is fat. Therefore, they could perform very poorly going in tail regions which are of substantial important in reinsurance. In addition, they are not flexible methods, but they have the advantage of being easy and quick to implement.

2.6.2 Numerical Quasi-Exact Methods

In Numerical quasi-exact methods, the aggregate claims amount distribution is defined by calculating the convolution integrals using discrete mathematics. Indeed, integrals can be approximated as a finite sum. It is necessary to discretize continuous distribution, respectively the severity distribution $f_Z(z)$ and the aggregate loss distribution $f_R(r)$ using steps of height h. In particular, f_k is the value of $f(z)$ evaluated in $z = hk$. In such a way, it is possible to approximate the aggregate loss distribution as a finite sum:

$$f_{R,k} = \sum_{n=0}^{k} Pr(N = n) \cdot f_{Z,k}^{*n}$$

The two typical numerical quasi-exact methods are the Fast Fourier Transform Method (FFT) and the Panjer recursion. In practice, regarding the FFT, it is necessary to make sure that numerical errors are sufficiently small, paying attention to the choice of the parameters in terms of number of subdivisions M and discretization step h. In particular, the step h must be defined sufficiently smaller than the typical loss amount and such that the difference between the statistics of the severity computed on that value of h and the true ones is not significant. M cannot be too large in order to avoid computational issues. In addition, $M \cdot h$ must be chosen such that the probability of obtaining total losses larger than $M \cdot h$ is negligible. Given the same number of steps (M), a too low value of h could lead to a poor approximation since it could not capture the full range of loss amount. On the other side, a value too high of h could lead to an insufficient granularity of the severity distribution. To sum up, the Fast Fourier method has an arbitrary degree of flexibility achieved according to the calibration of the discretisation step and the number of points. It is less flexible than a parameter approximation method, but it is very fast. In particular, the speed does not depend on the number of claims, but only on the number of subdivisions. If the frequency distribution belong to the (a,b,0) class, it is possible to apply the Panjer recursion. In particular, the density function of the aggregate claims amount can be written as:

$$g_k = \sum_{n=0}^{k} p_n \cdot f_k^{*n}$$

where $p_n = Pr(N = n)$. For a distribution belonging to the Panjer class, the following relationship holds:

$$p_k = \left(a + \frac{b}{k}\right) \cdot p_{k-1}, \;\; k \geq 1$$

with $p_0 \neq 0$. The recursive formula for the aggregate claims amount is:

$$g_k = \frac{1}{1 - af_0} \sum_{j=1}^{k} \left(a + \frac{bj}{k}\right) \cdot f_j \cdot g_{k-1}, \;\; k = 1, 2, ...$$

g_0 is the probability of having aggregate claims amount equal to 0 and it is given by:

$$g_0 = Pr(R = 0) \begin{cases} e^{-\lambda(1-f_0)} \; (Poisson \; Case) \\ (1 + q(f_0 - 1))^m \; (Binomial \; Case) \\ (1 - \beta(f_0 - 1))^m \; (Negative \; Binomial \; Case) \end{cases}$$

As for the FFT, the Panjer recursion has an arbitrary degree of accuracy achieved through the calibration of the discretisation step. With respect to the FFT, the Panjer recursion is slower and has a simpler implementation, but it can be fitted only for certain frequency distributions. In addition, there are not many constraints on the choice of the parameters and different levels of accuracy can be achieved by setting different discretization steps. However, if the number of claims is very large, a computational issue can arise. Indeed, $g = 0 = e^{-\lambda(1-f_0)}$ could be lower than the smallest number a computer is able to represent.

2.6.3 Montecarlo Simulation

The Monte Carlo simulations is the most typical way applied for calculating the aggregate claims amount distribution since it is flexible and easy to implement. In order to run Montecarlo simulations, two inputs are need:

- *Frequency model*, like the Negative Binomial, representing the number of claims in excess of the reporting threshold. Recall that the reporting threshold has to be lower than the deductible.
- *Severity model*, like a GPD or a Pareto distribution, representing the distribution of the claim size above the reporting threshold.

The algorithm for obtaining the distribution of the aggregate claims amount ceded to the reinsurer consists of the following steps:

1. Sample a number n of losses from the frequency model.
2. For each of these n losses, sample a random number from the severity model, obtaining the vector: $z_1, z_2, \ldots, z_n$.
3. Calculate the amount of losses to be ceded to the reinsurer.
4. Repeat steps 1 to 3 for S times, where S denotes the number of simulations. Higher is S, higher is the accuracy of the methodology.
5. The outcome of the S simulated scenarios, if ordered in ascending order, provides an approximation of the distribution of the aggregate losses to the layer.

In practice, the possible complexities arising from the application of Montecarlo Simulations are:

- A higher value of S implies a better precision in the calculation of quantiles, but also a slower methodology.
- The desired number of simulations is not known in advance.
- The sorting process requires the store of all the scenarios in the computer's memory which could be problematic if the number of simulations is very large.

To sum up, the accuracy of MC simulations can be easily improved by increasing the number of simulations. It is completely flexible, but it is very slow when the number of claims and the number of simulations is high.

In practice, typically Montecarlo simulation method is preferred where the maximum level of flexibility is needed, whereas the FFT should be used only when computational efficiency is prioritized. The Panjer recursion is typically applied when computational efficiency is less crucial and an ease of implementation is required.

After having calculated the entire distribution of the aggregate claims amount above the threshold, the computation of the technical premium is quite straightforward. Only some additional complexities with respect to insurance pricing need to be highlighted in case of high layers. First of all, the results of the model could be very unreliable due to the high volatility in that region of the distribution and the poor data availability. Due to the high volatility, the marginal cost of setting capital aside is important. Secondly, the presence of reinstatement premiums makes the premium paid over the course of the contract a stochastic variable. Indeed, the premium has to be calculated basing on the expected value of that random variable and on a correction factor driven by the estimated number of reinstatements.

2.7 Reinstatement Premiums

It is quite simple to calculate the expected losses to a layer through Montecarlo simulations also in case of a limited number of reinstatements. Especially in property line, a premium is charged to reinstate the layer which implies that the premium to be paid is not fixed in advance, but it needs to be simulated being itself a random variable. Analytical formulas have been developed in order to calculate the premium to be charged upfront according to the reinstatement structure. Since typically reinstatements are present for short-tail risks, it is not necessary to consider layer indexation, which is typical of liability line. Considering an XL $L\ xs\ D$ without aggregate deductible and without reinstatement and having claims $Z_1, \dots, Z_n$, the aggregate claims amount to the layer are defined as:

$$R(D,L) = \sum_{i=1}^{n} \mathcal{L}_{D,L}(Z_i) = \sum_{i=1}^{n} \min(Z_i, D+L) - \min(Z_i, D)$$

Its expected value is computed as:

$$E(R(D,L)) = \sum_{i=1}^{n} E(\mathcal{L}_{D,L}(Z_i))$$

Let's now introduce the constraint that there will be only k reinstatements to the layer, which is equivalent to assume an aggregate limit equal to: $AL = (1+k) \cdot L$ and a null aggregate deductible. Denote with P_0 is the initial premium paid for the cover of a first layer $\min(R(D,L),L)$. For each reinstatement j, there will be a reinstatement premium equal to:

$$P_j = \beta_j \cdot P_0$$

where β_j is a number between 0 and 1. For the k reinstatements, the sequence $(\beta_j)_{1\leq j\leq k}$ is fixed in advance and called premium plan. The values β_j can be fixed (pro-rata capita) or dependent on the time the reinstatements are paid (pro-rata temporis). The reasoning underlying the pro-rata temporis is that closed to the expiry date of a contract it will be less likely that the next layer will be used up. Easily, the expected loss to the layer is defined as:

$$E(R(D,L;0,(k+1)L)) = E\Big(\mathcal{L}_{0,(k+1)L}(\sum_{i=1}^{n} E(\mathcal{L}_{D,L}(Z_i)))\Big)$$

The expected overall premium is:

$$E(P) = P_0 \cdot \Big(1 + \sum_{j=0}^{k-1} \frac{\beta_j}{L} E\Big(\mathcal{L}_{jL,L}(\sum_{i=1}^{n} \mathcal{L}_{D,L} Z_i)\Big)\Big)$$

By inverting the previous analytical formula, the upfront premium can be easily obtained. In practice, it is much more common to calculate the expected losses and the premiums through Montecarlo simulations. Assume to determine the premium basing on the expected losses to the layer and not on the full aggregate claims amount distribution. Given an XL treaty, for each scenario, it is simulated the total losses to the layer before the aggregate limit: $R(D,L)$. Secondly, it is computed the number of required reinstatements defined as the ratio between the total losses to the layer and the limit L. It should be considered also the fact that the treaty could define a maximum number of possible reinstatements. In addition, the treaty defines for each reinstatement the related premium as a percentage of the upfront premium:

$$P_j = \beta_j \cdot P_0, \ with \ 0 \leq \beta_j \leq 1$$

The effective reinstatement premium required by each loss will be defined according to the proportion of layer "eaten" by the loss. For the h-th loss, the premium for the j-th reinstatement will be equal to:

$$P_{h,j} = \beta_j \cdot P_0 \cdot \frac{\min(reinstatement_j, L)}{L}$$

where $reinstament_j$ denotes the amount of the h-th loss exceeding the previous $j-1$ layers. If the reinstated layer L is used completely, the related reinstatement premium will be equal to P_j, otherwise it will be defined according to the percentage of layer used. For each simulated scenario, the total premium is computed as the sum of the upfront premium and the reinstatement premiums. To sum up, given a total loss X_i, a total premium P_i is calculated as a multiple (not necessarily an integer) of the upfront premium:

$$P_i = \rho_i \cdot P_0,\ with\ \rho_i \geq 1$$

At the end of the simulation process, it will be available an estimate of the expected losses $E\big(R(D, L; 0, (k+1)L)\big)$ and an estimate of the total expected premium defined as the product between the expected number of reinstatements and the upfront premium:

$$E(P) = E(\rho) \cdot P_0$$

What it is interesting for pricing purposes, it is the estimate of the necessary number of reinstatements, $E(\rho)$. Indeed, the upfront premium can be computed as:

$$P_0 = \frac{P}{\rho} = \frac{\big(E(\min(Z, D+L)) - E(\min(Z, D))\big) \cdot E(N)}{\rho}$$

3 Solvency II

As explained in the Chapter 1, one of the reasons why an insurance company is interested in underwriting a reinsurance treaty is the reduction of its capital requirement. In this Chapter, the Solvency II regime will be described, starting from an introduction regarding the general framework and moving towards the computation of the Solvency Capital Requirement (SCR) for the macro-risks most affected by a reinsurance treaty (respectively the Non-Life Underwriting Risk and the Counterparty Default Risk), looking also at the amendments introduced by the Review 2018 and at the possible changes according to the EIOPA Opinion on Review 2020.

3.1 Introduction

3.1.1 Before Solvency II

From the beginning of the 21st century, some discussions about the pertinence of the Solvency I framework started. In particular, the Required Solvency Margin (RSM) under the Solvency I framework considered essentially just the underwriting risk ignoring other risks as the operational risk, market risk, credit risk, liquidity risk and their dependence structure. In addition, the factor-based approach was not able to properly measure the complex forms of risks and of risk transfers. Those factors led to wrong and unreasonable estimation of the capital requirement. At that time, the Solvency Ratio was well above 2, since insurance undertakings held an Available Solvency Margin much larger than the RSM which was not properly estimated. In addition, there was not sufficient attention to the qualitative aspects of the supervision and no rules regarding insurance groups. Therefore, a new regulation and of a holistic risk management were surely necessary as the financial crisis of 2009 highlighted.

3.1.2 Why Solvency II

Before presenting the review of the prudential regime for insurance undertakings introduced by Solvency II, it could be interesting to make a comparison with respect to the previous solvency regime.

First of all, Solvency II has higher *risk sensitivity*. Indeed, the Required Solvency Margin under the Solvency I framework was computed by applying the same percentages to the volume of premiums or of claims of the total business without considering the different volatilities of the Lines of Business. Conversely the Solvency Capital Requirement regarding the Non-Life Underwriting Risk is computed separately for each segment considering the peculiar characteristics.

In addition, the SCR considers much more macro risks than the RSM of Solvency I by including the counterparty risk, the market risk, the credit risk, the operational risk, and much more proper measurements of the underwriting risks.

Another aspect properly improved in the passage from Solvency I to Solvency II regards the aggregation. Indeed, under Solvency I, the diversification was not properly measured, and it was considered only by computing the capital requirement on the total business. Differently, Solvency II prescribes the computation of the SCR for submodules which are aggregated through linear correlation (in the Standard Formula). In a (partial) internal model, the undertaking can use different aggregation tools (as copulae) which are more flexible and able to describe the real dependence structure existing between risk submodules.

As previously anticipated, a big news of the Solvency II framework is the introduction of an ad-hoc calculation of the capital requirement, through internal models.

3.1.3 Solvency II objectives

The main objective of Solvency II is the protection of policyholders and beneficiaries, which is reached through the introduction of a capital requirement computed under a risk-based approach (*more risks, more capital*). In particular it must consider all the risks the insurance undertaking is exposed to, overcoming the shortfalls of the Solvency I framework. The role of the capital requirement is to assure that insurance companies will be able to meet their commitments towards beneficiaries and policyholders. The protection of policyholders and of beneficiaries cannot be absolute, since it would imply an infinite capital, totally unfeasible for the undertaking. A proper level of confidence must be chosen in order to balance the trade-off between the policyholder's protection and the cost of capital. In particular, the higher is the level of confidence, the higher is the protection of the policyholders, but also the higher is the capital requirement. In the Solvency II framework, the chosen level of confidence is 99.5%, which is equivalent to accept that 0.5% of the insurance undertakings will go in ruin (in one year).

The Solvency II framework presents also secondary objectives which must be taken into account without undermining the main one. Examples of secondary objectives are the financial stability and the deepening of the single market, which can be achieved through the introduction of uniform rules (regulatory harmonization), by reducing the member states' options. The last objective, which is also an effect of the implementation of the Solvency II framework, is an improvement of the international competitiveness between the European insurance companies. Indeed, the amount of the capital requirement strictly depends on the quality of the risk management, implying that the insurers will carry out their business with the optimal amount of capital.

3.1.4 Lamfalussy Approach

One of the objectives of the Solvency II project is the regulatory harmonization which can be reached through a complex regulation system structured in different levels, denoted as the Lamfalussy approach. At the level 1, there is the Solvency II directive[9], adopted by the council of the European Union and by the European Parliament in November 2009. It defines only the general framework, structure, and principles of Solvency II (principle-based approach). At that time, it was not defined the implementation date, which was set only in December 2013 to be 1st January 2016. In addition, revisions to the Solvency II directive, including the application date, are contained in Omnibus II directive (April 2014). At the level 2, there are the implementing measures and regulations issued by the European Commission. They specify rules and detailed measures on the new solvency regime. The most important regulation is the Delegated Acts published in January 2015, explaining also in quantitative terms the details of the first pillar's calculation. At the level 2.5, there are the technical standards proposed by EIOPA and adopted by the European Commission. They include the Regulatory Technical Standards and the Implementing Technical Standards. At level 3, there are the guidelines by EIOPA aiming at ensuring consistent implementation and cooperation between Member States of the European Union. They regard specifications of rules directly issued by EIOPA and since EIOPA has not legislative power, a national implementation is needed. Finally, at level 4, there is the rigorous enforcement of Community legislation by the Commission.

3.1.5 Three Pillars Structure

One of the main characteristics of the Solvency II framework is the three-pillars structure:

- *Pillar I*

  The Pillar I involves the quantitative requirements needed to check whether the insurance company has sufficient resources in order to cover the risks to which it is exposed. In order to comply with the requirement, its Solvency Ratio (i.e., the ratio between Eligible Own Funds and the Capital Requirement) shall be not lower than 100%.

[9]Directive 2009/138/EC.

- *Pillar II*: governance requirements and supervisory review requirements.

  It aims at implementing a sound and prudent business management through a proper system of governance. In addition, it includes also an ORSA process.

- *Pillar III*

  The Pillar III regards the reporting and disclosure requirements which should strengthen the market discipline.

3.2 Pillar I

The calculation of the Solvency Ratio involves the computation of the Own Funds and of the capital requirement. In particular, the Solvency II framework prescribes two risk measures: the Solvency Capital Requirement and the Minimum Capital Requirement.
Before introducing the Own Funds (OF) and their different configuration, it is crucial the definition of evaluation criteria for assets and of liabilities since they affect the value of the OF.

3.2.1 Evaluation Criteria

The Solvency II directive prescribes the Full Fair Value approach, according to which assets and liabilities (except technical provisions) must be evaluated at their current exit price (Fair Value) using market-consistent information and methodologies (Article 75[10] of Directive 2009/138/CE). The regulation states that insurers should use IFRSs if IFRSs are consistent with the fair value and that insurers should use fair value if IFRSs are not consistent with the fair value.
The article 76 of the Solvency II directive proposes a technical provisions' evaluation very closed to the definition of the current exit value:
≪*The value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking*≫.
Conversely to assets, technical liabilities are not listed in the market and, consequently, a market value is not available. Therefore, it is not possible to follow a mark to market approach. Only a mark to model approach is feasible, by introducing a methodology consistent with the definition of current exit value. In addition, the evaluation shall be market-consistent, meaning that the undertaking must use all the information coming from the

[10]"Assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction≫. ≪Liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction".

financial markets and the underwriting risks data. The introduction of a market-consistent evaluation aims at limiting as much as possible the subjectivity and the discretion related to the hypotheses typically made in the Local GAAP technical provisions' evaluation. The methodology prescribed by the Solvency II framework in order to evaluate technical liabilities depends on the type of the liability itself. In particular, the distinction is made between:

- *Hedgeable liabilities.*
  If future cashflows related to the insurance or reinsurance obligations can be replicated reliably using financial instruments for which a reliable market value is observable, the value of technical provisions associated with those future cash-flows shall be determined on the basis of the market value of those financial instruments (replicating portfolio technique[11]). Unlucky, the majority of the insurance cashflows are not hedgeable and, therefore, the replicating portfolio technique can be applied only for very peculiar insurance contracts as unit-linked and index-linked without guarantees.

- *Non-hedgeable liabilities.*
  The value of technical provisions shall correspond to the sum of a best estimate and of a risk margin. The Best Estimate (BE) shall be equal to the probability weighted average of future cashflows, taking into account the time value of money by using the relevant risk-free rate term structure ($r(0,t)$), monthly calculated and published by EIOPA:
  $$BE = \sum_{t=1}^{T} \frac{E(F_t)}{(1 + r(0,t))^t}$$
  In addition, it has to be computed on the basis of credible and current information and realistic assumptions, by considering also the financial guarantees existing on the insurance contracts under evaluation. In the projection of cashflows, all the cash in and out flows needed for settling the insurance liability must be properly taken into account over its lifetime. In principle, the BE should be evaluated for each contract or claim, but the regulation allows to make the cashflow projection on groups of contracts suitably defined.

[11] If either a financial instrument or an insurance contract has a set of future random cash flows F that is hedgeable, then its fair value is equal to the market price of the correspondent replicating portfolio:
$$F = \sum_{i}^{N} x_i R_i \rightarrow V(F) = \sum_{i}^{N} x_i V(R_i)$$
It is a direct consequence of the Law of one price, which requires the hypothesis of absence of arbitrage opportunity.

Finally, the Best Estimate has to be calculated gross of reinsurance, without any deduction for recoverables arising from reinsurance contracts and special purpose vehicles. The recoverables shall take into account the expected losses due to default of the counterparty, defined as the loss given default of the counterparty weighted by the probability of default of the reinsurer/special vehicle. The loss given default corresponds to expected present value of the change in cash-flows underlying the recoverables, resulting from a default of the counterparty at a certain point in time. It is evaluated market consistently and the probability of default depends on the rating of the counterparty.[12] When the amount of the recoverable is estimated, it is possible to calculate a net value of the Best Estimate defined as the difference between the gross BE and the recoverables.

The second term involved in the computation of technical provisions is the Risk Margin (RM). There are different methodologies available for computing the RM, but in order to avoid discretion in the evaluation, the Solvency II directive imposes a Cost of Capital approach, as defined in article 77, being more in line with the definition of current exit value. In particular, the risk margin should be calculated by determining the cost of providing an amount of eligible own funds equal to the SCR necessary to support the insurance obligations over the lifetime thereof:

$$RM = \sum_{t=0}^{T} CoC \cdot \frac{SCR_t}{(1 + r(0, t+1))^{t+1}}$$

The first step in the computation of the RM is the projection of the Solvency Capital Requirement from the evaluation date (t=0) to the total run-off of the overall liabilities ($t = T$). SCR_t includes only a specific set of risks, respectively: the underwriting risk for existing business, the default risk[13], the operational risk and the material market risk. Secondly, the cost of capital, $CoC \cdot SCR_t$, for each year must be quantified. The cost of capital rate CoC is the lack of profitability a shareholder suffers by investing in the Own Funds of the insurance undertaking, since they cannot be invested freely but according to some constraints set by the regulation. It can be defined as the difference between the rate of return obtainable on the market through an investment with a risk profile like the insurer's one and the risk-free rate. In the Solvency II framework, the cost of capital rate is fixed at 6% for all the companies, regardless

[12] Worse is the rating, higher is the probability of default.

[13] with respect to reinsurance contracts and any other material exposures which are closely related to the insurance and reinsurance obligations

their risk profile. Finally, the cost of capital must be properly discounted at the basic risk-free rates without any volatility adjustment or matching adjustment. Notice that the discounting is applied for a year more than the period to which the SCR refers, since the SCR at the generic time t will be used for covering risks in t+1.

Finally, the RM must be computed separately for the Life business and the Non-Life business. A calculation segment[14] specific would be more in line with the current exit value's definition, but it is very time-consuming (a lot of assumptions, data, and computations) and it does not allow for any diversification benefit. The main complexity in the RM computation regards the SCR projection, especially for very long-term business. Therefore, the regulation allows for some proxies which make the RM computation more feasible:

1. Approximate the individual risks or sub-risks within some or all modules and sub-modules to be used for the calculation of future SCRs.
2. Estimate all future discounted SCRs "at once", as using an approximation based on the duration approach (mostly applied by Life insurance companies).
3. Approximate the risk margin by calculating it as a percentage of the best estimate defined by the regulation and segment specific.
4. Approximate the whole SCR for each future year, as using a proportional approach (mostly used by Non-Life insurers).

$$SCR_t = SCR_0 \cdot \frac{BE_t^{net}}{BE_0^{net}}$$

   The projected SCR at the generic time t is computed by applying to the current SCR a proportionality factor given by the ratio between the Best estimate net of reinsurance at time t and the best estimate net of reinsurance at the evaluation time. It is implicitly assumed that the SCR decreases in the same proportion the BE[15] does. Since the BE is a (present) expected value, whereas the SCR is a risk measure, it is like assuming that the risk decreases in the same proportion the expected value does. This assumption could be not met in practice, since typically during the first development years the less risky claims are settled and, as a result,

[14]The Annex I of the Delegated Acts list the 12 segments present in the Solvency II framework. The first nine segments regard the Non-Life obligations related to direct insurance business and to the proportional treaties as risk taker, while the last three regard Non-Life obligations, related to non-proportional treaties as risk taker.

[15]The Best Estimate presents a decreasing behaviour since in the technical provisions' evaluation, the existing business is considered.

the expected value decreases more than the risk leading to an underestimation of the RM.

According to the Opinion on Review 2020, EIOPA advices to introduce a new formulation for the calculation of the Risk Margin which tries to fix two unsolved issues:

- The Cost of Capital Approach is sensitive to changes in interest rates, especially for long-term products. Interest rates are involved in the projection of risks and in their discounting such that a decrease in interest rates will increase the risk margin. Consequently, wrong estimates of the long end of the risk-free curve can lead to overestimation or underestimation the risk margin. Therefore, the current Cost of Capital approach could be too much sensitive to interest rates changes and it could lead to unintended consequences and procyclical behaviors of undertakings.
- Due to its complexity, the projection of future SCR is typically approximated by taking a central scenario over the liabilities' lifetime. In this central scenario, an average emergence of risk is assumed, ignoring shock events. The consequence is that SCR_t could be not significantly different from SCR_{t-1}, other than in the run-off of liabilities, and that the dependence of risks over time is completely ignored. For instance, if a loss occurs in one period, the current approach ignores that the future SCRs may be expected to be lower[16]. On average it will take into account the loss also in the future periods, even though it could not happen in future, leading to an overestimation of the projected SCRs. Therefore, it is necessary that the economic approach applied for the projection of future SCRs considers the dependence of risks overtime even if its exact definition can be quite challenging[17].

In order to take into account the time dependence and to reduce the RM sensitivity to interest rates changes, EIOPA Opinion on Review 2020 amends the formula for the calculation of the Risk Margin as follows:

$$RM = CoC \cdot \sum_{t=0}^{T} \frac{\max(\lambda^t, floor) \cdot SCR_t}{(1 + r(0, t+1))^{t+1}}$$

[16] The effect of the time dependence among risks depends also on the characteristics of the undertaking's risk profile. For instance, for some kinds of risks, an emergence of a risk in a period may make further emergences of risks in future more likely.

[17] The estimation of time dependence would require a full stochastic projection of future SCRs throughout the whole lifetime of the insurance obligations.

The future projected SCRs computed under the current approach are adjusted in order to consider the time dependence:

- $SCR_0^{'} = SCR_0$: the current SCR is not adjusted since no risks have emerged.
- For $t \geq 1$, $SCR_t^{'} = \max\left(\lambda \cdot SCR_{t-1}^{'} \cdot \frac{SCR_t}{SCR_{t-1}}, floor \cdot SCR_t\right)$.

  The adjusted future SCRs ($SCR_t^{'}$) are determined under the assumption that the emergence of risk during the interval [t-1, t] leads to an annual reduction of the SCR by the factor:

$$\mu = 1 - \lambda$$

  The maximum function assures that the adjusted $SCR_t^{'}$ is at least equal to a floor percent of the unadjusted SCR. The parameter floor avoids excessive reductions of the adjusted future projected SCRs and it considers that for some risks (as expense risk) the emergence of risks generally does not lead necessarily to a reduction in the risk behavior in future periods.

By the annual reduction accumulated through time, the adjusted SCR decreases smoothly in an exponential way by the factor λ^t with respect to the unadjusted SCRs. From the outputs of the analyses performed, EIOPA notices that the introduction of the floored, exponential and time dependent element λ will reduce the sensitivity of the risk margin with respect to interest rates changes especially for long-term products. The calibration of the parameters λ and $floor$ is quite complicated due to the high volatility in the assessment of time dependence. Since typically the time dependence of risks is assumed to not exceed 2.5%, λ should not be lower than 97.5%. In addition, the accumulated reduction of the projected future SCRs should not exceed 50%. The final values set by EIOPA in its analysis are $floor = 50\%$ and $\lambda = 97.5\%$, which could lead to a reduction of the size and of the volatility of the Risk Margin.

To conclude this paragraph on the evaluation criteria, it could be useful to highlight the two motivations behind the Full Fair value approach:

- The FV is a uniform approach which limits subjectivity and discretion in particular for traded financial instruments. Conversely for the liabilities, the subjectivity is not completely deleted since not always a market value is available.

- The full FV approach allows for a correct measurement of the risks since the SCR is defined as the loss of the BOF in the worst-case scenario.

The main drawback of the full Fair Value approach is its volatility. Being a current value, the Fair Value is highly affected by the financial market conditions. If the financial conditions are very volatile, also the value of assets and liabilities will be very volatile, implying quite unstable value of BOF and consequently of the Solvency Ratio.

3.2.2 Own Funds

The Own Funds are the available resources held by an insurance company in order to cover possible future losses in the perspective of the policyholder. There are different configurations of Own Funds. The Basic Own Funds (BOF) are given by the sum of the excess of assets over liabilities and the subordinated liabilities. Ancillary Own Funds (AOF) consist of items other than BOF which can be called up to absorb losses. Typically, they include unpaid share capital, initial fund that has not been called up, letters of credit and guarantees and any other legally binding commitments received by the insurance undertaking. The amount of AOF to be taken into account in the computation of Own Funds is subject to the supervisory approval. The Total Own Funds (TOF) are simply given by the sum of Basic OF and Ancillary OF. The TOF must be reduced by the future dividends the company expects to pay in the next year since they will be no more available for covering possible losses in the perspective of the policyholders.

Once assessed the TOF, it is important to properly tier them in order to consider their ability to fully absorb future losses on a going basis. In particular, Own Funds are classified into three tiers:

- Tier 1: OF with high quality and fully absorbing losses.
- Tier 2: intermediate quality own fund items.
- Tier 3: low quality own funds items.

The excess of assets over liabilities belong to the Tier 1 since they are resources fully at disposal of the insurance undertaking. Subordinated liabilities can belong to a particular tier according to their contractual characteristics. Conversely, AOF can be only of Tier 2 or 3. The characteristics determining the tier are the permanent availability, the subordination, sufficient duration, absence of encumbrances, of incentives to redeem and of mandatory servicing costs. In addition, the sum of paid-in preference share and of the related share premium account, of paid-in subordinated mutual member accounts and of paid-in subordinated liabilities must be less than 20% of total Tier 1 Funds.

In order to move from the Total Own Fund to the Eligible Own Funds, the directive puts some quantitative limits. The eligible amount of own funds to cover the Solvency Capital Requirement shall be equal to the sum of the amount of Tier 1, the eligible amount of Tier 2 and the eligible amount of Tier 3, according to the following constraints:

- the proportion of Tier 1 items in the eligible own funds shall be higher than one third of the total amount of eligible own funds.
- the eligible amount of Tier 3 items shall be less than one third of the total amount of eligible own funds.
- the eligible amount of Tier 1 items shall be at least one half of the Solvency Capital Requirement.
- the eligible amount of Tier 3 items shall be less than 15% of the Solvency Capital Requirement.
- the sum of the eligible amounts of Tier 2 and Tier 3 items shall not exceed 50% of the Solvency Capital Requirement.

The eligible amount of Basic Own Funds to cover the Minimum Capital Requirement shall be equal to the sum of the amount of Tier 1 and the eligible amount of basic own fund items classified in Tier 2, subjected to all of the following quantitative limits:

- the proportion of Tier 1 items in the eligible basic own funds shall be higher than one half of the total amount of eligible basic own funds.
- the eligible amount of Tier 1 items shall be at least 80% of the Minimum Capital Requirement.
- the eligible amount of Tier 2 items shall not exceed 20% of the Minimum Capital Requirement.

3.2.3 SCR

The Solvency Capital Requirement "shall correspond to the Value at Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99,5% over a one-year period"[18]. In other words, it is the level of patrimonial resources that keeps the company solvent in a Worst-Case Scenario subject to a 99.5% confidence level over one-year period. The SCR takes into account all quantifiable risks and, at least, the Non-Life

[18] Art.101.3, Directive 2009/138/EC.

underwriting risk, the Life underwriting risk, the health underwriting risk, the market risk, the credit risk and the operational risk. It shall cover existing business, as well as the new business expected to be written over the following 12 months on a going concern. The SCR has to be computed by insurance undertaking at least once a year or every time the risk profile deviates significantly from the assumptions underlying the last reported Solvency Capital Requirement.

There are four possible approaches for calculating the Solvency Capital Requirement:

1. *Standard Formula.*

   The risk map, the aggregation approach, modules, and the aggregation parameters are fixed by the second level regulation. The vast majority of Italian insurance companies applies it.

2. *Full internal Model.*

   The risk map, the aggregation approach, modules, and the aggregation parameters are developed internally by the company according to its peculiar characteristics.

3. *Standard Formula with the Undertaking Specific Parameters (USP).*

   The risk map, the aggregation approach and the aggregation parameters are fixed by the second level regulation, but some parameters in the underwriting risk modules may be company specific.

4. *Partial Internal Model.*

   The company applies the Standard Formula for some modules and for others an Internal Model.

For applying USP, Full internal model or partial internal model, it is required an authorization by the Supervisory authority. From now on, the calculation of the SCR according to the Standard Formula is depicted.

SCR Computation - Standard Formula. The Solvency Capital Requirement in the Standard Formula is based on a modular structure. The Basic SCR is made up of six macro-modules, respectively the market risk, the health underwriting risk, the Non-Life underwriting risk, the Life underwriting risk, the counterparty default risk, and the SCR intangible. Each module is divided into submodules as Figure 11 shows. The SCR is computed for each submodule according to two possible approaches, the scenario-based approach, or the factor-based approach, with fixed parameters/shocks, as the Delegated

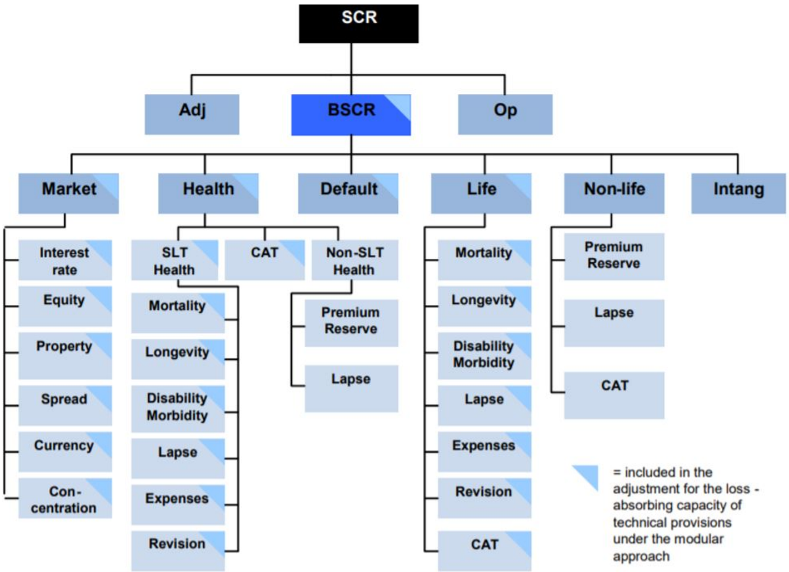

Figure 11: Modular structure of the Standard Formula, IVASS

Acts states. In a scenario-based approach, a stressed scenario for the specific submodule is calibrated at 99.5% confidence level. In order to compute the SCR, it is necessary to compute the value of assets, liabilities, and Basic Own Funds in the normal scenario and in the stressed one. The SCR is defined as the loss in the BOF, computed as the difference between the current BOF and the stressed BOF:

$$SCR = \max[\Delta BOF|Stress; 0]$$

For some risk submodules there could be more than one stressed scenario and the SCR is given by the scenario in which the variation in the BOF is higher. Life Underwriting risk, Market risk and Cat risk for Non-Life business are typical examples of a Scenario-Based Approach.

In a factor-based approach, the SCR is computed by applying specific percentages to volume of risk exposure as premiums and technical provisions:

$$SCR = f(TP_s; Premiums)$$

It is based on single risks exposures and risk factors, which are calibrating considering the tail of the distribution, the trend, and the volatility effect. Premium&Reserve risk and Operational risk modules are two typical examples of Factor-Based Approach.

The scenario-based approach is much more in line with the definition of Value at Risk, but, in some cases, it is too difficult to be applied (for instance for Non-Life premium and reserve risk). Therefore, the factor-based approach is more feasible since it takes into account the dimension of the company through some accounting variables without measuring with precision the risks. In some cases, like for the premium and reserve risk, parameters could be calibrated using internal data (see USP in paragraph 3.5.2).

In order to compute the Basic SCR, a two-step aggregation, based on linear correlation coefficients, is compulsory. The first aggregation regards the SCRs of submodules for the same risk module. The second-step aggregation allows to compute the Basic SCR as:

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j} + SCR_{intangibles}$$

Five risk modules (Life Underwriting risk, Non-Life underwriting risk, Health Underwriting risk, counterparty default risk and market risk) are aggregated by applying the aggregation formula mathematically proved for the standard deviation. All the linear correlation coefficients involved in the two-step's aggregation are provided by the second-level regulation (Table 1).

	Market	**Default**	**Life**	**Health**	**Non-Life**
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-Life	0.25	0.5	0	0	1

Table 1: Correlation Matrix for the macro-risks.

Since they are typically lower than 1, a diversification benefit is allowed. Conversely, notice that the SCR for intangibles is added, by assuming full correlation and no diversification benefit with respect to the other modules.
Finally, the SCR is defined as:

$$SCR = BSCR - adjustment + SCR_{operational}$$

Adjustments consider the loss absorbing capacity of profit sharing of Life insurance contracts and deferred taxes. In this last computation, no diversification benefit is allowed.

Non-Compliance with the SCR. If the insurance undertaking observes that the SCR is no longer complied with, or where there is a risk of non-compliance in the following three months, it shall immediately inform the supervisory authority. Within two months from the observation of non-compliance with the Solvency Capital Requirement, it must submit a realistic recovery plan to the supervisory authority. Finally, the compliance with the SCR shall be restored within six months, by increasing eligible own funds or by decreasing the risks. Typical measures applied for increasing the EOF are the issue of new equity, of new subordinated debts or new Ancillary Own Funds. The other possibility for getting a Solvency Ratio higher than 1 is to decrease the Solvency Capital Requirement, by reducing the risk exposure. In other words, the company can try to improve the diversification (quite difficult in practice) or to decrease market risks with a change in the asset allocation. The other possibility is to decrease the underwriting risks through a reinsurance contract, or by selling the insurance portfolio or by modifying the business plan and/or the underwriting policies.

In addition, in case of exceptional adverse insurance market situations, the supervisory authority may extend the recovery period from six months up to a maximum of seven years.

3.2.4 MCR

The MCR is defined as the minimum level of security below which the amount of financial resources should not fall[19]. Indeed, it corresponds to an amount of eligible basic own funds below which policyholders and beneficiaries are exposed to an unacceptable level of risk if undertaking will go on to carry out the business. It must be computed every three months in a clear and simple manner applying a factor-based approach. In addition, the MCR shall be calculated as a linear function of a set or sub-set of the following variables, measured net of reinsurance: the undertaking's technical provisions, written premiums, capital at risk, deferred tax, and administrative expenses. The linear function is calibrated according to the Value at Risk of the basic own funds subject to a confidence level of 85% over a one-year period. Further details are specified in the Delegated Act. The MCR cannot fall below 25% of the SCR and it cannot exceed 45% of the SCR.

Finally, the regulation sets some absolute floors, relevant only for very small companies:

- EUR 2.200.000 for Non-Life insurance undertakings, including captive insurance undertakings.
- EUR 3.200.000 for Life insurance undertakings, including captive insurance undertakings.
- EUR 3.200.000 for reinsurance undertakings, except in the case of captive reinsurance undertakings, in which case the Minimum Capital Requirement shall be not less than

[19] Art. 129, Directive 2009/138/EC.

EUR 1.000.000.

In case of non-compliance with the MCR, the measures imposed by the regulation are stricter than for the non-compliance with the SCR since the level of EOF is too low for offering sufficient protection to policyholders and beneficiaries. In particular, the insurance undertakings shall immediately inform the supervisory authority where they observe that the Minimum Capital Requirement is no longer complied with or where there is a risk of non-compliance in the following three months. Within one month from the observation of non-compliance with the Minimum Capital Requirement, the undertaking shall submit a short-term realistic finance scheme to the supervisory authority to restore the EOF within three months from that observation or to reduce its risk profile to ensure compliance with the Minimum Capital Requirement. The supervisory authority shall withdraw the authorisation in the event that the undertaking does not comply with the Minimum Capital Requirement, or if it considers that the finance scheme submitted is manifestly inadequate or if it fails to comply with the approved scheme within three months from the observation of non-compliance with the Minimum Capital Requirement.

3.3 Pillar II

As the financial crisis in 2009 has shown, setting only quantitative requirements is not sufficient in order to avoid crisis since a lot of moral hazard problems can arise from the exaggerate risk exposure assumed by the top management. In addition, not all the risks the company is exposed to can be measured in a quantitative way. Therefore, the Solvency II framework sets up also a second Pillar involving qualitative requirements, in order to encourage companies to develop and use better techniques for internal control and risk management.

A well-structured system of governance is crucial for getting a sound and prudent management of the business and it has to be proportionate to the nature, scale, and complexity of the operations of the insurance undertakings (proportionality principle). In particular, a great role is played by the key functions, respectively the risk management function, the compliance function, the actuarial function, and the internal audit function. The internal control and risk management system of an undertaking is structured on three levels. At level 1, there are operational units, which are monitored by three key functions at level 2: risk management function, actuarial function, and compliance function. The risk management must identify, measure, monitor, manage and report, on a continuous basis the risks (also the ones not included in the computation of the SCR), at an individual and at an aggregated level, to which the undertaking is or could be exposed, and their interdependencies. It must be effective and well-integrated into the organizational structure and in the decision-making processes of the insurance undertaking. The actuarial function monitors the computation of the Technical Provisions, provides an opinion on the overall underwriting policy and on the adequacy of reinsurance arrangements and checks the data quality. Finally, the compliance function identifies on a continual basis the regulations to be applied by the undertaking and assesses their impact on the company's processes and procedures in order to prevent the risk of non-compliance. The second level functions are monitored by the internal audit (third level) which evaluates the adequacy, the efficiency and effectiveness of the internal control system and other elements of the system of governance.

In addition, the Second Pillar involves an Own Risk and Solvency Assessment (ORSA) under which the company has to make a risk and solvency evaluation over a mid-term period (3 years). It has to be assessed the overall solvency requirement, taking into account the specific risk profile, the approved risk tolerance limits, and the business strategy of company. Every possible deviations of the risk profile of the company from the assumptions underlying the SCR calculated using the standard formula or an internal model (partial or full), must be considered. ORSA must be an integral part of the business strategy and it must be taken into account in strategic decisions. It has to be performed periodically and every time there is a significant change in the risk profile.

3.4 Pillar III

In order to strengthen the market discipline, insurance undertakings must disclose publicly on annual basis a report on their financial and solvency conditions. The great introduction of Solvency II is that the company has not only to disclose its financial and solvency position to the supervisory authority, as was made in the past, but also to the market. In such a way, the asymmetry of information typically present in the relationship insurer-insured should reduce (increasing the transparency). In addition to the private report submitted to the supervisory authority, denoted as Solvency Supervisory Report, the undertaking has to publish on its website publicly the solvency and financial conditions' report (SFCR). The latter shall contain a description of a list of information including the business, the performance, the system of governance, the capital management and much more details listed in the article 51 of the Solvency II directive. Since the company has to disclose its solvency situation to the market, the policyholders are able to clearly evaluate the company's security, making the market discipline much stronger. Therefore, it is desirable for the undertaking to show a favourable and solvent situation in order to favourably impress market participants and policyholders. By the Review 2020, EIOPA advices to simplify the content of the Regular Supervisory Report and of the Solvency and Financial Condition Report in order to reduce the burden related to the preparation of those reports. In addition, EIOPA introduces two different parts of the SFCR, one dedicated to policyholders and the other one addressed to other users.

After having introduced the general framework of Solvency II, the formulas for computing the SCR for the Non-Life Underwriting Risk and the Counterparty Default Risk are presented.

3.5 Non-Life Underwriting Risk

The Non-Life underwriting risk module must reflect the risk arising from Non-Life insurance obligations, related to the perils covered and to the processes used in the business conduct. It includes three risk submodules: Premium&Reserve Risk, the Lapse Risk, and the Cat Risk. The Lapse risk is very negligible for Non-Life business, conversely to the Life business, but it could be a bit more significant in case of multiannual coverages. According to the Quantitative Impact Study 5 (QIS5), the Cat risk and the Premium&Reserve risk were depicted by quite similar proportion (50% vs 70%[20]), whereas nowadays the situation is completely changed since Cat risk is much less relevant (15-20%) due to the spread of Cat reinsurance treaties. The SCR for Premium&Reserve submodule is computed according to a factor-based approach, whereas the SCR for Lapse risk and the Cat Risk on the basis of the scenario-based approach calibrated according to a VaR 99.5% on one-year time horizon. The premium and reserve risk regards "the risk of loss, or of adverse change in the value of insurance liabilities, resulting from fluctuations in the timing, frequency and severity of insured events and in the timing and amount of claim settlements"[21]. In particular, the premium risk represents the risk to have insufficient pricing coming from the policies which will be underwritten (renewals included) in the following year and of the existing business still in force in order to cover the claim amount and the expenses. EIOPA clarified that the premium risk also includes the expense risk (i.e., the risk that the real expenses afforded by the company are higher than the expense loading). The expense risk is quite negligible

[20] Proportions computed before diversification.
[21] Article 105, 2a, Directive 2009/138/EC.

for Non-Life coverages, since they are usually on annual basis, but it can become more significant in case of multiannual coverages. The Reserve risk represents the risk of claim reserve at the date of reference to be insufficient for the time horizon of one year.
The Lapse risk is “the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level or volatility of the rates of policy lapses, terminations, renewals and surrenders”[22].
Finally, the CAT risk represents the risk of losses or unfavorable variations in the insurance liabilities value coming from high volatility in assumptions for pricing and reserving due to exceptional or extreme events.

Once computed the SCR for each risk submodule, the total SCR for Non-Life Underwriting risk is computed by applying the usual aggregation formula and by assuming a small positive correlation between Cat Risk and Premium&Reserve risk (and uncorrelation among all the other submodules):

$$SCR_{NL} = \sqrt{SCR^2_{P\&R} + SCR^2_{CAT} + SCR^2_{Lapse} + 2 \cdot 0.25 \cdot SCR_{P\&R} \cdot SCR_{CAT}}$$

In the following paragraphs, only Premium&Reserve risk is depicted. The SCR for the Premium&Reserve Risk submodule is defined as:

$$SCR_{P\&R} = 3 \cdot \sigma_{nl} \cdot V_{nl}$$

where:

- σ_{nl} is the aggregated standard deviation for Non-Life premium and reserve risk.
- V_{nl} is a volume measure.
- 3 is a multiplier.

3.5.1 Volume Measure

The volume measure V_{nl} for Non-Life premium and reserve risk shall be equal to the sum of the volume measures for premium and reserve risk for each segment. The volume measure of a particular segment s shall be defined as:

$$V_s = \big(V_{(prem,s)} + V_{(res,s)}\big) \cdot (0.75 + 0.25 \cdot DIV_s)$$

[22]Article 105, 3f, Directive 2009/138/EC.

where $V_{(prem,s)}$ is the volume measure for premium risk for the segment s, $V_{(res,s)}$ is the volume measure for reserve risk for the segment s and DIV_s is an index considering the geographical diversification. The volume measure for the premium risk for the segment s shall be computed as:

$$V_{(prem,s)} = \max(P_s, P_{(last,s)}) + FP_{(existing,s)} + FP_{(future,s)}$$

P_s is defined as an estimate of the premiums to be earned by the insurance undertaking in the segment s during the following 12 months, whereas $P_{(last,s)}$ denotes the premiums earned by the insurance undertaking in the segment s during the last 12 months. In addition, $FP_{(existing,s)}$ involves the expected present value of premiums to be earned by the insurance undertaking in the segment s after the following 12 months for existing contracts. Finally, $FP_{(future,s)}$ denotes:

- for contracts which initial term is one year or less, the expected present value of premiums to be earned by the insurance company in the segment s, excluding the premiums to be earned during the 12 months after the initial recognition date.
- for all contracts with initial term more than one year, the amount equal to 30% of the expected present value of premiums to be earned by the insurance undertaking in the segment s after the following 12 months.

Insurance undertakings can adopt an alternative calculation for the volume measure of premium risk:

$$V_{(prem,s)} = P_s + FP_{(existing,s)} + FP_{(future,s)}$$

This possibility is admissible only if the undertaking meets the following conditions:

- According to decisions of its administrative, management or supervisory body, the undertaking's earned premiums in the segment s during the following 12 months will not exceed P_s.
- the undertaking has established effective control mechanisms to ensure that the limits on earned premiums will be met and it has informed its supervisory authority about this decision.

In both calculation methodologies, the premium must be net of the reinsurance premiums. The volume measure for the reserve risk for the segment s is defined as the best estimate for claims outstanding for that segment. This amount should be reduced by the recoverable from reinsurance contracts and special purpose vehicles.

For all segments, the factor for geographical diversification of a particular segment s shall be computed as:

$$DIV_s = \frac{\sum_r \left(V_{(prem,s,r)} + V_{(res,s,r)}\right)^2}{\left(\sum_r \left(V_{(prem,s,r)} + V_{(res,s,r)}\right)\right)^2}$$

The summations at the numerator and at the denominator involve all the geographical areas defined by the Delegated Acts. $V_{(prem,s,r)}$ and $V_{(res,s,r)}$ denote respectively the volume measure for premium risk and the volume measure for the reserve risk of the segment s and the region r. The factor for geographical diversification shall be equal to 1:

- for Credit and Suretyship (segment 6) and for non-proportional treaties (segments 10, 11 and 12).
- if insurance undertakings use an undertaking-specific parameter for the standard deviation for Non-Life premium risk and/or Non-Life reserve risk of the segment.

If $DIV_s = 1$, there is no geographical diversification, and the total volume is simply given by the sum of the premium volume estimated for the next year and the loss reserve. As the factor decreases (till the minimum value 0), the diversification increases up to a maximum benefit of 25%.

3.5.2 Standard Deviation

The aggregated standard deviation for the Non-Life premium and Reserve risk comes from a two-steps aggregation. The first step aims at computing σ_s by aggregating the volatility factors of the premium risk and of the reserve risk for the specific segment:

$$\sigma_s = \frac{\sqrt{\sigma^2_{(prem,s)} \cdot V^2_{(prem,s)} + \sigma_{(prem,s)} \cdot V_{(prem,s)} \cdot \sigma_{(res,s)} \cdot V_{(res,s)} + \sigma^2_{(res,s)} \cdot V^2_{(res,s)}}}{V_{(prem,s)} + V_{(res,s)}}$$

$V_{(prem,s)}$ and $V_{(res,s)}$ denote the volume measure for premium risk and the volume measure for reserve risk of segment s. Weighting the standard deviations by the volume measures is a way to consider which is the most relevant risk between the reserve risk and the premium risk for the specific segment. For instance, in long-tail segment as 4 and 8, the reserve risk is typically much more important than the premium risk, conversely to short-tail segment (as 5). $\sigma_{(prem,s)}$ and $\sigma_{(res,s)}$ are respectively the standard deviation for Non-Life premium risk and the standard deviation for the Non-Life reserve risk of segment s. For calculating them, insurance undertakings can refer to the market-wide volatility factors (Table 2), specified by the Delegated Acts or apply an Undertaking Specific Parameter Approach (USP) subject to the supervisory authority's approval.

Segment s	$\sigma_{(prem,s)}$	$\sigma_{(res,s)}$
1) Motor vehicle liability insurance and proportional reinsurance	10%	9%
2) Other motor insurance and proportional reinsurance	8%	8%
3) Marine, aviation and transport insurance and prop. reinsurance	15%	11%
4) Fire and other damage to property insurance and prop. reins.	8%	10%
5) General liability insurance and proportional reinsurance	14%	11%
6) Credit and suretyship insurance and proportional reinsurance	19%	17.2%
7) Legal expenses insurance and proportional reinsurance	8.3%	5.5%
8) Assistance and its proportional reinsurance	6.4%	22%
9) Miscellaneous financial loss insurance and prop. reinsurance	13%	20%
10) Non-proportional casualty reinsurance	17%	20%
11) Non-proportional marine, aviation and transport reinsurance	17%	20%
12) Non-proportional property reinsurance	17%	20%

Table 2: Market-wide volatility factors.

It is implicitly assumed a correlation coefficient equal to 0.5 between $\sigma_{(prem,s)}$ and $\sigma_{(res,s)}$ for all the segments which takes into account the diversification benefit.

For each segment, the standard deviation for Non-Life premium risk shall be equal to the product of the standard deviation for Non-Life gross premium risk and the adjustment factor for non-proportional reinsurance. For segments 1, 4 and 5, the adjustment factor for non-proportional reinsurance shall be equal to 80%, while for all the others to 100%. The regulation allows an Undertaking Specific Parameter approach also for the adjustment factor (see paragraphs 3.5.2).

The second aggregation is among volatility of different segments in order to compute the standard deviation of the Non-Life premium and reserve risk:

$$\sigma_{nl} = \frac{1}{V_{nl}} \cdot \sqrt{\sum_{r,c} Corr_{r,c} \cdot \sigma_r \cdot \sigma_c \cdot V_r \cdot V_c}$$

V_{nl} is the volume measure for Non-Life premium and reserve risk. The summation involves all the possible combinations (r, c) of the segments. The correlation $Corr_{r,c}$ between the segments r and c is defined by the second-level regulation. σ_r and σ_c denote respectively the standard deviations for Non-Life premium and reserve risk of segments r and c, whereas V_r and V_c are the volume measures of segments r and c.

Undertaking Specific Parameter (USP). The regulation allows insurance undertakings to replace the following subset of standard parameters by undertaking-specific parameters:

- the standard deviation for Non-Life premium risk.
- the standard deviation for Non-Life reserve risk.
- the adjustment factor for non-proportional reinsurance treaties provided that there is a recognizable excess of loss reinsurance contract or a recognizable stop loss reinsurance contract for that segment.

Data used to calculate undertaking-specific parameters must be declared as complete, accurate and appropriate according to the constraints set by the regulation. The insurance undertaking shall submit a written application for approval of the use of USP to the supervisory authority. In duly justified circumstances, they may revert to the standard parameters, by sending a request to the supervisory authority, stating the reasons for the inappropriateness of the USP and providing documentary evidence for this, but the revocation is of competence of the supervisory authority.

Regarding the standard deviations of premium risk and reserve risk, the final volatility factor to be applied is given by a weighted average between the market-wide parameter and the USP parameter estimated using company's empirical data:

$$\sigma_{(prem,s)} = c_s \cdot \sigma_{(U,prem,s)} + (1 - c_s) \cdot \sigma_{(M,prem,s)}$$

$$\sigma_{(res,s)} = c_s \cdot \sigma_{(U,res,s)} + (1 - c_s) \cdot \sigma_{(M,res,s)}$$

For estimating $\sigma_{(U,prem,s)}$ and $\sigma_{(U,res,s)}$, the Delegated Acts provides one method for the Premium Risk based on Loss Ratio volatility and two methodologies for the reserve risk, respectively one based on the run-off volatility and the other one on the Merz and Wuthrich formula. The company has to prove that the assumptions underlying the methodology are met by its data which must be of sufficient quality.

The weights c_s are the credibility factors defined by the regulation distinguishing between segments 1,5, 6 and the other ones (Tables 3 and 4).

Time lenghts in years	5	6	7	8	9	10
Credibility factor c	34%	43%	51%	59%	67%	74%

Time lenghts in years	11	12	13	14	≥ 15
Credibility factor c	81%	87%	92%	96%	100%

Table 3: Credibility factors for segments 1, 5 and 6.

The credibility factor is related to the time length in year of data used in the calibration which depends on the type of parameter to be replaced and on the methodology applied (if

Time lenghts in years	5	6	7	8	9	≥10
Credibility factor c	34%	51%	67%	81%	92%	100%

Table 4: Credibility factors for segments other than 1, 5 and 6.

the regulation allows for more than one methodology, as for the reserve risk). Larger is the time series, higher is the credibility c_s. In case of full credibility (given by at least 15 years of data for segments 1, 5 and 6 and by at least 10 years of data for the others), the parameter is completely company-specific, and the market-wide parameter does not play any role. For all the segment, at least 5 years of data are needed, in order to get a minimum credibility of 34%.

The USP for the adjustment factor can be applied only if the treaty is recognizable. An XL reinsurance contract or a SL reinsurance contract for a segment is defined recognisable if it meets the following conditions:

- the XL treaty shall provide for complete compensation up to a specified limit or without limit for losses, larger than a specified retention, related to single insurance claims or to all insurance claims under the same policy during a specified time period. The SL treaty must provide for complete compensation for aggregated losses of the ceding undertaking related to all insurance claims in the segment during a specified time period and larger than a specified retention (and up to a limit, if present).
- cover all insurance claims that the insurance undertaking may incur in the segment during the following 12 months.
- allow for a sufficient number of reinstatements such that it is possible to cover all claims of multiple events incurred during the following 12 months.
- meet the requirements of the techniques of risk mitigation.

The adjustment factor for non-proportional treaty coming from an USP approach should be computed as follows:

$$NP_{USP,s} = c_s \cdot NP_s^{'} + (1 - c_s) \cdot NP_s$$

It is a weighted average between a value estimated through insurer data ($NP_s^{'}$) by applying a predefined methodology and a market-wide value (NP_s) equal to 80% for segments 1, 4 and 5 and to 100% for all the other ones. c_s are the same credibility factor defined for premium risk and reserve risk (Tables 3 and 4). The data to be used in the calibration of the USP adjustment factor for an XL treaty consists of the ultimate claim amounts of insurance claims, denoted with Y_i, reported to the insurance undertaking in segment s during the last financial years, separately for each insurance claim. By considering an XL treaty with retention b_1 and limit b_2, $NP_s^{'}$ is computed as:

$$NP_s^{'} = \sqrt{\frac{\omega_1 - \omega_2 + \omega + 2 \cdot (b_2 - b_1) \cdot (\mu_2 - \mu_1)}{\omega}}$$

whereas, if the XL has no limit, the formula degenerates to:

$$NP_s^{'} = \sqrt{\frac{\omega_1}{\omega}}$$

μ and ω denote the first and second moment, respectively, of the claim amount distribution evaluated over the last n years gross of reinsurance:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

$$\omega = \frac{1}{n} \sum_{i=1}^{n} Y_i^2$$

ω_1 is the estimate of the quadratic mean of the claim amount net of reinsurance derived by evaluating the effect of reinsurance retention on a lognormal distribution. The formulas of $\mu_1, \mu_2, \omega_1, \omega_2$ are specified by the Delegated Acts. The main drawback of this formulation is the complete ignorance of the systematic volatility, which is typically present in practice, implying typically an overestimation of the benefits obtained by the undertaking from the reinsurance treaty.

The data involved in the estimation of $NP_s^{'}$ for a Stop Loss treaty shall consist of the aggregated annual losses of insurance claims that were reported to the insurance undertaking in segment s during the last financial years. In absence of priority, the $NP_s^{'}$ is defined as:

$$NP_s^{'} = \sqrt{\frac{\omega - \mu_1^2}{\omega - \mu^2}}$$

In presence of a limit, the formula becomes:

$$NP_s^{'} = \sqrt{\frac{\big(\omega_1 - \omega_2 + \omega + 2 \cdot (b_2 - b_1) \cdot (\mu_2 - \mu)\big) - (\mu_1 + \mu - \mu_2)^2}{\omega - \mu^2}}$$

The values of $\mu_1, \mu_2, \omega_1, \omega_2$ are computed according to the formulas provided by the Delegated Acts on the aggregated claim amount.

Review 2020. According to the Opinion on Review 2020, EIOPA advices to not change the current regulation on the adjustment factor for the recognition of the non-proportional reinsurance for the premium risk. Conversely, it advices to introduce an adjustment factor for the reserve risk in order to take into account the Adverse Development Covers (ADC). ADC are a form of retrospective reinsurance in which the insurer cedes the claims development risk associated with policies from past underwriting periods. In other words, they cover the reserve risk[23] for a defined portfolio or segment. According to the results of the numeric sensitivity analyses performed in 2016 and 2017, EIOPA advised in 2018 to not recognize adverse development covers in the Non-Life underwriting risk of the Standard Formula, since the proposal typically led to underestimation of the real risk and worked properly only for mono-line insurers. In particular, it seems that bigger is the part of the portfolio not covered by the ADC, more the risks are potentially underestimated. Since ADC covers policies from past underwriting periods in run-off, as time passes by, the part of the portfolio covered by ADC decreases and the possible under-estimation increases.

Even though the standard deviation for reserve risk has been calibrated including already the average effect of (non-proportional) reinsurance, stakeholders argued that, given the limited amount of ADCs existing in the market, the net of reinsurance calibration might be misleading. Indeed, further analyses have shown that the initial calibration of the volatility of the reserve risk net of reinsurance has no material difference with respect to a gross of reinsurance calibration. According to the outcomes coming from the numerous analyses later performed, EIOPA advices to recognise adverse development covers with the following limitations:

- Each ADC can only be applied on one specific group of policies (with the same risk characteristics within the same segment), with a separate attachment and detachment point.
- The attachment point shall not exceed $(1+\sigma)$ times Best Estimate reserves.
- The additional reinsurance premium (C) shall not be negative.

The limitation on attachment and detachment point have been introduced in order to reach a proper balance between risk transfer and capital relief. Undertakings should perform on an annual basis, the recalculation of the cover, possible reinsurance recoverable, the attachment point and premium in order to decrease the risk of underestimation due to the decreasing reserves' behaviour.

[23] In particular, the risk that existing claims reserves are not sufficient to cover the insurance obligations.

In addition, EIOPA advises to compute the standard deviation for Non-Life reserve risk of a particular segment as the product of the standard deviation for Non-Life gross reserve risk of the segment and the adjustment factor for non-proportional reinsurance, calculated as:

$$Adjustmentfactor = \frac{A - (B - C) \cdot D \cdot E}{A}$$

where:

- A is the impact on basic own funds of reserve risk scenario as defined under the Standard Formula.[24].
- B is the ADC recovery under reserve risk scenario, computed as the lower of the following:
  - $BE_N \cdot (1 + 3 \cdot \sigma_{(res,s)}) - AP$, where where BE_N denotes the nominal best estimate net reserves covered by the reinsurance structure and AP reinsurance structure attachment point.
  - Reinsurance structure cover size.
- C is the additional reinsurance premium.
- D is the cession to the reinsurer (expressed in percentage).
- E is the prudency factor set to 100%. It is added in order to overcome the risk of possible underestimation. It will be evaluated by EIOPA, based on reported data, on a bi-annual basis.

Finally, EIOPA advises to develop further level 3 guidance on the application of Adverse Development Covers in the Standard Formula in order to clarify the definition of the applicable covers, of the parameters involved and of the way to calculate the appropriate adjustment factor.

3.5.3 The multiplier

The Non-Life capital requirement could be derived as the difference between the VaR at 99.5% level of confidence and the expected value[25] of the probability distribution of the total losses. In the Standard Formula, this distance is approximated by taking a multiple (triple) of the standard deviation. Theoretically, the value of the multiplier should vary according

[24] It is computed as: $3 \cdot Nominal\ BE\ net\ reserves \cdot \sigma_{(res,s)}$

[25] ignoring the impact of the expected technical result.

to the probability distribution of aggregate claims amount. For instance, if the underlying distributional assumption were the Gaussian, the multiplier at level 99.5% would be 2.58. Surely, the normal distribution is not suitable since it ignores skewness, kurtosis, and fat tail, which typically characterize the empirical distribution of the aggregate claims amount. Indeed, in QIS5, it was assumed that the aggregate claims amount follows a Lognormal distribution for which it exists a formula for the multiplier measuring the exact distance between the 99.5-th percentile of a Lognormal distribution and its expected value[26]:

$$\rho(\sigma_{nl}) = \frac{\exp\left[N_{0.995} \cdot \sqrt{\ln(\sigma_{nl}^2 + 1)}\right]}{\sqrt{\sigma_{nl}^2 + 1}} - 1$$

This formula takes into account of the (relative) volatility and of the skewness of the probability distribution[27]. Therefore, when the volatility is closed to 0, also the skewness is closed to 0 and $\frac{\rho(\sigma)}{\sigma}$ is closed to the Gaussian multiplier (2.58). Conversely, larger is the coefficient of variation, higher is the skewness, higher is the multiplier and higher is the SCR. Even though, the QIS5 formula was quite good, a lot of controversies arose at that time in particular from small companies claiming that the capital requirement in that way computed was completely unsustainable. Therefore, the final solution of the regulation is a multiplier equal to 3 which comes from an underlying assumption of positively skewed distribution for the aggregate claims amount. Its shortfall is that it does not depend on the risk profile of the company and on the effective skewness of the distribution. The multiplier 3 is generally advantageous (disadvantageous) for company with very high (low) relative volatility since their capital requirement should be higher (lower) according to their real risk profile. In other words, there is no consideration of the size factor.

In addition, another shortfall of the Standard Formula is the complete ignorance of the expected technical result. Indeed, a technical profit, providing more resources to the reinsurer, should reduce the capital requirement, while a technical loss should increase it. Ignoring the technical result leads to an overestimation of the capital requirement in case of technical profit and to an its underestimation in case of technical loss, which can be quite dangerous.

[26]In QIS5, the capital requirement for Non-Life Premium&Reserve risk was computed as:

$$SCR_{NL} = \rho(\sigma_{nl}) \cdot V_{nl}$$

[27]Recall the relationship between skewness and volatility valid for a Lognormal distribution:

$$\gamma = CV \cdot (3 + CV^2)$$

3.6 Counterparty Default Risk

The other risk module quite affected by a reinsurance contract is the counterparty default risk. Indeed, the insurance company is exposed to it because, in case of the reinsurer's default, it will remain responsible for the whole amount with respect to policyholder. The counterparty default risk shall reflect possible losses due to unexpected default, or deterioration in the credit standing, of the counterparties and debtors of insurance undertakings over the following 12 months. Therefore, it regards risk-mitigating contracts, as reinsurance arrangements, securitisations and derivatives, receivables from intermediaries and all the credit exposures not covered in the spread risk sub-module. In addition, it shall be evaluated separately for each counterparty and regardless the legal form of its contractual obligations. The SCR for the counterparty default risk module is computed by aggregating the SCRs calculated separately for type 1 exposures $\left(SCR_{(def,1)}\right)$ and for type 2 exposures $\left(SCR_{(def,2)}\right)$ as follows:

$$SCR_{def} = \sqrt{SCR^2_{(def,1)} + 1.5 \cdot SCR_{(def,1)} \cdot SCR_{(def,2)} + SCR^2_{(def,2)}}$$

Type 1 include exposures where the counterparty is likely to be listed as for risk-mitigation contracts including reinsurance arrangements, cash at bank and other items[28]. Type 2 exposures shall consist of all credit exposures not covered in the spread risk sub-module and neither in type 1 exposures, as receivables from intermediaries and policyholder debtors. The following list of credit risk shall be not included in the counterparty default risk: the credit risk transferred by a credit derivative, the credit risk on debt issued by special purpose vehicles, the underwriting risk of credit and suretyship insurance or reinsurance and the credit risk on mortgage loans not fulfilling the requirements set by the Article 191 of the Delegated Acts. Here only the SCR computation for type 1 exposures is explored since reinsurance treaties belong to them.

3.6.1 Capital Requirement

The formula of the capital requirement for the counterparty default risk related to exposures of type 1 depends on the value of the standard deviation of the loss distribution of type 1 exposures (σ).

[28] defined in article 189.2 of Delegated Acts.

- If σ is lower than or equal to 7% of the total losses-given-default on all type 1 exposures:

$$SCR_{(def,1)} = 3\sigma$$

- If σ is between 7% and 20% of the total losses-given-default on all type 1 exposures:

$$SCR_{(def,1)} = 5\sigma$$

  Since Review 2018, in order to avoid computational burden, if the article 88 is complied with and if the standard deviation of the loss distribution of type 1 exposures lower than or equal to 20% of the total losses-given default on all type 1 exposures, the SCR for counterparty default risk may be calculated[29] as:

$$SCR_{(def,1)} = 5\sigma$$

- If σ is higher than 20% of the total losses-given-default on all type 1 exposures, the capital requirement for counterparty default risk on type 1 exposures shall be equal to the total losses-given-default on all type 1 exposures.

The standard deviation of the loss distribution of type 1 exposures shall be equal to:

$$\sigma = \sqrt{V}$$

where V denotes the variance of loss distribution of type 1 exposures:

$$V = V_{inter} + V_{intra}$$

V_{inter} is calculated as:

$$V_{inter} = \sum_{j,k} \frac{PD_k \cdot (1 - PD_k) \cdot PD_j \cdot (1 - PD_j)}{1.25 \cdot (PD_k + PD_j) - PD_k \cdot PD_j} \cdot TLGD_j \cdot TLGD_k$$

The summation involved regards all possible combinations (j, k) of different probabilities of default on single name exposures. $TLGD_j$ refers to the sum of losses-given-default on type 1 exposures from counterparty with a probability of default PD_j.

[29] Optional simplification.

V_{intra} is defined as:

$$V_{intra} = \sum_j \frac{1.5 \cdot PD_j \cdot (1 - PD_j)}{2.5 - PD_j} \sum_{PD_j} LDG_i^2$$

The first summation involves all different probabilities of default on single name exposures, whereas the second sum regards the loss-given-default on the single name exposure i LGD_i of all single name exposures that have a probability of default equal to PD_j.

3.6.2 Loss-given-to-default

The loss-given-default (LGD) on a single name exposure must be equal to the sum of the loss-given-default on each exposure to counterparties belonging to the single name exposure. It shall be net of the liabilities towards those counterparties if the liabilities and exposures are set off in the case of their default. The insurance undertaking has also the possibility to calculate the loss-given-default (including the risk-mitigating effect on underwriting and market risks and the risk-adjusted value of collateral), for a group of single name exposures and assign to them highest probability of default associated with single name exposures included in the group.
Regarding a reinsurance agreement, the LGD is defined as:

$$LGD = \max[50\% \cdot (Recoverables + 50\% \cdot RM_{re}) - F \cdot collateral, 0]$$

where:

- *Recoverables* are defined as the best estimate of amounts recoverable.
- RM_{re} denotes the risk mitigating effect on underwriting risk which represents the additional loss above the current value of the counterparty exposure expected to arise in a stressed situation. It is defined as the larger of 0 and the difference between the hypothetical capital requirement for underwriting risk the insurance undertaking would apply if the reinsurance arrangement did not exist and the capital requirement for underwriting risk of the insurance undertaking. In order to calculate it, it could be applied a simplified approach, provided that its use is proportionate to the nature

scale and complexity of the undertakings' counterparty risk profile (see article 88[30]):

$$RM_{re} = RM_{re,all} \cdot \frac{Recoverables_i}{Recoverables_{all}}$$

$RM_{re,all}$ denotes the risk mitigating effect on underwriting risk of the reinsurance arrangements for all counterparties calculated as the difference between the hypothetical capital requirement for underwriting risk the insurance undertaking would apply if none of the reinsurance arrangements exist and the capital requirement for underwriting risk of the insurance undertaking. $Recoverables_i$ and $Recoverables_{all}$ refer respectively to the best estimate of amounts recoverable from the reinsurance arrangement for counterparty i and to the best estimate of amounts recoverable from the reinsurance arrangement for all counterparties.

In addition, if the reinsurance arrangement for the counterparty i is proportional, the risk-mitigating effect on underwriting risk j can be computed as:

$$RM_j = SCR_j \cdot \frac{Recoverables_i}{BE - Recoverables_{all}}$$

where BE denotes the best estimate of obligations gross of the amounts recoverable, while SCR_j the capital requirements for underwriting risk j of the insurance or reinsurance undertaking.

According to EIOPA Opinion on Review 2020, the calculation of the risk mitigating effect is the most burdensome component of the counterparty default risk module. Therefore, EIOPA advices to introduce an additional optional simplification for its computation. The basic idea is to extend the current simplification $RM_{re} = RM_{re,all} \frac{Recoverables_i}{Recoverables_{all}}$ to derivatives in order to calculate the risk mitigating effect jointly for reinsurance arrangements and for derivatives[31]. The first step of the simplified calculation aims at computing the total risk mitigating effect related to all reinsurance agreements, derivatives exposure, and securitizations as:

$$RM_{total} = BSCR^{*,without} - BSCR^{*}$$

$BSCR^{*,without}$ is the Basic Solvency capital requirement excluding the counterparty

[30] A simplified calculation of a (sub)module is allowed if it is proportionate to the nature, scale, and complexity of the risks by assessing the nature, scale, and complexity of the risks of the undertaking falling within the relevant module or sub-module and the error introduced in the results of the simplified calculation.

[31] Consider that this simplification works properly for simple derivative structure, whereas it is quite inappropriate for complex derivative strategies.

default risk module which would result if no derivatives, reinsurance arrangements and insurance securitisations were in force. Conversely, $BSCR^*$ is the current Basic Solvency Capital Requirement if the counterparty default risk module is excluded.
Secondly, the total risk mitigating effect must be allocated towards the different counterparties in a simple and proportional way:

$$RM_i = \frac{\max|EAD_i|}{\sum_{i=1}^{n} \max|EAD_i|} \cdot RM_{Total}$$

It is assumed that the undertaking is exposed to the counterparty default risk towards n counterparties. $|EAD_i|$ denotes the absolute value of the exposure at default of the derivative, reinsurance arrangement, special purpose vehicles and insurance securitisations towards the counterparty i. If the risk mitigating instrument is a derivative, EAD_i will be the Fair Value of the derivative, whereas if the risk mitigating instrument is a reinsurance arrangement, it will be the best estimate of amounts recoverable from the reinsurance arrangement towards counterparty i. The absolute value ensures that derivatives and recoverables with negative values are properly considered in the risk mitigating effect calculation.
Finally, the risk mitigating effect for the specific counterparty i is given by a proportion of the total risk mitigating effect, where the proportion is given by the ratio between the maximum (absolute value of) exposure towards the counterparty i and the total exposure with respect to all n counterparties.

- $collateral$ refers to the risk-adjusted value of collateral.
  It shall be equal to the difference between the value of the assets held as collateral evaluated market-consistently and the adjustment for market risk, if criteria set out by Article 214 of the Delegated Acts are met. If at least one of the previous constraints is not fulfilled, different calculations are set by the second-level regulation. The adjustment for market risk is computed as the difference between the hypothetical capital requirement for market risk of the insurance undertaking that would apply if the assets held as collateral were not included in the calculation and the hypothetical capital requirement for market risk of the insurance undertaking if the assets held as collateral were included in the calculation.

- F is a factor which considers the economic effect of the collateral arrangement in relation to the reinsurance treaty in case of any credit event related to the counterparty.

If in case of insolvency of the counterparty, the insurance undertaking's proportional share of the counterparty's insolvency estate in excess of the collateral is determined without taking into account that the undertaking receives the collateral, the factor F shall be equal to 100%, otherwise it shall be equal to 50%.

Where the reinsurance arrangement is with an insurance or reinsurance undertaking or a third country insurance or reinsurance undertaking and 60% or more of that counterparty's assets are subject to collateral arrangements, the loss-given-default shall be calculated as:

$$LGD = \max[90\% \cdot (recoverables + 50\% \cdot RM_{re}) - F \cdot collateral, 0]$$

Review 2018 allows this computation of the LGD for reinsurance arrangement even without the constraint on the collateral arrangement[32], provided that the use of this simplified formula is proportionate to the nature scale and complexity of the undertakings' counterparty risk profile.

3.6.3 Probability of default

The probability of default on a single name exposure must be computed as the average of the probabilities of default on each exposure to counterparties belonging to the single name exposure, weighted by their loss-given-default. If for the single name exposure i is available a credit assessment by a nominated ECAI[33], the probability of default PD_i shall be defined in accordance with the table 5:

Credit Quality Step	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Probability of default	0.002%	0.01%	0.05%	0.24%	1.2%	4.2%	4.2%

Table 5: Probability of default.

Conversely, if a credit assessment is not available and if the undertaking meets its Minimum Capital Requirement, the probability of default PD_i is assessed according to the undertaking's Solvency Ratio[34] (Table 6). If the solvency ratio falls in between the solvency ratios above specified, the value of the probability of default must be linearly interpolated from the closest values of probabilities of default corresponding to the closest solvency ratios. Whereas if the SR is lower than 75%, the probability of default shall be 4,2%. Conversely, if

[32]Therefore, by applying this optional simplification, the LGD, and consequently the SCR for the counterparty default risk, is computed on the basis of the assumption that more than 60% of the counterparty's assets are subject to collateral arrangements.

[33]An ECAI (External Credit Assessment Institution) is an entity, recognized by the competent Supervisory Authority, which can produce external credit assessments.

[34]The solvency ratio is computed as the ratio of the eligible amount of own funds to cover the SCR and the Solvency Capital Requirement.

Solvency Ratio	196%	175%	150%	125%	122%	100%	95%	75%
Probability of default	0.01%	0.05%	0.1%	0.2%	0.24%	0.5%	1.2%	4.2%

Table 6: Probability of default.

the SR is higher than 196%, PD_i shall be 0,01%. Finally, if the insurance undertaking does not meet its Minimum Capital Requirement, the probability of default shall be assigned equal to 4,2%.

4 Multi-risk Reinsurance Treaties

A multi-risk product is an instrument which combines several exposures into a single contract, giving the insurer an efficient and cost-effective risk mitigation solution. The intervention of the multi-risk reinsurer is triggered by the occurrence of one (or more) of the several perils defined in the treaty. Therefore, a great role is played by the correlation (dependence) structure of the risks belonging to the portfolio and by their joint probability. In addition, multi-risk products offer a risk protection which is typically cheaper than the sum of the protections provided by per-peril basis products.
It is possible to split multi-risk products into two categories:

- *Multiple peril products.*

  They are contracts which cover multiple classes of (un)related events. Typical examples of multiple peril products are multi-line policies, commercial general liability policy and umbrella policies.

- *Multiple trigger products.*

  They are contracts which provide coverage only if multiple events occur. Typical examples of multiple trigger products are dual and triple trigger instruments with fixed, variable or switching trigger references.

4.1 Multiple peril products

The typical reinsurance treaties provide coverages on a per-peril basis, defining proper and distinct contract conditions in terms of deductible, limit, and premiums. Covers are typically added when exposure emerges or grows up and several forms of protection can be purchased from different reinsurers. A risk protection strategy defined as above could not be an efficient and cost-effective program and multiple peril products could be a more proper solution. Indeed, they gather all the exposures the insurance company wants to reinsure into a single comprehensive policy with an aggregate deductible, limit and premium. Since the design of multi-peril products is quite involved, most of them have a multiannual duration (from 3 to 7 years). Through a multi-peril product, all exposures are covered under one policy. Therefore, it is no more important which is the specific source of a loss because the policy will provide appropriate indemnification if the peril is under the scope of the contract.
The typical benefits arising from multiple peril contracts are:

- *Lower transaction costs.* The negotiation is no more necessary for each individual risk exposure.

- *Lower premium.* Since typically the exposures included in multiple perils contracts are not perfectly correlated, a diversification benefit arises and makes the overall premium to be lower than the sum of the premiums of the hypothetical single exposure policies.

- *Less chance of over-insurance.* In normal business scenarios, it is quite unlikely that the insurer will incur in simultaneous claims from each of the named perils.

Notwithstanding the risk reduction of possible over-insurance, the insurance company has always to pay attention to not under-insurer its business through a single policy cap. In order to avoid this undesired situation, typically multiple peril policies include reinstatements, allowing to refresh the limits if totally used prior to maturity. The reinstatement provision defines the details on the new granted limits and the premium to be paid.
The usual multiple peril products purchased on the market are the following:

- *Multi-line policy*, which contains common policy conditions and specific coverages with own declarations and causes of loss forms. If a loss occurs in any of the perils named in the policy, the cedant (i.e., the direct insurer) is covered up to a net amount reflecting the deductible and the cap.

- *Commercial general liability (CGL) policy.* It is typically purchased when the insurance company seeks to cover exposures related only to liabilities as products, environmental damage and director and officer fiduciary breaches (D&O).

- *Commercial umbrella policy.* It provides protection for very large amounts regarding a broad range of insurable risks, which are much higher than the one obtainable through typical property and casualty covers. It serves primary as an excess layer facility rather than a first loss cover since it pays out the ultimate net loss in excess of a retained limit. It may present some exclusions, failing to be truly comprehensive in scope.

In order to write the desired multiple peril policy, the direct insurer can choose between the attachment method and the single text method. Under the attachment method, different monoline policies are grouped together into a new-brand agreement. Via the single text method, the existing covers are drafted again into a new policy such that all the named perils are included into a single agreement. Generally, the attachment method is easier to define, but it could be subject to overlaps, conflicts, or gaps.

Finally, in order to define the aggregate deductible and limit of the multiple peril product, the insurer has to identify its retained risk appetite on a portfolio basis and, by making a cost-benefit scenarios analysis, determine the optimal value of limit and deductible. The results surely depend heavily on correlations and joint event probabilities.

4.2 Multiple Trigger products

Conversely to multiple peril policies, multiple trigger products are effective only if various events jointly occur. If only one of the named events occurs and the cedant suffers a loss, no pay-out is made. According to this definition, multiple peril policies can be considered as single trigger product since the indemnification occurs once the total aggregate claim amount exceeds the stated deductible.

Typically, multiple trigger products are dual triggers or triple triggers policies. Dual triggers contracts require the onset of two events before the pay-out occurs, whereas triple triggers three breaches. Since dual (triple) triggers provide payments if and only if the second (third) events occur, the likelihood of a pay-out is lower than for similar multiple peril contracts. As a result, the cedant obtains a cheaper protection also against risks that might have been defined as uninsurable. Therefore, it is possible to obtain a unique and manageable joint risk exposure which allows better risk diversification within the insurer's portfolio.

Typically, multiple perils products are created as multi-year insurance contracts with trigger annually reset which could be in several forms as:

- *Fixed trigger.* The trigger is simply a barrier which determines whether or not an event occurs, indicating whether the contract will pay out, without usually impacting on the value of the contract.

- *Variable trigger.* The value of the pay-out is defined according to the level of the trigger in relation to some defined events.

- *Switching trigger.* The trigger varies according to the performances of the individual risk exposures in the cedant's portfolio.

In addition, the contract may be created on a per occurrence basis or aggregate basis. Per occurrence triggers permit a reset of the trigger each time an event occurs, while aggregate triggers allow accumulation over multiple events.

In order to properly define a multiple trigger contract, the cedant must analyse the casual relationship between specific events and claims. The focus is on events which can create losses and, on the behaviour, and the characteristics of such claims (dimension with respect to time or severity of an event, static or dynamic). Once the casual relationships are properly analysed, triggers have to be structured such that they are able to provide an appropriate level of protection at a price which reflects a lower probability of pay-out.

Generally, the nature and the level of the trigger is negotiated between the cedant and the insurer but, in order to avoid moral hazard instances, one of the triggers must be based on an outside metric. In any case, the outside trigger must be sufficiently correlated with the cedant's underlying exposure. For instance, one trigger can refer to a financial or non-insurance event (as an equity index level or an interest rate), whereas another one a specific insurance hazard (as a business interruption loss or a property damage loss). In addition, the cedant has to prove an insurable interest for the multiple trigger structure to be considered insurance.

Although their advantages, multiple trigger products have certain drawbacks. Most transactions include a charge reflecting the cost of product development, since multiple trigger products are highly customized. Indeed, introducing standardizations could determine an excess of basis risk inside a given transaction which can remove the benefits offered. In addition, their accounting and legal treatment belong to a 'grey area', since a non-financial insurance trigger is surely an insurance component, whereas if the trigger is tied to a financial index, it could be seen as derivative (implying a mark-to-market treatment and less tax deductions). Typically, these products are treated entirely as insurance by proving explicitly that there is an insurable interest and a risk exposure's transfer. Indeed, the main difference between a multiple trigger and a derivative is that the multiple trigger guarantees post-loss financing if the event occurs, whereas the same does not hold necessarily for the derivative (for instance the loss occurs, but the derivative is out-of-the-money).

4.3 Medium-term growth prospects

Regarding the outlook for multi-risk products, it is needed to split between multiple peril policies and multiple trigger instruments. Multiple peril contracts are likely to expand in future years, but only gradually. Indeed, even if they are good at managing portfolios of similar risks, they are overcome by more comprehensive Enterprise Risk Management programs, which group several risks by using the most efficient vehicles/instruments.

Conversely, multiple trigger contracts should feature strong end-user demand. The relative pricing efficiencies and the recognition of the savings that can be achieved have become more widely broadcast.

To sum up, growth prospects for multiple peril policies might therefore be seen as moderate, while prospects for multiple triggers appear stronger.

5 Dependence structure

5.1 Introduction

When an insurance company faces a portfolio of risks, it has usually to manage two problems:

- Modelling the risk itself through a statistical model.
- Aggregating the risks.

The dependence structure according to which risks are aggregated affects the risk diversification which is a typical key indicator in insurance and investments. Stronger is the dependence structure, lower is the diversification and higher is the company's need for capital. Typically, risks within an insurance portfolio are rarely independent. For instance, the line of business Motor Own Damage (MOD) is typically correlated with Motor Third-Party Liability (MTPL). Indeed, they involve claims arising from motor accidents and in addition, both are affected by economic cycles. Also, the segment General Third Party Liability (GTPL) is influenced by economic cycles, and it is correlated with MTPL due to similar evaluation methods of personal injuries.
Therefore, the introduction and the choice of a dependence structure is crucial for a multiline insurer, in particular if it decides to cede risks via a multi-risk reinsurance treaty. If aggregate claims amounts among different lines of business are positively dependent and the insurance company models them simply as independent, there could be a possible underestimation of the effective capital requirement calculated inside its internal model. Indeed, a positive dependence implies a non-negative likelihood to get high aggregate claims amounts jointly from different LoBs, making the capital requirement increase.

Several dependence structures can be fit inside an internal model. The most used and simplest measure of dependence is the linear correlation coefficient. In the Solvency II framework, the dependence structure underlying the Standard Formula involves fixed correlation coefficients between segments, to be applied by all the companies regardless their real business. Conversely, if the company decides to apply for an internal model, it has to fit a dependence structure properly describing the existing relationships between risks going also beyond the simple Pearson linear correlation coefficient.

In the following paragraphs, a presentation of the possible different dependence structures will be carried out. In particular, in paragraph 5.2, a brief introduction about the linear correlation coefficient is provided in order to highlight its strengthens and shortfalls. Starting from paragraph 5.4, copulae functions will be introduced and presented as a possible improvement of the basis correlation structure.

5.2 Linear correlation coefficient

Typically, the first indicator calculated for assessing the dependence structure between random variables is the Pearson correlation coefficient ($\rho(X,Y)$). $\rho(X,Y)$ describes the linear dependence between variables by a unique real number. Given two non-degenerated[35] random variables X and Y, the Pearson linear correlation coefficient can be defined as:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

where

$$Cov(X,Y) = E[XY] - E[X]E[Y]$$

$\rho(X,Y)$ ranges from -1 to 1. Higher is its absolute value, stronger is the relationship between the random variables. If X and Y are independent, $\rho(X,Y) = 0$, since $E(XY) = E(X)E(Y)$. The opposite does not necessarily hold. In particular, if the linear correlation coefficient is null, X and Y could be also dependent (but not linearly correlated). If $\rho(X,Y) = \pm 1$, there is a perfect linear dependence between the two variables. Conversely, if $-1 < \rho(X,Y) < 1$, the linear correlation is not perfect, which does not mean that there is not any non-linear dependence.

Assuming to have at disposal two vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ of paired realisations of the non-degenerate variables X and Y, it is possible to compute the sample mean, variance, covariance through the following unbiased estimators:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{Var}(X) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\hat{Cov}(X,Y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

They can be used for defining the estimator of the linear correlation coefficient which is asymptotically unbiased:

$$\hat{\rho}(X,Y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

[35] A non-degenerated random variable has finite mean and finite non-null variance.

Some difficulties in the interpretation of the Pearson linear correlation coefficient can arise when:

- the expected value of one variable is not defined (as for the Cauchy distribution).
- the variance of one variable is not defined (as for the Pareto distribution, quite used in the reinsurance field).
- one variable has null variance.

Although its computation is quite straightforward, the Pearson correlation coefficient has some shortfalls:

- It estimates the dependence between variables on the assumption that their relationship is linear. In practice, dependences could be also non-linear and, by ignoring this possible evidence, the Pearson linear correlation coefficient could lead to misleading information.
- In addition, $\hat{\rho}(X, Y)$ is a good estimator of linear dependence when X and Y are correlated, but there is not a general criterion which determines whether there is linear correlation between two random variables based on its value (except when X and Y can be approximated by a Normal distribution).
- It is not invariant to non-linear strictly increasing transformations[36], with the exception of positive, affine relationships.
- Being a function of the marginal distributions, as the form of the marginals changes, also the value of the linear correlation coefficient will change.
- A well-defined risk measure must rank risks correctly. In other words, bigger are the risks, bigger should be the risk measure. If the random variables X and Y double, the Pearson correlation coefficient does not change:

$$\rho(2X, 2Y) = \frac{Cov(2X, 2Y)}{\sqrt{Var(2X)Var(2Y)}} = \frac{4Cov(X, Y)}{\sqrt{4Var(X)4Var(Y)}} = \rho(X, Y)$$

  Indeed, assuming a linear correlation between two variables, it is like assuming the same correlation coefficient in any point of the distribution. Therefore, corresponding percentiles of the distributions at different confidence levels share the same linear

[36] The translation invariance is one of the properties a risk measure shall meet since it assures a proper treatment of riskless cash flows.

dependence structure. This feature could be quite problematic in practice and in particular in insurance business, since typically the dependence increases when diversification is most needed: in case of stress.

As highlighted by its shortfalls, the Pearson correlation coefficient is not able to properly describe the non-linear behaviour of diversification and the tail dependence, particularly crucial in insurance field.

5.3 Tail dependence

Tail dependence is a phenomenon by which the percentiles related to the tail(s) of the distributions tend to be associated each other more strongly than the percentiles in the body of the distributions. It could be quite relevant in the insurance field, since risks may be reasonably independent except in the region of very large aggregate claims amounts which are of full interest in the Solvency Capital Requirement computation. Easily the upper tail dependence between two random variables X and Y with distribution function absolutely continuous can be defined as:

$$\tau(u) = Pr\big(Y > F_Y^{-1}(u) | X > F_X^{-1}(u)\big)$$

It is the conditional probability that Y is above a given percentile at level u given that also X is above its percentile at the same level. Being a conditional probability, it ranges between 0 and 1. Typically, it is of interest the behaviour of the dependence in the tail corresponding at the extremes of the interval (0,1). Therefore, it is possible to say that it exists an asymptotic tail dependence if and only if $\tau^{+} = \lim_{u \to 1^{-}} \tau(u) > 0$. In addition, the (asymptotic) tail dependence is function of the copula linked to the joint distribution and it does not depend on the marginals.
Graphically, it is possible to detect the tail dependence by drawing the rank scatterplot of the random variables. It is appropriate to model the dependence by a linear correlation coefficient only if the rank scatterplot shows a linear dependence. Otherwise, it is suggested to fit a generalized non-linear dependence structure. A suggestion could be to apply a mathematical tool: copula functions.

5.4 Copula functions

Copula is a multivariate cumulative distribution function which margins are standard uniform. Copula functions are able to overcome the shortfalls of the Pearson linear correlation coefficient, building joint cumulative function from the CDFs (Cumulative Distribution Functions) of the marginals and taking into account the potentially different dependence structures in different points of the distribution. Their popularity is given by the Sklar's Theorem.

Assume to have a generic random vector $(X_1, X_2, \ldots, X_d)$ of known marginal distribution $F_1(x_1), F_2(x_2), \ldots, F_d(x_d)$. According to the Sklar's theorem, it exists a function $C : [0,1]^d \to [0,1]$ (satisfying the properties of multivariate distribution function[37]) such that the multivariate cumulative distribution function can be written as:

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$$

If the CDFs of the marginals $F_1(x_1), F_2(x_2), \ldots, F_d(x_d)$ are continuous, the copula C is unique. In case at least one CDF associated to a marginal is non-continuous or not uniformly continuous, it is always possible to separate the marginals from the dependence structure through a copula, but it is no more uniquely identified.

Therefore, the Sklar's theorem states that the joint cumulative distribution function can be decomposed into:

- the copula, which contains all information about the dependence structure among the random variables.
- the univariate marginal cumulative distribution functions which contain all information about the marginal distribution of the random variables.

Therefore, it is possible to split the characteristics of each marginal and the effect of the dependence and to analyse and calibrate them separately. The decomposition provided by the Sklar's theorem is based on the probability integral transform. According to this statistical result, having a vector of random variables, it is possible to define uniform random variables $U_1, U_2, \ldots, U_d$ such that $U_1 = F_1(x_1), U_2 = F_2(x_2), \ldots, U_d = F_d(x_d)$. The copula function C can be represented as a multivariate distribution function of uniform marginal

[37] The copula function shall satisfy the following conditions:

1. For $k = 1, \ldots, d$:

$$C(u_1, \ldots, u_{k-1}, 0, u_{k+1}, \ldots, u_d) = 0, \forall\ u_1, \ldots, u_{k-1}, 0, u_{k+1}, \ldots, u_d \in [0,1]$$
$$C(1, \ldots, 1, u, 1, \ldots, 1) = u, \forall\ u \in [0,1]$$

   These equalities assure that the marginals are uniform.

2. The function C is non-decreasing in order to obtain a measure of probability linked to the copula which is positive in each subset of the domain. In other words, it assures that the joint cumulative distribution function associated to the copula is continuous and non-decreasing in each component.

distributions:

$$C(u_1, \dots, u_d) = P(U_1 \leq u_1, U_2 \leq u_2, \dots, U_d \leq u_d)$$

Since the function C describes the dependence structure, by applying it to the marginals of interest, it is possible to get the joint cumulative multivariate distribution $F(x_1, \dots, x_d)$. In addition, through copulae, given the same marginals, it is possible to change the dependence structure by choosing a different copula and obtaining a completely different joint distribution.

Here three basic examples of copulae are provided:

- *Product Copula.*

  $$\Pi(u_1, ..., u_d) = \prod_{i=1}^{d} u_i$$

  It is the dependence structure underlying mutually independent random variables $X_1, X_2, ..., X_d$. Therefore, the resulting joint distribution is only given by the product of the marginals:

  $$F(x_1, ..., x_d) = \prod_{i=1}^{d} F_i(x_i)$$

  In a two-dimensional case, the sample p-th percentile of X_1 can be matched with any percentile of X_2 with equal probability (i.e., at random).

- *Full dependence copula.*

  $$M(u_1, \dots, u_d) = \min(u_1, \dots, u_d)$$

  In the bivariate case, it is the dependence structure underlying fully dependent and concordant variables, implying that the p-th percentile of X_1 will be always matched by the p-th percentile of X_2. Applied to an insurer's internal model, claims from each LoB occur jointly, implying a very large total aggregate claims amount.

- *Full Negative dependence copula.*

  $$W(u_1, ..., u_d) = \max\left(\sum_{k=1}^{d} u_k + d - 1, 0\right)$$

  It is the dependence structure of variables which are fully dependence and discordant. In the two-dimensional case, the p-th percentile of X_1 will be always matched with the (1-p)-th percentile of X_2. Applied to the capital requirement computation, the aggregate claims amount of one LoB compensate the ones from other LoBs each other.

In the literature, there are a lot of possible copulae in between the two extreme cases of $M(u_1, u_2)$ and $W(u_1, u_2)$. Indeed, the Frechet-Hoeffding bounds inequality states that:

$$M(u_1, \ldots, u_d) \leq C(u_1, \ldots, u_d) \leq W(u_1, \ldots, u_d)$$

$M(u_1, \ldots, u_d)$ and $W(u_1, \ldots, u_d)$ are denoted respectively as the lower bound and the upper bound of Frechet-Hoeffding. $\Pi(u_1, u_2)$ could be considered to be right in the middle between the full dependence and the full negative dependence cases. If there is at least some kind of dependence, there will be some degree of regularity in the way the percentiles are associated. For instance, for dependent and concordant distributions, the top percentiles of X_1 will tend to be associated with the top percentiles of X_2. Due to the surely present degree of randomness, some top percentiles of X_1 will associate with the bottom percentiles of X_2, and vice versa.
In addition, conversely to the Pearson correlation coefficient, copulae functions are invariant with respect to monotonic strictly increasing transformation of marginals. Since the copula links the ranks of random variables, transformations which preserve the ranks of random variables will also preserve the copula. Therefore, if the transformation is strictly monotonic, the functional form of the copula is unchanged. For instance, it is possible to associate to the copula function C, the function $\bar{C} : [0, 1]^d \to [0, 1]$, named survival copula, defined as:

$$\bar{C}(u_1, ..., u_d) = \sum_{k=1}^{n} u_k + 1 - d + C(1 - u_1, ..., 1 - u_d)$$

It is the function obtained by inverting all the arguments of the copula function and it has tails with opposite orientation with respect to the original copula C.

Copulae are quite powerful function to describe dependence, but they require the estimation of their parameters. Therefore, for obtaining a reliable estimate, a sufficiently long dataset is needed. In actuarial fields, data availability is quite restricted, implying that the use of copula function is a bit trickier than in finance. A possible (practical) solution is to calibrate the parameters according to a sort of combination between the empirical observations and priors which are based on the subjective judgement of actuaries and of underwriters (Bayesian approach).

In the following paragraphs, copulae belonging to two different copula families, respectively the Gaussian copula and the Gumbel copula, are presented, highlighting their points of strength and shortfalls. Before presenting copulae, it could be useful to introduce another measure of dependence, the Kendall's tau.

5.5 Kendall's tau

The Kendall's tau is typically computed as a concordance index measuring general forms of dependence which could be not necessarily linear. Given two vectors of realizations of the random variables X and Y of size n, the Kendall's tau is calculated as:

$$\tau = \frac{2}{n(n-1)} \sum_{i<j} sign(x_i - x_j) sign(y_i - y_j)$$

$$i = 1, \ldots, n; \; j = 1, \ldots, n$$

A pair of values (x_i, y_i) is said to be concordant (discordant) if the movement with respect to the following value (x_j, y_j) is in the same (opposite) direction for the two time series. The Kendall's tau increases due to concordant pairs and decreases in presence of discordant pairs[38].

Being a measure of concordance, the Kendall's tau can be defined also as:

$$\tau = \frac{number\ of\ concordant\ pairs\ -\ number\ of\ discordant\ pairs}{\frac{n(n-1)}{2}}$$

As the Pearson correlation coefficient, it ranges between -1 and 1 and higher is its absolute value, stronger is the relationship between the two random variables. If it takes on value of (-)1, there is a perfect, not necessarily linear, positive (negative) dependence between the two random variables. Therefore, after having observed an increasing behaviour in the values of X, it is expected to observe an increasing behaviour in the values of Y too.

The main difference between the Pearson correlation coefficient and the Kendall's Tau is that the first one is a value correlation, whereas the second one a rank correlation. Therefore, conversely to the Pearson linear correlation coefficient, the Kendall's Tau is not affected by the values of the variables X and Y, but by only their rank.

In addition, τ does not depend on the shape of the marginals, but exclusively on the copula function linking them. It is used for calibrating the dependence parameter in Archimedean copulae where it is not possible to refer to linear correlation, since those copulae are good at catching tail dependence and non-linear dependence.

[38] Indeed, if the pair is concordant, $sign(x_i - x_j) sign(y_i - y_j) = 1$, if discordant, $sign(x_i - x_j) sign(y_i - y_j) = -1$.

According to the Skar's theorem, on the same set of marginals, it is possible to fit different copulae, obtaining a different joint probability distribution. In order to allow comparisons, it is necessary to calibrate the copulae on consistent measures of dependence. Therefore, it could be useful to highlight the relationship existing between the Pearson linear correlation coefficient (ρ) and the Kendall's Tau (τ):

$$\rho = \sin\left(\frac{\pi}{2}\tau\right)$$

5.6 Gaussian copula

The Gaussian copula is an elliptical copula[39] corresponding to the dependence structure underlying a multivariate normal distribution with a given correlation matrix Σ. According to the Sklar's theorem, it can be defined as:

$$C_{\Sigma}^{Ga}(u) = \Phi_{\Sigma}^{d}\big(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_d)\big)$$

Φ_{Σ}^{d} is the distribution function of a multivariate standard normal of dimension d with correlation matrix Σ, whereas Φ^{-1} is the inverse of a univariate normal distribution. The Gaussian copula is symmetric, and it does not have any tail dependence (with the exception of $\rho = \pm 1$). Whatever is the value of the correlation, going far enough in the tails, extreme events tend to occur independently. In other words, the occurrence of an extreme event for one of the variables does not allow to obtain information on the occurrence of extreme events for the other variables. Therefore, if from the rank scatterplot of the data of interest, it is evident the presence of a tail dependence, it is not recommended to use a Gaussian copula, but a copula with tail dependence. Indeed, fitting a Gaussian copula to the aggregate claims amount distribution in presence of right tail dependence will lead to an overestimation of the diversification benefit and to an underestimation of the capital requirement. This result is justified from the fact that in case of bad performances from a LoB, due to tail dependence, there is a non-negative probability that also the other LoBs will perform badly. By holding the capital requirement obtained by fitting a Gaussian copula, the company could not have sufficient resources to cover the very high total aggregate claim amount.

The Gaussian copula depends on a set of parameters depicted by the correlation matrix Σ. Higher is the correlation/dependence, lower is the diversification benefit. Fitting a Gaussian

[39] Elliptical copulae are a family of copulae defined on elliptical probability distributions. Elliptical multivariate distributions are a family of distributions including the multivariate standard distribution and the t-Student multivariate distribution.

copula, the diversification benefit decreases linearly as the parameters of the correlation matrix increase (equivalent to a stronger dependence between risks). In addition, in absence of correlation, the correlation matrix coincides with the identity matrix ($\Sigma = \mathbb{I}$) and the multivariate Gaussian copula degenerates to the product copula.

5.6.1 Simulations out of a Gaussian copula

The procedure followed for generating samples out of the vector of Gaussian-copula correlated random variables $\mathbf{X} = (X_1, \ldots, X_d)$ with given marginals is quite straightforward. The inputs are:

- The correlation matrix Σ of dimension d. Σ must be squared of dimension dxd (in the bivariate case 2x2), symmetric, positive-defined, with elements in between (-1,1) and unitary diagonal elements. In the bivariate case, only if $\rho = \pm 1$, the matrix is not positive-defined and the generation of samples out of the Gaussian distribution follows the algorithms for the full dependence copula ($\rho = +1$) and the full negative dependence copula ($\rho = -1$).
- The cumulative distribution function $F_1, F_2, \ldots, F_d$ of the d marginals.
- m, the number of simulations.

The first step for obtaining simulations out of a Gaussian copula involves the determination of the Cholesky matrix A associated to Σ, such that $\Sigma = AA'$. In the bivariate Gaussian copula, the dependence (correlation) structure existing between the random variables X_1 and X_2 is given by the correlation matrix Σ:

$$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

where ρ is the linear (Pearson) correlation coefficient. In this case, the Cholesky's decomposition is quite simple to calculate, and it is equal to the following lower left triangular matrix:

$$A = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}$$

Secondly, m vectors of d elements $\mathbf{n}$ must be generated out of d independent standard normal distributions (mean equal to 0 and variance equal to 1): $n_i \sim N(0,1), \; for \; i = 1, \ldots, d$. For obtaining a set of correlated normally distributed variates, it is necessary to create the vector $\mathbf{z}$ of m elements given by the product between the matrix A and $\mathbf{n}_i$:

$$\mathbf{z}_i = A\mathbf{n}_i, \ i = 1, ..., m$$

Then, $\mathbf{z}_i$ must be transformed uniform variables:

$$\mathbf{u}_i = \Phi(\mathbf{z}_i), \ i = 1, ..., m$$

where Φ is the CDF of the standard normal distribution. The vectors $\{\mathbf{u}\}_{i=1,..,m}$ represent samples out of the Gaussian copula.

Once the dependence structure among the variables of the random vector $\mathbf{X}$ has been defined, it is possible to introduce the marginal distributions in order to determine the joint distribution of $\mathbf{X}$. It is necessary to associate the quantiles $\mathbf{u}_i$ to the ones of the marginals applying the inverse probability integral transform, creating the vectors $\mathbf{x}_i$:

$$x_{ij} = F_j^{-1}(u_i)$$

$$i = 1, ..., m, \ j = 1, ..., d$$

where $= F_j$ is the marginal cumulative distribution functions of the random variable X_j. The vector $\{\mathbf{x}_i\}_{i=1,..,m}$ represents a sample of m elements of the distribution with marginals $X_1, \ldots, X_d$ and Gaussian dependence structure.

The inverse probability integral transform cannot be applied when the marginal cumulative distributions are unknown. In this case, assuming to have realizations of the random vector $\mathbf{X}$, it is possible to build their joint probability distribution through a sorting process of their realizations. In particular, the realizations of X_1 will be sorted according to the rank of u_1, the ones of X_2 according to the rank of u_2 and so on.

5.7 Gumbel copula

As highlighted in paragraph 5.6, the Gaussian copula does not fit any tail dependence. In the computation of the capital requirement, the tail dependence could play a quite relevant role since it will increase the likelihood of joint occurrence of large aggregate claims amount from different LoBs. The Gumbel copula, belonging to the Archimedean copula family, can help in modelling tail dependence. Archimedean copulae are characterized by the fact that one parameter θ, denoted as dependence parameter, controls the dependence between marginals. The domain of θ depends on the type of copula, but, in general, the closer is to its lower (upper) bound, the more the marginals tend to independence (total dependence). In order to estimate θ, a measure of association is needed. Typically, it is used the Kendall's Tau since it is related to θ, by the following mathematical equality:

$$\tau = 1 - \theta^{-1}$$

The Gumbel copula stresses the right tail dependence. Therefore, it could be applied to model the dependence structure between aggregate claims amounts of different Lines of Business. In other words, when high aggregate claims amount has been observed for one segment, it is quite probable that also in the other line of business high aggregate claims amount will be observed. In addition, it has a very small left tail dependence for finite quantile, but asymptotically, there is not left tail dependence.
The Gumbel copula in the bivariate case is defined as:

$$C(F_1(x_1), F_2(x_2)) = \exp\left[-\left[\left(-\ln\left(F_1(x_1)\right)\right)^\theta + \left(-\ln\left(F_2(x_2)\right)\right)^\theta\right]^{1/\theta}\right]$$

$$\theta \in [1, +\infty)$$

A value of θ equal to 1 implies independence between random variables. Higher is the value of θ, stronger is the dependence and the right tail dependence.

5.7.1 Simulations out of a Gumbel copula

The procedure for simulating realizations of two random variables X and Y related by a Gumbel copula is based on the conditional distribution method. Assume to have two standard uniform random variables (U_1, U_2), which joint distribution function is C, the copula between X and Y. Take a generic observation of (U_1, U_2), represented by the pair (u_1, u_2). For applying the conditional distribution method, it is necessary to compute the conditional distribution function of U_2, given $U_1 = u_1$, denoted by $F_{u_1}(u_2)$:

$$F_{u_1}(u_2) = P[U_2 \leq u_2 | U_1 = u_1] =$$

$$= \lim_{\Delta u_1 \to 0} \frac{C(u_1 + \Delta u_1, u_2) - c(u_1, u_2)}{\Delta u_1} = \frac{\partial C(u_1, u_2)}{\partial u_1}$$

For obtaining draws of a copula, samples out of two independent standard uniform distributions (u_1, v_2) must be generated. Easily, u_2 can be computed as:

$$u_2 = F_{u_1}^{-1}(v_2)$$

where $F_{u_1}^{-1}$ is the quasi-inverse of F_{u_1}.
F_{u_1} under a Gumbel copula is not invertible. Therefore, an acceptance-rejection method must be applied, implying a higher computational time. Once obtained the draws out of the Gumbel copula, the same sorting procedure explained in paragraph 5.6.1 must be applied to the realizations of the two random variables X and Y.

The Gumbel copula is very good in stressing right tail dependence, but, being an Archimedean copula, it fits only pairwise dependence. In a multidimensional case higher other the bivariate one, variables must be aggregated two-by-two, whereas through a Gaussian copula, it is possible to aggregate all of them at once. Therefore, for Gumbel copula, and in general for Archimedean copulae, some issues can arise in aggregating more than two marginals. Indeed, they have only one communal measure of dependence θ, which could be not able to describe the potentially different dependence structures between pairs of marginals. Vine copulae overcome these shortfalls.

6 Internal model

In order to choose whether to underwrite or not a reinsurance treaty, the insurer has to perform a risk-profitability analysis. In particular, in this chapter, a model is introduced in order to study the impact of the reinsurance cover on the risk profile and on the profitability of the cedant. The insurer's risk profile is measured by its capital requirement computed according to an internal model, based on a collective approach. Moreover, its performances are assessed through the Return on Equity.

6.1 Profitability

Before introducing a measure for the insurer's performances, a brief reminder of the risk reserve in one-year time horizon is needed. The stochastic risk reserve at the end of the year 1 for a mono-line insurer is given by the following relationship:

$$\tilde{U}_1 = U_0(1+j) + \left[B_1 - \tilde{X}_1 - \tilde{E}_1\right](1+j)^{1/2}$$

where:

- j is the annual rate of investment return. Since the investment risk is ignored, j corresponds to the risk-free rate.
- $\tilde{E}_1$ are the general and acquisition expenses.
- $\tilde{X}_1$ is the stochastic aggregate claim amount.
- B_1 are the gross premium volume, ignoring the effect of the premium reserve (i.e., by assuming the coincidence between written premiums and earned premiums).

In addition, it is assumed that claims, premiums, and expenses are realized, on average, in the middle of the year. Therefore, they are invested at the risk-free rate j for half a year. The gross premium amount is supposed to be composed as follows:

$$B_1 = P_1 + \lambda P_1 + cB_1$$

The risk premium is computed as the expected aggregate claims amount:

$$P_1 = E(\tilde{X}_1)$$

To P_1, it is added a safety loading (expected technical profit) and an expense loading corresponding to the expected value of expenses:

$$cB_1 = E(\tilde{E}_1)$$

Under this assumption, the technical profit $B_1 - \tilde{X}_1 - \tilde{E}_1$ can be easily rewritten as $P_1(1+\lambda) - \tilde{X}_1$. In addition, the gross volume premium is assumed to increase yearly by the claim inflation (i) and real growth (g):

$$B_1 = (1+i)(1+g)B_0$$

Notice that i, g and j are assumed to be constant (and not time-dependent) during the one-year time period.

A measure for the performances of the direct insurer is the expected Return on Equity (RoE), which can be defined in function of the risk reserve. Indeed, it can be computed as:

$$\bar{R}(0,1) = E\left(\frac{\tilde{U}_1 - U_0}{U_0}\right) = (1+g)(1+i)\frac{ru_0 + \lambda p}{u_0} - 1$$

where r is a synthetic index calculated as:

$$r = \frac{1+j}{(1+i)(1+g)}$$

Moreover, p represents the incidence of the risk premium on gross premium increased by the investment return for half a year:

$$p = \frac{P_1}{B_1}(1+j)^{1/2} = \frac{1-c}{1+\lambda}(1+j)^{1/2}$$

$\bar{R}(0,1)$ is a measure of profitability gross of any reinsurance treaty. In order to define the efficiency and the convenience of underwriting a reinsurance cover, the gross ROE must be compared with the ROE net of reinsurance. Therefore, the risk reserve must be extended to the case where the insurance company underwrites a reinsurance treaty:

$$\tilde{U}_1 = U_0(1+j) + \left[B_1 - \tilde{X}_1 - \tilde{E}_1 - \left(B_1^{RE} - \tilde{X}_1^{RE} - C_1^{RE}\right)\right](1+j)^{1/2}$$

With respect to the formula of the gross risk reserve, the technical result for the reinsurer, $(B_1^{RE} - \tilde{X}_1^{RE} - C_1^{RE})$, is introduced. In particular, B_1^{RE} represents the gross premiums paid to the reinsurer, whereas C_1^{RE} denotes the commissions typically characterizing the pricing process of proportional treaties. In particular, for a quota share treaty with a retained quota equal to α, the premiums paid to the reinsurer and the reinsurance commissions can be calculated as:

$$B_1^{RE} = (1-\alpha)B_1$$

$$C_1^{RE} = c^{RE} B_1^{RE}$$

The coefficient c^{RE} is fixed in advance in case of fixed commissions or it could change in accordance with the Loss Ratio in case of scaling commissions.

$\tilde{X}_1^{RE}$ is the stochastic aggregate claim amount charged to the reinsurer, which is computed for an Excess-of-Loss treaty with priority M and no limit as:

$$\tilde{X}_1^{RE} = \sum_{k=1}^{\tilde{K}_1} \max\big[0, \tilde{Z}_{k,1} - M\big]$$

Therefore, the risk premium charged by the reinsurer shall be:

$$P_1^{RE} = E\big(\tilde{X}_1^{RE}\big) = n_1 E\big(\tilde{Z}_1^{RE}\big)$$

Notice that in the formula it is involved the total expected number of claims (and not only the ones exceeding the priority). $E\big(\tilde{Z}_1^{RE}\big)$ can be calculated through the stop-loss transform as:

$$E(\tilde{Z}_1^{RE}) = \int_M^{+\infty} (Z-M) dS_1(Z) = \int_M^{+\infty} (1 - S_1(Z)) dZ$$

where $S_1(Z)$ is the cumulative distribution function of the claim amount Z in the year 1.

Easily, the gross premium to be paid to the XL reinsurer can be computed:

$$B_1^{RE} = \big(1 + \lambda^{RE}\big) P_1^{RE}$$

$$c^{RE} = 0$$

where λ^{RE} denotes the safety loading coefficient applied by the reinsurer. It increases as the priority increases since the part of the distribution ceded is more volatile.

6.2 Solvency Capital Requirement

In this paragraph, a possible internal model for the premium risk and expense risk is introduced. Denote with $\tilde{Y}_1$ the random variable representing the one-year (0,1) stochastic technical result, evaluated at the end of the year 0 for a multi-line Non-Life insurer. It is possible to define $\tilde{Y}_1$ as:

$$\tilde{Y}_1 = \Bigg[\sum_{h=1}^{L}(B_{1,h}^{writt} + PR_{0,h} - \tilde{PR}_{1,h}) - \sum_{h=1}^{L}\tilde{E}_{1,h} - \\ \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{X}_{1,h}^{paid,PY} + \tilde{PCO}_{1,h}^{CY} + \tilde{PCO}_{1,h}^{PY} - PCO_{0,h}\big)\Bigg]$$

where:

- L is the number of LoBs.
- $B_{1,h}^{writt}$ are the estimated gross written premiums for the h-th LoB. The company has to consider the budget plan, approved by the board, for the next year in terms of volume of the written premiums. For conservative reasons, if the expected premiums' volume is decreasing, the company will take the (known) volume of the premiums of the previous year.
- $PR_{0,h}$ and $\tilde{PR}_{1,h}$ are respectively the initial and deterministic premium provision and the final and stochastic premium provision. Their difference allows to switch from written premiums to earned premiums.
- $\tilde{E}_{1,h}$ are the expenses of the year for the h-th LoB, excluding settlement expenses which are already included in the payments or in claims provisions.
- $\tilde{X}_{1,h}^{paid} = \tilde{X}_{1,h}^{paid,CY} + \tilde{X}_{1,h}^{paid,PY}$ represents the payments made in year 1. It is split into two components, respectively:
  - $\tilde{X}_{1,h}^{paid,CY}$: the payments in the year 1 for claims incurred in the current year.
  - $\tilde{X}_{1,h}^{paid,PY}$: the payments in the year 1 for claims incurred in previous years.

  This distinction is quite relevant since these claims are covered by different amounts. Payments for claims occurred in the current year are covered by the premiums (premium risk), whereas the other ones by the claim reserve (reserve risk).
- $PCO_{0,h}$ and $\tilde{PCO}_{1,h}^{PY}$ are respectively the initial and the final provision for outstanding claims.

Since the insurer is supposed to be multi-line, a proper dependence structure should be introduced in terms of payments and expenses. Typically, payments and expenses are assumed independent. Much more relevant is the dependence assumption among aggregate claim amounts arising from different LoBs. Only in the case those amounts are independent, the total aggregate claim amount can be calculated as the simple sum of the LoBs' ones. Otherwise, a proper aggregation procedure, as through copula functions, must be fit and well calibrated.

By ignoring Cat risk (and the related extreme events) and lapse risk, it is possible to split the effect on the technical result of the premium risk component and the reserve risk component:

$$\tilde{Y}_1 = \tilde{Y}_1^{pr} + \tilde{Y}_1^{res}$$

where

$$\tilde{Y}_1^{pr} = \left[\sum_{h=1}^{L}(B_{1,h}^{writt} + PR_{0,h} - \tilde{PR}_{1,h}) - \sum_{h=1}^{L}\tilde{E}_{1,h} - \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{PCO}_{1,h}^{CY}\big)\right]$$

$$\tilde{Y}_1^{res} = \sum_{h=1}^{L}(PCO_{0,h} - \tilde{X}_{1,h}^{paid,PY} - \tilde{PCO}_{1,h}^{PY})$$

Since in the case study, the reserve risk is not taken into account, here only the premium risk is considered and analysed. The contribution of the premium to the technical profit arises from the comparison between premiums, claims and expenses for claims occurred in the current year. Recalling that the claims outstanding provision under the Solvency II is defined as the sum of the Best Estimate and the Risk Margin, the premium component of the stochastic technical result can be rewritten as:

$$\tilde{Y}_1^{pr} = \Bigg[\sum_{h=1}^{L}(B_{1,h}^{writt} + PR_{0,h} - \tilde{PR}_{1,h}) - \\ \sum_{h=1}^{L}\tilde{E}_{1,h} - \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY} + \tilde{RM}_{1,h}^{CY}\big)\Bigg]$$

As in paragraph 6.1, the gross premiums of the LoB h can be defined in function of the risk premiums as:

$$B_{1,h}^{writt} = P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}$$

where $\lambda_h P_{1,h}$ is the safety loading and $c_h B_{1,h}$ is the expense loading. In particular, the risk premium is defined as the expected amount of payments and reserve for claims incurred in the current year:

$$P_{1,h} = E\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY}\big)$$

The expense loading is computed as the expected value of expenses:

$$c_h B_{1,h} = E(\tilde{E}_{1,h})$$

Substituting this definition of the gross premium, $\tilde{Y}_1^{pr}$ can be rewritten as:

$$\tilde{Y}_1^{pr} = \Bigg[\sum_{h=1}^{L}(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h} + PR_{0,h} - \tilde{PR}_{1,h}) - \sum_{h=1}^{L}\tilde{E}_{1,h} - \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY} + \tilde{RM}_{1,h}^{CY}\big)\Bigg]$$

Recall that the claims outstanding provision in the year 1 is not affected by the reserve risk, which is already considered in its proper risk submodule (respectively, the reserve risk). Therefore, the additional volatility coming from the run-off is not considered here.

In order to neglect the impact of the premium reserve, the written premiums are assumed to be equal to the earned premiums. It follows that the stochastic technical result's premium risk component can be defined as:

$$\tilde{Y}_1^{pr} = \Bigg[\sum_{h=1}^{L}(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}) - \sum_{h=1}^{L}\tilde{E}_{1,h} - \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY} + \tilde{RM}_{1,h}^{CY}\big)\Bigg]$$

It could be interesting to calculate the expected value of the random variable $\tilde{Y}_1^{pr}$:

$$E(\tilde{Y}_1^{pr}) = E\Bigg[\sum_{h=1}^{L}(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}) - \sum_{h=1}^{L}\tilde{E}_{1,h} - \sum_{i=1}^{L}\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY} + \tilde{RM}_{1,h}^{CY}\big)\Bigg]$$

By recalling the definition of the risk premium and of the expense loading:

$$P_{1,h} = E\big(\tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY}\big)$$

$$c_h B_{1,h} = E\big(\tilde{E}_{1,h}\big)$$

and applying the linearity property of the expected value, it is possible to calculate the expected technical result (only for the premium component) as:

$$E(\tilde{Y}_1^{pr}) = \sum_{h=1}^{L}\lambda_h P_{1,h}$$

By definition, the safety loading is the expected technical profit introduced in the pricing process by the insurer in order to get a remuneration for the risk borne and to have a capital buffer.

In order to compute the capital requirement, a risk measure shall be introduced. Under the Solvency II framework, a VaR risk measure at 99.5% level of confidence shall be considered. Therefore, the SCR for premiums and expense risk can be calculated as:

$$SCR_1^{prem\&exp} = -VaR_{0.5\%}(\tilde{Y}_1^{pr})$$

Notice that the VaR is calculated at a level of confidence of 0.5% which is exactly the complement to 100% of 99.5% level defined by the Solvency II directive. It is taken the opposite of the 0.5% percentile since the risk for the stochastic technical result is on the left tail (risk of decrease in value).
Equivalently, the SCR can be defined also as:

$$SCR_1^{prem\&exp} = VaR_{99.5\%}\left\{\sum_{h=1}^{L} \tilde{X}_{1,h}^{CY} + \sum_{h=1}^{L} \tilde{E}_{1,h}\right\} - \sum_{h=1}^{L}\big(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}\big)$$

where $\tilde{X}_{1,h}^{CY} = \tilde{X}_{1,h}^{paid,CY} + \tilde{BE}_{1,h}^{CY}$. The Risk Margin is excluded from the SCR's computation in order to avoid circularity problems, since in turn the RM is a function of the SCR.
Finally, the SCR for premium and expense risk for the single LoB h can be calculated as:

$$SCR_{1,h}^{prem\&exp} = VaR_{99.5\%}\big\{\tilde{X}_{1,h}^{CY} + \tilde{E}_{1,h}\big\} - \big(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}\big)$$

In order to evaluate the convenience of underwriting or not a reinsurance treaty, the gross SCR for premium and expense risk shall be compared with the net SCR. Therefore, in case the direct insurer underwrites reinsurance contracts, measures must be considered net of reinsurance. In particular, premiums and payments must be net of the premiums paid to the reinsurer and of the claims of competence of the reinsurance company. In addition, reinsurance commissions, if present, must be properly taken into account. Therefore, the Solvency Capital Requirement for premium&expense risk in presence of reinsurance treaties becomes:

$$SCR_1^{prem\&exp} = VaR_{99.5\%}\left\{\sum_{h=1}^{L} \tilde{X}_{1,h}^{CY} + \sum_{h=1}^{L} \tilde{E}_{1,h} - \sum_{h=1}^{L} \tilde{C}_{1,h}^{RE}\right\} - \sum_{h=1}^{L}\big(P_{1,h} + \lambda_h P_{1,h} + c_h B_{1,h}\big)$$

6.3 Collective Risk Model for the aggregate claims amount

It should be noticed that both the Solvency Capital Requirement and the Return of Equity depend on the aggregate claims amount for the h-th LoB. Therefore, in the last paragraph of chapter 6.2, an analysis on $\tilde{X}_{1,h}$ is carried out. For obtaining a simpler and lighter notation, the subscript h is hidden. Following the logic of the Collective Risk Model (CRM), the aggregate claims amount $\tilde{X}_1$ is given by a mixed compound process:

$$\tilde{X}_{1,h} = \sum_{k=1}^{\tilde{K}_1} \tilde{Z}_{k,1}$$

Aggregate claims amounts in year 0 and 1 are assumed to be uncorrelated (no long-term cycles).

$\tilde{K}_1$ denotes the random number of claims occurred in the year 1 and its distribution is typically assumed to be the Poisson law. In practice, the simple Poisson law fails in describing properly the claim number distribution, because, in addition to pure random fluctuations, there are other type of fluctuations. In particular, short-period fluctuations affect only in the short-term the probability distribution without any time-dependence and they could be well represented by a structure variable. Therefore, the parameter of the Poisson becomes stochastic ($n_1\tilde{q}$), where $\tilde{q}$ is the structure variable with unitary expected value, which assures to not affect the expected number of claims. The expected number of claims is given by the expected number of claims in the basic portfolio (at time 0) increased by the real growth rate for one year:

$$n_1 = n_0(1+g)$$

The implicit assumption is that the additional policyholders entering in the portfolio due to the growth rate g have the same claim frequency of the ones inside the basic portfolio. A typical distribution of $\tilde{q}$ is a Gamma distribution with equal parameters. A mixed Poisson-Gamma turns out to be a Negative Binomial and $\tilde{X}_1$ is typically denoted as compound Polya process. In addition, the number of claims distribution $\tilde{K}_1$ is assumed to be independent with respect to any claims size amount $\tilde{Z}_{k,1}$, for $k = 1, 2, \ldots$.

$\tilde{Z}_{k,1}$ represents the random amount of the k-th claim in the year 1. The claim size amounts $\tilde{Z}_{k,1}$ are assumed to be independent and identically distributed random variables, which follow a continuous distribution scaled yearly by the inflation rate i. Therefore, their simple moment of the q-th order is simply defined as:

$$E(\tilde{Z}_{k,1}^{q}) = (1+i)^{q} E(\tilde{Z}_{k,0}^{q})$$

Since the claim amount is only rescaled (yearly), the shape of its distribution is assumed to be constant over time.

Under these assumptions, the moments of the aggregate claim amount distribution can be easily derived:

$$E(\tilde{X}_1) = n_1 m_1 = n_0 m_0 (1+i)(1+g)$$

Notice that the introduction of the structure variable does not impact on the expected aggregate claim amount. Conversely, $\tilde{q}$ introduces an additional source of volatility:

$$\sigma^2(\tilde{X}_1) = n_1 a_{2,Z_1} + n_1^2 m_1^2 \sigma_q^2$$

$$\gamma(\tilde{X}_1) = \frac{n_1 a_{3,Z_1} + 3 n_1^2 m_1 a_{2,Z_1} \sigma_q^2 + n_1^3 m_1^3 \gamma_q \sigma_q^3}{\left(n_1 a_{2,Z_1} + n_1^2 m_1^2 \sigma_q^2\right)^{3/2}}$$

In addition, it could be useful to compute the standard deviation of the loss ratio:

$$\sigma\left(\frac{\tilde{X}_1}{B_1}\right) = \frac{1-c}{1+\lambda} \sqrt{\frac{r_{2,Z_0}}{n_0(1+g)} + \sigma_q^2}$$

Great attention is paid to its asymptotic limit:

$$\lim_{n_0 \to \infty} \sigma\left(\frac{\tilde{X}_1}{B_1}\right) = \frac{1-c}{1+\lambda} \sigma_q$$

Considering the market as a single portfolio, n_0 can be assumed to be close to infinite. The increasing dimension will not eliminate the Loss ratio's variability, since there will be still the impact of the structure variable, which represents a systematic risk eliminable only by reinsurance covers (as quota share or stop loss). Therefore, from the asymptotic volatility of the Loss Ratio, it is possible to derive the volatility of the structure variable. σ_q can be estimated as the standard deviation of the market Loss Ratio, properly adjusted for the proportion of risk premiums with respect to the gross ones. The shortfall of this estimation method is that the volatility of the Loss Ratio also includes the effect of the underwriting cycles, which is quite difficult to be isolated. Due to this additional source of randomness, the value of σ_q, coming from the volatility of the market Loss ratio, could be overestimated.

6.4 Expenses

Inside the Internal Model, the other random variable to be properly modelled in order to calculate the SCR regards expenses. Typically, the Non-Life expense volatility is very low, due to the annual maturity of coverages. Therefore, taking them as deterministic could be a not so strong assumption. If the very low volatility is taken into account, expenses are assumed to be stochastic, and three items must be defined:

- The distribution of expenses. Lognormal or normal distributions could be good choices.
- The parameters of the distribution, which could be calibrated according to the historical time series of the expense ratio.
- The dependence structure between expenses and the aggregate claim amount. Typically, X and E are assumed to be independent, but their dependence structure depends effectively on the particular reality of the company. If the insurer recognizes some fees in addition to acquisition costs to the agencies in case they underwrite portfolios of good quality, expenses will be a function of the aggregate claims amount and the independence assumption will be violated. Since the entity of these additional fees is generally contained, the independence assumption is typically admitted.

In any case, due to the lower relevance of the expense risk with respect to the premium risk, the distributional and the dependence assumptions on E have usually not a strong impact on the capital charge, except for some lines of business as pecuniary losses.

7 Case Study

7.1 Parameters' Calibration

The aim of this case study is to carry out a risk-profitability analysis of several reinsurance treaties, in order to determine which is the optimal risk mitigation solution for the direct insurer. In particular, the cedent is assumed to be a big size multi-line insurance company, which business is made up of the segments Motor Third Party Liability (MTPL), General Third-Party Liability (GTPL) and Motor Own Damage (MOD) according to the following proportions:

- MTPL: 65%
- GTPL: 20%
- MOD: 15%

Since the risk profile is assessed through the computation of the Solvency Capital Requirement for premium risk, the time period considered is one year. A collective Risk Model is fitted, as described in paragraph 6.3, in order to model the next year aggregate claims amount. In particular, some assumptions related to the dynamic portfolio are needed, regarding the claim inflation rate i (3%), the financial rate r (1%) and the portfolio's growth rate g (1.9%). The random variable number of claims has been modelled via a mixed Poisson, where the structure variable is a Gamma (i.e., Negative Binomial), whereas for the single claim amount and expenses through Lognormal distributions. The parameters of these random variables are briefly summarized in table 7.

Segment	λ	c	c_a	$\sigma(c_a)$	c_m	$\sigma(c_m)$
MTPL	-1.36%	21.3%	15.8%	0.3%	5.5%	0.13%
GTPL	10.69%	32.5%	27.3%	0.4%	5.2%	0.16%
MOD	10.63%	30.9%	25.8%	0.7%	5.1%	0.14%

Segment	σ_q	m_0	c_Z	n_0
MTPL	5.93%	4500	5	80000
GTPL	7.96%	10000	10	8225
MOD	4.74%	2500	2	25900

Table 7: Parameters of the Internal Model

The safety loading coefficient (λ) has been calibrated for each segment according to market data published by Ania[40]. In particular, it has been obtained as the complementary to 1 of the average of the last five Combined Ratios (excluding the run-off component)

[40]Data from 2015 to 2019.

and then rescaled by the incidence of risk premiums with respect to gross premiums. It is implicitly assumed that the current safety loading is still in line with the ones observed in the last 5 years. Taking a longer time series could not improve the estimate, since the pricing strategy could be changed over time, introducing bias in the estimation. The segment MTPL has a small negative safety loading, implying, on average, technical losses. Indeed, it is a quite competitive line of business and, in order to acquire more market shares, the insurance company can adopt aggressive tariff strategies, by inserting a negative safety loading. Conversely, segments GTPL and MOD have a positive safety loading coefficient, determining an expected technical profit.

Acquisition and management expenses of the segment h are supposed to be stochastic and described by two Lognormal random variables with mean and standard deviation equal respectively to $(c_h^A B_{1,h}, \sigma_h^A B_{1,h})$ and $(c_h^M B_{1,h}, \sigma_h^M B_{1,h})$. In addition, they are supposed to be independent with respect to the aggregate claims amount. The expense loading coefficient c and its volatility have been calculated respectively as the average and the standard deviation of the last five Expense Ratios, computed separately for acquisition expenses and management expenses. As expected, the acquisition expenses of the segment MTPL are lower than the ones of the other lines of business, because, being a compulsory insurance, there is not so much need to have a distributional channel. On average, the management expenses are quite similar among the three segments, and they have typically a lower incidence on the premiums than acquisition expenses. Notice also that the volatility factors for both acquisition and management expenses are much lower than the corresponding claims' volatility factors, showing the lower relevance of expense risk with respect to the premium risk for a Non-Life company. Indeed, typically Non-Life contracts have an annual maturity, implying a lower expenses volatility and a consequent lower complexity related to their estimation.

The volatility of the structure variable $\tilde{q}$ has been calibrated according to the Loss Ratio volatility (see paragraph 6.3). In order to obtain a more reliable estimate, the standard deviation of the market Loss Ratio has been calculated on the whole time series available (respectively data from 1998 to 2019). The most volatile structure variable is for the segment GTPL since its technical result is typically subject to high volatility.

We assume that the insurance company could try to reduce its risk exposure through the following reinsurance treaties:

- *A quota share ceding 10% of the business, with fixed reinsurance commissions.*

  The fixed commissions coefficient is assumed to be equal to the total expense loading coefficient set by the insurer:

$$c_{RE}^{fixed} = c$$

- *A quota share ceding 10% of the business, with scaling reinsurance commissions.*

  The scaling commissions have been calculated in accordance with the expected Loss Ratio. The expected Loss ratio, defined as the incidence of the risk premiums with respect to the gross premiums, returns to be equal to 79.82% (MTPL), 60.98% (GTPL) and 62.48% (MOD). The highest expected loss ratio is recorded for MTPL segment, due to its lower safety loading coefficient (lower is the expected profitability, higher is the expected loss ratio).

  Five classes of width 5 percentage points define the effective amount of the scaling commissions' coefficient:

$$c_{RE} = \begin{cases} c + 2\delta c, \ if \ LR \leq E(LR) - 7.5\% \\ c + \delta c, \ if \ E(LR) - 7.5\% < LR \leq E(LR) - 2.5\% \\ c, \ if \ E(LR) - 2.5\% < LR \leq E(LR) + 2.5\% \\ c - \delta c, \ if \ E(LR) + 2.5\% < LR \leq E(LR) + 7.5\% \\ c - 2\delta c, \ if \ LR > E(LR) + 7.5\% \end{cases}$$

  where $\delta = 10\%$. Notice that the middle class is centred on the expected loss ratio, and it recognizes exactly the expense loading coefficient c. Lower is the Loss ratio, more profitable is the ceded portfolio, higher will be the commissions retroceded from the reinsurer to the insurer.

- *An excess of loss (XL) treaty, with priority M and without cover.*

  In order to get comparable values among the different segments, it is assumed that the priority M is defined in accordance with the moments of the claim size distribution as:

$$M = E(Z) + k \cdot \sigma(Z)$$

  The multiplier k has been calculated by taking a typical priority value for the MTPL segment, which is 500000. Therefore, the rounded multiplier k is 21. The resulting priorities are respectively 491310 for MTPL, 2173300 for GTPL and 110725 for MOD. More or less, they correspond to the 99.98-th percentile of the claim size distribution.

- *XL treaty, with priority M and limit 2M.*

- *Multi-Line treaty.*

  An umbrella policy acting on the aggregate claim amounts arising from all the segments is introduced. Its priority has been calibrated as described in paragraph 7.4.1.

In terms of risk measure, the capital requirement for premium risk will be computed as stated by the Standard Formula (see paragraph 3.5) and by applying an internal model (see paragraph 6.2). The measure of profitability applied is the Return on Equity (ROE).

7.2 Mono-line Analysis

First of all, it is analysed the aggregate claims amount distribution and the reinsurance impact on the profitability and on the risk profile separately for each segment.

7.2.1 Distributions

The main characteristics of the aggregate claims amount distributions simulated[41] separately for each line of business are summarised in Table 8.

Segment	Mean	Variance	CV	Skewness
MTPL	377 903 102	$5.41153 \cdot 10^{14}$	6.156%	0.128
GTPL	86 312 956	$1.34996 \cdot 10^{14}$	13.461%	2.971
MOD	67 957 302	$1.10905 \cdot 10^{13}$	4.9%	0.097

Table 8: Simulated moments of X

The main comments are:

- The segment MTPL has the highest expected aggregate claims amount due to the highest expected number of claims.

- Only referring to the absolute variance could be misleading, since the lines of business have different dimension due to the different expected number of claims. Therefore, it is better to refer to the coefficient of variation (absolute volatility divided by the mean). The most variable segment is General Third-Party Liability, due to the high coefficient of variation of the claim size distribution ($c_Z = 10$).

- The skewness of the GTPL aggregate claims amount distribution is highly above the threshold level of 1. Therefore, from Figure 12, it is possible to notice the long right tail. In addition, consider that the high value of the coefficient of variation of the claim size distribution makes the convergence process of the simulated moments to their theoretical values much slower. Indeed, the exact skewness for the segment GTPL,

[41] 600000 Monte Carlo simulations are run.

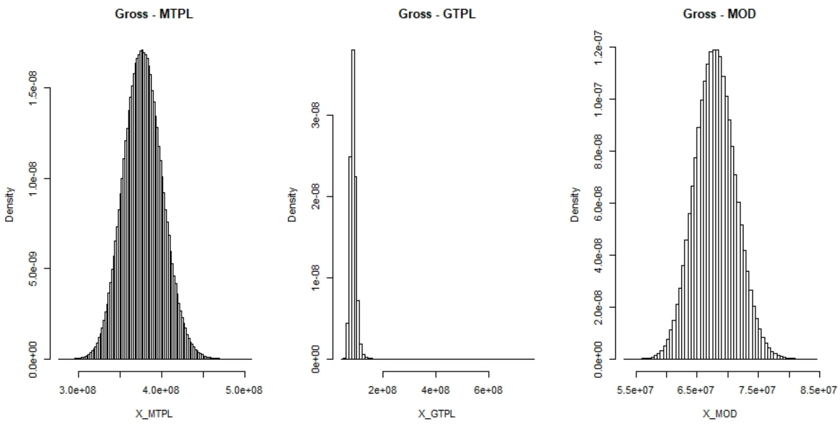

Figure 12: Comparison between Gross Aggregate Claims amount distributions for each segment

given the current parametrization, should be 5.977. A better convergence could be obtained by increasing the number of simulations (due to the law of large numbers), but it will require a higher computational time. Conversely, the MTPL and MOD distributions are almost symmetrical due to the low value of the skewness index.

The same analysis can be performed on the aggregate claims amount distribution net of the reinsurance treaty. Results related to the quota share treaty are summarised in Table 9.

Segment	**Mean**	**CV**	**Skewness**
MTPL	340 112 792	6.156%	0.128
GTPL	77 681 660	13.461%	2.971
MOD	61 161 571	4.9%	0.097

Table 9: Simulated moments of net aggregate claims amount – Quota Share

Since the quota share treaty cedes losses in the same proportion regardless their magnitude, it is expected a reduction in terms of expected value and standard deviation equal to the ceded quota. The coefficient of variation and the skewness are exactly the same of the gross aggregate claim amounts distribution because the treaty only rescales it. Therefore, the treaty is able to reduce the risk only in absolute terms, but it is not able to protect the insurer from extreme events (by reducing the relative volatility). In Figure 13, it is possible to notice that the shape of the distribution is totally unchanged.

Table 10 summarises the simulated moments of the aggregate claims amount distribution net of the XL treaty without limit. Conversely to the quota share treaty, the XL is able to reduce also the relative volatility and the skewness. Indeed, the XL cuts the right tail of the claim size distribution, covering only extreme claims. Therefore, also the aggregate claims amount distribution will be less (relatively) volatile and less skewed (but not tail-cut, since the treaty does not affect the number of claims). The strongest effect is recorded for

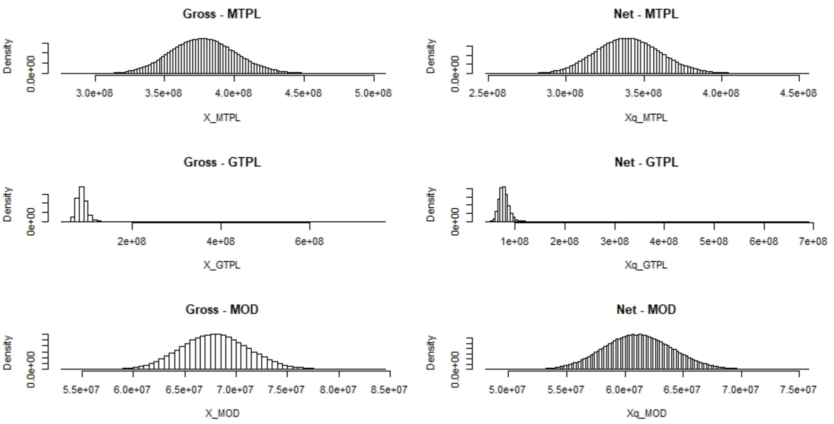

Figure 13: Graphical comparison between net and gross distributions – Quota Share

Segment	Mean	CV	Skewness
MTPL	370 201 287	6.046%	0.117
GTPL	82 874 038	10.236%	0.201
MOD	67 751 827	4.885%	0.096

Table 10: Net and gross simulated moments of aggregate claims amount – XL (no limit)

GTPL business. Even if the priority M corresponds more or less for all the segments to the 99.98-th percentile of their claim size distribution, the skewness index decreases strongly from 2.971 to 0.201. Figure 14 shows how the XL is able to reduce the very long right tail, characterizing the GTPL business. Similar effects but smaller in magnitude are observed also for MTPL and MOD.

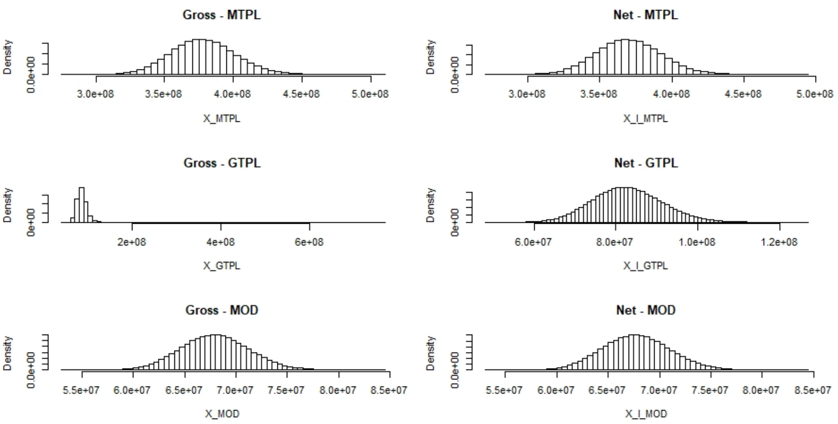

Figure 14: Graphical comparison between net and gross aggregate claims amount distribution – XL (no limit)

In order to limit its risk exposure, the reinsurer can impose a limit on the claim amount it is responsible for. In Table 11, the results are summarised. The expected net aggregate claims amount is higher than in the XL without limit as expected. Indeed, without any limit

Segment	Mean	CV	Skewness
MTPL	371 821 915	6.089%	0.126
GTPL	83 942 921	12.268%	3.678
MOD	67 764 039	4.887%	0.096

Table 11: Net and gross simulated moments of aggregate claims amount – XL (with limit)

the reinsurer will cover for each claim the entire claim size above the priority, whereas this value is upper bounded at the limit if present. By introducing a cover, the right tail of the claim size distribution is not cut, but it is still of competence of the direct insurer. Therefore, the reduction of the skewness and of the coefficient of variation coming from the XL with limit is lower than the one obtained from the corresponding XL without limit. The result in terms of skewness with respect to the gross case does not hold for the segment GTPL. Since the GTPL gross distribution is characterized by a very volatile and skewed claim size, by only ceding a portion of the tail of its distribution, extreme claims are still of competence of the insurer, making the skewness of the net aggregate claims amount distribution increase strongly.

7.2.2 Reinsurance Premiums

In order to quantify the impact of reinsurance treaties on the cedent's profitability and risk profile, it should be assessed how reinsurance coverages are priced. Premiums paid to the quota share reinsurer are summarized in Table 12, separately for each segment.

Segment	B_{RE}	Fixed Commissions	Average Scaling Commissions
MTPL	47 338 674	10 083 138	10 084 953
GTPL	14 157 650	4 601 236	4 638 819
MOD	10 877 479	3 361 141	3 361 197

Table 12: Premiums paid to the quota share reinsurer.

On average, the scaling commissions corresponds to the fixed commissions, for the way in which they are defined. Therefore, fixed and scaling commissions should have a similar impact on the net profitability, but a different effect on the cedent's risk profile due to the additional volatility arising from the scaling commissions.

The Excess of Loss treaties have been priced as the aggregate claims amount the reinsurer expects to pay increased by a safety loading. The safety loading coefficient has been calculated as (absolute value of) the safety loading coefficient applied by the direct insurer adjusted for the ratio between the gross and the net relative volatility. It is a way to increase the remuneration for the risk borne since the reinsurer covers only extreme events. Results are depicted respectively in Table 13 for the XL without limit and in Table 14 for the XL with limit.

Segment	B_{RE} - XL (no limit)	λ_{RE} - XL (no limit)
MTPL	7 811 604	1.425%
GTPL	3 922 815	14.071%
MOD	227 323	10.633%

Table 13: Premiums of the XL treaty without limit and the related safety loading coefficient.

Segment	B_{RE} - XL (with limit)	λ_{RE} - XL (with limit)
MTPL	6 167 258	1.415%
GTPL	2 648 300	11.741%
MOD	213 807	10.63%

Table 14: Premiums of the XL treaty with limit and the related safety loading coefficient.

The introduction of a limit in the XL cover makes the expected aggregate claims amount ceded to the reinsurer decrease, implying a lower risk premium. In addition, it leads a lower reduction of the relative volatility, determining lower safety loading. Due to both effects, the premium charged by the reinsurer for the XL with limit is lower than the one for the XL without limit.

Comparing the proportional treaties with the non-proportional ones, it is possible to say that for all the segments the quota share treaties have a stronger impact on the profitability due to the higher premiums paid to the reinsurer. This result does not hold for the XL treaty without limit, which implies a very strong reduction of the relative volatility and of the skewness, leading to a very high premium.

7.2.3 Profitability per segment

Before making a risk-profitability analysis for the whole multi-line insurer, it is assessed the impact of the different reinsurance treaties on the profitability separately for each segment. The gross and net expected returns on equity are summarized in Table 15.

	MTPL	GTPL	MOD
Gross	-3.766%	28.57%	28.951%
Net quota (fixed)	-3.29%	25.813%	26.156%
Net quota (scaling)	-3.288%	25.925%	26.156%
Net XL (no limit)	-3.846%	27.128%	28.867%
Net XL with limit	-3.843%	27.741%	28.872%

Table 15: Net and gross profitability differentiated per segment

For all the segments, the XL treaties determines a reduction of the profitability. Notice that for MOD, the difference between gross ROE and net ROE is very limited, due to the small effect on the relative volatility and the skewness given by the reinsurance cover. Conversely, the highest reduction in profitability due to the XL treaty is recorded for GTPL. In all the cases, the introduction of a limit leads to a lower profitability reduction since the reinsurer's intervention is upper bounded.

The quota share treaty has a different impact according to the segment to which it is applied. It implies a profitability reduction for GTPL and MOD, since a portion equal to α of the expected technical profit is ceded. Since the segment MTPL has a negative safety loading coefficient, a proportion α of the expected technical losses (i.e., the safety loading) is ceded, implying an improvement of the profitability. In addition, for all the segments, the introduction of scaling commissions with respect to fixed ones does not have a very strong impact on the expected Return on Equity.

7.2.4 Premium risk Solvency Capital Requirement

Internal Model. The capital requirement for premium risk has been calculated gross and net of the reinsurance treaties according to the internal model. The results are summarised in Figure 15. For obtaining a better comparison between the different segments, it should be more reasonable to calculate the capital absorption, i.e., the ratio between SCR and (gross) premiums.

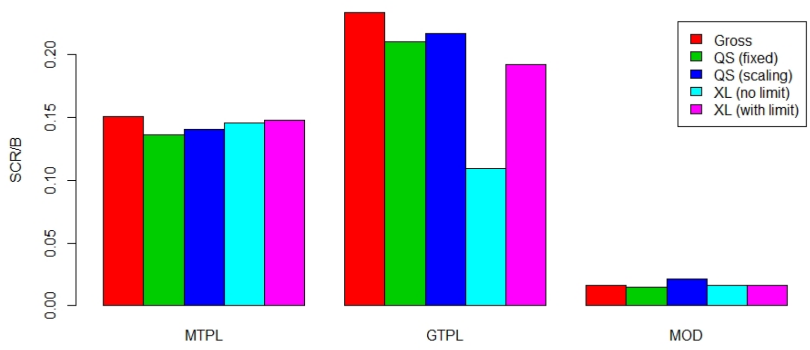

Figure 15: SCR/B per segment

The following general comments can be made:

- all reinsurance treaties are effective in all the segments in reducing the cedent's risk profile.
- the introduction of scaling commissions leads to an increase in the volatility, which makes the capital requirement (and consequently the capital absorption) increase.

- The introduction of a limit inside the XL makes the solvency capital requirement increase since there is a lower risk protection provided by the reinsurer. This result does not hold for the segment MOD, since the introduction of the limit in the XL cover does not impact so much on the characteristics of net aggregate claims amount, due to the very low skewness of the gross distribution. Therefore, the lower premiums of the XL treaty with limit are able to compensate the slightly higher skewness and relative volatility with respect to the XL without limit, making the two resulting capital requirements be very similar.

In addition, it should be noticed that the capital absorption strongly decreases with the introduction of the XL treaty without limit in the GTPL segment. The reinsurance cover is able to make the aggregate claims amount distribution less skewed and more insurable, determining a quite low ratio between capital requirement and premiums.

The introduction of scaling commissions determines a consistent increase of SCR for the MOD. Since the segment is characterized by a quite low (relative) volatility, the incidence of the scaling commission on the aggregate claims amount's volatility is higher for MOD than for the other lines of business (7% versus 4-5%). Therefore, scaling commissions makes the MOD's volatility increase dramatically, determining that the capital absorption net of quota share (21.12%) becomes also higher than the gross one (16.25%).

Standard Formula. In the Solvency II framework, it is possible to compute the capital requirement also by applying the Standard Formula. In order to show its shortfalls and limits, it is possible to make a comparison between the per-segment capital requirements obtained by applying the Standard Formula and the Internal Model. Since the Standard Formula does not consider the expected profit, a sensitivity analysis inside the Internal Model is performed by removing the safety loading and the expenses in order to get more comparable results.

	MTPL	**GTPL**	**MOD**
Gross - no safety loading	62 802 013	40 707 567	8 888 074
Net quota (fixed) - no safety loading	56 523 955	36 638 161	8 000 045
Net quota (scaling) - no safety loading	58 540 582	37 558 408	8 672 273
Net XL (no limit) - no safety loading	60 298 434	23 447 632	8 837 389
Net XL with limit - no safety loading	61 158 358	34 859 558	8 838 356
Gross - no safety loading and expenses	62 792 108	40 732 146	8 890 177

Table 16: Sensitivity analysis per segment on the SCR

As Table 16 shows, the ignorance of the safety loading makes the SCR decrease (increase) in case the expected technical profit is negative (positive). Indeed, in case of positive safety loading, the company will have more resources to cover the risks and its need for capital goes down. Therefore, the Standard Formula is overestimating the premium risk SCR when the safety loading is positive and underestimating it when λ is negative.

In addition, by removing the expenses, the capital requirement should decrease since the risk borne is lower. This does not happen for the segments MOD and GTPL and it could be justified by particular behaviour in terms of skewness and of standard deviation. Therefore, by ignoring expenses, it is possible to say that:

- the standard deviation decreases, making the SCR decrease.
- The skewness increases, making the implicit multiplier and the SCR increase.

For segments GTPL and MOD, the second effect prevails on the first one, making the SCR increase in absence of expenses.

Comparison. Finally, a brief comparison between the results of the Internal model and the Standard Formula is provided. The SCR excluding the safety loading and expenses is assumed to be the most comparable with the SF one. In order to simplify the comparison, also the multipliers and the volatility factors implicit in the Internal Model are reported in Table 17.

	Internal Model	**Standard Formula**
MTPL - SCR	62 792 108	142 016 023
GTPL - SCR	40 732 146	59 462 129
MOD - SCR	8 890 177	26 105 950
MTPL - multiplier	2.697	3
GTPL - multiplier	3.507	3
MOD - multiplier	2.670	3
GTPL - SCR	40 732 146	59 462 129
MOD - SCR	8 890 177	26 105 950
MTPL - volatility factor	4.914%	10%
GTPL - volatility factor	8.207%	14%
MOD - volatility factor	3.062%	8%

Table 17: Comparison between Standard Formula and Internal Model in terms of SCR, volatility factor and multiplier.

For each segment, it is possible to notice how the Standard Formula is strongly overestimating the capital requirement. The result can be justified by a market-wide volatility factor higher than the one implicit in the Internal Model. At the same time, for segments MTPL and MOD also the multiplier is overestimated, whereas for GTPL is underestimated. Indeed, underlying the Standard Formula, there is a general assumption of skewed aggregate claims amount distribution, but not dependent on the risk profile and on the size of the insurance company.

Up to now the comparison involves only gross SCRs, but the Standard Formula will present other limits in the computation of the capital requirement net of reinsurance treaties, as the results in Table 18 shows. There is no consideration of the additional volatility introduced by scaling commissions since it is obtained the same SCR for the quota share with fixed commissions and for the quota share with scaling commissions. In addition, for the non-proportional treaties, the Standard Formula allows for a non-proportional factor equal to 80% only for some segments, which reduces the volatility factors, but it does not consider the effective risk reduction coming from the XL covers.

	MTPL	**GTPL**	**MOD**
Gross	31.487%	44.082%	25.19%
Net quota (fixed)	28.338%	39.674%	22.671%
Net quota (scaling)	28.338%	39.674%	22.671%
Net XL (no limit)	24.774%	34.288%	25.137%
Net XL with limit	24.862%	34.606%	25.140%

Table 18: Gross and net capital absorption – Standard Formula

7.3 Aggregation

Once properly analysed the risk-profitability effect of the reinsurance treaties on the single segments, it is possible to carry out the same analysis on their aggregation, considering the insurer as a multi-line company. Therefore, an assumption of dependence structure is needed and crucial since it will impact on the overall solvency capital requirement and on the diversification benefit. In terms of profitability, the dependence structure does not play any role. Indeed, the expected return on equity is based on an expectation, which is not affected by the dependence. Therefore, the total ROE can be easily computed as the sum of the ROEs per segment, properly weighted for their relevance in the insurer business, measured by the ratio between the premiums of the segment and the total gross premiums. Results gross and net of reinsurance treaties are summarized in Table 19. It is possible to make similar comments to the ones reported in the profitability analysis per segment.
Conversely, the dependence structure strongly affects the aggregated SCR. In the following paragraphs, the overall premium risk Solvency Capital Requirement is computed as prescribed by the Standard Formula and by applying the Internal Model three different dependence structures: independence, Gaussian copula and Gumbel copula.

	ROE
Gross	7.477%
Net quota (fixed)	6.829%
Net quota (scaling)	6.852%
Net XL (no limit)	7.188%
Net XL with limit	7.252%

Table 19: gross and net ROE

7.3.1 Linear Correlation

The Standard Formula prescribes an aggregation of the volatility factors according to the mathematical formula of standard deviation's aggregation with given correlation coefficients. The resulting Solvency ratios[42] are summarized in Table 20.

	Solvency Ratio
Gross	89.243%
Net quota (fixed)	99.159%
Net quota (scaling)	99.159%
Net XL (no limit)	111.312%
Net XL with limit	110.826%

Table 20: Aggregated Solvency ratios - Standard Formula

Without any reinsurance treaties, the insurance company has not sufficient own funds to meet the capital requirement if computed according to the Standard Formula. Only underwriting the XL treaties, its Solvency Ratio results to be above the threshold 100%. Results in terms of risk-profitability are summarized in Figure 16.

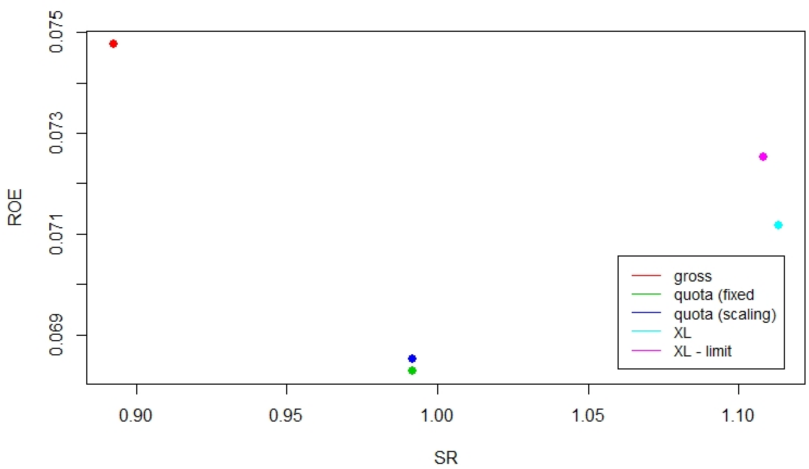

Figure 16: Risk-profitability analysis – Standard Formula

On the y-axis, it is drawn the expected return on equity, whereas on the x-axis, the Solvency

[42]The initial capital available is assumed to be equal to 25% of the gross premiums.

ratio. The desired reinsurance treaty should maximize the Solvency Ratio and minimize the profitability reduction. It seems that the XL treaties are better than the quota share treaties, since they are able to make the SR increase more and to lower the ROE less. In addition, the insurance company has to decide whether or not to introduce the limit. The choice depends on which are the guidelines of the cedent in terms of profitability promised to the shareholders and of its risk appetite. If the company wants to improve as much as possible its solvency position, the XL without limit seems to be the best solution. If it wants to protect as much as possible the gross profitability, it is better to sacrifice a small portion of Solvency Ratio in change of a smaller reduction of ROE.

7.3.2 Independence

By simply assuming an independence structure, the total aggregate claims amount is given by the sum of the aggregate claims amounts of each single segment. The empirical density function and the main characteristics of its distribution are depicted in Figure 17 and in Table 21. The distribution is quite symmetric, and it is not particularly highly volatile, since aggregate claims amounts from different segments occur independently and tend to compensate each other. In addition, it is possible to notice that the coefficient of volatility and the skewness are quite in line with the one of MTPL, being the predominant segment in the portfolio.

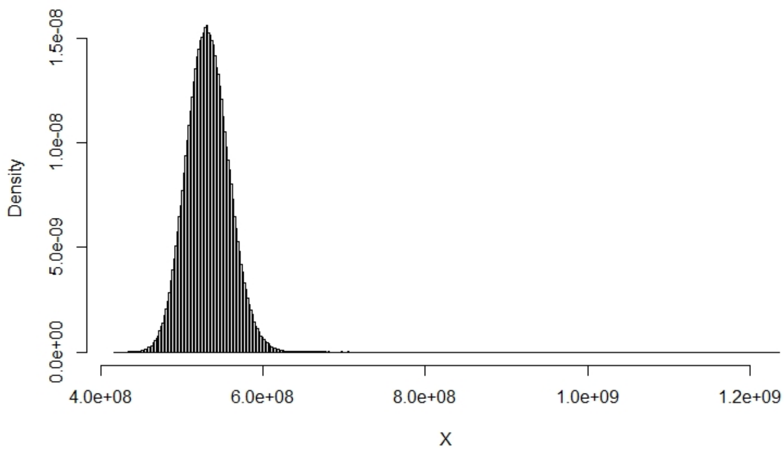

Figure 17: Total Aggregate Claims amount distribution

Results in terms of Solvency Capital Requirement are summarized in Table 22. As expected, all the reinsurance treaties are able to reduce the aggregate capital requirement.

Mean	532 131 990
Variance	$6.88233 \cdot 10^{14}$
CV	4.93%
Skewness	0.356

Table 21: Total Aggregate Claims Amount moments

	SCR
Gross	61576713
Net quota (fixed)	55408875
Net quota (scaling)	57926695
Net XL (no limit)	54198895
Net XL with limit	58386718

Table 22: SCR – Internal Model

The XL treaty without limit is the most effective in reducing the SCR. Conversely to the results obtained from the Standard Formula, the introduction of scaling commissions makes the Solvency Ratio decrease due to the additional volatility.

As made for the single segments, it is possible to carry out some sensitivity analyses in order to make a much proper comparison between the Internal model and the Standard Formula results. The Solvency Ratios are summarized in Table 23.

	Solvency Ratios
Gross, no λ	237.034%
Net quota (fixed), no λ	263.412%
Net quota (scaling), no λ	253.653%
Net XL (no limit), no λ	266.3030%
Net XL with limit, no λ	249.288%
Gross, no λ and expenses	237.015%

Table 23: SRs ignoring the effect of the safety loading and of expenses – Internal Model

By removing expenses, there is still a bit inconsistent result since the SCR increases. As justified in paragraph 7.2.4, the remotion of expenses implies a reduction of the standard deviation and an increase of the skewness (and of the implicit multiplier), which effect prevails.

It is possible to say that in gross and net terms, the Standard Formula is overestimating the Solvency Capital requirement, by assuming that the Internal Model is perfectly able to describe the insurer's risk exposure. By ignoring the safety loading and expenses, the different results can be explained by:

- *Multiplier.* The Standard Formula prescribes a multiplier equal to 3, whereas the implicit multiplier in the Internal Model is 2.771 (lower skewness).

- *Volatility Factor.* From the two-step aggregation, the aggregate volatility factor underlying the Standard Formula computation is equal to 8.897%, whereas in the Internal model it is much lower and equal to 3.625%.

- *Dependence structure.* Underlying the Standard Formula, there is an assumption of linear correlation between segments, which makes the SCR increase with respect to the independence assumption.

Once assessed the Solvency Capital Requirement including the effect of the safety loading and of expenses, it is possible to define which could be the optimal reinsurance treaty for the cedent (Figure 18).

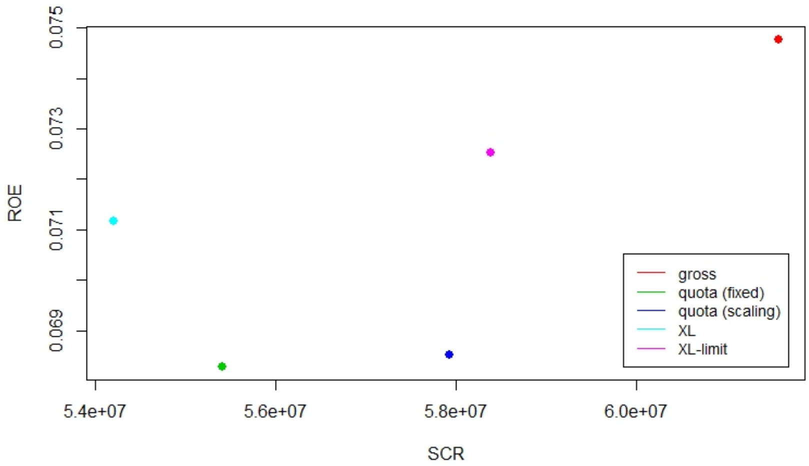

Figure 18: Risk-profitability analysis – Internal Model

The XL without limit is the treaty which reduces the most the Solvency Capital Requirement. A similar risk reduction could be obtained via a quota share with fixed commissions, but with a higher profitability reduction. In addition, if the company does not want to sacrifice too much ROE, the introduction of a limit inside the XL coverage could be a good solution.

7.3.3 Gaussian Copula

An independence structure among aggregate claims amount of the segments could be a not so realistic assumption. Therefore, a Gaussian copula has been fitted and calibrated according to the (Pearson) linear correlation coefficients provided by the Delegated Acts[43]. The aggregation process has been made on the sum of the random variables aggregate

[43] In particular, the correlation coefficient between MTPL and GTPL and MTPL and MOD is 0.5, whereas the one between GTPL and MOD is 0.25.

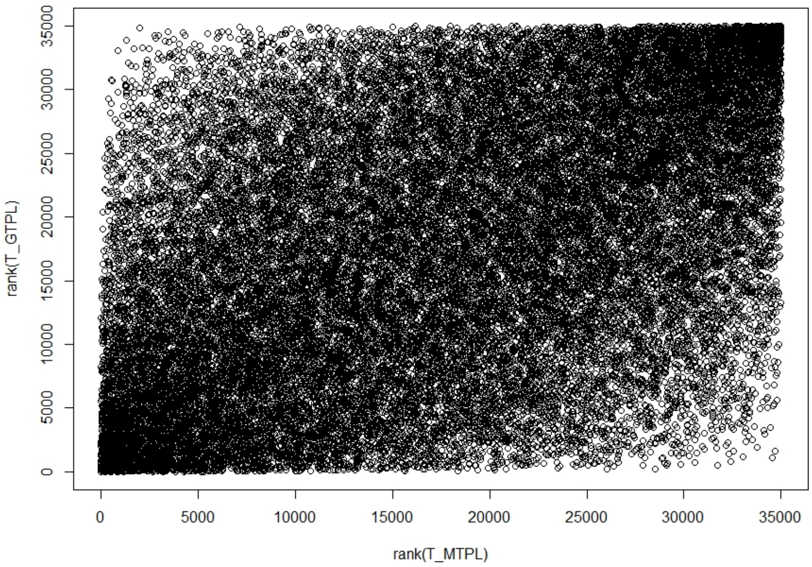

Figure 19: rank scatterplot between MTPL and GTPL – Gaussian copula

claims amount and expenses ($X + E$) of each segment. Since their joint distribution is not known in closed form, a sorting process has been introduced. Therefore, for each segment, the realizations of $X + E$ (denoted as T) have been sorted according to the rank of the simulations out of the Gaussian copula. Secondly, to the sorted realizations of $X + E$, the related aggregate claims amount ceded to the reinsurer and the reinsurance commissions are associated with. In order to verify the appropriateness of the sorting process, the following plots (respectively Figures 19, 20 and 21) are drawn, linking the rank of variables T for all the possible pairs of combinations of segments[44]. From the plots, it is possible to notice the linear behaviour in the body of the distribution since the Gaussian copula is the dependence structure of a multivariate linearly correlated Normal distribution. In the tails' regions, there is totally independence (no tail dependence). In addition, higher is the correlation coefficient between the pairs of marginals, stronger is their linear relationship. Indeed, there is a stronger relationship between segments MTPL-GTPL and MOD-MTPL than between MOD-GTPL.

It is also good practice to check the convergence of the simulated (rank) linear correlation with respect to the original parameters provided by the Delegated Acts. From Table 24, it is possible to notice a quite good convergence towards the theoretical value, which can be improved by increasing the number of simulations.

[44]In order to get a clearer graph, only 35000 simulations are drawn.

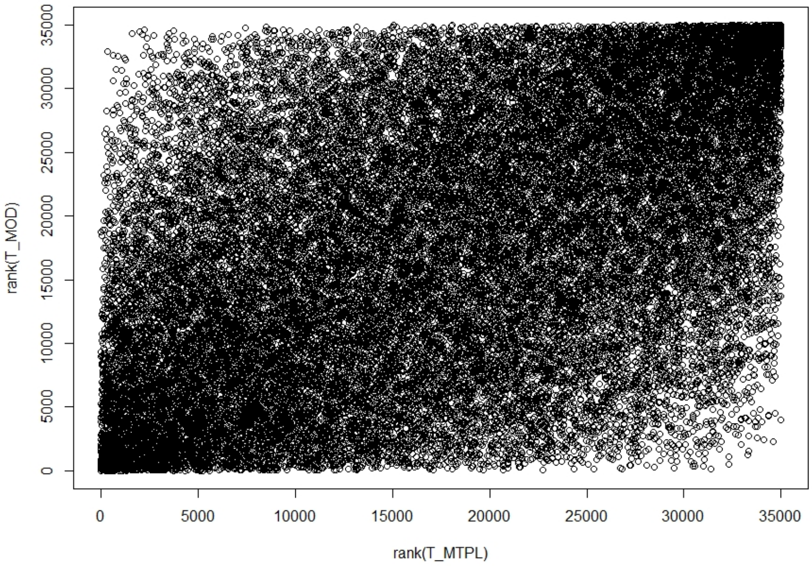

Figure 20: rank scatterplot between MTPL and MOD – Gaussian copula

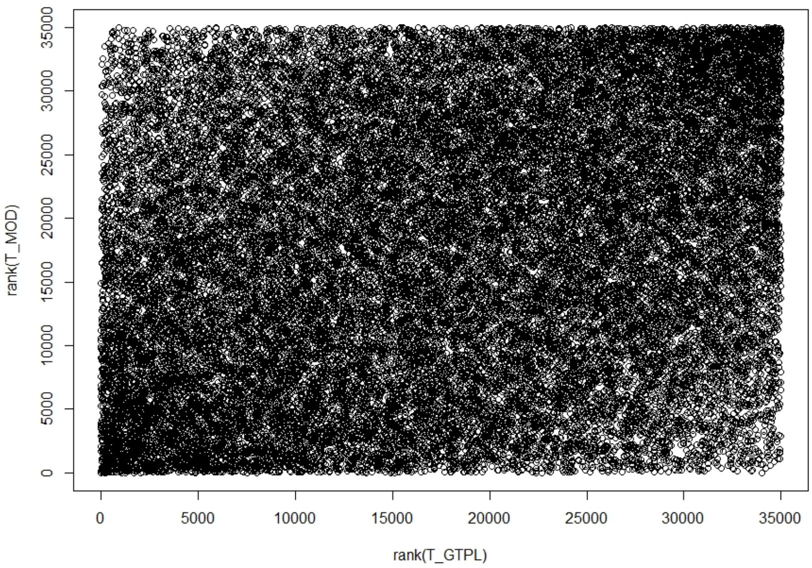

Figure 21: rank scatterplot between GTPL and MOD – Gaussian copula

	MTPL	GTPL	MOD
MTPL	1	0.4820515	0.4810494
GTPL	0.4820515	1	0.2391731
MOD	0.4810494	0.2391731	1

Table 24: Linear correlation coefficient – Gaussian copula

The capital requirements obtained under the independence assumption and the Gaussian copula are summarised and compared in Figure 22.

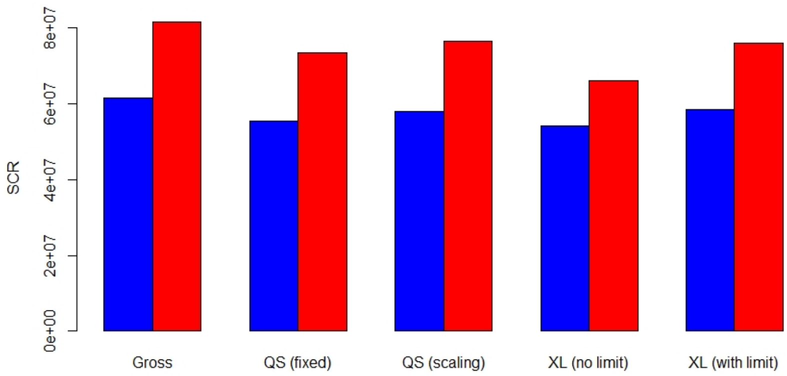

Figure 22: SCR – Internal model, Independence (blue) and Gaussian copula (red)

It is possible to notice that the SCR is significantly higher with respect to the independence case, implying a higher capital absorption and a lower Solvency Ratio, given the same gross premiums and initial capital available. This result could be explained recalling the relationship existing between dependence structure and the diversification benefit. Even though the Gaussian copula does not have any tail dependence, it fits a dependence stronger than the independence case, implying a lower diversification benefit and, consequently, a higher Solvency Capital Requirement. Since the Gaussian copula fits a linear dependence structure calibrated according to the coefficients stated in the Delegated Acts, these results are more comparable to the ones of the Standard Formula with respect to the independence assumption. In any case, the Standard Formula is still providing higher results than the internal model, showing its inappropriateness. In particular, excluding expenses, the multiplier, and the volatility factor implicit in the Internal model under the Gaussian copula assumption are respectively 2.874 and 6.059%, which are higher than in the independence case, but lower than the ones provided by the Standard Formula.

Once assessed the SCR to be met by the insurance company, a risk profitability analysis on the reinsurance contracts is needed and depicted in Figure 23. From the plot, it should

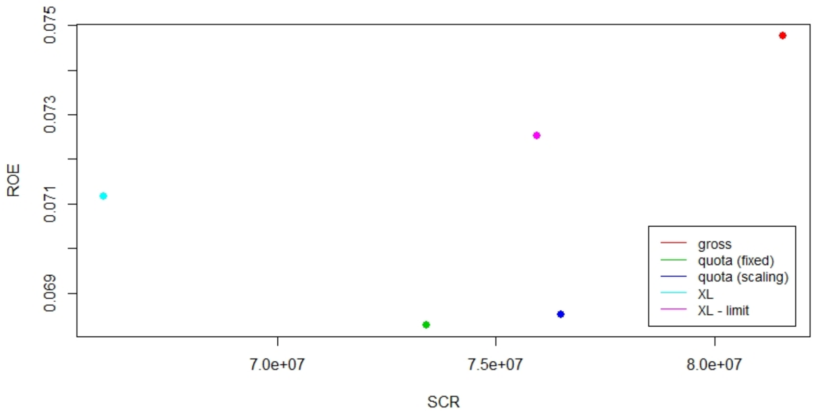

Figure 23: Risk-profitability analysis – Gaussian copula

be noticed that the reinsurance treaties are quite good in decreasing the Solvency capital requirement, but not as in the independence case. The XL without limit is still the contract providing the highest improvement of Solvency Ratio, with a quite limited reduction in terms of profitability. In this case, there is a quite relevant sacrifice in terms of Solvency Ratio, by introducing the limit in the XL treaty. The quota share treaties are not so able to provide a consistent increase in the Solvency Ratio and they imply the highest reduction of profitability. They should be avoided.

7.3.4 Gumbel Copula

The main limit of the Gaussian copula is the absence of tail dependence, which should be considered, if present, in the solvency capital requirement computation. Therefore, a Gumbel copula is introduced. In order to make consistent comparisons, the value of the Kendall's tau has been computed in function of the parameters of the Gaussian copula according to the formula specified in paragraph 5.5. Once assessed the Kendall's Tau, easily the parameter θ can be calculated. Since the Gumbel copula is able to fit only pairwise dependence, the following two-step aggregation is needed.

First of all, the segments GTPL and MOD are aggregated. Therefore, simulations out of a Gumbel copula of parameter theta equal to 1.191698 (coming from a linear correlation coefficient equal to 0.25) are run. Then, for each segment, the realizations of $X + E$ are sorted according to the rank of the copula simulations. Aggregate claims amounts ceded to the reinsurer and scaling commissions are properly associated with them. For the second aggregation, a bit of reasoning is needed since the Delegated Acts do not provide any correlation coefficient between MTPL and the aggregated GTPL-MOD. Therefore, in order to allow proper comparisons, an idea could be to estimate the linear correlation coefficient between aggregate claims amounts and expenses (variable T) for MTPL and aggregate claims amounts and expenses for GTPL and MOD aggregated through the Gaussian copula. The procedure is the following:

- Denote with T_h, the random variable representing the sum of aggregate claims amount and expenses for a specific segment h. Therefore, T_h^{gauss} is the sum of expenses and aggregate claims amount for the segment h after the application of the Gaussian copula.
- Estimate the variance of the following random variables:

$$Z^{gauss} = T_{MTPL}^{gauss} + T_{GTPL}^{gauss} + T_{MOD}^{gauss}$$

$$Y^{gauss} = T_{GTPL}^{gauss} + T_{MOD}^{gauss}$$

$$W^{gauss} = T_{MTPL}^{gauss}$$

Since Z^{gauss} can be seen as the sum of Y^{gauss} and W^{gauss}, the variance of Z^{gauss} can be calculated as:

$$var(Z^{gauss}) = var(Y^{gauss}) + var(W^{gauss}) + 2\rho\sigma(Y^{gauss})\sigma(W^{gauss})$$

Therefore, ρ can be calculated as:

$$\rho = \frac{var(Z^{gauss}) - var(Y^{gauss}) - var(W^{gauss})}{2\sigma(Y^{gauss})\sigma(W^{gauss})}$$

The resulting correlation coefficient is 0.5603546, which implies a value of θ equal to 1.609451. As for the first aggregation, simulations out of a Gumbel copula with parameter θ equal to 1.609451 are run. Secondly, simulations of MTPL and of the aggregate GTPL-MOD are sorted according to the rank of the copula's simulations. Finally, the correspondent aggregate claims amounts of GTPL and MOD are computed.

Table 25 summarises the estimated rank correlation between pairs of marginals out of the Gumbel copula. Notice that the correlation between MTPL and MOD is quite far from the initial Pearson correlation coefficient (0.5) defined by the Delegated Acts, due to the aggregation procedure adopted.

	MTPL	GTPL	MOD
MTPL	1	0.5156009	0.2707733
GTPL	0.5156009	1	0.2378412
MOD	0.2707733	0.2378412	1

Table 25: Linear correlation coefficient – Gumbel copula

In Figures 24, 25 and 26, the rank scatterplots of simulations out of the copula-correlated marginals are drawn[45]. With respect to a Gaussian copula, it is possible to notice the additional tail dependence. In particular, it is emphasised the right tail dependence, which is stronger, higher is the parameter theta. A higher value of theta is determined by a stronger dependence structure (higher Pearson correlation coefficient).

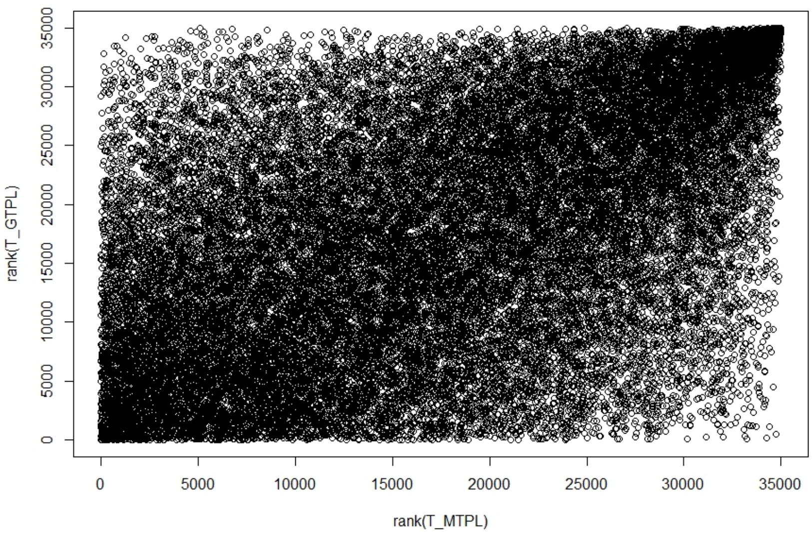

Figure 24: rank scatterplot between MTPL and GTPL – Gumbel copula

[45]In order to get a clearer graph, only 35000 simulations are drawn.

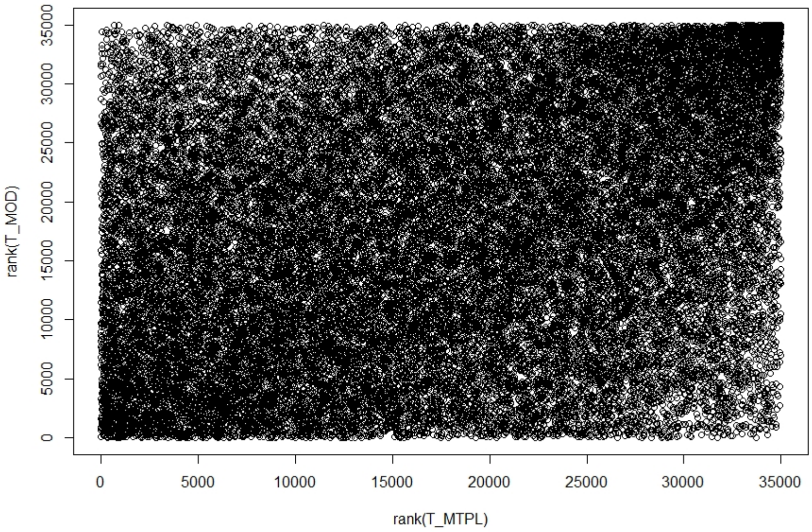

Figure 25: rank scatterplot between MTPL and MOD – Gumbel copula

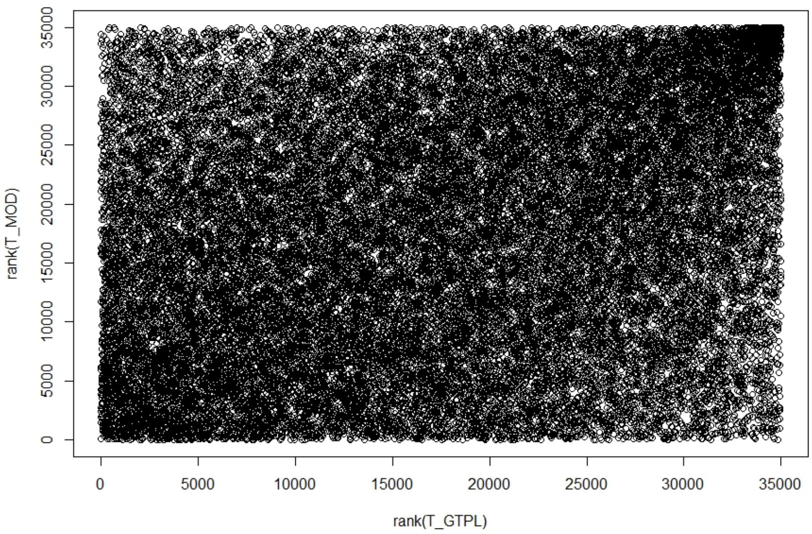

Figure 26: rank scatterplot between GTPL and MOD – Gumbel copula

Before computing the capital requirement, it could be interesting to make a comparison in terms of aggregate claims amount distribution under the three dependence structures. Their main characteristics are summarised in Table 26. As expected, the mean is totally not affected by the dependence structure. Conversely, stronger is the dependence, higher is the variance. Indeed, with respect to the independence case, the Gaussian copula introduces a linear correlation between aggregate claims amounts of different segments. In addition, the Gumbel copula also considers a right tail dependence, which is an additional source of dependence among lines of business. Finally, the skewness is the highest under the Gumbel copula, due to tail dependence.

	Mean	**Standard Deviation**	**Skewness**
Independence	532 173 360	26 234 201	0.3555567
Gaussian Copula	532 173 360	32 244 548	0.3840831
Gumbel Copula	532 173 360	32 401 943	0.7205448

Table 26: Main characteristics of the aggregate claims amount distribution under three different dependence assumptions.

Once properly analysed the distribution, it is possible to compute the premium SCR as made under the Gaussian copula. Solvency capital requirements net and gross of reinsurance are summarized in Table 27.

	Solvency Capital Requirement	**Solvency Ratios**
Gross	90 691 059	190.084%
Net quota (fixed)	81 619 207	211.212%
Net quota (scaling)	84 870 652	203.120%
Net XL (no limit)	68 686 281	250.981%
Net XL with limit	83 661 930	206.055%

Table 27: SCRs and Solvency Ratios – Gumbel copula

It could be interesting to notice that each Solvency Capital Requirement is higher than the corresponding one under the Gaussian copula, which is at the same time higher than the one under the independence assumption. Indeed, the right tail dependence fitted by the Gumbel copula increases the likelihood to get jointly high aggregate claims amounts for more than one segment with respect to the Gaussian copula. Stronger is the tail dependence, higher should be the capital requirement. Therefore, by introducing tail dependence, the capital requirement increases and the Solvency Ratio of the cedent decreases, increasing the need of risk mitigation techniques as reinsurance treaties. In any case, even assuming a Gumbel copula as dependence assumption, also in the gross case, the insurance company is meeting the capital requirement.

In order to compare more properly the results of the different dependence assumptions, it is possible to compute the implicit volatility factor and multiplier. They are respectively 6.089% and 3.143. With respect to the Standard Formula, under the Gumbel copula assumption, the multiplier should be higher than the one stated by the Delegated Acts, due to tail dependence. Nevertheless, the volatility factor is significantly lower than 8.89%, implying a lower capital requirement. Compared with the Gaussian copula and the independence assumption, both the multiplier and the volatility factor result to be higher than the other ones, due to the additional volatility and skewness introduced by the right tail dependence.

As made for the two dependence structures assumed before, it is proposed a final plot summarising the trade-off between risk and profitability (Figure 27).

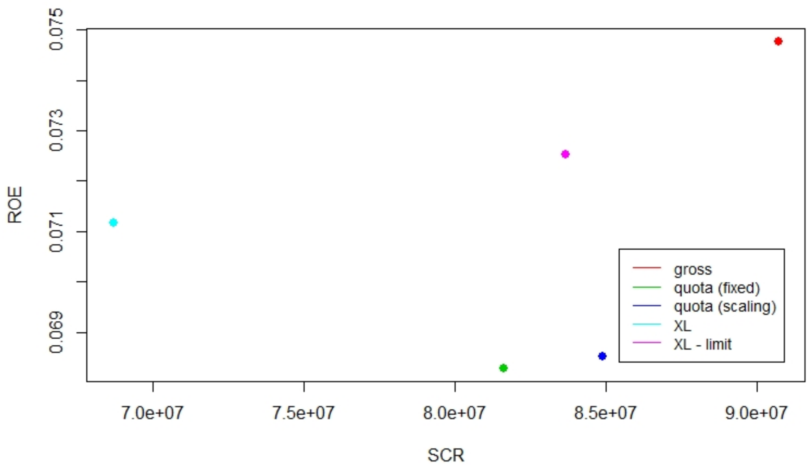

Figure 27: Risk-profitability analysis – Gumbel copula

It seems that the XL without limit is still the best choice for the direct insurance, since by suffering a quite small reduction in terms of profitability, it gets the highest improvement of Solvency Ratio. The quota share treaties are too much unfavourable in terms of profit with respect to the quite restrained SR's increase.

From the analyses carried out up to now, it should be clear the big role played by the dependence structure. Assuming an unrealistic dependence structure could lead to wrong estimations of the capital requirement and of the diversification benefit the company is able to get. In any case, under all the dependence structures, the reinsurance treaty optimal for the cedent seems to be the XL without limit, which tries to solve the risk-profitability trade-off.

In paragraph 7.4, a similar risk-profitability analysis is carried out of a multi-line reinsurance treaty acting on the three segments carried out by the direct insurer.

7.4 Multi-line Reinsurance treaty

Being a multi-line company, the direct insurer can decide to underwrite a multi-risk product, which could be more efficient than per-peril basis coverages.

7.4.1 Calibration

Regarding an umbrella treaty, a big role is played by the calibration of its priority. Therefore, it is important to study the impact of different values of the priority on the risk profile and on the profitability of the direct insurer. The calibration has been made considering the aggregate claims amounts of the segments MTPL, GTPL and MOD assumed to occur independently each other. The priority has been calculated as a percentage of the gross premiums and it could be useful to compute which percentile of the total aggregate claims amount distribution it corresponds to. The main characteristics of the total aggregate claims amount distribution net of the multi-line treaty are depicted in Table 28.

Priority	Percentile Level	Mean	Std Dev.	CV	Skewness
80%B	0.9599	531 645 883	24 897 954	4.683%	-0.08163
80.5%B	0.9695	531 772 779	25 146 495	4.729%	-0.03669
80.53%B	0.97	531 779 346	25 159 796	4.731%	-0.03421
80.75%B	0.9735	531 824 344	25 252 276	4.748%	-0.01666
81%B	0.9770	531 869 061	25 346 759	4.766%	0.001794
82%B	0.9871	531 995 419	25 632 158	4.818%	0.061815
82.5%B	0.9904	532 035 872	25 731 658	4.837%	0.084833
83.5%B	0.9948	532 087 980	25 869 548	4.862%	0.119602

Table 28: main characteristics of the net aggregate claims amount – Multi-line treaty

The following comments can be made:

- Higher is the priority, higher is the expected net aggregate claims amount, because the tail of X distribution is cut more on the right.
- As the priority decreases, the standard deviation decreases since the part of the distribution ceded to the reinsurer is more volatile.
- Higher is the priority, smaller is the ceded right tail and smaller is the reduction of the skewness. Notice that if the proportion of premiums is very low (lower or equal than 80.75%), the skewness of the aggregate claims amount distribution becomes negative. Therefore, the distribution of X presents a left tail, which is quite favourable for the direct insurer.

A graphical comparison between the gross and net aggregate claims amount distribution is proposed in Figure 28, for a value of priority equal to 80.53%B.

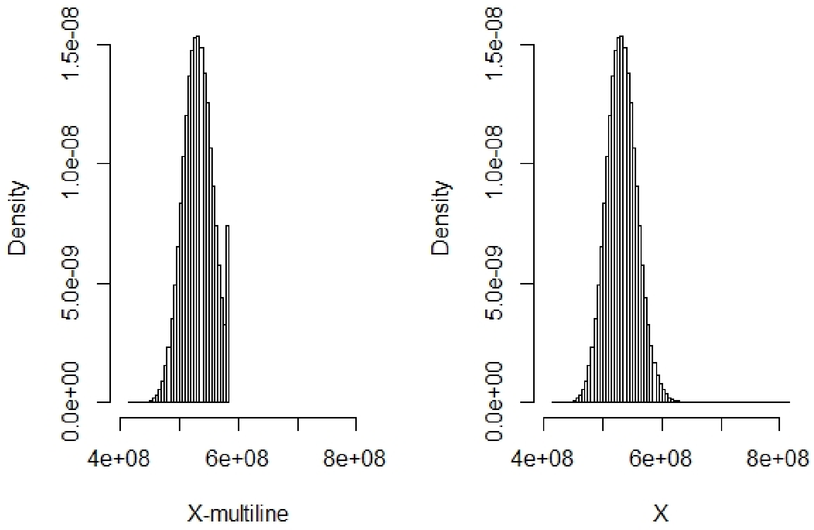

Figure 28: Net and gross aggregate claims amount distribution

Priority	$E(X_R)$	**Saved Capital**	**Safety loading**	B_{RE}
80%B	527 477	25 654 213	1 539 253	2 066 729
80.5%B	400 581	22 067 279	1 324 037	1 724 618
80.53%B	394 013	21 852 743	1 311 165	1 705 178
80.75%B	349 016	20 275 802	1 216 548	1 565 564
81%B	304 299	18 484 750	1 109 085	1 413 384
82%B	177 940	11 337 366	680 242	858 182
82.5%B	137 488	7 780 432	466 826	604 314
83.5%B	85 380	819 508	49 170	134 550

Table 29: Decomposition of the premiums paid to the reinsurer - Multi-line treaty

In order to select the priority according to its risk-profitability impact, it is necessary to define a pricing principle for the multi-risk product. The umbrella policy can be priced as the expected aggregate claims amount paid by the reinsurer increased by a safety loading. The safety loading can be computed as the cost of the capital saved by underwriting the reinsurance treaty (ignoring the effect of the safety loading), where the cost of capital rate is assumed to be 6%. The resulting premiums (and their composition) are summarised in Table 29. As the priority increases, the reinsurer expects to cover a lower aggregate claims amount since less right tail is ceded, implying a lower risk premium. At the same time, the saved capital resulting from the multi-risk product decreases, leading to a lower safety loading. Both effects determine that an increasing priority makes the premiums to be paid to the multi-line reinsurer to be lower.

Finally, in Figure 29, a risk-profitability analysis is carried out. In particular for the previous set of priorities, the capital requirement for premium risk and the ROE is calculated.

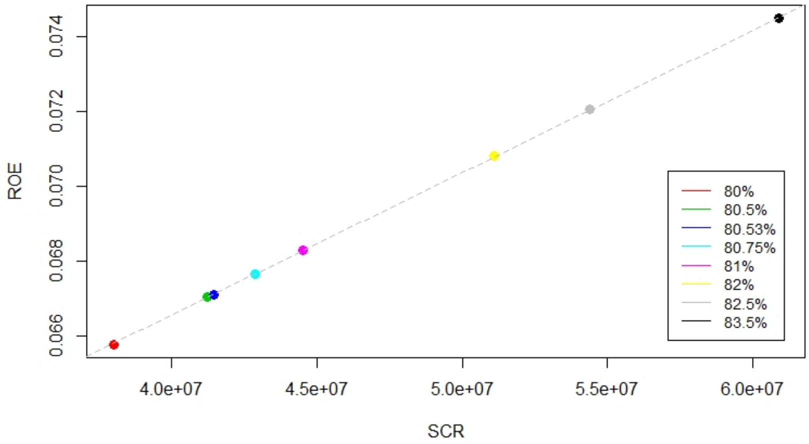

Figure 29: ROE versus SCR for different levels of priority

As expected, as the priority decreases, the SCR and the ROE decrease linearly. Therefore, the usual risk-profitability trade-off must be solved. In this case study, 80.53% of the gross premiums has been chosen as priority of the multiline treaty.

Once defined the characteristics of the multi-risk product, it is possible to introduce it in the risk-profitability analysis of the reinsurance treaties assuming the different dependence structure.

7.4.2 SCR computation

Once defined the characteristics of the multi-risk product, it is possible to compute the resulting net capital requirement in order to evaluate its ability to reduce the cedent's risk exposure.
As expected, Figure 30 shows how the umbrella policy, in all the dependence structures assumed in the Internal model, is able to strongly decrease the capital requirement. In absolute terms, the highest capital reduction with respect to the gross case is obtained by fitting a Gumbel copula between aggregate claims amounts of different segments. Indeed, under this dependence structure, the priority corresponds to the lowest level of percentile. Therefore, the right tail of the aggregate claims amount distribution is cut more on the left determining a stronger reduction of the relative volatility and of the skewness and

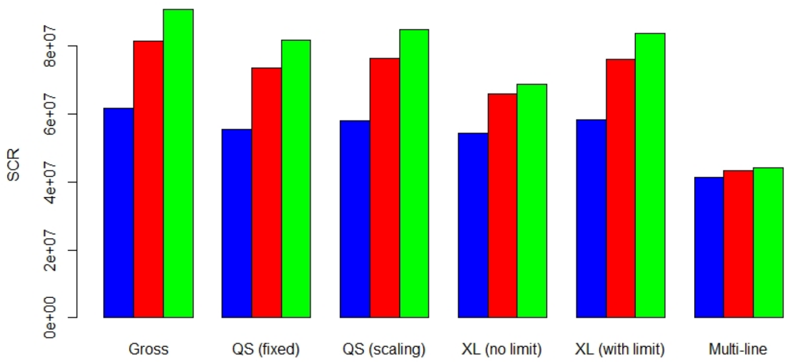

Figure 30: SCR – Internal model, under Independence (blue), Gauss Copula (red) and Gumbel copula (green)

a consequent higher reduction of capital requirement. By comparing the risk mitigation effect of the multi-line product with respect to the one of the other reinsurance treaties, it is possible to notice that the umbrella policy always determines the lowest capital requirement. Therefore, if the goal of the cedent is to significantly improve its solvency position, the multi-risk product seems to be the best solution. Before taking a risk mitigation decision, it is always better to check its impact on the profitability.

7.4.3 Profitability

Conversely to the reinsurance treaties previously introduced, the dependence structure has an influence on the expected return of equity net of the umbrella policy, due to the way the premiums charged by the multi-line reinsurer are priced. The risk premium (i.e., expected aggregate claims amount for the reinsurer) changes as the dependence structure changes, since the chosen priority corresponds to percentiles of different level. In addition, the safety loading, being a function of the saved capital, is strictly dependent on the dependence assumption. The gross and net expected returns on equity are summarised in Table 30.

	ROE
Gross	7.477%
Net multi-line - Independence	6.712%
Net multi-line - Gaussian copula	6.012%
Net multi-line - Gumbel copula	5.693%

Table 30: gross and net ROE – umbrella policy

The highest profitability reduction occurs under the Gumbel copula assumption, since it is the dependence structure determining the highest premiums to be paid to the reinsurer.

This ROE reduction is the highest recorded among all the reinsurance treaties and all the dependence structures considered in this case study. In addition, also by assuming independence or a Gaussian copula between aggregate claims amounts of different segments, the profitability reduction is quite high and superior with respect to the other coverages.

Therefore, in order to make an efficient decision about the optimal reinsurance treaty, it is not only possible to choose the contract providing the highest reduction of capital requirement, because it could lead to a dramatic profitability reduction. In paragraph 7.4.4, the multi-risk product is introduced in the risk-profitability analysis carried out under the several dependence structures.

7.4.4 Risk-profitability analysis

The following figures show the effect of the several reinsurance treaties jointly on the risk profile and on the profitability of the cedent, by assuming three different dependence structures.
If it is assumed independence between aggregate claims amounts of different segments (Figure 31), a good choice could be the multi-risk product.

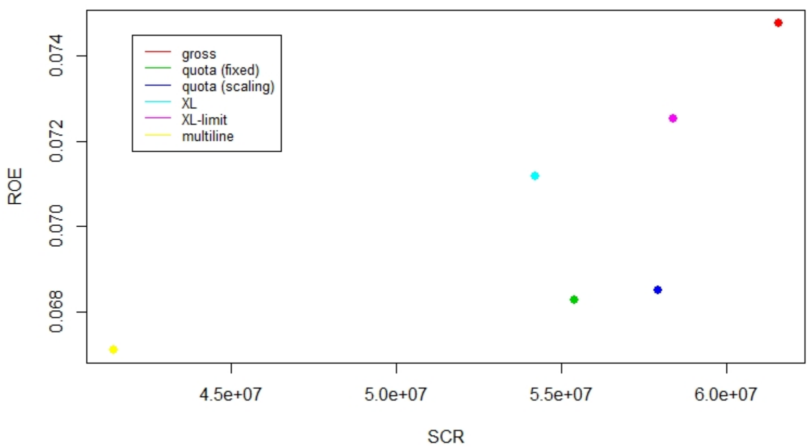

Figure 31: Risk-profitability analysis – Independence

Indeed, it implies a profitability reduction a little higher than the XL treaty without limit, which is more than compensated by the much lower net capital requirement. Therefore, the umbrella policy is able to strongly improve the solvency position of the cedent, by implying a not too high ROE's sacrifice.

Completely different comments can be made by fitting the copulas as dependence structure (Figures 32 and 33). Indeed, it is true that the multi-line product strongly decreases the capital requirement, but the profitability reduction could be too high and excessive. Therefore, the XL treaty without limit seems still to be the optimal contract, finding a compromise between risk reduction and net profitability.

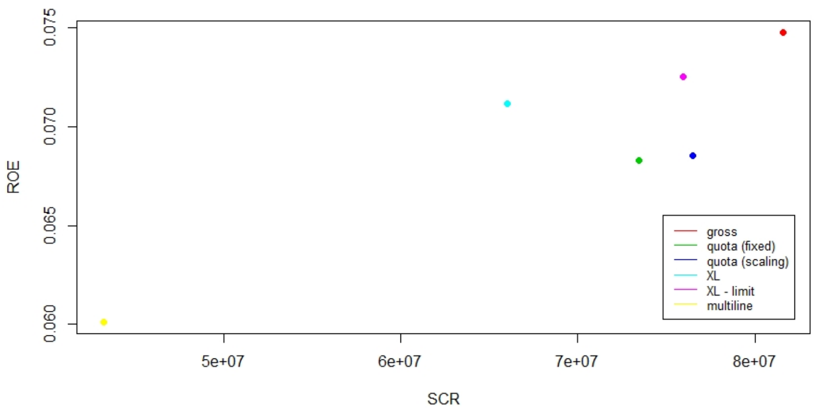

Figure 32: ROE vs Solvency Capital Requirement – Gaussian copula

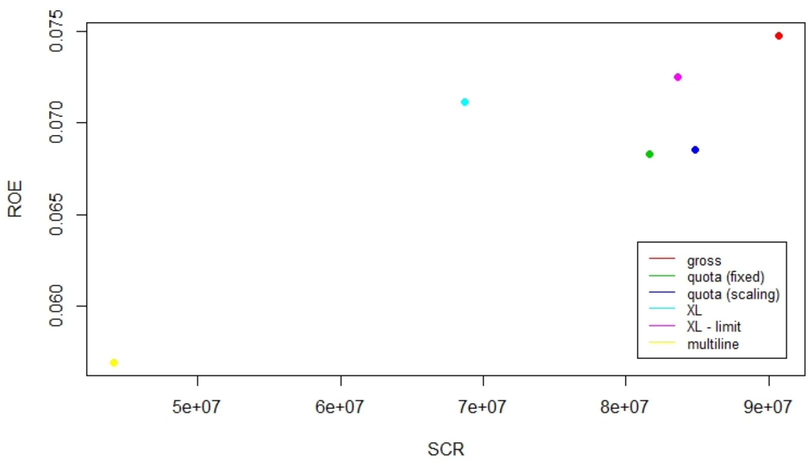

Figure 33: Risk-profitability analysis – Gumbel copula

7.5 Counterparty Default Risk

Underwriting a reinsurance treaty does not lead only to a lower underwriting risk, but it increases also the cedent's exposure to counterparty default risk. Therefore, in the last section of this case study, the choice of the optimal reinsurance treaty is made by looking at the net profitability and net total SCR, computed by aggregating the premium risk SCR and the counterparty default risk SCR.

The counterparty default risk SCR has been calculated, for simplicity, as stated by the Standard Formula. Recoverables are assumed to be equal to the reinsurance premiums. The risk mitigating effect on the underwriting risk, RM_{re}, has been defined as the capital requirement reduction (computed inside the Internal model) arisen from underwriting the reinsurance cover. It is assumed that the treaties are underwritten by the same reinsurer. This last assumption will strongly affect the value of the counterparty default risk SCR, since the variability arising from all possible combinations (j, k) of different probabilities of default on single name exposures is null (i.e., $V_{inter} = 0$). In order to get a range of values, Credit Quality Step of the reinsurer is assumed to be equal both to its maximum and its minimum values.

Regarding the basis reinsurance treaties, the probability of default and loss-given-to-default of the treaties of the same kind acting on the single segments have been aggregated as stated by the Standard Formula. Since the risk mitigating effect on the underwriting risk of treaties acting on the single segment is not affected by the dependence structure, the resulting counterparty default risk SCRs for quota share treaties and XL treaties are totally independent on the dependence assumption made inside the Internal Model. In Table 31, the extremes of the range between which varies the counterparty default risk SCR according to the reinsurer's credit quality step are reported.

	$SCR_{default}$, **CQS=0**	$SCR_{default}$, **CQS=6**
Net Quota (fixed)	281 856	21 249 597
Net Quota (fixed)	275 911	20 801 329
Net XL (no limit)	79 276	5 976 733
Net XL (with limit)	45 838	3 455 774

Table 31: Counterparty Default Risk SCR – basis treaties

As expected, worse is the credit quality step of the reinsurer (closer to 6), higher is the probability of default and higher is the capital requirement. Comparing the two proportional treaties, it is possible to notice how the SCR in case of scaling commissions is lower with respect to the case with fixed commissions. Indeed, the treaties have the same reinsurance premiums, but the quota share with fixed commissions determines a lower capital reduction (and a consequent lower risk mitigating effect) with respect to the quota share with fixed commissions. In addition, the introduction of a limit in the XL treaty makes the counterparty default risk SCR decrease. Indeed, the lower premiums and lower capital reduction leads to a lower loss-given-to-default. Finally, the proportional treaties have a higher counterparty default risk SCR with respect to the non-proportional ones due to the higher reinsurance premiums, which lead to a higher LGD.

Once analysed the basis reinsurance treaties, it is possible to calculate the counterparty default risk SCR for the multi-line policy under the three dependence structures (Table 32).

Dependence Structure	$SCR_{default}$, **CQS=0**	$SCR_{default}$, **CQS=6**
Independence	61 205	4 614 334
Gaussian copula	118 052	8 900 087
Gumbel copula	143 839	10 844 234

Table 32: Counterparty default risk SCR – Multi-line policy

Conversely to basis treaties, the counterparty default risk SCR for multi-line products depends on the dependence assumption made. Stronger is the dependence structure, higher are the reinsurance premiums and higher is the capital reduction for the underwriting risk module. Therefore, higher are Loss-given-to-default and the consequent higher SCR.

Once computed for each reinsurance treaty the resulting counterparty default risk SCR, it is possible to aggregate it with the Non-Life underwriting risk SCR in order to compute the total capital requirement. The aggregation procedure is the one stated by the Standard Formula:

$$SCR = \sqrt{SCR^2_{Non-Life} + SCR^2_{default} + 2 \cdot 0.5 \cdot SCR_{Non-Life} \cdot SCR_{default}}$$

In Figure 34, it is evaluated the relevance of the Non-Life underwriting risk and the Counterparty default risk on the total Solvency capital requirement (including the diversification), by assuming the worst credit quality step for the reinsurer. In order to properly take into account the diversification benefit, the SCR UWR and default risk has been recalculated after aggregation as the relevance of each risk module on the total SCR before aggregation (i.e., without diversifications) applied to the total SCR after diversification. The major role is played by the Non-Life Underwriting risk module. Among the different reinsurance treaties, the counterparty default risk SCR is the most relevant for quota share treaties with respect to the non-proportional ones.

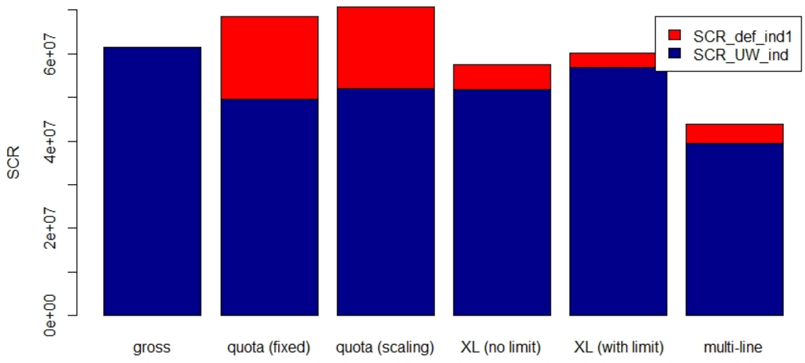

Figure 34: SCR decomposition, worst CQS – Independence

Conversely, by assuming the best credit quality step for the reinsurer, the relevance of the counterparty default risk module on the total SCR is infinitesimally small.

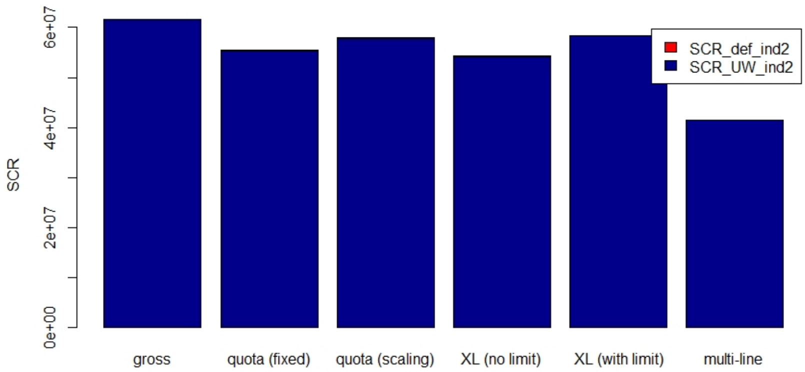

Figure 35: SCR decomposition, best CQS – Independence

The same comments hold for the other two dependence structures.

7.6 Optimal Reinsurance Treaty

Once calculated the total capital requirement net of the reinsurance treaties, it is possible to evaluate their effect on the cedent's risk profile and profitability under each dependence structure.

7.6.1 Independence

From the following plots, it is possible to evaluate if the insurance company has to review its risk mitigation decision, by introducing the counterparty default risk module. If the reinsurer has a very good rating (i.e., best credit quality step), the introduction of the counterparty default risk module does not affect too much the value of the final SCR (see Table 33). Therefore, it does not influence the choice of the optimal reinsurance treaty (Figure 36).

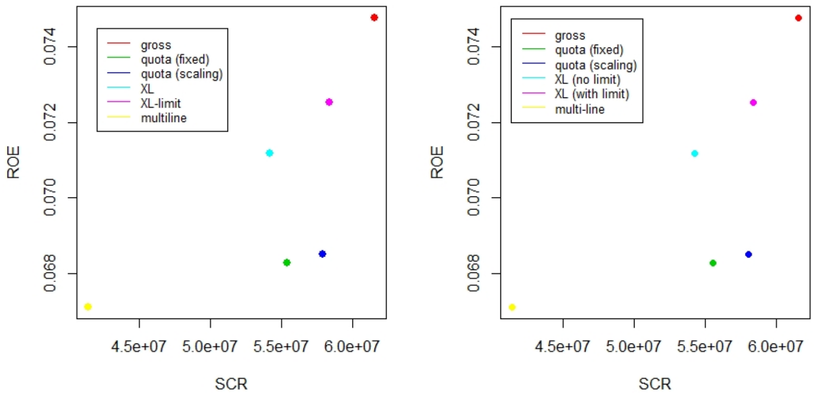

Figure 36: Risk-profitability analysis gross and net of the counterparty default risk module (best CQS) – Independence

The contract able to reduce at most the capital requirement is the umbrella policy. If the consequent profitability reduction is too high, the XL treaties could be a good compromise.

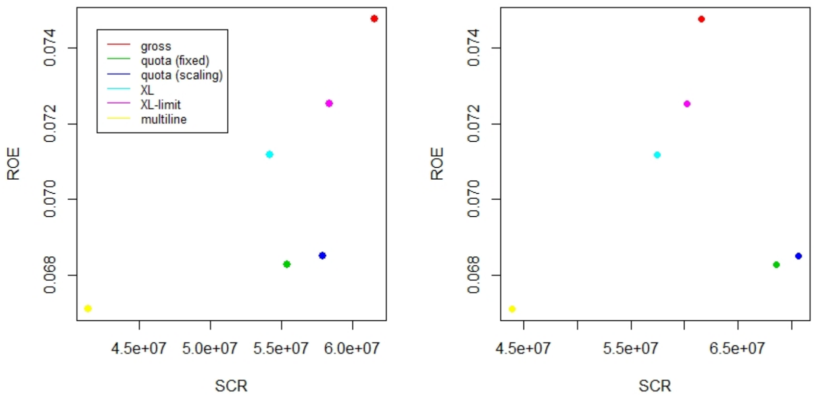

Figure 37: Risk-profitability analysis gross and net of the counterparty default risk module (worst CQS) – Independence

In case the reinsurer has the worst credit quality step (Figure 37), the counterparty default risk increases, making also the total SCR increase. In particular, the effect of this risk module is quite strong for proportional treaties, since the net aggregate SCR results to be higher than the gross one. Therefore, quota share treaties should be avoided.

Finally, in Table 33, a brief summary of the capital requirements and of ROE for the several reinsurance treaties resulting from the Internal model with the independence assumption is provided.

	SCR UWR	**Total SCR (best CQS)**
Gross	61 576 713	61 576 713
Net Quota (fixed)	55 408 875	55 550 339
Net quota (scaling)	57 926 695	58 065 142
Net XL (no limit)	54 198 895	54 238 576
Net XL (with limit)	58 386 718	58 409 650
Net Multi-line	41 429 148	41 459 784

	Total SCR (worst CQS)	**ROE**
Gross	61 576 713	7.477%
Net Quota (fixed)	68 550 018	6.829%
Net quota (scaling)	70 662 221	6.852%
Net XL (no limit)	57 421 023	7.118%
Net XL (with limit)	60 189 056	7.252%
Net Multi-line	43 918 496	6.712%

Table 33: SCR and ROE – Independence

7.6.2 Gaussian Copula

Table 34 summarises the SCRs and the ROEs obtained by fitting a Gaussian copula between aggregate claims amounts of different segments.

	SCR UWR	**Total SCR (best CQS))**
Gross	81 569 236	81 569 236
Net Quota (fixed)	73 416 264	73 557 597
Net quota (scaling)	76 483 138	76 621 466
Net XL (no limit)	66 030 763	66 070 437
Net XL (with limit)	75 946 512	75 969 441
Net Multi-line	43 275 463	43 334 609

	Total SCR (worst CQS)	**ROE**
Gross	81 569 236	7.477%
Net Quota (fixed)	86 032 315	6.829%
Net quota (scaling)	88 731 711	6.852%
Net XL (no limit)	69 212 941	7.118%
Net XL (with limit)	77 732 033	7.252%
Net Multi-line	48 343 900	6.012%

Table 34: SCR and ROE – Gaussian copula

As under the independence assumption, if the reinsurer has the best Credit Quality Step, the introduction of the counterparty default risk does not affect too much the SCR. Conversely, if the CQS is 6, the increase of capital requirement due to the possible counterparty default for quota share treaties overcompensates the benefits provided by the reinsurer covers. The umbrella policy strongly reduces the capital requirement (around 50%), but it implies the highest reduction in terms of profitability. Therefore, a good alternative could be an XL without limit. The decision is confirmed by looking at Figure 38.

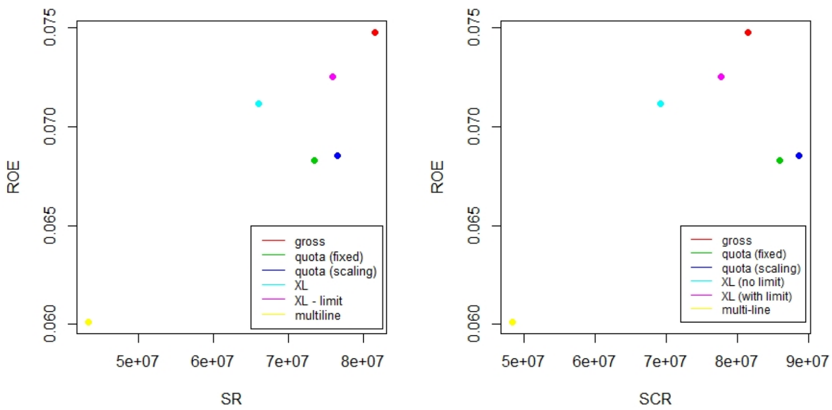

Figure 38: Risk-profitability analysis with and without counterparty default risk, best credit quality step – Gauss copula

7.6.3 Gumbel copula

Finally, the analysis is carried only under the last dependence structure assumed: Gumbel copula. As results in Table 35 and Figure 39 show, even if the reinsurer has the worst CQS, the choice of the reinsurance treaty is not affected by the introduction of the counterparty default risk. The umbrella policy determines a reduction of 2% of the ROE, which could be not easily accepted by the stakeholders. Therefore, the other non-proportional per-peril treaties provide a satisfactory risk reduction and a higher net profitability.

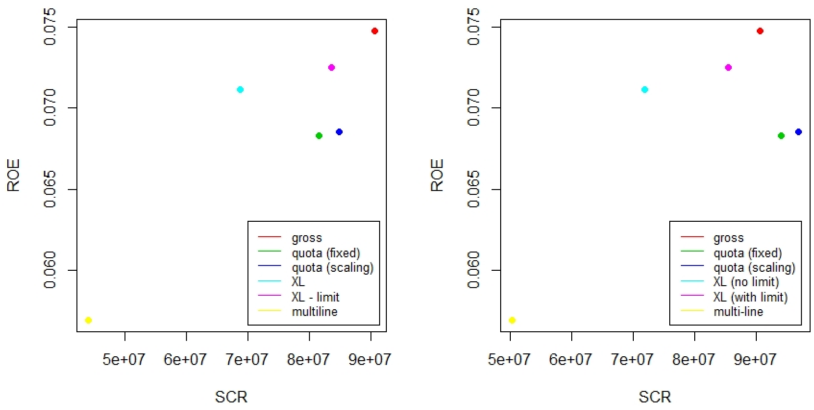

Figure 39: Risk profitability analysis – Gumbel copula

	SCR UWR	Total SCR (best CQS))
Gross	90 691 059	90 691 059
Net Quota (fixed)	81 619 207	81 619 207
Net quota (scaling)	84 870 652	84 870 652
Net XL (no limit)	68 686 281	68 686 281
Net XL (with limit)	83 661 930	83 661 930
Net Multi-line	44 086 156	44 086 156

	Total SCR (worst CQS)	ROE
Gross	90 691 059	7.477%
Net Quota (fixed)	94 061 765	6.829%
Net quota (scaling)	96 959 503	6.852%
Net XL (no limit)	71 861 298	7.118%
Net XL (with limit)	85 442 247	7.252%
Net Multi-line	50 391 141	5.693%

Table 35: SCR and ROE – Gumbel copula

Therefore, to sum up, whatever is the dependence structure assumed, if the principal purpose of the insurance company is to improve its solvency position, the umbrella policy is able to strongly decrease the capital requirement, with a profitability reduction varying according to the dependence structure assumed. Conversely, if the firm wants to find a compromise between risk and profitability reduction, the XL treaties seem to be a much proper solution.

Conclusions

The aim of this thesis is to understand the effects of several reinsurance treaties on the risk profile and on the profitability of a multi-line insurance company. A detailed case study has been provided. All the parameters involved in the internal model has been calibrated according to market data.

First of all, the impact of basis reinsurance treaties on the aggregate claims amount distribution has been analysed for each line of business. Results are quite different according to the segment and to the kind of reinsurance treaty introduced.

Secondly, the aggregate claims amount and expenses of each line of business have been aggregated under the simplest dependence assumption (independence) in order to compute the premium risk in a partial Internal model. In both gross and net terms, the Standard Formula is overestimating the capital requirement, due to the higher (implicit) multiplier, market-wide volatility factors and dependence assumption.

By introducing a measure of profitability (the return on equity), it is possible to define which is the optimal reinsurance treaty under the independence assumption. The XL treaties provide a satisfactory capital reduction, and the introduction of a limit could contain the profitability's sacrifice.

Modelling the dependence between aggregate claims amounts of different segments via a Gaussian copula leads to capital requirements higher than under the independence assumption, without modifying the choice of the optimal reinsurance treaty.

The tail dependence introduced by the Gumbel copula leads to a strong increase of the SCR. Nevertheless, the insurance company, also without risk mitigation tools, is able to meet the quantitative requirements. The non-proportional treaties are still the contracts to be preferred.

Under these three dependence structures, an umbrella policy has been introduced. Surely, the multi-risk product is the most efficient in improving the solvency position, but the profitability reduction could be quite high in particular when the dependence is modelled via a Gumbel copula. Therefore, the choice of the optimal treaty can be reviewed according to the guidelines in terms of risk appetite and profitability fixed by the insurance company.

Finally, the counterparty default risk SCR is introduced in order to understand whether it can affect the choice of the risk mitigation solution. By assuming the worst and the best credit quality step, it is possible to have a range in which the corresponding capital requirement varies. If the reinsurer's rating is very high, the counterparty default risk has a completely negligible effect on the total SCR. Therefore, the choice of the optimal reinsurance treaty is not at all affected. By assuming the worst credit quality step for the reinsurer, the capital requirement net of the quota share treaties strongly increases also above the gross SCR, because of higher recoverables. In this case study, results suggest to avoid proportional treaties, whatever is the dependence assumption. Therefore, the insurer has to choose between the XL treaties and the umbrella policy. XL treaties are quite good in finding a good compromise between risk and profitability reduction, but they are not able as the umbrella policy to improve the insurer's solvency position.

In conclusion, reinsurance treaties can be considered very crucial risk mitigation tools if they are coherent with the risk appetite and the target return on equity defined by the insurance company. Multi-risk products, if properly calibrated, seem to be very effective and efficient in reducing the capital requirement, implying that their diffusion is likely to grow over time.

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