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Systematic Risk in Long-Term Insurance Business

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Doctor in Business Economics

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Preface

What the future has in store no one ever knows before,
Nina Simone (1965)

The Law of Large Numbers (LLN) provides the groundwork for pricing and managing insurance risks. Its basic idea is that the sample mean of an endlessly repeated experiment with independent and identically distributed random outcomes approaches the theoretical mean of the individual experiments. Jakob Bernoulli initially formulated the *weak* LLN for a sequence of experiments with two outcomes, a win or a loss. As the number of experiments increases, the probability that the sample mean falls closely to the theoretical mean increases too. A century later, Pafnuty Chebyshev proved a more general *strong* LLN stating that the sample mean converges, almost surely, towards the theoretical mean when the number of repetitions increases.

To illustrate the role of the LLN in an insurance context, we consider a portfolio of n independent and identically distributed insurance policies. We denote by X_i the i-th insurance claim payment, which is equal to b with probability p, or to 0 with probability $1-p$, i.e. the insurer pays a benefit b with probability p. The theoretical mean of X_i is equal to $p \times b$. Thus, the claim payment per-policy $\bar{X}_n$ (which is the sample mean of the claim payments at portfolio level, or average loss) can be approximated by $p \times b$, for large n. Formally, the weak LLN states that:

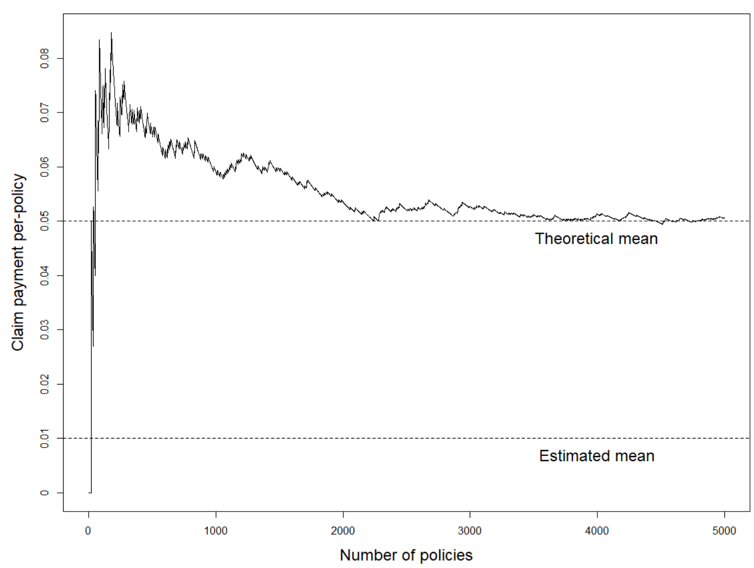
$$\lim_{n\to\infty} \mathbb{P}\left[|\bar{X}_n - p \times b| \geq \varepsilon\right] = 0,$$

for some small $\varepsilon > 0$, whereas the strong LLN states that:

$$\mathbb{P}\left[\lim_{n\to\infty} \bar{X}_n = p \times b\right] = 1.$$

This means that the *pure* premium (i.e. excluding safety loadings, administration costs, etc.) for each policy can be set equal to $p \times b$. The insurance company is exposed to the random variations of $\bar{X}_n$ around the mean $p \times b$. Provided the random payments of the n policies are independent and identically distributed, the LLN ensures that the risk associated to these variations can be diversified by increasing the size of the portfolio.

Suppose that $p \times b = 0.05$. The figure below displays the claim payments per-policy $\bar{X}_n$ as a function of the portfolio size n. For a low number of policies, the variations of $\bar{X}_n$ around $p \times b$ have a higher amplitude. As the size of the portfolio grows, the claim payment per-policy converges towards the theoretical mean 0.05.



The task is thus to determine the probability p of paying the claim, as well as the claim amount b. The insurer can use the available information to estimate these quantities. Again, a greater amount of available information reduces the estimation error. Nevertheless, this task becomes challenging in a realistic setting, where the probability p and/or the claim amount b change over time, and hence, their future values are random at the time the premium is set. In such cases, the diversifiable risk due to the random variation of $\bar{X}_n$ is not the only risk born by insurers.

For instance, suppose that, using the information available at time 0, an estimate $p^{(0)}$ is obtained for the probability p and an estimate $b^{(0)}$ is obtained for the claim amount b, such that $p^{(0)} \times b^{(0)} = 0.01$. The above figure shows that, as the number of policies increases, the claim amount per-policy $\bar{X}_n$ converges towards its theoretical mean, but the estimate $p^{(0)} \times b^{(0)} = 0.01$ of the theoretical mean is different from its actual realization 0.05. It turns out then that increasing the size of the portfolio is not sufficient. Indeed, in a setting where future realizations of the quantities p and b are random, the insurer is also exposed to a residual risk which remains after increasing the size of the portfolio, i.e. *systematic risk*.

In an attempt to reduce this risk, the actuarial stream of research on designing sophisticated forecasting models for p and b has flourished over the past decades. However, even the most sophisticated models may fail to consider all possible future scenarios. This does not mean that forecasting is a vain exercise, but rather that the use of models has to be combined with other risk management tools in order to reduce the exposure of the insurer to systematic risk.

The present thesis endeavors to study this residual risk, with a focus on its presence in long-term insurance business. We consider separately the systematic risk stemming from the uncertainty on the claim payment b, and that stemming from the uncertainty on the probability p.

In the first part of this thesis, we consider systematic risk arising from the uncertainty on the benefit, which is typically the case for health insurance contracts. This risk exacerbates when contracts are lifelong, and when the insurer has to estimate the evolution of health claim amounts in the future. These constraints are imposed to Belgian private health insurers by law, and raise a number of challenges which are addressed in Part I. This part is based on three contributions. Namely, Denuit, Dhaene, Hanbali, Lucas and Trufin (2017), "*Updating mechanism for lifelong insurance contracts subject to medical inflation*", European Actuarial Journal; Dhaene and Hanbali (2019) "*Measuring medical inflation for health insurance portfolios in Belgium*", European Actuarial Journal; Hanbali, Claassens, Denuit, Dhaene and Trufin (2019), "*Once covered, forever covered: The actuarial challenges of the Belgian private health insurance system*", Health Policy.

In the second part of the thesis, we focus on the the systematic risk arising from the uncertainty on the survival probabilities. More specifically, the systematic risk in this setting is essentially a longevity risk, where the insurer is exposed to the risk that the survival index of the portfolio exceeds its estimated value. Thus, Part II investigates a risk management technique based on risk-sharing between the insurer and policyholders. The content of Part II is adapted from the work performed in Hanbali, Denuit, Dhaene and Trufin (2019), "*A dynamic equivalence principle for systematic longevity risk management*", Insurance: Mathematics and Economics.

In the third part of the thesis, we focus on the offsetting relationship between longevity risk and mortality risk. The former is present in insurance contracts paying survival benefits, whereas the latter is present in insurance contracts paying death benefits. Part III is based on our working paper Hanbali and Villegas (2019), "*Pricing insurance contracts with offsetting relationship*". We study the situations where offsetting contracts are priced either separately or jointly, and evaluate the competitiveness argument supporting joint pricing.

Contents

Part I

Medical inflation risk management

CHAPTER 1

Private health insurance in Belgium

1.1 Introduction

Belgian mandatory health insurance is included in the social security system and provides only a partial cover of prescription drugs and medical services. Supplementary health insurance gives access to a more comprehensive spectrum of covered medical services, especially for hospitalization claims, which can be very expensive mostly due to the supplementary fees charged by medical practitioners in case of single room hospitalizations (Lecluyse et al., 2009). The yearly premiums of health insurance contracts sold by private companies take into account the different characteristics of the insured risk profile. Major factors at play in the pricing process are the health status and the age of the insured at contract inception; see Pitacco (2014). In case the contract is renewed on a yearly basis, as this is the case in many countries, the expected annual medical costs underlying the calculation of the yearly risk premiums will increase over time and may even become unaffordable at higher ages; see Figure 5.4.1. This is generally also the case for policyholders with chronic diseases or with disabilities.

Health expenditure in Belgium represents 10.4% of GDP (Health Prospect-ING, 2018). In 2015, Belgian out-of-pocket expenditures was one of the highest among the major European countries, with 17.7% of the total health

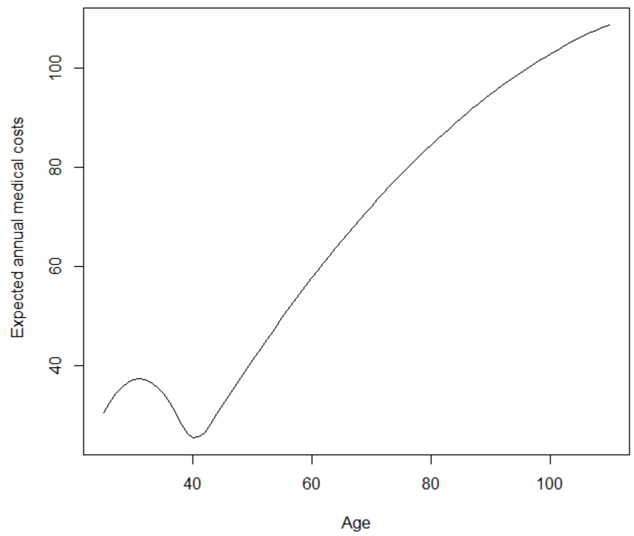


Figure 1.1.1: Typical shape of the expected annual medical costs in function of age obtained from the Belgian Mutualité Chrétienne. These costs are increasing with respect to the age, except for some local phenomena such as the accident and childbearing hump.

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expenditures. Together, in-patient, out-patient and dental care medical services represent about 46% of the out-of-pocket contributions (Health Prospect-ING, 2018). On the other hand, voluntary private insurance in 2015 represented 4.9% of the total health expenditure in Belgium (OECD, 2018). Hospitalization insurance, which is provided either by private health insurance companies or as an additional insurance by mutual insurers, allows to reduce the bill after a stay in a hospital. Based on the official figures of the National Bank of Belgium (NBB), 9.4 million health insurance contracts were in force in 2015 (Mutualité Chrétienne, 2018). The total premiums of private health insurance amounted to 1.7 billion euros, with 28.08% coming from individual private contracts. The union of insurers *Assuralia* reports that 80% of the Belgian population has a hospitalization insurance, subscribed either individually, or through employers (Assuralia, 2018).

The purpose of the Belgian Law of 20 July 2007 is to ensure the accessibility of individual supplementary health coverage.[1] Two important features have

[1]Loi du 20 juillet 2007 modifiant, en ce qui concerne les contracts privés d'assurance maladie, la loi du 25 juin 1992 sur le contrat d'assurance terrestre, Moniteur Belge du 10/08/2007 (FR); Wet van 20 juli 2007 tot wijziging, wat de private ziektev-erzekeringsovereenkomsten betreft, van de wet van 25 juni 1992 op de landverzeker-

been included in order to protect policyholders from discrimination and exclusion, essentially when these operate on the basis of age (article 138*bis*). More specifically, these contracts must be lifelong (with the exception of disability covers which extend to the end of the professional activity), with a technical actuarial basis. The legislator allows an adjustment of the premiums using the Consumer Price Index (CPI). In case the main characteristics of the insured risk change over time in a way that threatens the solvency of the insurer, it is allowed to adapt the premiums with the agreement of the relevant authority (i.e. the National Bank of Belgium).

Once covered, forever covered. This maxim which emerges from the Belgian Law has two technical consequences for insurers. First, from an actuarial perspective, the lifelong commitment combined with leveled premiums (over all ages, or over some age ranges) implies that these contracts have to be treated similarly to life insurance products. This means that the insurer has to build reserves using the surpluses accumulated in the early years of the contract to cover the increasing costs for higher ages. Second, on top of the modeling of future survival probabilities which follows from the analogy with life insurance products, the insurer has to perform a sound forecast of the future evolution of health claims. However, these evolutions are practically difficult (not to say impossible) to forecast, meaning that insurers have to cope with a risk which is specific to the private health insurance sector, that is, *medical inflation* risk.

Medical inflation, or the unpredictable systematic changes in medical claim payments over the years, is driven by several inter-related causes; see e.g. André-Dumont and Devoet (2012). A primary cause comes from the reimbursement structure of health expenditures in the Belgian context. Private health insurance is meant to cover part of the expenses which are not covered by Social Security. Therefore, in case the intervention threshold of the mandatory insurance decreases, e.g. due to some political decisions, claim expenses will inevitably increase for private health insurers. It is to be noted however that for many Belgian insurers, the extent of the covers is expressed as a multiple of this intervention threshold. Medical inflation is also caused by the interplay between demographic and socio-economic factors. On the one hand, there is a decrease of the contributions to Social Security and an increase of the volume of health benefits for elderly due to population aging. On the other hand, medical progress might lead to the discovery of

ingsovereenkomst, Belgish Staatsblad 10/08/2007 (NL). This Law has been replaced by: Loi du 4 avril 2014 relative aux assurances, Moniteur Belge du 30/04/2014 (FR); Wet van 4 april 2014 betreffende de verzekeringen, Belgish Staatsblad, 30/04/2014 (NL).

new diseases which have to be covered, or to the discovery of new cures for known diseases which are often expensive. All these consequences of medical progress have an impact on life expectancy, taking us back again to the population aging issue; see Lichtenberg (2017) and references therein.

Therefore, Belgian insurers have to anticipate a lifelong risk which can hardly be quantified mathematically. The solution proposed by the legislator is to transfer medical inflation risk back to policyholders by allowing an adjustment of the contract elements on a yearly basis. Note that our main focus here is on the adjustment of level premiums, but the discussion remains relevant to other components of the contract such as deductibles, and to some extent to the case where premiums are leveled by thresholds. This solution appeared to be a compromise between insurers and consumers' representatives, provided the adjustments are standardized across the sector. In fact, proposing an updating mechanism applicable at the sector level requires two distinct tasks. The first task is to construct medical inflation indices which capture the cost evolutions of the four main medical services covered in Belgian private health insurance (hospitalization with stay in a private room, hospitalization with stay in shared room, dental care and ambulatory care). The second task is to define a sound updating mechanism which accounts for the lifelong nature of these contracts. Achieving these two tasks is not straightforward, and the challenges underlying each of them is the topic of Part I of this thesis.

1.2 Research questions and structure

1.2.1 Construction of medical inflation indices

The medical inflation discuss here is specific to the private health insurance sector, and its related indices differ from the CPI. Constructing them using a basket-based approach would suffer from technical limitations. One of these limitations is the fact that quantifying some of the drivers of medical inflation in the insurance sector may not be feasible. Additionally, for the drivers that can be quantified, determining their weights in the basket can be challenging. Another limitation is that medical indices constructed in such a manner would not be age-dependent, whereas medical inflation is likely to be age-dependent. In order to overcome these drawbacks, Devolder et al. (2008) suggest an *actuarial* approach based on the claim costs experienced in the sector, which has been adopted for Belgian private health insurance; see Figure 1.2.2 which displays the evolution of medical inflation indices over time from the method of Devolder et al. (2008), for the five age categories defined in the Belgian Law (0–19, 20–34, 35–49, 50–64 and 65+). Although this method has shown its merits, the evolution of the health insurance market raises some shortcomings in both the calculation of the indices and their application.

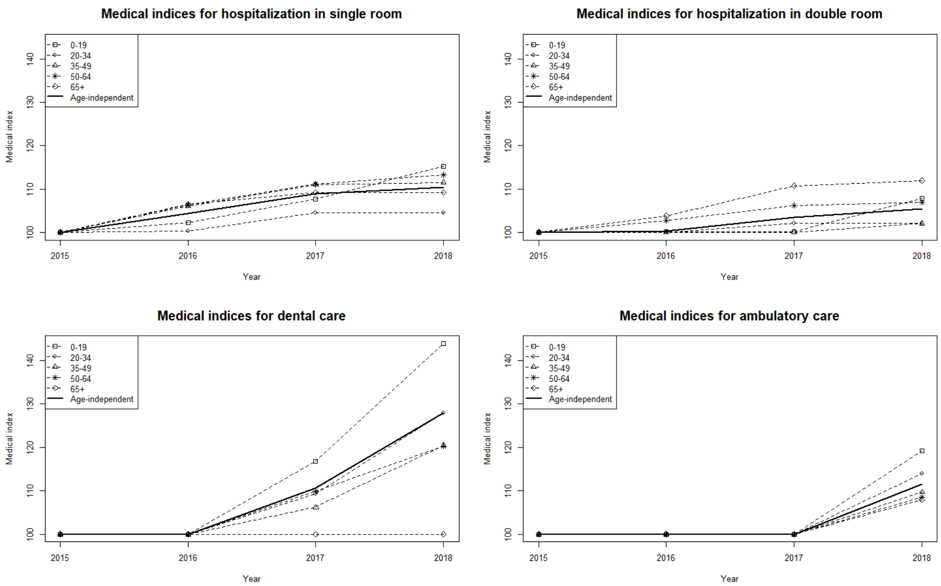

Figure 1.2.2: Official medical inflation indices for each medical service by age category from 2015 to 2018, obtained from https://statbel.fgov.be/.

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Concerning the calculation of the market-wide medical inflation indices for each type of covered medical service, the current Belgian approach allocates the full data of any health insurance product to a single category, namely the type of cover which has the highest weight in the claims over the past year. This implies that the market-wide index for each type of cover might be 'polluted' by claims not corresponding to that cover.

Concerning the index that can be used to adjust the premiums of a product, the current Belgian approach requires using the market index that corresponds to the most expensive cover in that product. This means that the choice of the index for a particular product does not take into account the proportion of costs related to the different medical services covered in that product.

In this context, the first research question, addressed in Chapter 2, is how to improve the current Belgian methodology for constructing medical inflation indices.

1.2.2 Updating mechanism for lifelong health insurance covers

The amending Law of 17 June 2009 states that insurers are allowed to adjust the premium using the medical indices or the CPI, whichever is higher. A first mechanism for premium adjustments was provided in the subsequent Royal Decree of 1 February 2010 that implements the Law in practice.[2] One month before the publication of this Royal Decree, the legislator solicited an advice from a committee made up of representatives of both insurers and consumers.[3] However, the concerns of the committee were not fully taken into account and the content of the Royal Decree could not meet the demands of neither of the parties. In 2011, the Council of State canceled the Royal Decree of 1 February 2010 in response to an action by Assuralia, an institution which represents insurance companies.[4] The main reason of this cancellation was that the legislator has omitted the reserves from the updating mechanism without providing any justification.

The legislator introduced the Royal Decree of 18 March 2016 which is similar to its predecessor in a number of points, but the reserves have been included in the adjustments.[5] The main difference between the new Royal Decree of 18 March 2016, which is the one currently implemented in practice, and its

[2] Arrêté Royal déterminant les indices spécifiques visés à l'article 138bis-4 de la loi du 25 juin 1992 sur le contract d'assurance terrestre, Moniteur Belge du 08/02/2010 (FR); Koninklijk besluit tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Belgish Staatsblad 08/02/2010 (NL).

[3] Avis de la Commission du 7/01/2010 sur le projet d'Arrêté Royal déterminant les indices spécifiques visés à l'article 138bis-4, de la loi du 25 juin 1992 sur le contrat d'assurance terrestre, Commission des assurances DOC C/2009/11 (FR); Over het ontwerp van koninklijk besluit tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Commissie voor Verzekeringen, DOC C/2009/11 (NL).

[4] Arrêt du Conseil d'état numéro 217.085 du 29 décembre 2011, numéro de rôle A. 196.306/VII-38.279.

[5] Arrêté Royal portant modification de l'Arrêté Royal du 1 février 2010 déterminant les indices spécifiques visés à l'article 138bis-4 de la loi du 25 juin 1992 sur le contract d'assurance terrestre, Moniteur Belge du 25/03/2016 (FR); Koninklijk besluit tot wijziging van het koninklijk besluit van 1 februari 2010 tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Belgish Staatsblad 25/03/2016 (NL).

predecessor is the introduction of an *updating factor* α. This updating factor α allows insurers to transfer back to policyholders the impact of medical inflation on both remaining premiums and accumulated reserve. Based on the approximation of Vercruysse et al. (2013), which builds upon some specific assumptions, the updating factor α is set equal to 1.5. Roughly speaking, if the official medical index is equal to $f\%$, the insurer is allowed to adjust the remaining premiums with an index up to $1.5 \times f\%$.

The '1.5 rule' relies on a set of assumptions, and hence, questioning its validity in a general setting is legitimate. The validity of the rule does not necessarily mean that it should provide a good approximation for the actual required factor α. In practice, this will very likely not be the case. As a matter of fact, the Royal Decree of 18 March 2016 states that the '1.5' approximation is an upper bound for premium adjustments. Therefore, what matters most, at least from the point of view of the insurer, is that the '1.5 rule' should provide a conservative estimate of the required updating factor. Thus, the second research question of Part I, investigated in Chapter 3, is whether the '1.5 rule' allows for the necessary adjustments for lifelong health insurance contracts.

In order to answer this question, a necessary step is to define an actuarially fair updating mechanism which serves as a benchmark. We will consider in Chapter 3 three different approaches. The first approach is the *individual* updating mechanism, where each contract is treated individually. The second approach is the updating mechanism based on pooling the risk of *new entrants*: policyholders starting their contracts in the same year will share the medical inflation risk, and thus, will have the same premium adjustments (without necessarily having the same level premium). The third approach generalizes the setting to an updating mechanism based on pooling the risk at the level of an *existing portfolio*, i.e. the updating factor is the same for all policyholders in the portfolio, regardless of their age and the seniority of their contracts.

Therefore, the assessment of the '1.5' approximation across the three methods will also allow us to investigate a third question, namely, what is the effect of introducing intergenerational solidarity in the updating mechanism?

Behind this question lies an important issue which requires some explanation. In 2005, consumers' representatives association Test-Achat/Test-Aankoop filed a complaint against the leading Belgian provider of private health insurance DKV before the Court of Trade. Test-Achat/Test-Aankoop was claiming that DKV applied unfair premium adjustments which were increasing with respect to age.[6] Note that a similar conflict occurred in 2010, shortly after the publication of the first Royal Decree.[7] DKV was ordered in 2005 to repay the extra-payments because the Court has ruled that the increases were discriminatory. The insurer appealed against this decision in 2006, and ultimately, the Court held in favor of the insurer.[8]

The distinction between the medical index f and the updating factor α is crucial in order to understand the challenge. Concerning the medical index, the legislator does not promote age-independent adjustments by defining different indices for each age category. Therefore, as long as the medical index f is different for each age category, the updating mechanism will be age-dependent. However, the medical index is not necessarily *increasing* with respect to the age; see Figure 1.2.2. Moreover, age-independent indices are published by the Belgian Statistical Office, and their application seems to be supported by both parties.[9]

Throughout Chapter 3, we will rule out the potential age-discrimination that may arise from the medical index by assuming that f is age-independent. It turns out that the analysis carried out in Chapter 3 allows to identify some important issues in the current Belgian system for adjusting premiums in private health insurance contract. This will be the focus of Chapter 4, which concludes the first part of this thesis. This chapter is devoted to a discussion on the challenges related to the initial and main purpose of the Law of 20 July 2007, i.e. advocating a system protecting policyholders against age discrimination in health insurance by ensuring affordable covers to all ages. We argue in this chapter that the problem of age-discrimination is strongly related to that of the transferability of the reserves, where the latter problem arises from the analogy between Belgian health insurance contracts and life insurance contracts.

[6] ASBL Test-Achats contre SA DKV Belgium, 7 mars 2005, Tribunal de Commerce de Bruxelles.

[7] `https://www.lecho.be/entreprises/services-financiers-assurances/dkv-condamnee-a-rembourser-des-primes/10073871.html`.

[8] Arrêt de la Cour (quatrième chambre), affaire C-577/11, 7 mars 2013, Recueil de la jurisprudence.

[9] Avis de la Commission du 09/09/2015 sur le projet d'Arrêté Royal portant modification de l'Arrêté Royal du 1 février 2010 déterminant les indices spécifiques visés à l'article 138bis-4, de la loi du 25 juin 1992 sur le contrat d'assurance terrestre, Commission des assurances DOC C/2015/3 (FR); Over het ontwerp van koninklijk besluit tot wijziging van het koninklijk besluit van 1 februari 2010 tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Commissie voor Verzekeringen, DOC C/2015/3 (NL).

CHAPTER 2

Construction of medical inflation indices

2.1 Introduction

The aim of this chapter is to evaluate the current method for the construction of medical inflation indices for private health insurance contracts in Belgium, and to propose an improved method. Constructing these indices is the first step towards a sustainable framework for premium adjustments. We compare the accuracy of the medical indices currently applied in Belgium for private health insurance contracts with product-specific experience-based indices. The latter enable to better capture product-specific systematic deviations due to medical inflation, but their application might raise some practical problems. Therefore, we propose an alternative way to construct medical inflation indices. Several numerical examples are used to compare the performance of the newly proposed indices as well as the current Belgian approach with the experience-based indices. These numerical examples show that the newly proposed indices provide good approximations for the product-specific experience-based indices without having their practical limitations.

This chapter is based on the work performed in Dhaene and Hanbali (2019), and is structured as follows. Section 2.2 contains the main notations. In Section 2.3, we discuss the current Belgian method for constructing medical inflation indices. We highlight some of its limitations and we further motivate the need of an alternative construction. In Section 2.4, we introduce indices capturing the experienced medical inflation for each type of cover in each product. As it is discussed in the same section, applying these indices might not be desirable from the viewpoint of the regulator, nor from that of insurance companies. However, these product-specific experience-based indices will be helpful to compare the Belgian indices and the proposed indices. In Section 2.5, we introduce alternative medical inflation indices and compare their accuracy with respect to the current Belgian ones. Finally, we conclude the chapter in Section 2.6 and discuss some possible topics for future research.

2.2 Notations and assumptions

The methodology proposed in Devolder et al. (2008) for the construction of medical indices is the one currently applied in practice. Based on this methodology, the Belgian government provides market-wide medical indices, which are published every year for 5 different age categories (0-19, 20-34, 35-49, 50-64 and 65+) and 4 different types of covered medical services (hospitalization with stay in a private room, hospitalization with stay in a shared room, dental case costs and ambulatory care costs). However, this method may fail to provide appropriate adjustments, especially because products may cover more than one medical service.

We will consider in this chapter a general setting with J products and K types of covers. We will assume that the set of products as well as the set of covered medical services is time-independent over the two consecutive periods $(t-2, t-1)$ and $(t-1, t)$, over which we determine the medical indices. Newly launched products or newly introduced covers require a special treatment, which will not be considered here. We will also assume that each medical service is covered by at least one product, and that the types of covers included in a product do not change over the observation period.

An insurance company in the private health insurance market may sell different products, implying that J is larger than or equal to the number of insurance companies in the market. Insurance products may cover one or more medical services. For instance, in order to reduce the level of the premium, a product may only cover shared room hospital stays. A product covering stays in a private room will always include a cover for stays in a shared room, as private rooms may not be available at hospitalization. Note however, that insurers should treat the corresponding invoiced amount as if the policyholder has stayed in a private.

The medical indices that we will investigate can be determined for different age-categories. In the following derivations, we consider a single age category, as the methodology is identical for each category.

For any product $j = 1, 2, \ldots, J$ and any type of cover $k = 1, 2, \ldots, K$, we introduce the following notations:

- $C_j^{(k)}(t)$: total claims resulting from cover k of product j in year $(t-1, t)$.
- $C_j(t) = \sum_{k=1}^{K} C_j^{(k)}(t)$: total claims of product j in year $(t-1, t)$.
- $l_j(t)$: total number of insureds of product j in year $(t-1, t)$.

Similarly, we introduce the corresponding quantities $C_j^{(k)}(t-1)$, $C_j(t-1)$ and $l_j(t-1)$, which are all related to year $(t-2, t-1)$. In practice, the method used to determine $l_j(t-1)$ and $l_j(t)$ should account for exits and entries during the year, and should be fixed in order to ensure coherence in the data collected from the different insurance companies.

We also use the notation $I^{(k)}$ for the set of products in the market which include cover k, i.e.

$$I^{(k)} = \{j \in \{1, 2, ..., J\} \mid \text{medical service } k \text{ is covered by product } j\},$$

for any $k \in \{1, 2, \cdots, K\}$, where the letter I is chosen to remind that cover k is 'included' in a product belonging to this set. For any product j in $I^{(k)}$, we will assume that $C_j^{(k)}(t-1) > 0$ and $C_j^{(k)}(t) > 0$. Furthermore, when $j \notin I^{(k)}$, we set $C_j^{(k)}(t-1) = C_j^{(k)}(t) = 0$. Our previous assumption that each medical service is covered by at least one product ensures that all sets $I^{(k)}$ are non-empty. Finally, we will assume that $l_j(t-1) > 0$ and $l_j(t) > 0$.

The setting allows for two possible interpretations of the total claims. The first interpretation is to consider all the $C_j^{(k)}$ as *gross claims*, which are defined as total invoiced amounts minus payments from Social Security[1]. The second interpretation consists in considering all $C_j^{(k)}$ as *net claims*, which correspond to the amounts effectively paid by insurers, i.e. the gross claims minus claims not covered by the insurer. Gross and net claims will

[1] Interested readers can find more information at `https://www.socialsecurity.be/`

differ e.g. in case the insurer covers only a proportion of the gross claims, or in case of a maximum claim payment per hospitalization day. In the current Belgian system, each $C_j^{(k)}(t)$ is a gross claim payment.

In what follows, we will suppose that we have arrived at time t, and that we want to measure the increase of medical costs from year $(t-2, t-1)$ to year $(t-1, t)$, for each type of cover and each product, as well as for all types of covers in the whole market. These observed increases can then be used as the market indices at time t to update premiums for the coming year $(t, t+1)$. We refer to the following Chapter 3 for more details on how to adjust premiums using the medical index.

2.3 The official Belgian medical inflation indices

This section considers a simplified version of the current Belgian system for level premiums of lifelong private insurance contracts. Recall that in this system, one distinguishes 4 types of covered medical services: *private room stays* (or private room inpatient), *shared room stays* (or shared room inpatient), *dental care* and *ambulatory care* (or outpatient), meaning that $K = 4$. A health insurance product may cover more than one medical service. In case a product covers more than one type of cover, current Belgian law requires to classify that product into a single category. The appointed category of a product corresponds to the type of cover with the highest weight in the overall claims of that product, where this categorization is done at product level. Therefore, product j is classified as a 'type of cover k' - product in year $(t-1, t)$ if $C_j^{(k)}(t)$ is such that:

$$C_j^{(k)}(t) = \max\left\{C_j^{(1)}(t), \ldots, C_j^{(K)}(t)\right\}.$$

For $k = 1, 2, \ldots, K$, the set of 'type of cover k' - products in year $(t-1, t)$ is denoted by $M^{(k)}(t)$, where the letter M is chosen to remind that the claim is a maximum, i.e.

$$M^{(k)}(t) = \left\{j \in I^{(k)} \mid j \text{ is a type of cover } k \text{ - product in year } (t-1, t)\right\}.$$

The set of 'type of cover k' - products in year $(t-2, t-1)$, which is denoted by $M^{(k)}(t-1)$, is defined in a similar way.

Every year the medical index is determined for each type of cover k, based on aggregated data from the market. Determining the index for a given product is thus a two step procedure. The first step is to determine the medical inflation index at market level. The second step is to choose the appropriate index for the product under interest.

For the first step, we introduce the time-t index for 'type of cover k' at the market level, which is denoted by $i_m^{(k)}(t)$, where the subscript 'm' refers to the *market*. This index follows from the equation:

$$\frac{\sum\limits_{j\in M^{(k)}(t-1)} C_j(t-1)}{\sum\limits_{j\in M^{(k)}(t-1)} l_j(t-1)} \times \left(1+i_m^{(k)}(t)\right) = \frac{\sum\limits_{j\in M^{(k)}(t)} C_j(t)}{\sum\limits_{j\in M^{(k)}(t)} l_j(t)}. \tag{2.3.1}$$

Notice that the index is well-defined, provided neither $M^{(k)}(t-1)$ nor $M^{(k)}(t)$ is empty. The right hand side of definition (2.3.1) corresponds to the total amount of gross claims for 'type of cover k' - products in the market, divided by the total number of insurance contracts of 'type of cover k' - products, in the year $(t-1,t)$. On the left hand side, the same average appears for the previous year $(t-2,t-1)$. The index $i_m^{(k)}(t)$ can be interpreted as an estimate for the observed medical inflation of type of cover k in the market, from year $(t-2,t-1)$ to $(t-1,t)$.

For the second step, Belgian law stipulates that in the coming year $(t,t+1)$, the premiums for product j can be updated taking into account the index $i_j(t)$, which is defined by:

$$i_j(t) = i_m^{(k)}(t), \qquad \text{if } j \in M^{(k)}(t). \tag{2.3.2}$$

This means that the premiums for 'type of cover k' - product j can be updated taking into account the 'type of cover k' - index $i_m^{(k)}(t)$. Notice that the index $i_j(t)$ is not defined in case $M^{(k)}(t-1)$ is empty.

From (2.3.1), we observe that the total claim payments of 'type of cover k' - product j are entirely used to calculate the market index $i_m^{(k)}$ for category k. Moreover, premiums of this product can be indexed by the category k index $i_m^{(k)}(t)$ in year $(t,t+1)$. In case each product only covers a single type of cover, the Belgian updating mechanism is definitely appropriate. In practice however, many products cover more than a single type of cover, which leads to situations where the current system provides inappropriate indices. The issues are at the level of the construction of the indices as well as at the level of their application.

To illustrate the problem, suppose that a certain product covers multiple medical services. In general, medical costs evolve differently in each category. Hence, taking into account costs for, say, private room covers when calculating the market index for shared room covers is likely to lead to wrong figures for the index of shared room covers. Moreover, dental care in a product which combines several types of covers is not necessarily the cover with the highest claims payments. Therefore, the market index for dental care covers is in practice based only on the claim payments of 'pure' dental care products, ignoring dental care data coming from products with several types of covers.

For the choice of the medical index to be applied for a particular product, suppose that a certain product is only covering stays in private or shared rooms, and that the proportions of claims related to these types of covers are 49% and 51%, respectively. The premiums of this product can then be indexed using the market index for shared rooms, which may not be appropriate for this portfolio. On the other hand, in case the claim payments related to the insurance product reveal proportions 51% / 49%, the medical index that can be applied is the private room medical index, which again might not correspond to the medical inflation observed in this portfolio.

In the following example, we illustrate the calculation of medical inflation indices according to the current Belgian approach.

Example 1. Consider a market with 4 lifelong health insurance products ($J = 4$) and two types of covers ($K = 2$). The claim amounts and number of policyholders observed in the previous two periods are given in Table 2.1.

	Product 1			**Product 2**			**Product 3**			**Product 4**		
t	$C_1^{(1)}(t)$	$C_1^{(2)}(t)$	$l_1(t)$	$C_2^{(1)}(t)$	$C_2^{(2)}(t)$	$l_2(t)$	$C_3^{(1)}(t)$	$C_3^{(2)}(t)$	$l_3(t)$	$C_4^{(1)}(t)$	$C_4^{(2)}(t)$	$l_4(t)$
0	400	600	10	900	600	15	1800	0	30	0	2400	60
1	880	1260	20	1800	1000	25	3600	0	50	0	4000	100

Table 2.1: Claims and number of insureds for the market in Example 1.

In a first step, each product available in the market is classified as either a 'type of cover 1' or 'type of cover 2'- product, based on the type of cover of the product that leads to the largest proportion of claim payments. In this example we find for both observation years that products 2 and 3 are 'type of cover 1' - products, while products 1 and 4 are 'type of cover 2' - products.
Next, medical inflation indices of the market for both types of covers are determined according to (2.3.1):

$$\frac{(900+600)+1800}{15+30} \times \left(1 + i_m^{(1)}(1)\right) = \frac{(1800+1000)+3600}{25+50}$$

and

$$\frac{(400+600)+2400}{10+60} \times \left(1+i_m^{(2)}(1)\right) = \frac{(880+1260)+4000}{20+100}.$$

This leads to

$$i_m^{(1)}(1) = 16.4\% \text{ and } i_m^{(2)}(2) = 5.3\%.$$

Therefore, we find that $i_2(1) = i_3(1) = 16.4\%$, while $i_1(1) = i_4(1) = 5.3\%$. This means that in the coming year $(1, 2)$, premiums of products 2 and 3 are updated using the index $i_m^{(1)}(1)$, whereas premiums of products 1 and 4 are updated using the index $i_m^{(2)}(1)$. $\triangledown$

2.4 Product-specific experience-based medical inflation indices

Before introducing an alternative way of calculating market-based indices for all types of covers offered in the market, we first introduce product-specific experience-based indices for each medical service covered by the product under consideration. These indices are constructed such that they capture the experienced health claim increases which are specific to the product and to the type of cover. We introduce these indices to study in the subsequent section the accuracy of the newly proposed indices with respect to the current Belgian ones.

The product-specific experience-based index for product j and type of cover k observed in year $(t-1, t)$ is denoted by $e_j^{(k)}(t)$. It is defined by

$$\frac{C_j^{(k)}(t-1)}{l_j(t-1)} \left(1+e_j^{(k)}(t)\right) = \frac{C_j^{(k)}(t)}{l_j(t)}, \qquad \text{for } j \in I^{(k)}. \tag{2.4.1}$$

The factor $\left(1+e_j^{(k)}(t)\right)$ in (2.4.1) connects the 'type of cover k' - claim cost per insured of product j, over two consecutive years. Furthermore, in case $j \notin I^{(k)}$, we set $e_j^{(k)}(t)$ equal to 0, by convention.

The overall product-specific index for product j in year $(t-1, t)$ is denoted by $e_j(t)$. It follows from:

$$\frac{C_j(t-1)}{l_j(t-1)} \left(1+e_j(t)\right) = \frac{C_j(t)}{l_j(t)}. \tag{2.4.2}$$

Here, the factor $(1 + e_j(t))$ connects claim costs per insured for product j, over two consecutive years.

It is straightforward to prove the following relation between the 'type of cover k' indices $e_j^{(k)}(t)$ and the overall index $e_j(t)$ of the product:

$$1 + e_j(t) = \sum_{k=1}^{K} \left(1 + e_j^{(k)}(t)\right) \times w_j^{(k)}(t) \tag{2.4.3}$$

with weights $w_j^{(k)}(t)$ given by

$$w_j^{(k)}(t) = \frac{C_j^{(k)}(t-1)}{C_j(t-1)} \tag{2.4.4}$$

The product-specific index $e_j(t)$ can be defined for gross as well as for net claim amounts. The index has the advantage of enabling the insurer to account for product-specific systematic risk due to medical inflation. However, it could also capture diversifiable risk, which is supposed to be borne by the insurer, and hence should not trigger the indexing mechanism. Another disadvantage of applying this index for updating premiums is that it reflects the (gross or net) claim increases that were experienced by the insurer, which could constitute competitive sensitive information. Finally, using product-specific indices $e_j(t)$'s could suffer from a lack of transparency towards policyholders, and would not lead to uniform premium increases across the market.

2.5 A new class of medical inflation indices

In this section, we propose a new method to construct market medical inflation indices for the different types of covers, as well as for product-specific indices. In the construction of the index for type of cover k, we take into account all products offering that cover. In the application of the index for product j, we take into account all types of covers included in that product. Thus, compared to the current Belgian indices, the proposed method provides market indices which are more representative of the experienced market medical inflation for each type of cover, and allows for an indexing at the product-level which captures more accurately the medical inflation for that product.

In a first step, we define $f_m^{(k)}(t)$, the time-t medical index of the market for type of cover k, as follows:

$$\frac{\sum\limits_{j\in I^{(k)}} C_j^{(k)}(t-1)}{\sum\limits_{j\in I^{(k)}} l_j(t-1)} \times \left(1+f_m^{(k)}(t)\right) = \frac{\sum\limits_{j\in I^{(k)}} C_j^{(k)}(t)}{\sum\limits_{j\in I^{(k)}} l_j(t)}. \tag{2.5.1}$$

Our previous assumptions ensure that $f_m^{(k)}(t)$ is well-defined for each type of cover k. The right hand side of this equation is the total market claims paid for 'type of cover k' in year $(t-1,t)$, divided by the total number of insurance contracts offering that cover. On the left hand side, the same average appears for the previous year $(t-2,t-1)$. The factor $\left(1+f_m^{(k)}(t)\right)$ connects both average claim amounts.

In a second step, we propose that product j is appointed a product-specific index, based on the relative importance of the different medical services covered by that product. Inspired by formula (2.4.3) and using the weights defined in (2.4.4), we suggest the following calculation:

$$1+f_j(t) = \sum_{k=1}^{K} \left(1+f_m^{(k)}(t)\right) \times w_j^{(k)}(t). \tag{2.5.2}$$

As opposed to the market indices $i_m^{(k)}(t)$ defined in (2.3.1), the market indices $f_m^{(k)}(t)$ defined above are not biased by the claim amounts of other categories that do not contribute to the claim increases of category k. Moreover, the product-specific index $f_j(t)$ takes into account the weight of each type of cover in the total claims of the product. Therefore, it is to be expected that the newly proposed method will give rise to a more accurate indexing mechanism than the one currently used in Belgium.

Comparing formulas (2.4.3) for the experience-based indices and (2.5.2) for the proposed indices, we see the calculation of the indices $e_j^{(k)}(t)$ is performed at product level, whereas the $f_m^{(k)}(t)$ are determined at market level. Hence, $e_j(t)$ is derived from product-specific evolution of the claims related to the covered type of medical service, whereas for $f_j(t)$ market averages are used for it. As the market portfolio is a larger pool than the portfolio of an individual product, the index $f_j(t)$ will be superior to the index $e_j(t)$ in terms of capturing the medical inflation due to systematic risk.

Concerning the claim amounts $C_j^{(k)}$ in formulas (2.4.4), (2.5.1) and (2.5.2), both the 'gross claims' and the 'net claims' interpretation are possible. In case the medical indices $f_m^{(k)}(t)$, $f_j(t)$ and the weights $w_j^{(k)}(t)$ are based on gross claims, the market indices $f_m^{(k)}(t)$ capture the increase of gross claims in the market, while the weights $w_j^{(k)}(t)$ might not correctly capture the relative weights of the different types of covers in the product under consideration, leading to a wrong figure for the medical inflation $f_j(t)$. On the other hand, using the net claims to define the medical indices and the weights, is an appropriate approach for the weights $w_j^{(k)}(t)$, but might give a wrong picture of the medical inflation indices $f_m^{(k)}(t)$ for the different types of covered medical services in the market. One possible rule of thumb consists in determining the market indices $f_m^{(k)}(t)$ using formulas (2.5.1), with the $C_j^{(k)}$ defined as gross claims, while the weights $w_j^{(k)}(t)$ are determined according to (2.4.4) with the $C_j^{(k)}$ interpreted as net claims.

Hereafter, we will numerically illustrate the validity of our approach. In the examples that we will consider, we always assume that all products have an unlimited cover, i.e. that gross claims and net claims are identical. Throughout the remaining numerical illustrations, we use the product-specific experience-based indices (2.4.3) as benchmark. In the next example, we revisit the market considered in Example 1 and compare its indices $i_j(1)$, $e_j(1)$ and $f_j(1)$.

Example 2. Consider the market with 4 products and 2 types of covers observed at time 1, as described in Table 2.1 of Example 1. The experience-based indices for each product follow directly from (2.4.2), such that:

$$\begin{aligned}
\frac{400+600}{10} \times (1+e_1(1)) &= \frac{880+1260}{20}, \\
\frac{900+600}{15} \times (1+e_2(1)) &= \frac{1800+1000}{25}, \\
\frac{1800}{30} \times (1+e_3(1)) &= \frac{3600}{50}, \\
\frac{2400}{60} \times (1+e_4(1)) &= \frac{4000}{100},
\end{aligned}$$

and we find $e_1(1) = 7\%$, $e_2(1) = 12\%$, $e_3(1) = 20\%$ and $e_4(1) = 0\%$. For the proposed index $f_j(1)$ of product j, the calculation is carried out in two steps. The first step is to determine market medical indices for each type of cover, i.e. the indices $f_m^{(k)}(1)$ defined in (2.5.1). For type of cover 1, we

have:
$$\frac{400+900+1800}{10+15+30}\left(1+f_m^{(1)}(1)\right)\frac{880+1800+3600}{20+25+50},$$
which leads to $f_m^{(1)}(1) \approx 17.28\%$. For type of cover 2, we have:
$$\frac{600+600+2400}{10+15+60}\left(1+f_m^{(2)}(1)\right)\frac{1260+1000+4000}{20+25+100},$$
which leads to $f_m^{(2)}(1) \approx 1.93\%$. We clearly see that medical inflation is much higher for type of cover 1 compared to that of type of cover 2. The second step is to determine the indices for each product j using the weighted sum in (2.5.2). The couple of weights $\left(w_j^{(1)}(1), w_j^{(2)}\right)$ for each product $j = 1, 2, 3$ and 4 is given by $(0.4, 0.6)$, $(0.6, 0.4)$, $(1, 0)$ and $(0, 1)$, respectively. Therefore, we find:
$$\begin{aligned}
1+f_1(1) &\approx (1+17.28\%)\times 0.4+(1+1.93\%)\times 0.6,\\
1+f_2(1) &\approx (1+17.28\%)\times 0.6+(1+1.93\%)\times 0.4,\\
1+f_3(1) &\approx (1+17.28\%)\times 1+(1+1.93\%)\times 0,\\
1+f_4(1) &\approx (1+17.28\%)\times 0+(1+1.93\%)\times 1.
\end{aligned}$$
The indices $i_j(1)$, $e_j(1)$ and $f_j(1)$ are reported for all products in Table 2.2.

	Product 1	**Product 2**	**Product 3**	**Product 4**
Belgian indices $i_j(1)$	5.3%	16.4%	16.4%	5.3%
Proposed indices $f_j(1)$	8.1%	11.1%	17.3%	1.9%
Experience-based indices $e_j(1)$	7.0%	12.0%	20.0%	0.0%

Table 2.2: Comparison of the different medical indices for the market of Example 1.

In this market, the proposed indices $f_j(1)$ provide a good approximation for the experience-based indices $e_j(1)$, whereas the Belgian indices $i_j(1)$ perform worse. This phenomenon is in particular observed for product 4 with a single cover, which did not experience any medical inflation. For this product, the current Belgian system suggests to update the premiums based on a medical inflation of $i_4(1) = 5.3\%$, whereas our approach leads to $f_4(1) = 1.9\%$, which is much closer to the experience-based inflation $e_4(1) = 0$. $\triangledown$

In the following example, we consider another market to illustrate the performance of the different medical inflation indices.

Example 3. Consider a market with 4 products ($J = 4$) and 2 types of covers ($K = 2$) with data displayed in Table 2.3.

	Product 1			**Product 2**			**Product 3**			**Product 4**		
t	$C_1^{(1)}(t)$	$C_1^{(2)}(t)$	$l_1(t)$	$C_2^{(1)}(t)$	$C_2^{(2)}(t)$	$l_2(t)$	$C_3^{(1)}(t)$	$C_3^{(2)}(t)$	$l_3(t)$	$C_4^{(1)}(t)$	$C_4^{(2)}(t)$	$l_4(t)$
0	400	600	10	900	600	15	1800	0	30	0	2000	50
1	880	1500	20	1650	1250	25	3300	0	50	0	5000	100

Table 2.3: Observed claims for the market of Example 3.

In this example, we find that all experience-based type-of-cover-specific indices are equal, in the whole market:

$$e_j^{(1)}(1) = 10\% \quad \text{and} \quad e_j^{(2)}(1) = 25\%, \qquad \text{for } j = 1, 2.$$

Hence, the claim amounts per-policy for types of covers 1 and 2 increase by 10% and 25%, respectively, for all products. The values of the indices $i_j(1)$, $e_j(1)$ and $f_j(1)$ are summarized in Table 2.4. We observe again that the newly proposed index $f_j(1)$ outperforms the Belgian index $i_j(1)$, when compared to the experience-based index $e_j(1)$. $\triangledown$

	Product 1	**Product 2**	**Product 3**	**Product 4**
Belgian indices $i_j(1)$	23.0%	12.7%	12.7%	23.0%
Proposed indices $f_j(1)$	18.7%	15.4%	8.9%	25.3%
Experience-based indices $e_j(1)$	19.0%	16.0%	10.0%	25.0%

Table 2.4: Comparison of the medical indices for the market in Example 3.

In the (theoretical) special case of a market with a single product, i.e. $J = 1$, the current Belgian index, the experience based index and the newly proposed index are identical:

$$i_1(t) = e_1(t) = f_1(t).$$

The proof of these equalities follows in a straightforward way from (2.3.1), (2.4.3) and (2.5.2). In the following theorem, we move to the more realistic case of a multiple product market. We consider conditions under which the product-specific index $e_j(t)$ and our newly proposed index $f_j(t)$ are equal.

Theorem 2.1. *Consider a market at time t and suppose that in the period $(t-1, t)$, the observed average claims for any given type of cover are equal* *for all products which include that cover, i.e.*

$$\frac{C_j^{(k)}(t)}{l_j(t)} = c^{(k)}(t), \quad \textit{for any } k \in \{1, 2, \dots, K\} \textit{ and } j \in I^{(k)}. \tag{2.5.3}$$

Furthermore, suppose that the corresponding observation also holds for the period $(t-2, t-1)$*:*

$$\frac{C_j^{(k)}(t-1)}{l_j(t-1)} = c^{(k)}(t-1), \quad \textit{for any } k \in \{1, 2, \dots, K\} \textit{ and } j \in I^{(k)}. \tag{2.5.4}$$

Then for any product j *in the market, the indices* $e_j(t)$ *and* $f_j(t)$ *are equal:*

$$e_j(t) = f_j(t), \quad \textit{for any } j = 1, 2, \dots, J. \tag{2.5.5}$$

Proof. Taking into account (2.5.3) and (2.5.4), we immediately find from (2.4.1) that

$$1 + e_j^{(k)}(t) = \frac{c^{(k)}(t)}{c^{(k)}(t-1)}, \quad \text{for any } k \in \{1, 2, \dots, K\} \text{ and } j \in I^{(k)}. \tag{2.5.6}$$

Hence, for any product j which includes type of cover k, the index $e_j^{(k)}(t)$ is independent of j. From (2.5.1), (2.5.3), (2.5.4) and (2.5.6) it follows then that

$$e_j^{(k)}(t) = f_m^{(k)}(t), \quad \text{for any } k \text{ in } \{1, 2, \dots, K\} \text{ and } j \in I^{(k)}.$$

Observing that $w_j^{(k)}(t) = 0$ in case $j \notin I^{(k)}$ and comparing definitions (2.4.3) and (2.5.2) of the indices $f_j(t)$ and $e_j(t)$, the equalities derived above lead to (2.5.5). □

Although conditions (2.5.3) and (2.5.4) imply the equality of the indices $f_j(t)$ and $e_j(t)$ for any product j, in general these conditions do not imply that the indices $i_j(t)$ and $e_j(t)$ are equal too. This phenomenon is illustrated in the following example.

Example 4. At time 1, we consider a market with 2 products and 2 types of covers, with claim amounts in the previous two periods given in Table 2.5. Product 1 only includes type of cover 1, whereas product 2 offers both types of covers. Therefore, we have $I^{(1)} = \{1, 2\}$ and $I^{(2)} = \{2\}$. Furthermore, from Table 2.5, we find that $M^{(1)}(t) = \{1, 2\}$ for $t = 0$ and 1, whereas

$M^{(2)}(0)$ and $M^{(2)}(1)$ are empty sets. One can easily verify that in this market, conditions (2.5.3) and (2.5.4) of Theorem 2.1 are fulfilled.
The indices for this market are reported in Table 2.6. Clearly, the Belgian indices fail to properly capture the experienced medical inflation in this case. In particular, the Belgian indices $i_1(1)$ and $i_2(1)$ are equal, while the other indices lead to different values for both products. $\bigtriangledown$

	Product 1			**Product 2**		
t	$C_1^{(1)}(t)$	$C_1^{(2)}(t)$	$l_1(t)$	$C_2^{(1)}(t)$	$C_2^{(2)}(t)$	$l_2(t)$
0	100	0	10	200	150	20
1	120	0	10	240	200	20

Table 2.5: Observed claims for the market of Example 4.

	Product 1	**Product 2**
Belgian indices $i_j(1)$	24.4%	24.4%
Proposed indices $f_j(1)$	20.0%	25.7%
Experience-based indices $e_j(1)$	20.0%	25.7%

Table 2.6: Comparison of the medical indices for the market in Example 4.

The example above illustrates the fact that the conditions of Theorem 2.1 are not sufficient to ensure that the Belgian indices $i_j(t)$ of the different products are equal to the corresponding product-specific indices $e_j(t)$. This has to be considered as a weakness of the current Belgian medical inflation indices. Our newly introduced indices $f_j(t)$ do not exhibit this weakness.

Finally, consider a product j for which some of the experience-based indices $e_j^{(k)}(t)$ are very different from the newly introduced indices $f_m^{(k)}(t)$. For such a product also the global experience-based index $e_j(t)$ might be very different from the newly introduced index $f_j(t)$, indicating that product j is very different from the average product in the market. In this case, the product might still need an approach different from the one mentioned above. Note that the Belgian Law allows for a personalized updating upon approval from the regulating authority.

2.6 Concluding remarks

Portfolios of lifelong health insurance contracts are subject to systematic medical inflation risk. In order to cope with this unpredictable risk, the Belgian regulation allows private insurers to update level premiums of lifelong health insurance contracts, using specific medical inflation indices, which are based on aggregated market data. The construction of these indices remains a challenge. Although such medical inflation indices are so far a Belgian originality, as pointed out in Devolder et al. (2008), their construction remains relevant to other countries, provided there is a sufficient number of contracts with lifelong cover and the need for premium adjustments. One example of countries where lifelong health insurance products can be found is Germany; see Schneider (2002). Another example is the Indian health insurance market where some contracts also provide a lifelong health cover. Besides their application for premium adjustments, these indices can also be useful in other applications such as the study of the drivers of medical inflation in a governmental context.

In this chapter, we described the new methodology proposed in Dhaene and Hanbali (2019) for constructing medical inflation indices when different products with multiple types of covers are sold in the market. We compare the accuracy of the newly proposed indices with the current Belgian approach on the basis of some numerical examples, where company-specific experience-based indices are taken as benchmark. Although the latter indices allow for a tailor-made updating for each product sold in the market, their application might be not desirable due to several reasons which were discussed above. The newly proposed medical indices can be considered as improved versions of the current indices prescribed by the Belgian regulator, as they better reflect the experienced medical inflation of any particular health insurance portfolio. It is worth noting that the proposed method as well as the experience-based method may lead to noisy medical inflation indices across the different insurance companies. One approach which can be investigated in the future is to implement smoothing mechanisms that help to reduce this variability.

A relevant issue when determining market-wide medical inflation indices, which is not considered in this chapter, is how to take into account waiting periods in the calculation of the medical indices. These waiting periods are typical for newly underwritten contracts. The problem could arise e.g. in case the market experiences a substantial growth in contracts. Not carefully taking into account the waiting period may lead to medical inflation indices which are not appropriate. One simple solution consists of not taking into account policies which are still in the waiting period when calculating the medical indices.

We conclude this chapter with a discussion on some practical issues related to measuring medical inflation which may define topics for future research.

The experience-based index $e_j^{(k)}(t)$ introduced in (2.4.3) can be expressed as a function of an experience-based medical *frequency* index and an experience-based medical *severity* index. Let us introduce the notation $d_j^{(k)}(t)$ for the number of insureds of product j in year $(t-1, t)$ with a 'type of cover - k' claim in that year. The medical frequency index $n_j^{(k)}$ for product j and category k is defined by

$$\frac{d_j^{(k)}(t-1)}{l_j(t-1)}\left(1+n_j^{(k)}(t)\right)=\frac{d_j^{(k)}(t)}{l_j(t)}, \tag{2.6.1}$$

while the corresponding medical severity index $y_j^{(k)}$ follows from

$$\frac{C_j^{(k)}(t-1)}{d_j^{(k)}(t-1)}\left(1+y_j^{(k)}(t)\right)=\frac{C_j^{(k)}(t)}{d_j^{(k)}(t)}. \tag{2.6.2}$$

Obviously, the medical frequency index measures the 'frequency inflation' in two consecutive years, whereas the medical severity index measures the 'severity inflation'. From (2.4.2), (2.6.1) and (2.6.2), we find the following relation between the experience-based indices $e_j^{(k)}(t)$, $n_j^{(k)}(t)$ and $y_j^{(k)}(t)$ of product j and category k:

$$\left(1+e_j^{(k)}(t)\right)=\left(1+n_j^{(k)}(t)\right)\left(1+y_j^{(k)}(t)\right). \tag{2.6.3}$$

This decomposition of the medical inflation in frequency and severity components can be useful to analyze the drivers of medical inflation. However, such an analysis that could build on earlier work of e.g. Bachler et al. (2006) is out of the scope of this thesis.

CHAPTER 3

Updating mechanisms for lifelong insurance covers

3.1 Introduction

In this chapter, we present the work performed in Denuit et al. (2017), which investigates practical ways for indexing of level premiums in lifelong medical insurance contracts. We assume that medical inflation indices are available, and can be used to adjust premiums for a given contract. It is shown that ex-post indexing can be achieved by considering only premiums, without explicit reference to reserves. This appears to be relevant in practice as reserving mechanisms may not be transparent to policyholders. Moreover, some insurers do not compute contract-specific reserves, managing the whole portfolio in a collective way. Three different updating mechanisms are introduced. The first one is the individual updating where the adjustment accounts for medical inflation risk contract-per-contract. The second is based on solidarity among new entrants, regardless of the age of policyholders. The third updating mechanism consists in pooling contracts at portfolio level, for all entry years and all ages.

This study provides two main insights. On the one hand, it allows to assess the '1.5 rule' introduced by the Belgian legislator with the Royal Decree of 18 March 2016. This approximation is compared with the proposed *actuarially fair* updating mechanisms. On the other hand, the comparison of the three updating schemes allows to determine the effect of introducing intergenerational solidarity between policyholders with different ages. Recall that one of the main purposes of the Belgian Law of 20 July 2007 is to allow access to private health insurance covers for elderly, and to protect against sudden premium increases.

The approach of this chapter is related to other updating mechanisms for lifelong health insurance contracts. For instance, a one-step version of the formula in the individual case used here has been derived in Schneider (2002) in the particular case of no reserve update; see also Vercruysse et al. (2013). Here, this formula is extended to a multi-period setting, allowing for premium and/or reserve revisions. The case where the reserve is also updated is studied in Section 5.4 of Pitacco (2014). Dhaene et al. (2017) derived updating mechanisms for lifelong health insurance contracts in case the reserves are transferable. To the best of our knowledge, the present work is the first to provide a comprehensive study with sensitivity analysis of updating mechanisms for lifelong health insurance covers, and a comparison between individual and aggregate methods.

The remainder of this chapter is organized as follows. In Section 3.2, we describe the actuarial model for health insurance contracts considered in this paper. Section 3.3 presents the individual updating mechanism, where we first start with the one-step adjustments of premium and/or reserves and then generalize to periodic revisions during the coverage. In Section 3.4, we replace the individual revision formula with a collective one, considering all policyholders who entered the portfolio during a given year (i.e. a cohort of new contracts). In Section 3.5, the indexing is performed for the whole portfolio, accounting for new businesses and exists. Section 3.6 consists in a case study where we simultaneously illustrate and compare the three updating mechanisms, and we assess the simple rule introduced in Belgium after the Royal Decree of 18 March 2016. The final Section 3.7 concludes the paper, revisiting some assumptions.

3.2 Actuarial model

The origin of time is chosen at policy issue. Time k stands for the seniority of the policy (i.e. the time elapsed since policy issue). The policyholder's (integer) age at policy issue is denoted by x. We denote the ultimate integer age by ω, assumed to be finite. This means that survival until integer age ω has a positive probability, whereas survival until integer age $\omega + 1$ has probability zero.

The contracts under interest stipulate that no surrender value is paid out in case of policy cancellation, which is the case of Belgian private health insurance contracts. A discussion on the transferability of the reserves under the current Belgian legislation is provided in the following Chapter 4. The expected annual health cost at age $x+j$, which is denoted by b_{x+j}, is random due to medical inflation. Let $b_{x+j}^{(0)}$ be an estimate at time 0 for the expected medical expenses in year $(0,1)$ for a person aged $x+j$ at time 0. At time 0, determining level premiums for these contracts requires an estimate of future costs. Starting from the current expected cost $b_{x+j}^{(0)}$ for individuals aged $x+j$, the estimates of future costs are assumed to be equal to $b_{x+j}^{(0)}$, increased by the assumed inflation rate. Suppose that the insurer assumes a constant yearly medical inflation $f \geq 0$ over the coming years. Hence, $b_{x+j}^{(0)}\ (1+f)^j$ is an estimate at time 0 of the expected medical expenses in year $(j, j+1)$ for a person aged $x+j$ in the beginning of that year. The results that we present hereafter can easily be generalized to the case of non-constant but deterministic estimates for future inflation in the coming years.

The tariff $\pi_{x,0}^{(0)}$ is determined from a technical basis, i.e. from assumptions about mortality, surrenders, interest and medical inflation. We focus on the medical inflation risk only by assuming that the realizations of the technical basis follow the assumptions. This means that the assumptions about mortality, surrenders and interest rates are not subject to revision. In addition, we assume that adverse selection has been ruled out by the insurer using an appropriate underwriting policy. Technical aspects on adverse selection in the health insurance market and its impact on the updating mechanism are out of our scope; see Newhouse (1996) and Handel (2013) for a discussion.

The level yearly premium $\pi_{x,0}^{(0)}$ for a health insurance contract underwritten at current time 0 on an insured aged x is determined by means of the equivalence principle. Let $v(0,j)$ be the discounting factor over the period $(0,j)$. The expected present value (or actuarial value) $B_x^{(0)}$ of the benefits paid by the insurer is then given by:

$$B_x^{(0)} = \sum_{j=0}^{\omega-x} b_{x+j}^{(0)}\ (1+f)^j\ {}_jE_x. \tag{3.2.1}$$

The actuarial discounting factor ${}_jE_x$ accounts for mortality, lapses and interest, over the period $(0, j)$, i.e.

$$ {}_jE_x = v(0,j)\,{}_jp_x, $$

where ${}_jp_x$ is the sojourn, or non-exit, probability, i.e. probability that a policyholder aged x at policy issue does not leave the portfolio due to death or lapse for instance. Furthermore, let $\ddot{a}_x$ be the actuarial value of an annuity-due paying a unit amount per year, as long as the policy is in force, i.e.

$$ \ddot{a}_x = \sum_{j=0}^{\omega-x} {}_jE_x. \tag{3.2.2} $$

We then have

$$ \pi_{x,0}^{(0)} = \frac{B_x^{(0)}}{\ddot{a}_x}. \tag{3.2.3} $$

The (unpredictable) increase of medical costs in the future generates a systematic risk for health insurance providers. In this setting, it is assumed that medical inflation is not guaranteed when setting the level premiums at policy issue. Instead, premiums and eventually also reserve (also known as mathematical reserve or policy value) are regularly updated, accounting for observed medical inflation over the previous years. The premium-updating mechanism is based on a medical inflation index which may be different from the assumed medical inflation f used to determine the level premium. In the following sections, we present an actuarially sound methodology for revising the level of the premium as inflation emerges over time.

Note that at time 0, the superscript "$^{(0)}$" indicates that the corresponding quantity is estimated at policy issue. Analogously, a superscript "$^{(k)}$", for $k = 0, 1, 2, \ldots$, will be used throughout this chapter to indicate that the quantity under consideration is based on information about medical costs available at time k. Hereafter, $\pi_{x,j}^{(k)}$ denotes the revised level premium to be paid at time k for a contract that was underwritten at time $j \leq k$ at age x. Moreover, the observed medical inflation index in year k is denoted by $f^{(k)}$. Based on the discussion provided in Chapter 2, the medical inflation index $f^{(k)}$ may correspond to one of the indices $i_j(k)$ (Belgian approach), $f_j(k)$ (proposed approach) or $e_j(k)$ (product-specific experience-based approach) for year k and product j. In this chapter, the subscript j identifying the contract becomes redundant, and is thus omitted. Moreover, the time index for the evolution of the contract is used as a superscript.

3.3 Individual updating mechanism

3.3.1 Adapting the premium and/or the reserve at time 1

Suppose that we have arrived at time 1 and that the policy that was underwritten at age x at time 0 is still in force. This means that at time 1, a positive prospective reserve

$$V_{x+1}^{(0)} = (1+f)\, B_{x+1}^{(0)} - \pi_{x,0}^{(0)}\, \ddot{a}_{x+1}, \tag{3.3.1}$$

is required for the policyholder now aged $x+1$, where $B_{x+1}^{(0)}$ and $\ddot{a}_{x+1}$ are defined similarly to (3.2.1) and (3.2.2), respectively. Taking into account that the premium $\pi_{x,0}^{(0)}$ was determined via the equivalence principle (3.2.3), the prospective expression (3.3.1) for $V_{x+1}^{(0)}$ at time 1 can be transformed into a retrospective expression, or the available reserve of the policyholder. In particular, we have:

$$V_{x+1}^{(0)} = \left(\pi_{x,0}^{(0)} - b_x^{(0)}\right) \left({}_1E_x\right)^{-1}. \tag{3.3.2}$$

Suppose that the inflation for medical expenses observed during the first year is given by $f^{(1)}$, which is assumed to be age-independent; see Section 3.7 on how to relax this assumption. This means that at time 1, due to the observed medical inflation in the past year, the expected annual medical expenses $b_{x+1+j}^{(0)}$ have to be updated to

$$b_{x+1+j}^{(1)} = \left(1+f^{(1)}\right) b_{x+1+j}^{(0)}, \tag{3.3.3}$$

for $j = 0, 1, 2, \ldots$. The assumption of uniformity of medical inflation over all ages implies that at time 1, the actuarial value of future benefits estimated at that time becomes:

$$B_{x+1}^{(1)} = (1+f^{(1)}) B_{x+1}^{(0)}, \tag{3.3.4}$$

instead of $(1+f)\, B_{x+1}^{(0)}$ which was estimated at time 0. Suppose that the insurer pays out to the policyholder the updated value of the benefit $B_{x+1}^{(1)}$. In the context of health insurance contracts, this implies that the insurer covers the necessary amount to the policyholder. Therefore, due to the change of the benefit estimate, the required (prospective) reserve becomes

$$B_{x+1}^{(1)} - \pi_{x,0}^{(0)}\, \ddot{a}_{x+1},$$

which coincides with the available (retrospective) provision $V_{x+1}^{(0)}$ in (3.3.1) only if the observed inflation $f^{(1)}$ in the first year is equal to the assumed inflation f at time 0. In case $f^{(1)} > f$, the available provision $V_{x+1}^{(0)}$ is insufficient to cover future liabilities. In order to restore the actuarial equivalence, the premium $\pi_{x,0}^{(0)}$ and/or the available provision $V_{x+1}^{(0)}$ will have to be updated to levels $\pi_{x,0}^{(1)}$ and $V_{x+1}^{(1)}$, respectively. Any of the infinite pairs $\left(V_{x+1}^{(1)},\ \pi_{x,0}^{(1)}\right)$ satisfying the equality

$$V_{x+1}^{(1)} = B_{x+1}^{(1)} - \pi_{x,0}^{(1)}\ \ddot{a}_{x+1} \tag{3.3.5}$$

will perform the task of resetting the actuarial equivalence. Notice that (3.3.5) is the prospective reserve at time 1, based on updated benefits and premiums. Subtracting (3.3.5) from (3.3.1), we find that the new premium level $\pi_{x,0}^{(1)}$ at time 1 is given by

$$\pi_{x,0}^{(1)} = \pi_{x,0}^{(0)} + \left(f^{(1)} - f\right)\ \pi_{x+1,0}^{(0)} - \frac{V_{x+1}^{(1)} - V_{x+1}^{(0)}}{\ddot{a}_{x+1}}, \tag{3.3.6}$$

where

$$\pi_{x+1,0}^{(0)} = \frac{B_{x+1}^{(0)}}{\ddot{a}_{x+1}} \tag{3.3.7}$$

is the level premium at time 0 for a contract underwritten at age $x+1$.

Remark 3.1. *Consider the special case where $f = 0$ and the insurer updates the premium according to the observed medical inflation $f^{(1)}$, i.e.*

$$\pi_{x,0}^{(1)} = \left(1 + f^{(1)}\right) \pi_{x,0}^{(0)}.$$

This case corresponds to what has initially been prescribed in the Royal Decree of 1 February 2010. Under these assumptions, we find from (3.3.1), (3.3.4) and (3.3.5) that:

$$V_{x+1}^{(1)} = \left(1 + f^{(1)}\right) V_{x+1}^{(0)}.$$

This means that in case no inflation is taken into account to determine the initial premium level $\pi_{x,0}^{(0)}$, indexing the premium according to the observed medical inflation $f^{(1)}$ requires the same proportional update of the available reserve. Thus, the Royal Decree of 1 February 2010 allowed insurers to adjust the premiums to the impact of medical inflation on future premiums only, but not on the already built up reserves.

It appears that an infinite number of updating mechanisms can be implemented to restore the actuarial equivalence at individual level. Nevertheless, we can identify some special cases. The first case is when the insurer takes all medical inflation risk, i.e. $\pi_{x,0}^{(1)} = \pi_{x,0}^{(0)}$. This means that the reserve is updated by adding the amount $\left(f^{(1)} - f\right) B_{x+1}^{(0)}$. The second case is when the deviation of observed inflation $f^{(1)}$ from assumed inflation f is completely financed by the policyholder, i.e. $V_{x+1}^{(0)} = V_{x+1}^{(1)}$. In the sequel, we will focus on the second case only.

Let us assume that the level of the available provision is left unchanged, i.e.

$$V_{x+1}^{(0)} = V_{x+1}^{(1)}. \tag{3.3.8}$$

From (3.3.6) it follows then that the new premium level at time 1 is given by:

$$\pi_{x,0}^{(1)} = \pi_{x,0}^{(0)} + \left(f^{(1)} - f\right) \pi_{x+1,0}^{(0)}, \tag{3.3.9}$$

which is similar to the formula obtained in Schneider (2002) in the particular case of $f = 0$. Formula (3.3.9) shows that the premium increase $\pi_{x,0}^{(1)} - \pi_{x,0}^{(0)}$ at time 1 can be interpreted as the level premium corresponding to a new insurance contract underwritten at time 1 offering benefits with actuarial value equal to the benefit increase $\left(f^{(1)} - f\right) B_{x+1}^{(0)}$. This can be intuitively explained as follows: due to the increase in future medical costs from $(1 + f)\, B_{x+1}^{(0)}$ to $(1 + f^{(1)})B_{x+1}^{(0)}$, the policyholder now aged $x + 1$ must virtually buy at time 1 a supplementary insurance policy, covering the benefit increase $\left(f^{(1)} - f\right) B_{x+1}^{(0)}$, whose price $\left(f^{(1)} - f\right) \pi_{x+1,0}^{(0)}$ adds to $\pi_{x,0}^{(0)}$. The premium formula (3.3.9) can be rewritten in the following form:

$$\pi_{x,0}^{(1)} = \left(1 + \frac{\pi_{x+1,0}^{(0)}}{\pi_{x,0}^{(0)}} \left(f^{(1)} - f\right)\right) \pi_{x,0}^{(0)},$$

and hence, the actual indexing for the original premium is $\frac{\pi_{x+1,0}^{(0)}}{\pi_{x,0}^{(0)}} \left(f^{(1)} - f\right)$. In case no inflation assumption is made at policy issue, i.e. $f = 0$, the proportional increase of the premium will be different (and usually higher) than the observed medical inflation $f^{(1)}$ over the first year. Also notice that in case the inflation assumption in the first year was too conservative, i.e. $f^{(1)} < f$, the premium level may be reduced at time 1.

3.3.2 Adapting the premium level at time k

Suppose that we have arrived at time $k = 2, 3, \ldots$ and that the policy that was underwritten on the policyholder aged x at time 0 is still in force. The observed medical inflation up to time $k-1$ has been taken into account by restoring the actuarial equivalence and updating the premium levels at times $1, 2, \ldots, k-1$. Suppose that the deviations of observed inflation from assumed inflation f are completely financed by the policyholder, which means that the available provisions are not updated. Let $V_{x+k-1}^{(k-1)}$ and $\pi_{x,0}^{(k-1)}$ be the available provision and the premium level determined at time $k-1$. They were set such that the actuarial equivalence at time $k-1$ was restored:

$$V_{x+k-1}^{(k-1)} = B_{x+k-1}^{(k-1)} - \pi_{x,0}^{(k-1)} \ddot{a}_{x+k-1}. \tag{3.3.10}$$

In this formula, $B_{x+k-1}^{(k-1)}$ is the actuarial value at time $k-1$ of future health benefits related to a policyholder aged $x+k-1$ at that time, i.e.

$$B_{x+k-1}^{(k-1)} = \sum_{j=0}^{\omega-x-k+1} b_{x+k-1+j}^{(k-1)} \ (1+f)^j \ {}_jE_{x+k-1},$$

where $b_{x+k-1+j}^{(k-1)}$ is the expected health benefit in year $(k-1, k)$ for a person aged $x+k-1+j$ in the beginning of that year, based on the information available at time $k-1$, such that:

$$b_{x+k-1+j}^{(k-1)} = b_{x+k-1+j}^{(0)} \prod_{l=1}^{k-1} (1+f^{(l)}).$$

The available provision at time k for this policy is then given by:

$$V_{x+k}^{(k-1)} = \left(V_{x+k-1}^{(k-1)} + \pi_{x,0}^{(k-1)} - b_{x+k-1}^{(k-1)} \right) \left({}_1E_{x+k-1} \right)^{-1}. \tag{3.3.11}$$

Taking into account the restored actuarial equivalence (3.3.10) at time $k-1$, the available reserve $V_{x+k}^{(k-1)}$ at time k can be expressed in the following prospective form:

$$V_{x+k}^{(k-1)} = (1+f)\, B_{x+k}^{(k-1)} - \pi_{x,0}^{(k-1)} \ \ddot{a}_{x+k}, \tag{3.3.12}$$

with

$$B_{x+k}^{(k-1)} = \sum_{j=0}^{\omega-x-k} b_{x+k+j}^{(k-1)} \ (1+f)^j \ {}_jE_{x+k}.$$

Due to the observed medical inflation during the year k, the actuarial value of future health benefits $(1+f)\,B_{x+k}^{(k-1)}$ based on an evaluation at time $k-1$ has to be updated to $B_{x+k}^{(k)}$ which is given by:

$$B_{x+k}^{(k)} = (1+f^{(k)})B_{x+k}^{(k-1)},$$

under the age-uniform medical inflation. At time k, the premium level $\pi_{x,0}^{(k-1)}$ and/or the available provision $V_{x+k}^{(k-1)}$ have to be replaced by $\pi_{x,0}^{(k)}$ and $V_{x+k}^{(k)}$, respectively, in order to restore the actuarial equivalence as follows:

$$V_{x+k}^{(k)} = B_{x+k}^{(k)} - \pi_{x,0}^{(k)}\;\ddot{a}_{x+k}. \tag{3.3.13}$$

From (3.3.12) and (3.3.13), we find that for any pair $\left(V_{x+k}^{(k)}, \pi_{x,0}^{(k)}\right)$ which restores the actuarial equivalence, the updated premium $\pi_{x,0}^{(k)}$ can be expressed as:

$$\pi_{x,0}^{(k)} = \pi_{x,0}^{(k-1)} + \left(f^{(k)} - f\right)\,\pi_{x+k,k-1}^{(k-1)} - \frac{V_{x+k}^{(k)} - V_{x+k}^{(k-1)}}{\ddot{a}_{x+k}}, \tag{3.3.14}$$

where $\pi_{x+k,k-1}^{(k-1)}$ is given by:

$$\pi_{x+k,k-1}^{(k-1)} = \frac{B_{x+k}^{(k-1)}}{\ddot{a}_{x+k}}, \tag{3.3.15}$$

which is the initial level premium for a lifelong health insurance contract underwritten at time $k-1$ on a person aged $x+k$ at that time.

Assuming that the observed inflation $f^{(k)}$ is solely financed by the policyholder, i.e. $V_{x+k}^{(k)} = V_{x+k}^{(k-1)}$, the premium updating formula (3.3.14) reduces to:

$$\pi_{x,0}^{(k)} = \pi_{x,0}^{(k-1)} + \left(f^{(k)} - f\right)\,\pi_{x+k,k-1}^{(k-1)}, \tag{3.3.16}$$

of which the interpretation is similar to that of (3.3.9). The updated premium $\pi_{x,0}^{(k)}$ can also be written as:

$$\pi_{x,0}^{(k)} = \left(1 + \alpha_{x,0}^{(k)}\left(f^{(k)} - f\right)\right)\pi_{x,0}^{(k-1)}, \tag{3.3.17}$$

where

$$\alpha_{x,0}^{(k)} = \frac{\pi_{x+k,k-1}^{(k-1)}}{\pi_{x,0}^{(k-1)}}. \tag{3.3.18}$$

The proportional premium increase depends on the age x at policy issue as well as on the number k of years that the contract has been in force so far. The proportional increase of the premium will usually be larger for policies that are longer in force.

3.4 Aggregate updating mechanism for a group of new entrants

The exact premium indexing mechanism (3.3.17) that we considered so far is based on yearly restoring the actuarial equivalence at an individual level. In this section, we will present an *aggregate* premium indexing mechanism, where the yearly restoring of the actuarial equivalence is performed at aggregate level for all insureds entering the portfolio at the same time. This approach can be useful since some insurers treat the reserves in a collective way.

Let us consider a portfolio of new entrants at times 0. For each age x, let us denote by $l_{x,0}^{(k)}$ the number of policyholders who entered the portfolio at age x at time 0 and who are still in the portfolio at time $k = 0, 1, 2, \ldots$. Of course, we have that $l_{x,0}^{(k)} = 0$ for $x + k > \omega$.

3.4.1 Adapting the premium level at time 1

Suppose that at time 1, the equivalence between available provision and required provision is restored on an aggregate level, i.e. we replace the individual equivalence relation $V_{x+1}^{(0)} = V_{x+1}^{(1)}$ by the following aggregate equivalence relation:

$$\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ V_{x+1}^{(0)} = \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ V_{x+1}^{(1)}, \tag{3.4.1}$$

where x_0 is the youngest age in the portfolio. Taking into account (3.3.1) and (3.3.5), we find that (3.4.1) can be rewritten as

$$\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ \pi_{x,0}^{(1)}\ \ddot{a}_{x+1} = \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ \pi_{x,0}^{(0)}\ \ddot{a}_{x+1} + \left(f^{(1)} - f\right) \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ B_{x+1}^{(0)}. \tag{3.4.2}$$

An infinite number of premiums $\pi_{x_0,0}^{(1)}$, $\pi_{x_0+1,0}^{(1)}, \ldots,$ $\pi_{\omega-1,0}^{(1)}$ can satisfy this aggregate equivalence condition. In order to specify the new tariff, we assume now that at time 1, the premium of each new entrants at time 0 is adapted by the same factor $\alpha_{ne,0}^{(1)}$, namely

$$\pi_{x,0}^{(1)} = \Big(1 + \alpha_{ne,0}^{(1)}(f^{(1)} - f)\Big)\pi_{x,0}^{(0)}, \qquad x = x_0,\ x_0+1, \ldots, \omega - 1. \tag{3.4.3}$$

Inserting these expressions for the $\pi_{x,0}^{(1)}$'s in relation (3.4.2) leads to:

$$\alpha_{ne,0}^{(1)} = \frac{\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ B_{x+1}^{(0)}}{\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ \pi_{x,0}^{(0)}\ \ddot{a}_{x+1}},$$

which can be transformed using (3.2.3) into:

$$\alpha_{ne,0}^{(1)} = \frac{\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ \pi_{x+1,0}^{(0)}\ \ddot{a}_{x+1}}{\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)}\ \pi_{x,0}^{(0)}\ \ddot{a}_{x+1}}.$$

The updating factor $\alpha_{ne,0}^{(1)}$ is applied at time 1 to the deviation $f^{(1)} - f$ in order to adjust the premiums of all policies that were underwritten at time 0, regardless of the age of the new entrants. Typically, the numerator exceeds the denominator, so that $\alpha_{ne,0}^{(1)} > 1$, and for $f^{(1)} \geq f$, all premiums $\pi_{x,0}^{(1)}$ are increased by the factor $\alpha_{ne,0}^{(1)}(f^{(1)} - f)$, which is larger than the difference between experienced and assumed inflations.

3.4.2 Adapting the premium level at time k

Let us suppose that we have arrived at time k and that at any times $1, 2, \ldots, k-1$, we have reset premiums on an aggregate level according to a similar procedure as the one performed at time 1. The time-k aggregate equivalence relation between available and required provisions can now be expressed as follows:

$$\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)}\ V_{x+k}^{(k-1)} = \sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)}\ V_{x+k}^{(k)}. \tag{3.4.4}$$

This means that the available provision at time k, aggregated over all policies that were underwritten at time 0 which are still in force at time k, is set equal to the required aggregate provision for this same set of policies. From (3.3.12) and (3.3.13), the equivalence relation (3.4.4) can be restated as follows:

$$\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)}\ \pi_{x,0}^{(k)}\ \ddot{a}_{x+k} = \sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)}\ \pi_{x,0}^{(k-1)}\ \ddot{a}_{x+k} + \Big(f^{(k)} - f\Big) \sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)}\ B_{x+k}^{(k-1)}. \tag{3.4.5}$$

Let us now again assume a uniform updating mechanism for all policies under consideration, i.e. the premiums $\pi_{x,0}^{(k)}$ are determined via

$$\pi_{x,0}^{(k)} = \left(1 + \alpha_{ne,0}^{(k)} \left(f^{(k)} - f\right)\right)\pi_{x,0}^{(k-1)}.$$

Inserting these expressions in (3.4.5) leads to

$$\alpha_{ne,0}^{(k)} = \frac{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \; B_{x+k}^{(k-1)}}{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \; \pi_{x,0}^{(k-1)} \; \ddot{a}_{x+k}},$$

or, taking into account (3.3.15), we find that:

$$\alpha_{ne,0}^{(k)} = \frac{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \; \pi_{x+k,k-1}^{(k-1)} \; \ddot{a}_{x+k}}{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \; \pi_{x,0}^{(k-1)} \; \ddot{a}_{x+k}} \tag{3.4.6}$$

for the updating factor $\alpha_{ne,0}^{(k)}$ that is applied to premiums of all policies that were underwritten at time 0 and are still in force at time k.

3.5 Aggregate updating mechanism for an existing portfolio

In this section, we consider an aggregate premium indexing mechanism at portfolio level. Each year, the actuarial equivalence is restored by imposing an equality between available and required reserves for the whole existing portfolio at that moment. The related proportional increase of the premiums is chosen to be equal for all members of the portfolio at that moment. Hereafter, we will denote by $l_{x,j}^{(k)}$ the number of policyholders observed in the portfolio at time k, who entered that portfolio at age x at time $j \leq k$. At time k, these policyholders have reached age $x + k - j$. Obviously, we have $l_{x,j}^{(k)} = 0$ for $x > \omega - k + j$.

3.5.1 Adapting the premium level at time 1

Suppose that we have arrived at time 1. According to the aggregate premium indexing mechanism at portfolio level, the premiums $\pi_{x,j}^{(1)}$ are chosen such that the available and the required aggregate provisions are equal:

$$\sum_{j\le 0}\sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\left((1+f)\,B_{x+1-j}^{(0)}-\pi_{x,j}^{(0)}\,\ddot{a}_{x+1-j}\right)$$
$$=\sum_{j\le 0}\sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\left((1+f^{(1)})B_{x+1-j}^{(0)}-\pi_{x,j}^{(1)}\,\ddot{a}_{x+1-j}\right). \qquad (3.5.1)$$

We impose a uniform updating mechanism for all insureds in the portfolio at time 1. This means that the premiums $\pi_{x,j}^{(1)}$ satisfy

$$\pi_{x,j}^{(1)}=\left(1+\alpha_{ptf}^{(1)}(f^{(1)}-f)\right)\pi_{x,j}^{(0)} \qquad (3.5.2)$$

for an aggregate factor $\alpha_{ptf}^{(1)}$. Inserting (3.5.2) in equation (3.5.1) leads to the following expression for the updating factor $\alpha_{ptf}^{(1)}$:

$$\alpha_{ptf}^{(1)}=\frac{\sum_{j\le 0}\ \sum_{x=x_0}^{\omega-1+j}\ l_{x,j}^{(1)}\ B_{x+1-j}^{(0)}}{\sum_{j\le 0}\ \sum_{x=x_0}^{\omega-1+j}\ l_{x,j}^{(1)}\ \pi_{x,j}^{(0)}\ \ddot{a}_{x+1-j}}.$$

Taking into account that $B_{x+1-j}^{(0)}=\pi_{x+1-j,0}^{(0)}\ \ddot{a}_{x+1-j}$, we can rewrite the previous expression in terms of the premium structure at time 0:

$$\alpha_{ptf}^{(1)}=\frac{\sum_{j\le 0}\ \sum_{x=x_0}^{\omega-1+j}\ l_{x,j}^{(1)}\ \pi_{x+1-j,0}^{(0)}\ \ddot{a}_{x+1-j}}{\sum_{j\le 0}\ \sum_{x=x_0}^{\omega-1+j}\ l_{x,j}^{(1)}\ \pi_{x,j}^{(0)}\ \ddot{a}_{x+1-j}}. \qquad (3.5.3)$$

3.5.2 Adapting the premium level at time k

Suppose that we have arrived at time k and that we have restored the actuarial equivalence at times $1,2,\ldots,k-1$ on an aggregate portfolio level, applying a procedure similar to the one applied at time 1. This has lead to the aggregate updating factors $\alpha_{ptf}^{(1)},\alpha_{ptf}^{(2)},\ldots,\alpha_{ptf}^{(k-1)}$. Now, having arrived at time k, the updated premiums $\pi_{x,j}^{(k)}$ are chosen such that the available and the required aggregate provisions for the whole portfolio are again equal at time k:

$$\sum_{j\le k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\left((1+f)\ B_{x+k-j}^{(k-1)}-\pi_{x,j}^{(k-1)}\ \ddot{a}_{x+k-j}\right)$$
$$=\sum_{j\le k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\ \left((1+f^{(k)})\ B_{x+k-j}^{(k-1)}-\pi_{x,j}^{(k)}\ \ddot{a}_{x+k-j}\right).$$

Assuming a uniform updating factor $\alpha_{ptf}^{(k)}$ for the premiums, i.e.

$$\pi_{x,j}^{(k)} = \left(1 + \alpha_{ptf}^{(k)}(f^{(k)} - f)\right) \pi_{x,j}^{(k-1)}, \tag{3.5.4}$$

for all j and k, the equivalence relation above leads to

$$\alpha_{ptf}^{(k)} = \frac{\sum_{j \leq k-1} \ \sum_{x=x_0}^{\omega-k+j} \ l_{x,j}^{(k)} \ B_{x+k-j}^{(k-1)}}{\sum_{j \leq k-1} \ \sum_{x=x_0}^{\omega-k+j} \ l_{x,j}^{(k)} \ \pi_{x,j}^{(k-1)} \ \ddot{a}_{x+k-j}},$$

or equivalently, taking into account that $B_{x+k-j}^{(k-1)} = \pi_{x+k-j,k-1}^{(k-1)} \ \ddot{a}_{x+k-j}$, we find:

$$\alpha_{ptf}^{(k)} = \frac{\sum_{j \leq k-1} \ \sum_{x=x_0}^{\omega-k+j} \ l_{x,j}^{(k)} \ \pi_{x+k-j,k-1}^{(k-1)} \ \ddot{a}_{x+k-j}}{\sum_{j \leq k-1} \ \sum_{x=x_0}^{\omega-k+j} \ l_{x,j}^{(k)} \ \pi_{x,j}^{(k-1)} \ \ddot{a}_{x+k-j}}. \tag{3.5.5}$$

3.6 Case study: Assessing the Belgian '1.5 rule'

3.6.1 Indexing rule imposed by the Belgian law

The *actuarially fair* updating mechanisms derived in Sections 3.3, 3.4 and 3.5 provide the following indexing rule for the remaining premiums at time k of a policyholder underwriting the contract at aged x:

$$\pi_{x,0}^{(k)} = \left(1 + \alpha^{(k)} \left(f^{(k)} - f\right)\right) \pi_{x,0}^{(k-1)}. \tag{3.6.1}$$

In case the adjustment is performed at the individual level, we have that $\alpha^{(k)} = \alpha_{x,0}^{(k)}$, where $\alpha_{x,0}^{(k)}$ is given in (3.3.18). This updating factor depends on the age of the policyholder and the seniority of the contract. In case the adjustment is uniform across all ages of new entrants at time 0, we have that $\alpha^{(k)} = \alpha_{ne,0}^{(k)}$, which is given in (3.4.6). Then, the updating factor depends on the age distribution of new entrants and on the seniority of the contracts. In case the adjustment is uniform across all contracts in force at the time this adjustment is performed, we have that $\alpha^{(k)} = \alpha_{ptf}^{(k)}$, which is given in (3.5.5). The updating factor depends on the age distributions and the number of new entrants at any time prior the updating.

The Belgian Royal Decree dated March 18, 2016 provides the following updating mechanism:

$$\pi_{x,0}^{(k)} \leq \left(1 + \alpha_{BE}^{(k)} \ f^{(k)}\right) \pi_{x,0}^{(k-1)}, \tag{3.6.2}$$

where
$$\alpha_{BE}^{(k)} f^{(k)} = \min\left\{1.5 f^{(\bar{k})}, f^{(\bar{k})} + 2\%\right\},$$
and $f^{(\bar{k})}$ is the latest released non-negative medical index. Thus, Belgian private insurers can adjust the premium with the medical inflation index $f^{(k)}$ observed during the year k, scaled with an updating factor 1.5. The premium increase resulting from this indexation cannot exceed $f^{(k)} + 2\%$. For instance, if the observed medical inflation is equal to 5%, the insurer has to limit the increase to 7%, instead of $1.5 \times 5\% = 7.5\%$. In order to compensate for the limit $f^{(k)} + 2\%$, insurers are allowed to use the latest non-negative medical inflation index if the index for year k is negative.

The updating rule proposed by the Belgian legislator differs from the actuarially fair updating rule described above. One of the differences is that the Belgian updating mechanism takes into account the medical index $f^{(k)}$, and not the deviation between the assumed and realized medical indices, i.e. $f^{(k)} - f$. This implies that if the insurer has already included medical inflation in the tariff, the Belgian updating rule still allows for an adjustment. For instance, if the insurer assumes $f = 2\%$ and observes $f^{(k)} = 2\%$, future premiums can be updated by 3% according to the Belgian rule. Clearly, the updating mechanisms described in this chapter do not allow for an adjustment in case $f^{(k)} = f$. Another difference is that unlike the proposed updating rules, the Belgian system does not allow for premium decreases, even in the case of negative observed medical inflation.

In this section, we focus on a third difference between the proposed and the Belgian updating mechanisms. In particular, we analyze the approximation $\alpha^{(k)} \approx 1.5$ for all $k = 1, 2, ...$, and we do not consider other restrictions which apply only to very special cases. Since the Belgian Law provides with the '1.5 rule' an upper bound for premium adjustments, we mainly adopt the insurer's point of view and examine whether this rule may threaten its solvency.

We assume in this numerical analysis that the time-0 estimate of level premiums does not include medical inflation, i.e. we always take $f = 0$. Henceforth, premiums calculated according to (3.6.2) are called the '1.5 rule premiums' and denoted by $\pi_{x,0}^{(k)}(150\%)$. We compare these premiums with the actuarially fair premiums, which follow from (3.3.17):
$$\pi_{x,0}^{(k)} = \left(1 + \alpha^{(k)}\ f^{(k)}\right) \pi_{x,0}^{(k-1)}, \tag{3.6.3}$$
for some specification of $\alpha^{(k)}$, i.e. contract-per-contract, new entrants or existing portfolio. We sometimes denote the actuarially fair premiums (3.6.3)

by $\pi_{x,0}^{(k)}$ (exact), in order to distinguish them from the premiums derived from the '1.5 rule'.

3.6.2 Technical basis

General assumptions

The assumed discount factors follow from a constant yearly interest rate $i = 1\%$. Mortality is assumed to obey the first Heligman-Pollard law, in the sense that the 'independent mortality rates' for ages $x = 25, 26, \ldots, 109$ are given by:

$$\frac{q_x}{1-q_x} = A^{(y+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x, \tag{3.6.4}$$

with $A = 0.00054$, $B = 0.017$, $C = 0.101$, $D = 0.00013$, $E = 10.72$, $F = 18.67$, $G = 1.464 \times 10^{-5}$ and $H = 1.11$. Furthermore, we fix the ultimate age to $\omega = 110$. For a justification of this mortality law, we refer to Pitacco (1999) and Vercruysse et al. (2013).

The corresponding 'dependent mortality rates' p_x^{ad} in the two-decrement model satisfy the relation:

$$p_x^{ad} = q_x \left(1 - \frac{p_x^{aw}}{2-q_x}\right),$$

which holds under the assumption of a uniform distribution of decrements in any year for each of the two single decrement models involved, see Section 8.10.2 in Dickson et al. (2013). The 'dependent lapse rates' p_x^{aw} are assumed to be given by:

$$p_x^{aw} = 0.1 - 0.002(x-20),$$

at age $x = 25, 26, \ldots, 70$ and 0 otherwise. The sojourn probability p_x at age x follows from:

$$p_x = 1 - p_x^{aw} - p_x^{ad}.$$

The severity of medical claims is based on age-specific annual claim amounts including an accident-childbearing hump and a concave behavior near the end of the lifetime. The data have been normalized to fit the annual expected hospitalization costs provided by the Belgian *Mutualité Chrétienne*; see Figure 1.1.1. The minimum age to underwrite the contract is assumed to be 25.

As mentioned above, no medical inflation is taken into account when setting level premiums, i.e. $f = 0$. In Figure 3.6.1, the resulting insurer's tariff $\pi_{x,0}^{(0)}$, for $x = 25, 26, ..., 109$, is depicted. We observe that the accident-childbearing hump and the concave behavior for higher ages visible on Figure 1.1.1 are impacting the tariff structure, causing the break right before age 40.

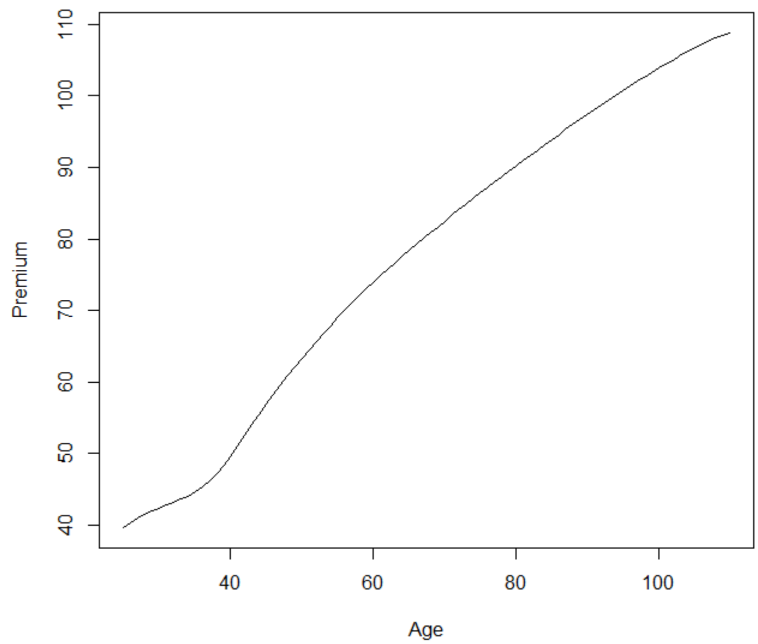


Figure 3.6.1: Tariff at policy issue: $\pi_{x,0}^{(0)}$ for $x \in \{25, 26, ..., 109\}$.

Assumptions for aggregate portfolios

In the calculation of the updating factor $\alpha_{ne,0}^{(k)}$, which is uniform across new entrants, some further assumptions are required on the composition of the portfolio. We consider two different portfolio compositions, see Figure 3.6.2. Portfolio 1 has a concentration of young new entrants at time 0, which is likely to be the case given the non-transferability of the reserves in the Belgian context. In Portfolio 2, new entrants are uniformly distributed over the ages 20 to 50, followed by a decreasing number of new entrants up to age 55. The distributions of ages in these two portfolios are displayed in Figure 3.6.2. The number of new entrants in each portfolio at time 0 does not impact the updating factor.

Additional assumptions are used in the calculation of the updating factor $\alpha_{ptf}^{(k)}$ at the level of an existing portfolio. We consider two sets of portfo-

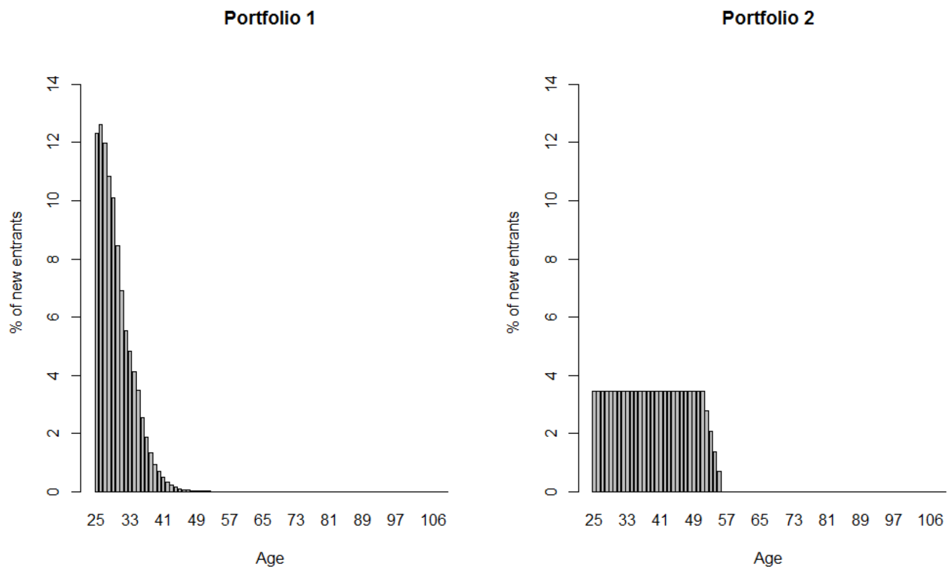


Figure 3.6.2: Relative numbers of new entrants $l_{x,0}^{(0)}$ as a function of age x.

lios, and we assume that they have all existed for 20 years, i.e. since time -20. Within each set of portfolios, we suppose constant proportions of new entrants over time, and in any year these proportions are as described in Figure 3.6.2. Past observed inflation, i.e. the inflation in the years before time 0, is taken equal to 2%. In each set of portfolios, we consider three scenarios for the absolute number of new entrants in any year. In the first scenario, we assume a stable number of 10 000 new entrants per year. In the second scenario, we assume a linearly increasing number of new entrants over time, from 10 000 at time -20 to 100 000 at time 65. In the third scenario, we assume a linearly decreasing number of new entrants over time, from 100 000 at time -20 to 10 000 at time 65.

In what follows, we analyze the '1.5 rule' under different assumptions, and conduct sensitivity analysis for further insights. All numerical computations are performed using `R` software (R Core Team, 2018).

3.6.3 Assessing the '1.5 rule' in the base case scenario

In the remainder of this section, we investigate the evolution over time of the updating factor $\alpha^{(k)}$, as well as the corresponding successive premiums to be paid by policyholders aged 25, 35 and 50 at policy issue. For the evolution over time, we assume that observed medical inflation is $f^{(k)} = 2\%$ for all $k = 1, 2, \ldots$.

We start by looking at the evolution of the updating factor. In order to provide a clearer comparison between the three methods (i.e. contract-per-contract, new entrants and existing portfolios), we report their results for the base case scenario in two figures. Figure 3.6.3 displays the evolution over the time index k of the individual updating factors $\alpha_{x,0}^{(k)}$ for ages $x = 25, 35$ and 50, as well as the aggregate updating factors for new entrants for Portfolio 1 and Portfolio 2. Figure 3.6.4 displays the evolution over k of the updating factors $\alpha_{ptf}^{(k)}$ determined at the level of the existing portfolios, for each assumption on the distribution over time of new entrants and their numbers.

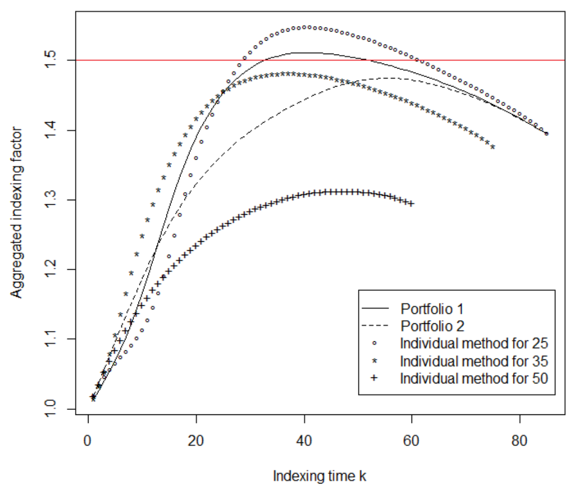


Figure 3.6.3: Individual indexing factor $\alpha_{25,0}^{(k)}$, $\alpha_{35,0}^{(k)}$ and $\alpha_{50,0}^{(k)}$, as well as aggregate indexing factors $\alpha_{ne,0}^{(k)}$ for two portfolios.

We observe from Figure 3.6.3 that the updating factors $\alpha_{x,0}^{(k)}$ and $\alpha_{ne,0}^{(k)}$ as a function of k have similar patterns. In particular, these factors are initially close to 1, and then increase until they reach a maximum value, after which the factors start to decrease. For the updating factor $\alpha_{ptf}^{(k)}$, Figure 3.6.4 suggests a slightly different behavior. This comes from the influence of the number of new entrants in each year. For instance, when this number is stable, we observe that the updating factor becomes constant after some years. Another reason explaining these stable patterns comes from the time-independent sojourn probabilities. One observation on the behavior of $\alpha_{ptf}^{(k)}$

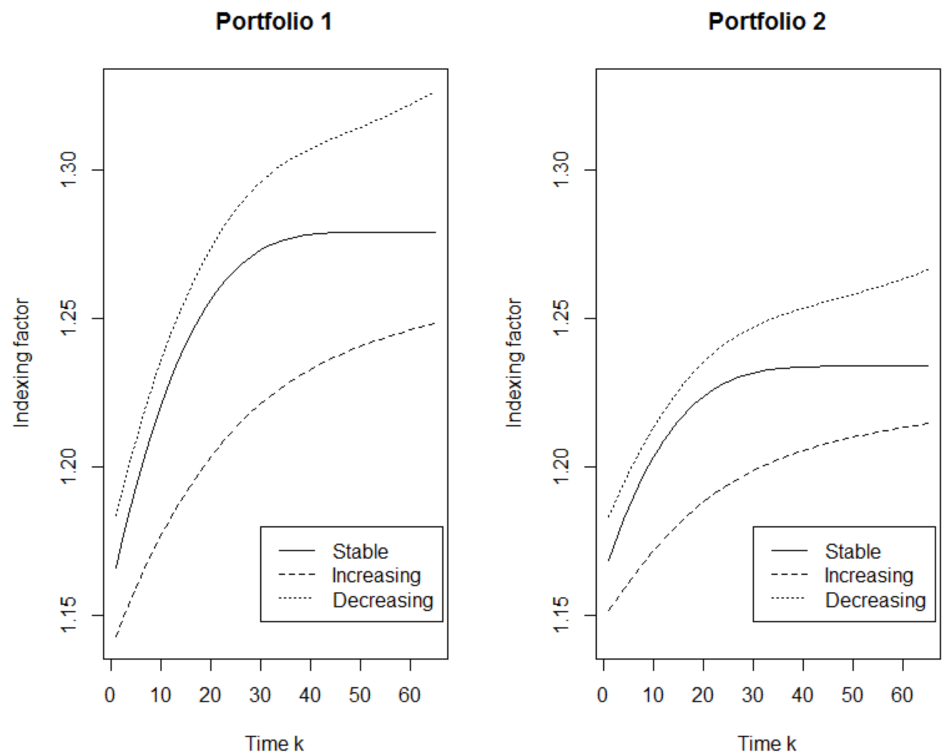


Figure 3.6.4: Aggregate indexing factors $\alpha_{ptf}^{(k)}$ at portfolio level for two portfolios and three assumptions on the number of new entrants.

is that an increasing absolute number of entrants leads to lower indexing factors than a decreasing absolute number. This behavior is meaningful, since in the former case, medical inflation is risk is spread over a greater number of policyholders every year.

In general, the 1.5 approximation appears to be very conservative during periods where the available reserves are relatively small. We observe on Figure 3.6.3 for young new entrants (i.e. aged 25 at policy issue) as well as for portfolios with high concentrations of young new entrants (i.e. Portfolio 1) that the updating factor becomes slightly larger than 1.5 during some years. However, these intermediate periods are more than compensated in the initial period, where the exact updating factors are substantially lower than 1.5.

These two figures provide also insights on the effect of age on the updating factor. We observe on Figure 3.6.3 that $\alpha_{25,0}^{(k)}$ is initially smaller than both

$\alpha_{35,0}^{(k)}$ and $\alpha_{50,0}^{(k)}$, but later, and for most of the time, becomes greater. This implies that for two policyholders starting their contracts at the same time, the younger policyholder is likely to experience higher adjustments. This has a direct consequence on the value of the updating factor at the level of new entrants. Indeed, we clearly observe that aggregating the reserves for new entrants leads to higher values of the updating factor when the concentration of young new entrants is high. In other words, elderly new entrants are likely to pay for the adjustments of young new entrants. We conclude in this particular numerical illustration that younger entrants are better off with the aggregate method than older ones, unless the percentage of elderly new entrants is sufficiently high.

Figure 3.6.3 and Figure 3.6.4 provide information on the updating factors, but not necessarily on the actual adjustments of the premiums. In order to investigate how the adjusted premiums evolve over time, we shift the focus toward a set of three figures.

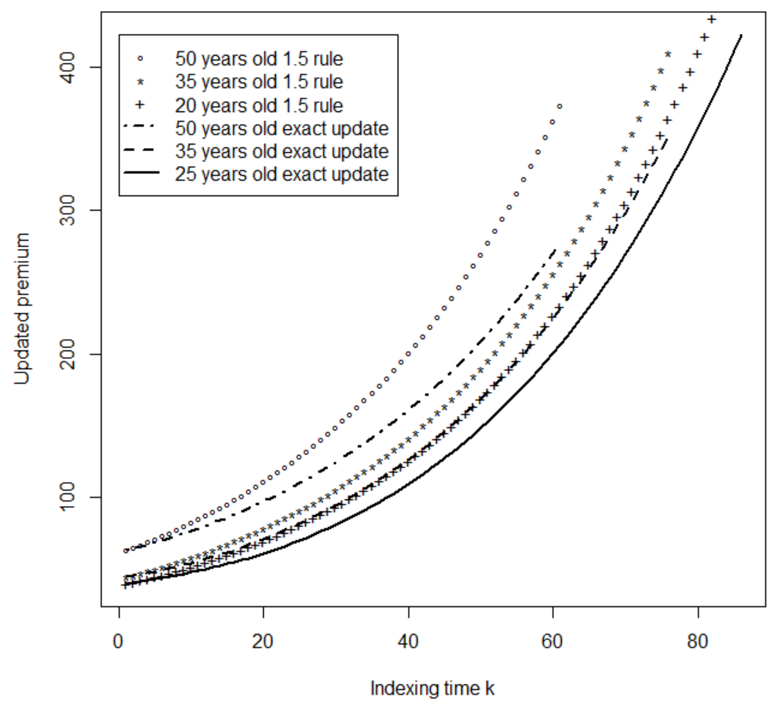


Figure 3.6.5: $\pi_{x,0}^{(k)}$ (exact) and $\pi_{x,0}^{(k)}$ (150%) for $x = 25, 35$ and 50 as a function of k.

First, Figure 3.6.5 displays $\pi_{x,0}^{(k)}$ (exact) for $x = 25, 35$ and 50 obtained from

$\alpha_{x,0}^{(k)}$, with a comparison with their corresponding $\pi_{x,0}^{(k)}\,(150\%)$ from the 1.5 rule. Clearly, the '1.5 rule premiums' are uniformly larger than their corresponding exact premiums, confirming that the 1.5 approximation is a conservative rule in these three cases.

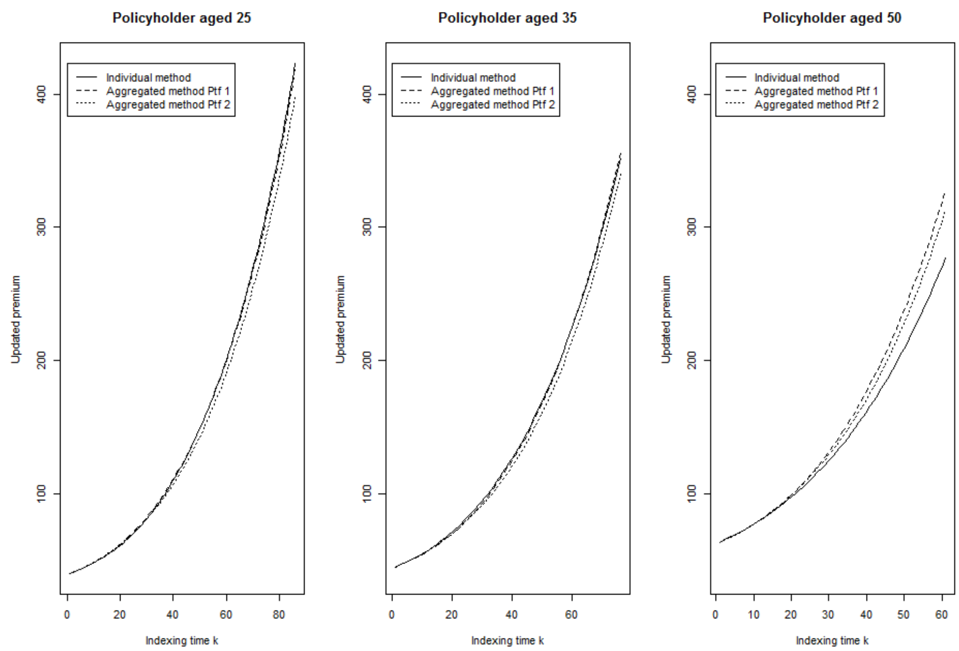


Figure 3.6.6: Updated premiums $\pi_{x,0}^{(k)}$ (individual) and $\pi_{x,0}^{(k)}$ (aggregate) for different ages and groups of new entrants.

Second, Figure 3.6.6 displays the individual updated premium curve as well as the corresponding aggregate updated premium curves from each portfolio of new entrants, for ages $x = 25, 35$ and 50. The main observations is that for new entrants aged 25, we find that

$$\pi_{25,0}^{(k)}\,(\text{individual}) \approx \pi_{25,0}^{(k)}\,(\text{portfolio 1}) > \pi_{25,0}^{(k)}\,(\text{portfolio 2})\,,$$

whereas for new entrants aged 50, we find that

$$\pi_{50,0}^{(k)}\,(\text{individual}) < \pi_{50,0}^{(k)}\,(\text{portfolio 2}) < \pi_{50,0}^{(k)}\,(\text{portfolio 1})\,.$$

This confirms that for young entrants ($x = 25$), the aggregate indexing method is to be preferred, in particular when there is a substantial number of older new entrants. On the other hand, for older policyholders ($x = 50$), the individual indexing mechanism always leads to lower premiums. The implication of these findings on the Belgian regulation will be discussed in the following Chapter 4 of this thesis.

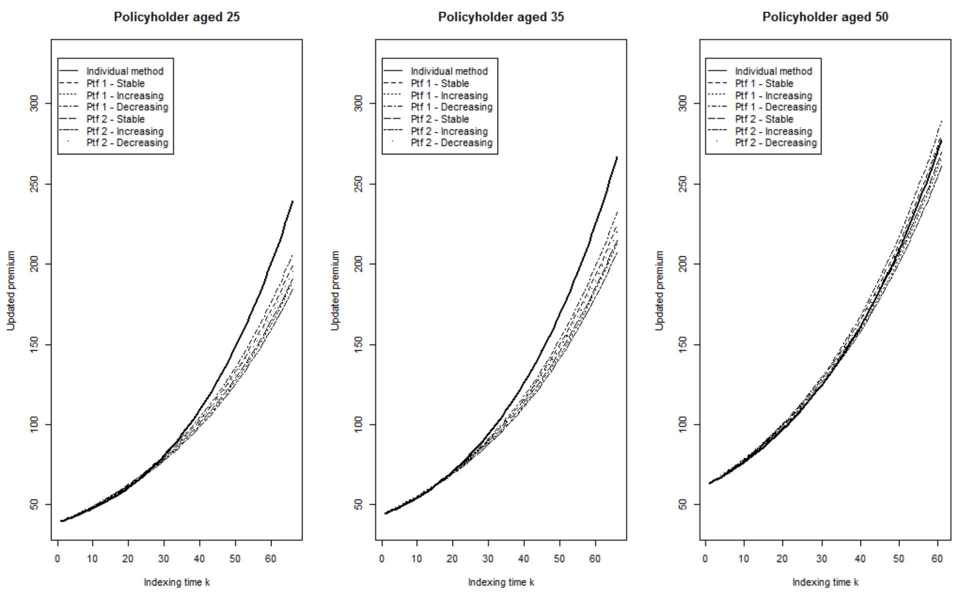


Figure 3.6.7: Updated premiums $\pi_{x,0}^{(k)}$ (individual) and $\pi_{x,0}^{(k)}$ (aggregate) for different ages and existing portfolios.

Third, Figure 3.6.7 displays the exact adjusted premiums obtained from the aggregate method for the existing portfolios, compared to the exact adjusted premiums from the individual indexing factors. For policyholders aged 25 and 35 at policy issue, any of the aggregate indexing methods leads to lower premiums than the individual indexing method. For policyholders aged 50 at policy issue, the lowest premiums are obtained when the number of new entrants is increasing with an age distribution of Portfolio 2, while the highest premiums occur when the number of new entrants is decreasing following the age distribution of Portfolio 1.

3.6.4 Sensitivity analysis

In this subsection, we perform a sensitivity analysis by varying the constant interest rate and observed future medical index level. We pursue the analysis with the individual indexing mechanism for a policyholder aged 25 at policy issue.

Let us first consider the sensitivity of the updating mechanism with respect to the interest rate. Recall that in the base case scenario, we assume a constant interest rate equal to $i = 1\%$. Now, we consider two additional scenarios, namely $i = 0.5\%$ and $i = 5\%$. In Figure 3.6.8 we depict the behavior of the exact indexing factor $\alpha_{25,0}^{(k)}$ for the three interest rate scenarios considered. Overall, the indexing factor increases with i. The 1.5 rule (3.6.2) seems to be more conservative in a context of low interest rates. Figure 3.6.9 displays the ratio $\frac{\pi_{25,0}^{(k)}(150\%)}{\pi_{25,0}^{(k)}(\text{exact})}$ for the three different interest rate assumptions. The ratio is always greater than 1, meaning that the 1.5 rule is conservative in all considered cases.

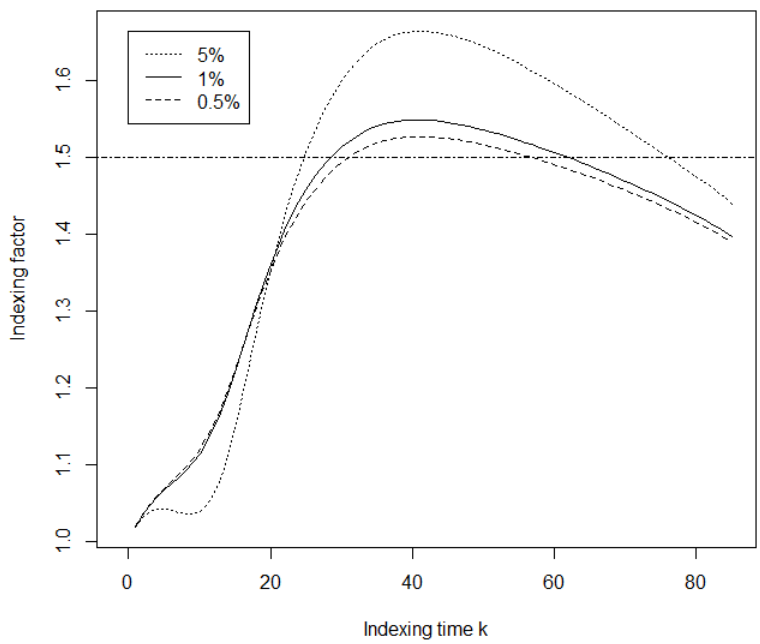


Figure 3.6.8: Indexing factor $\alpha_{25,0}^{(k)}$ for different technical interest rates.

Let us now examine the effect of different assumptions of future medical inflation on the updating mechanism. Returning to the base case for the interest rate, i.e. $i = 1\%$, we vary the observed medical inflation according to the following scenarios: either $f^{(k)} = 0.5\%$ for all $k = 1, 2, \ldots$, or $f^{(k)} = 3\%$ for all $k = 1, 2, \ldots$.

Figure 3.6.10 shows the evolution of the indexing factors $\alpha_{25,0}^{(k)}$ for the two different medical inflation scenarios considered. We compare in Figure 3.6.11

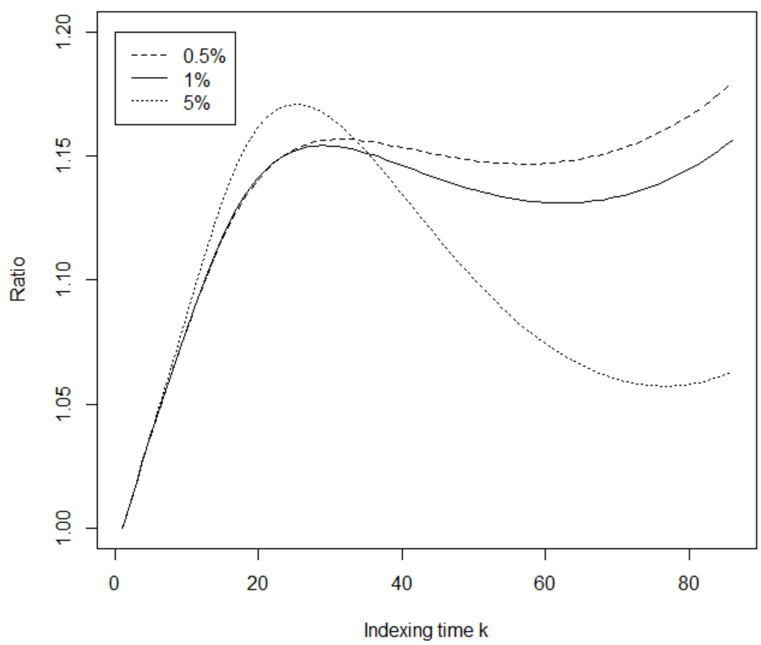


Figure 3.6.9: Ratio $\frac{\pi_{25,0}^{(k)}(150\%)}{\pi_{25,0}^{(k)}(\text{exact})}$ for three different technical interest rate assumptions.

the corresponding premiums $\pi^{(k)}_{25,0}$ (exact) and $\pi^{(k)}_{25,0}$ (150%). We observe that the '1.5 rule' is more conservative for higher medical inflation levels. Indeed, we can see in Figure 3.6.10 that for an observed inflation of 0.5%, the 1.5 approximation is under-estimating the exact update after the first 25 years of the coverage.

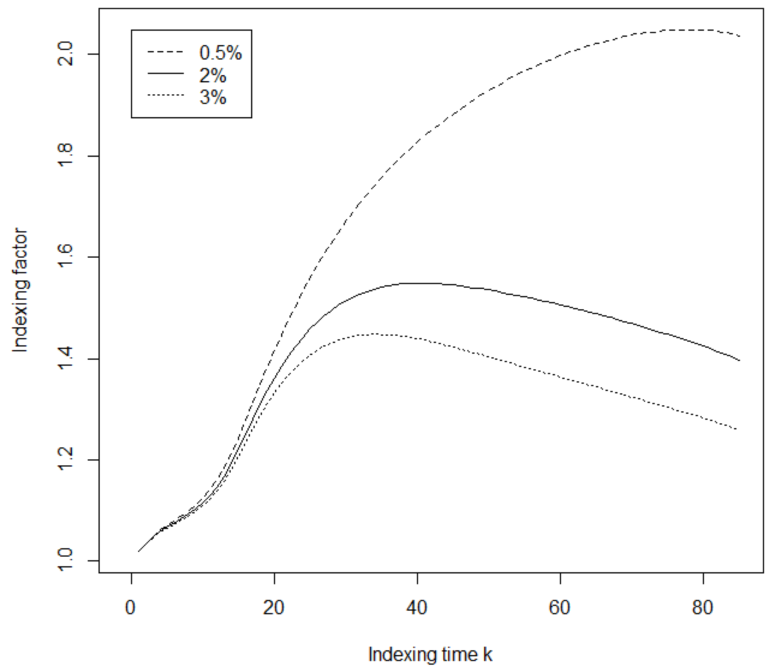


Figure 3.6.10: Updating factor $\alpha^{(k)}_{25,0}$ as a function of k, for different inflation scenarios.

3.7 Concluding remarks

In this chapter, we investigate how to derive updating mechanisms for lifelong health insurance contracts at three levels, namely, at the level of the individual contract, at the level of new entrants, at the level an existing portfolio. Besides the influence of the technical basis, we find that the required proportional increase of the premium depends on the difference between observed and assumed medical inflation in the previous year, and also on whether the adjustments are performed at an individual or aggregate level. Adjustments at the individual level imply that the required proportional increases depend on the age at policy issue and on the time since policy issue,

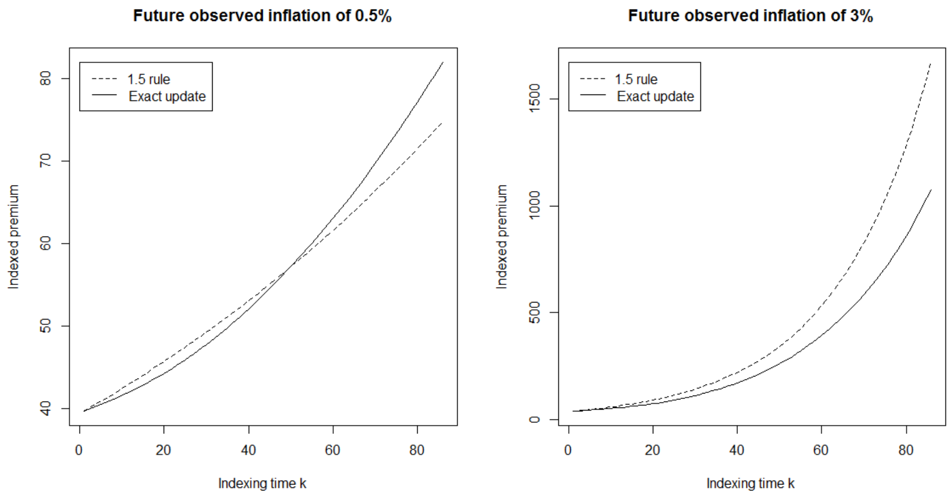


Figure 3.6.11: Premiums $\pi_{25,0}^{(k)}$ (exact) and $\pi_{25,0}^{(k)}$ (150%) as a function of k, for the two different medical inflation scenarios.

whereas adjustments at the aggregate level lead to required proportional increases which depend on the age-distribution in the portfolio.

Our assessment of the '1.5 rule' adopted in the Belgian system suggests that this approximation can be conservative for insurers, but not always. Moreover, the dependency phenomena that we observe for the 'exact' updating mechanism might not be easy to explain to consumers. Thus, there are understandable reasons to adopt such mechanism. Nevertheless, since future economic scenarios cannot be modeled in an accurate way, Belgian private health insurance providers may face situations in the long-term, when the approximation $\alpha^{(k)} \approx 1.5$ is not sufficient. For instance, this could be the case if observed medical inflation is persistently low. In this regard, applying an indexing mechanism that causes less opposition from consumers but is at the same time less sound is highly questionable, in particular in case of lifelong contracts.

One important remark is that the updating mechanism described in this chapter does not completely eliminate the risk of the insurer. Indeed, since the adjustments are implemented ex-post, the insurer remains exposed to the systematic risk before these adjustments. Nevertheless, compared to the case without adjustment, here medical inflation risk is transformed from a long-term to a short-term risk, which allows for a significant enhancement of the insurer's solvency situation. The impact of contract adjustments over time on the solvency of the insurer will be discussed in Chapter 5.

Throughout this chapter, we made the assumption that in any year $k = 1, 2, \ldots$, observed medical inflation is uniform over all ages, i.e.

$$b_{x+j}^{(k)} = (1 + f^{(k)})\, b_{x+j}^{(k-1)}, \qquad j = 1, 2, \ldots,$$

for some age-independent inflation index $f^{(k)}$. This assumption was useful in order to isolate the effect of age coming from the updating factor. Notice however that the results presented here can in a straightforward way be adapted to the case of age-specific medical inflation by replacing the inflation factor $f^{(k)}$ in the formula above by an age-dependent factor, such that:

$$b_{x+j}^{(k)} = (1 + f_{x+j}^{(k)})\, b_{x+j}^{(k-1)}, \qquad j = 1, 2, \ldots$$

Once the age-specific inflation factors $f_{x+j}^{(k)}$ have been set, we can determine the global inflation factors $\overline{f}_{x+k}^{(k)}$ from

$$B_{x+k}^{(k)} = (1 + \overline{f}_{x+k}^{(k)})\, B_{x+k}^{(k-1)}.$$

In this setting, the general premium updating rule (3.6.1) has to be replaced by

$$\pi_{x,0}^{(k)} = \left(1 + \left(\overline{f}_{x+k}^{(k)} - f\right) \alpha^{(k)}\right) \pi_{x,0}^{(k-1)},$$

which is similar to the general updating rule under the assumption of age-independent medical inflation.

CHAPTER 4

Further thoughts on the Belgian updating mechanism

4.1 Introduction

In Belgium, the introduction of a medical inflation index and an updating mechanism for lifelong private health insurance is a compromise between insurers and policyholders. On the one hand, insurers can cope with the unpredictable future evolution of health claims due to medical inflation by transferring this risk back to policyholders. On the other hand, policyholders are guaranteed a lifelong cover and protected from sudden and sharp increases that could have happened before the introduction of the Law of 20 July 2007. It should however be noted that this latter protection affects increases only to some extent. As mentioned in the introductory Chapter 1, insurers are also allowed to adjust their premiums upon approval of the regulating authority. For instance, the Belgian DKV increased in 2017 its level premiums by 9%, on top of the medical index.[1] Thus, whether the current Belgian system really protects policyholders against sudden and sharp increases is subject to debate, and is out of the scope of the present chapter.

Throughout this first part of the thesis, we have discussed different challenges arising under this new constraint imposed by the Belgian Law. Medical inflation is at the core of these challenges. The risk that its unpredictable

[1] http://www.standaard.be/cnt/dmf20170816_03019772.

nature poses was taken into account by the legislator at two levels. First, by introducing medical inflation indices, whose construction was the topic of Chapter 2. Second, by proposing an updating mechanism allowing insurers to adjust premiums over time. Chapter 3 was devoted to the analysis of the updating mechanism provided by the legislator in the Royal Decree of 18 March 2016. In the present chapter, we conclude this first part with a discussion on some remaining issues in the current Belgian system, based on the work of Hanbali et al. (2019a).

The main difference between the Royal Decree of 18 March 2018 and its first version of 1 February 2010 is that now, insurers are allowed to adjust future premiums taking into account the impact of medical inflation on both premiums and reserves. Nevertheless, for the current version, several concerns expressed about the first Royal Decree by the representatives of both parties in the appointed insurance committee remain valid[2]. Some of these are related to the principle of pricing freedom and the data collection methodology. However, there are two other issues in the current Belgian system which we deem to be of major importance in the debate about the coherence between the initial goal of the Law and the practical constraints of the actuarial techniques.

The first issue is discussed in the following Section 4.2, and comes from the technical analogy between Belgian health insurance contracts and life insurance contracts. In particular, this analogy raises new challenges related to the transferability of the reserves. The second issue, which is the topic of Section 4.3, is related to the initial and main purpose of the Law of 20 July 2007 advocating a system protecting policyholders against age discrimination in health insurance by ensuring affordable covers to all ages. Moreover, we argue that these two issues are interrelated, which implies that reconciling the initial goal of the Belgian Law and its actuarial implications is not straightforward under the current system.

The present chapter does not provide a quantitative analysis of the Belgian system for premium adjustments. Moreover, we do not discuss possible conflicts between the Laws governing the Belgian system for private health

[2] Avis de la Commission du 09/09/2015 sur le projet d'Arrêté Royal portant modification de l'Arrêté Royal du 1 février 2010 déterminant les indices spécifiques visés à l'article 138bis-4, de la loi du 25 juin 1992 sur le contrat d'assurance terrestre, Commission des assurances DOC C/2015/3 (FR); Over het ontwerp van koninklijk besluit tot wijziging van het koninklijk besluit van 1 februari 2010 tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Commissie voor Verzekeringen, DOC C/2015/3 (NL).

insurance and European Union regulation. For such discussion, we refer to e.g. Calcoen and van de Ven (2017). Instead, our purpose is to highlight some aspects of the current Belgian system for private health insurance contracts which deserve a careful attention.

4.2 Transferability of the reserves

One of the concerns expressed by consumers' representatives is that a systematic adjustment of premiums might lead to business practices diverging from the objectives of the lifelong nature. This issue is particularly sensible given that the reserves of private health insurance contracts are currently not considered as surrender values. Consumers' representatives provided in the report of the committee of 2010 the example of contracts sold to young policyholders against attractive premiums, but these premiums would increase too much over time.[3] Moreover, policyholders at higher ages for whom the premium becomes too expensive would lose their accumulated reserves in case they lapse. Therefore, in order to provide a better protection for policyholders, consumers' representatives propose to treat the reserve as a cash surrender value, further pushing the analogy with life insurance contracts. In this sense, the current legislation for lifelong health insurance contracts which does not impose the transferability of the reserves differs from that of life insurance business where the insurer is required to be transparent on the evolution of the surrender value over time.[4]

On the other hand, representatives of insurers claim that their reserving techniques are based on a collective solidarity principle, and thus, the reserves are assigned to groups of policyholders rather than to individuals. In fact, a substantial issue which arises from collectivizing the reserves is adverse selection. Young, and likely to be healthier, policyholders who leave the insurer's portfolio for another company would increase the risk of that insurer. The latter which would then be exposed to a higher insolvency risk

[3] Avis de la Commission du 7/01/2010 sur le projet d'Arrêté Royal déterminant les indices spécifiques visés à l'article 138bis-4, de la loi du 25 juin 1992 sur le contrat d'assurance terrestre, Commission des assurances DOC C/2009/11 (FR); Over het ontwerp van koninklijk besluit tot vaststelling van de specifieke indexcijfers bedoeld in artikel 138bis-4 van de wet van 25 juni 1992 op de landverzekeringsovereenkomst, Commissie voor Verzekeringen, DOC C/2009/11 (NL).

[4] Article 16 de l'Arrêté Royal relatif à l'activité d'assurance sur la vie, Moniteur Belge du 14/11/2003 (FR); Artikel 16 van de koninklijk besluit betreffende de levensverzekeringsactiviteit, Belgish Staatsblad, 14/11/2003 (NL).

will therefore be forced to claim from the relevant authority an adjustment of the premiums. However, even if this adjustment is granted, increasing premiums may draw other young policyholders away from the portfolio.

These considerations on the reserves lead to the following question: to whom should they belong? In the current situation, the reserves belong to the insurer, and policyholders who do not cancel their contract benefit from the collective scheme at the expense of those who do. This situation is in contrast with that in place in Germany since 2007 through the GKV-Wettbewerbsstärkungsgesetz act, which imposes the transferability of the reserves for private insurers; see e.g. Richter (2009). However, both viewpoints can be argued. The main drawback of the non-transferability is that policyholders become in some sense binded to their initial insurer. Nevertheless, this non-transferability has the advantage that insurers might include in their tariff information about the expected surrender rates, which eventually results in a discount for policyholders. Note however that estimating surrender rates can be challenging due to the potential influence of future economic factors. On the other hand, a drawback of the transferability is that only healthy policyholders can benefit from it, since it may be difficult for unhealthy policyholders to find another cover. Thus, policy cancellation may have a negative impact on the principle of pooling at portfolio level. In order to cope with this problem, a possible solution is to let the reserve depend on the health status such that policyholders in poor health can be allocated a higher surrender value, e.g. by linking the allocation of the reserves to the experience of the insurer via a credibility system.

4.3 Age discrimination

The problem of age discrimination in premium adjustments has surfaced first in 2005, when Test-Achat/Test-Aankoop pointed out the age-dependent adjustments applied by the Belgian health insurance provider DKV. At the end of this case, the Court held in favor of the insurer, stating that the adjustments were not discriminatory.

In order to understand the challenge of age discrimination, we go back again to the distinction between the medical index (i.e. f) and the updating factor (i.e. α). As this has been said earlier, age-dependent premium adjustments are partly due to the use of age-specific medical inflation indices. However, Figure 1.2.2 suggests that these indices are not increasing with respect to the age, and age-independent indices can be applied too. Note also that based on Section 3.7 of Chapter 3, using age-specific indices may lead to more complex implementation of the updating mechanism. Concerning the updating factor α, the analysis carried out in Subsection 3.6.3 of Chapter 3 shows that its expression depends on how the reserves are treated. The factor α is necessarily age-dependent in case the reserves are treated in an individual way. Nevertheless, some insurers claim that they are treating the reserves in a collective way, implying that the updating factor α can be set equal for all ages. Additionally, the legislator introduced a constant updating factor through the '1.5 rule'. Thus, why setting age-independent medical indices and updating factor does not close the debate?

Attempting to answer this question turns the discussion towards the updating factor α and highlights the overlap with the discussion on the transferability of the reserves. Chapter 3 allowed to compare the individual and the aggregate updating rules, while assuming an age-independent medical inflation index f. For the individual updating mechanism, each contract is treated individually and the updating factor inevitably depends on the age and seniority of the contract. One important finding here is that the individual method often leads to higher adjusted premiums for young policyholders. For the aggregate updating mechanism (say, at the level of new entrants), policyholders starting their contracts in the same year will share medical inflation risk, and thus, will have the same premium adjustments. This situation introduces solidarity between a group, regardless of their age. The consequence is that the updating factor does not depend on the age, but on the distribution of all ages among new entrants, as well as on the seniority of their contracts.

The comparison of the evolution of the adjustments in these cases questions the use of methods based on pooling in order to protect policyholders from age discrimination. Indeed, it is found in Chapter 3 that in the individual scheme, policyholders who start the contract early tend to have higher future adjustments compared to those who start at an advanced age. In the collective scheme, this implies that young policyholders could contribute more in the medical inflation risk than policyholders with higher ages. Thus, elderly can benefit from the collective adjustments only in case of a high concentration of old policyholders in the pool. However, since the reserves are not transferable, it is likely that new entrants in portfolios of private insurers will often be relatively young.

4.4 Discussion and recommendations

To sum up, the discussion about discrimination on the basis of age seems to be closely linked to that of the transferability of the reserves. Managing the reserves in a collective way leads to more favorably priced contracts and supports the uniform adjustments across different age groups. However, this solidarity implies that elderly are likely to pay for the medical inflation risk of young policyholders. This effect exacerbates when we take into account that the collective reserving constrains the transferability of the reserves, and thus, is likely to lead to a concentration of young new entrants. In the opposite case where the reserves are managed individually, policyholders bear only their own medical inflation risk. However, this leads to non-uniform adjustments across ages, and opens room for the type of business practices pointed out by consumers' representatives. In particular, with the individual reserving scheme, some insurers could exclude policyholders with a high risk profile (typically elderly or individuals with chronic diseases) from their portfolio by applying too high adjustments.

Theoretically, a potential solution to the problem stated above would consist in applying a uniform tariff for all ages. This solution echoes the risk-equalization scheme adopted in Slovenia after the 2005 reform (Albreht et al., 2009). However, such restrictive regulation might violate the principle of pricing freedom. Moreover, the European Court of Justice ruled against the 2005 Slovene Law in 2013 (Calcoen and van de Ven, 2017).

Another solution would consist in taking advantage of the Belgian *Twin Peaks* regulatory framework introduced in 2011. Under this framework, the NBB is in charge of the prudential supervision of insurance companies, and on the basis of the Belgian Law of 9 July 1975, can grant or require premium adjustments beyond what the medical indices suggest. On the other hand, for private health insurance companies, the *Financial Services and Markets Authority* (FSMA) has, among others, the competence to monitor the products in order to ensure fairness towards consumers. The analogue of the FSMA for mutual insurers selling hospitalization covers is the OCM/CDZ (*Office de Contrôle des Mutualités*, in French, and *Controledienst voor de Ziekenfondsen*, in Dutch). Whereas the role of the NBB is unambiguously specified in the current Belgian Law on premium adjustments, the role of the FSMA and the OCM/CDZ should be defined more clearly in the application of the index. In the current situation, the FSMA and the OCM/CDZ are in charge of collecting the data underlying the calculation of the medical indices. The task of monitoring premium adjustments could be entrusted to the FSMA and the OCM/CDZ, and their role in the updating mechanism could be crucial in order to avoid discrimination on the basis of age.

Part II

Longevity risk management

CHAPTER 5

A dynamic equivalence principle for longevity risk management

5.1 Introduction

The evolution of the overall mortality pattern of a population is impacted by factors which can be either positive, due to medical advances and developments in health care, or negative, due to epidemics and other natural disasters. In the context of long-term insurance business, the fact that these factors are common to all individuals in the population induces positive dependence between the remaining lifetimes of policyholders, implying that the independence assumption necessary for the Law of Large Numbers (LLN) is violated. Although increasing the number of identically distributed policies may help to hedge against the diversifiable part of the risk, insurers remain exposed to a systematic longevity part that requires alternative hedging techniques.

The solutions proposed in the existing literature to cope with systematic longevity risk can be summarized in two categories: internal or external hedging. Internal hedging essentially consists in implementing natural hedging strategies. This solution is however not sufficient, since insurance companies in practice cannot treat their business lines as financial assets; see Chapter 6 for a discussion. External hedging, on the other hand, involves a third party to which systematic risk is transferred. The third party can for instance be a reinsurer or a pension insurer who is better able to perform the internal hedge. We refer to Blake et al. (2013) for a discussion on the pension (re)insurance market, and to Denuit et al. (2011) for some limitations related to reinsurance for life insurance, the most important one being related to credit risk. The third party can also be an investor seeking for diversification opportunities. This implies that the risk is transferred in the form of a longevity-linked derivative; see Blake et al. (2013) and Blake et al. (2017), among others, for more details on the growing life market.

In this chapter, which is based on the work performed in Hanbali et al. (2019b), we investigate how to design a product that allows to manage unpredictable longevity risk throughout the life of the contract. This is achieved by transferring this risk back to policyholders via an agreed-upon risk-sharing scheme. The contribution of the present work is threefold. First, inspired by previous work of e.g. Milevsky et al. (2006), we analyze the performance of the LLN when policyholders' future lifetimes are only conditionally independent. One of the (obvious) findings is that increasing the size of the portfolio is still beneficial for the insurer. However, the risk stemming from the uncertainty on the survival index of a portfolio cannot be eliminated, which highlights the risk of specifying the elements of the insurance contract in absolute terms. Second, we propose a framework in which the information stream can be used to update the contract elements over time. Under a pricing scheme which we label the *dynamic equivalence principle*, the insurer and policyholders agree upon how the experienced loss is shared between both parties on, say, a yearly basis. Third, we provide indexing formulas for the updated premium plan and benefit package for pure endowment contracts and term annuity contracts. It is shown that an appropriate updating mechanism has to comply with two conditions. The first condition aims at enhancing the solvency situation of the insurer. The second condition follows from the fact that the premium of the contract has to be lower compared to the one from the corresponding classical contract, in order to remain sufficiently appealing for policyholders. Taking into account these two constraints enables us to derive a viable updating mechanism.

Updating mechanisms shifting (part of) the burden of systematic risk back to policyholders are not new in the literature. The main idea is to design the product such that the insurer is less exposed to systematic deviations by adapting premiums, benefits and/or the date of the first benefit payment. Particularly well-known examples of such products are unit-linked policies which consist in linking the benefits to the performance of a fund, and in this way, putting the down- as well as upside systematic investment risk on the shoulders of the policyholders. Another example of products where the benefits are adapted to the experienced survival is a tontine scheme or a survival fund; see e.g. Milevsky and Salisbury (2016), Forman and Sabin (2017) as well as the tonuity proposal of Chen et al. (2019) which combines a tontine and an annuity. Contracts where systematic longevity risk is coped with by regularly updating the benefit have also been considered in Dahl (2004). Indexing may as well be related to group self-annuitization, in which retirees pool and form a fund to provide protection against longevity; see Piggott et al. (2005) and Valdez et al. (2006). Moreover, Denuit et al. (2011) and Richter and Frederik (2011) advocate indexing mechanisms for life annuities. We also refer to the participating variable investment-linked deferred annuities (VILDAs) studied in Mauer et al. (2013). The updating mechanism discussed in Chapter 3 is also an example where premiums are adjusted on a regular basis. However, unlike in Chapter 3 where the medical inflation risk transferred back to policyholders is present in the value of the benefit, here the longevity risk is related to the payment of the benefit, and arises from the uncertainty on the survival probabilities.

This chapter provides a setting for updating the various contract elements. It may offer a superior protection to policyholders, since the insurer can contribute to restore the break of the actuarial equivalence. An additional advantage of working under the proposed framework is that it enables to design a risk-sharing scheme which enhances the insurer's solvency. Furthermore, the proposed setting allows us to derive conditions on the yearly share and on the longevity risk loadings such that contracts priced with the dynamic equivalence principle lead to lower premiums than their classical counterparts. Note that Mauer et al. (2013) find, under a lifecycle portfolio choice model with CRRA utility function, that policyholders would be keen to purchase participating contracts provided the loading of this contract is below a certain threshold; see also Weale and van de Ven (2016). In the same spirit, Boon et al. (2018) compare the CRRA-based preferences of policyholders between annuity contracts and GSA plans and include the perspective of equity holders. The results presented here differ and generalize those mentioned above at different levels. The proposed setting goes beyond the no-transfer/full-transfer binarism and provides a scheme where systematic risk is shared among the two parties. Additionally, the conditions under which each party has an advantage in engaging in the dynamic contract are given in closed-form expression. Moreover, the constraint imposed from the viewpoint of the policyholder is that the risk-sharing scheme should lead to a contract where, taking into account the potential future payments, the premium has to be lower than that of the corresponding classical contract priced in a classical way. This implies that the results hold for any utility function describing the choice of a profit-seeking decision maker, and thus, no assumption is required on its functional form. Also, the loading considered here is not restricted to a given choice of the loading function, and includes, e.g. the quantile-based loading considered in Mauer et al. (2013) as a particular case. Last but not least, our work reconciles both the solvency constraint of the insurer and the price constraint of policyholders, leading to a viable risk-sharing scheme.

The remainder of the chapter is organized as follows. Section 5.2 addresses the systematic longevity risk and the consequences of replacing the independent remaining lifetimes assumption by a conditional independence assumption. In Section 5.3, we introduce the dynamic equivalence principle as a possible solution to reduce the insurer's exposure to systematic risk. We apply the approach to a portfolio of pure endowments and a portfolio of term annuities in Section 5.4 and Section 5.5, and show how to design a viable contract which is appealing to both the insurer and policyholders. Finally, Section 5.6 concludes the paper.

We end this introduction by noting that although the angle taken here is from the longevity aspect of the risk, all results can readily be generalized to include other elements of the technical basis such as interest rates and lapse rates.

5.2 Systematic longevity risk

Consider a portfolio of l_x policyholders all aged x at time 0 whose remaining lifetimes are denoted by $T_1, \ldots, T_{l_x}$. At time 0, the survival-or-not upon time t of policyholder i is represented by the following survival indicator:

$$I_i(0,t) = \begin{cases} 1 & \text{if } T_i > t \\ 0 & \text{otherwise} \end{cases}, \tag{5.2.1}$$

and the corresponding survival index of the portfolio is defined as follows:

$$I(0,t) = \frac{1}{l_x} \sum_{i=1}^{l_x} I_i(0,t). \tag{5.2.2}$$

Throughout the paper, as in most models for projecting mortality and survival, the conditional remaining lifetimes of the policyholders, given a particular mortality scenario $\Theta = \theta$, are assumed to be mutually independent. Unconditionally however, all remaining lifetimes depend on the change of survival over time and hence, are mutually dependent. We introduce the notations ${}_tp_x(\theta)$ and ${}_tq_x(\theta)$ for the conditional survival and death probabilities, respectively:

$$ {}_tp_x(\theta) = \mathbb{P}\left[I_i(0,t) = 1 \mid \Theta = \theta\right] = 1 - {}_tq_x(\theta). \tag{5.2.3} $$

The unconditional survival probability, which is denoted by ${}_tp_x$, is given by:

$$ {}_tp_x = \mathbb{P}\left[I_i(0,t) = 1\right] = \mathbb{E}\left[{}_tp_x(\Theta)\right], \tag{5.2.4} $$

and a similar expression holds for the corresponding death probability. This set up allows for an 'urn of urns' interpretation. The survival-or-not of the different policyholders until time t is a two-stage process. First, the mortality scenario $\Theta = \theta$ unfolds. Next, the outcome of $I_i(0,t)$ for each insured i is drawn from the conditional distribution of $I_i(0,t)$, given $\Theta = \theta$.

In this setting, we find that the joint survival probability for insureds i and j, for $i \neq j$, equals

$$ \mathbb{P}\left[I_i(0,t) = 1,\ I_j(0,t) = 1\right] = \mathbb{E}\left[\left({}_tp_x(\Theta)\right)^2\right]. \tag{5.2.5} $$

Furthermore, from (5.2.4) and (5.2.5), we find that the covariance between the variables $I_i(0,t)$ and $I_j(0,t)$, for $i \neq j$, is given by:

$$ \mathrm{Cov}[I_i(0,t),\ I_j(0,t)] = \mathrm{Var}\left[{}_tp_x(\Theta)\right], \tag{5.2.6} $$

which implies a non-negative dependence between the survival indicators. This expression shows that the degree of our ignorance about the future survival probability drives the dependence between the survival indicators: the more ${}_tp_x(\Theta)$ is uncertain, in the sense that it has a larger variance, the more the survival indicators are correlated. In case ${}_tp_x(\Theta)$ is deterministic, meaning that the survival probability is known with certainty, the covariance between $I_i(0,t)$ and $I_j(0,t)$ is zero and the survival indicators are independent (since zero correlation is equivalent to independence in the Bernoulli case). This latter case is in line with the standard actuarial assumption of independence.

Furthermore, for $i \neq j$, we have that:

$$ \begin{aligned} \mathbb{P}[I_j(0,t) = 1 \mid I_i(0,t) = 1] &= \frac{\mathbb{E}[I_i(0,t) \times I_j(0,t)]}{\mathbb{E}[I_i(0,t)]} \\ &= \mathbb{P}[I_j(0,t) = 1] + \frac{\mathrm{Cov}[I_i(0,t),\ I_j(0,t)]}{\mathbb{E}[I_i(0,t)]}. \end{aligned} $$

Taking into account (5.2.6), we find that the t-year conditional survival probability of policyholder j, given that policyholder i is alive at that time, can be expressed as

$$\mathbb{P}[I_j(0,t)=1 \mid I_i(0,t)=1] = \mathbb{P}[I_j(0,t)=1] + \frac{\mathrm{Var}\left[{}_tp_x(\Theta)\right]}{\mathbb{E}\left[{}_tp_x(\Theta)\right]} \geq \mathbb{P}[I_j(0,t)=1]. \tag{5.2.7}$$

From this expression, we see that the knowledge that policyholder i survives increases the probability that policyholder j survives. Additionally, the more uncertain ${}_tp_x(\Theta)$, the more the conditional survival probability of j exceeds the unconditional one.

The dependence between remaining lifetimes implies that the strong LLN is not applicable. In other words, increasing the portfolio size will not fully diversify the longevity risk. The classical strong LLN has to be replaced by the conditional strong LLN which states that

$$\lim_{l_x \to \infty} \frac{1}{l_x} \sum_{i=1}^{l_x} I_i(0,t) = \mathbb{E}\left[I_1(0,t)|\Theta\right] = {}_tp_x(\Theta), \tag{5.2.8}$$

almost surely; see e.g. Majrek et al. (2005). The limiting survivor proportion, or equivalently the average benefit, is a random variable which is related to the systematic part of the portfolio's risk per pure endowment policy.

Using the law of total variance, we find that $\mathrm{Var}[I(0,t)]$ can be split into two parts:

$$\mathrm{Var}\left[I(0,t)\right] = \frac{1}{l_x}\mathbb{E}\left[{}_tp_x(\Theta)\ {}_tq_x(\Theta)\right] + \mathrm{Var}\left[{}_tp_x(\Theta)\right]. \tag{5.2.9}$$

The diversifiable part of the insurance risk is captured by the first term in (5.2.9), which depends on the size of the portfolio. The second term is related to the uncertainty on the survival probabilities, and hence, captures the systematic part. This two-components representation implies that the variance of the average benefit payment is a decreasing function of the number of insureds. The cases where l_x goes to infinity and to 1 lead to the following lower and upper bounds:

$$\mathrm{Var}\left[{}_tp_x(\Theta)\right] \leq \mathrm{Var}\left[I(0,t)\right] \leq \mathrm{Var}\left[I_i(0,t)\right]. \tag{5.2.10}$$

implying that it is beneficial to increase the size of the portfolio.

The benefit of diversification can also be illustrated using the tail value-at-risk, which we denote for the random variable X and the level $\epsilon \in (0,1)$ by $\mathrm{TVaR}_\epsilon[X]$, such that:

$$\mathrm{TVaR}_\epsilon[X] = \frac{1}{1-\epsilon}\int_\epsilon^1 \mathrm{VaR}_q[X]\ \mathrm{d}q,$$

where $\mathrm{VaR}_q[X]$ is the value-at-risk at the level $q \in (0,1)$.

The inequality

$$\frac{1}{l_x+1}\mathrm{TVaR}_\epsilon\left[\sum_{i=1}^{l_x+1} I_i(0,t)\right] \leq \frac{1}{l_x}\mathrm{TVaR}_\varepsilon\left[\sum_{i=1}^{l_x} I_i(0,t)\right]$$

holds for all probability levels ϵ, and for any possible conditional dependence between the $I_i(0,t)$'s; see e.g. Denuit et al. (2005) and Feng and Shimizu (2016). Thus, the relative contribution of each endowment policy to the solvency capital (determined as the TVaR minus the best estimate) decreases as the size of the portfolio increases.

The main conclusion of this section is that in case of conditional independence, only part of the insurance risk can be diversified by pooling. Taking into account this observation, it seems that specifying the elements of the insurance contract in absolute terms at policy issue can be extremely dangerous, because of the substantial systematic longevity risk captured in $\mathrm{Var}\left[{}_tp_x(\Theta)\right]$.

5.3 Dynamic equivalence principle

5.3.1 The actuarial equivalence principle at contract initiation

The main idea behind the proposed risk-sharing scheme is similar to that of Chapter 3. At the end of each period, the retrospective reserve is compared to the required liabilities (determined prospectively, taking into account new information on the insured risks). In contrast with the setting of Chapter 3 where the risk is fully transferred, here the experienced loss (i.e. the difference between the retrospective and prospective reserves) is shared between the policyholder and the insurer, according to an agreed-upon risk-sharing scheme which is determined at policy issue. We focus in this section on deriving rules for a risk-sharing scheme for insurance contracts with survival benefits, but the approach consisting in sharing the experienced loss is applicable to other types of insurance contracts, including those with a death benefit.

We consider an insurance contract sold to l_x policyholders aged x at time 0. The policy stipulates that upon survival at times $k+1$, $k = 0, 1, 2, \ldots$, the beneficiary will receive the amount $b_{k+1}^{(0)} \geq 0$, while upon survival at time k, the policyholder will pay the premium $\pi_k^{(0)} \geq 0$. Given the information available at time 0, hence, the superscript "$^{(0)}$", we denote by $\mathcal{C}_0^{(0)}$ the contract elements agreed-upon at time 0 :

$$\mathcal{C}_0^{(0)} = \left\{ \underline{b}_0^{(0)}, \underline{\pi}_0^{(0)} \right\},$$

where $\underline{b}_0^{(0)} = (b_1^{(0)}, b_2^{(0)}, ...)$ and $\underline{\pi}_0^{(0)} = \left(\pi_0^{(0)}, \pi_1^{(0)}, ... \right)$ are the benefit package and the premium plan, respectively. The benefit package can be considered as a guaranteed benefit, or as an estimate of future random benefit as in the setting presented in Chapter 3. In this latter case, we can reconcile the updating mechanisms incorporating information on survival probabilities or on medical inflation. The present value at time 0 of future benefit payments per-policy is denoted by the random variable $B_0^{(0)}$:

$$B_0^{(0)} = \sum_{j=0}^{\infty} b_{j+1}^{(0)} v^{j+1} \ {}_{j+1}p_x(\Theta),$$

and the present value of future premiums is given by the random variable $\Pi_0^{(0)}$:

$$\Pi_0^{(0)} = \sum_{j=0}^{\infty} \pi_j^{(0)} v^j \ {}_jp_x(\Theta),$$

where v is the constant yearly discounting factor. The loss random variable at time 0 is denoted by L_0 and is defined as the present value of future benefits minus future premiums, i.e.:

$$L_0 = B_0^{(0)} - \Pi_0^{(0)}. \tag{5.3.1}$$

Recall that the j-year survival of policyholder i is characterized by the conditional probability ${}_jp_x(\Theta)$. Based on the knowledge available at time 0 and by using the notation $\mathcal{F}_0$, the actuary attaches the following values to these probabilities:

$$\mathbb{E}\left[{}_jp_x(\Theta)|\mathcal{F}_0\right] = {}_jp_x^{(0)}, \tag{5.3.2}$$

for $j = 1, 2, ...$ Similarly to the notations of Chapter 3, we will also use the superscript "$^{(k)}$" to indicate quantities that are based on information and expert opinion available at time k, $k = 1, 2, ...$, and we replace the notation $\mathbb{E}\left[.|\mathcal{F}_k\right]$ by $\mathbb{E}_k\left[.\right]$. Note that besides the probabilities introduced above, at time 0, the actuary also has to determine other factors of the technical basis.

The expected present value of the benefit and premium cash flow streams are then given by the following expected present values:

$$\mathbb{E}_0\left[B_0^{(0)}\right] = \sum_{j=0}^{\infty} b_{j+1}^{(0)} v^{j+1} \ {}_{j+1}p_x^{(0)},$$

and

$$\mathbb{E}_0\left[\Pi_0^{(0)}\right] = \sum_{j=0}^{\infty} \pi_j^{(0)} v^j \ {}_j p_x^{(0)},$$

respectively. We assume that premiums and benefits of the contract are set such that they fulfill the actuarial equivalence principle, i.e. such that the expected value of the loss in (5.3.1), conditionally on the information available at time 0, is equal to 0 at that time:

$$\mathbb{E}_0\left[L_0\right] = 0 \iff \mathbb{E}_0\left[B_0^{(0)}\right] = \mathbb{E}_0\left[\Pi_0^{(0)}\right]. \tag{5.3.3}$$

The valuation approach based on the actuarial equivalence principle (5.3.3) is the one commonly used in practice for classical life insurance, where the benefit and premium cash flow streams $\underline{b}_0^{(0)}$ and $\underline{\pi}_0^{(0)}$ are fixed at policy issue and remain, in principle, unchanged during the life of the contract. This means that the risk of having chosen a wrong technical basis is fully taken by the insurer. In order to cope with this risk, the insurer commonly charges a (implicit) loading which is used to cover the diversifiable as well as the undiversifiable parts of the risk. In this section, we discard this loading, but we will include it in Section 5.4.

5.3.2 The dynamic equivalence principle

Suppose that we have arrived at time 1 and that the policy is still in force. Assuming that the technical interest rate is guaranteed, the available provision (or reserve) at that time is given by:

$$V_1^{(0)} = \frac{\pi_0^{(0)}}{vI(0,1)},$$

where $I(0,1)$ is the observed survival index over the first year which is assumed to be strictly greater than 0. This assumption means that there is at least one survivor at time 1. In case $I(0,1)=0$ then all contracts are terminated at time 1 and no updating is needed anymore. Note that throughout the paper and for any $k=1,2,...$, the reserves are calculated before benefit and premium payment.

Having arrived at time 1, the realization $I(0,1)$ provides the insurer with additional information. Moreover, assuming that the insured i is still alive, we know that $I_i(0,1)=1$. Based on this and on other information available at time 1, we obtain the following updated survival probabilities:

$$\mathbb{E}_1\left[\ {}_jp_{x+1}(\Theta)\right] = \ {}_jp^{(1)}_{x+1}, \qquad j=1,2,\ldots \tag{5.3.4}$$

From (5.3.4), it follows that the required reserve based on the new information is given by:

$$\mathbb{E}_1\left[B_1^{(0)}-\Pi_1^{(0)}\right] = \sum_{j=0}^{\infty} b^{(0)}_{j+1} v^j \ {}_jp^{(1)}_{x+1} - \sum_{j=0}^{\infty} \pi^{(0)}_{j+1} v^j \ {}_jp^{(1)}_{x+1}.$$

In general, the actuarial equivalence will be broken because the realization of the retrospective reserve deviates from its assumption and the estimate of the technical basis has changed. In order to restore it, the following capital is required:

$$\mathbb{E}_1\left[B_1^{(0)}-\Pi_1^{(0)}\right]-V_1^{(0)},$$

which represents the deviation between the required and the retrospective reserve at time 1. We suppose now that this required amount is shared among the insurer and the policyholder. For $\alpha_1\in[0,1]$, let $1-\alpha_1$ be the share of the loss covered by the insurer whereas the contribution of the policyholder is given by $\alpha_1\left(\mathbb{E}_1\left[B_1^{(0)}-\Pi_1^{(0)}\right]-V_1^{(0)}\right)$. On the one hand, the retrospective reserve after having been increased by the insurer's participation is given by:

$$V_1^{(1)} = V_1^{(0)} + (1-\alpha_1)\left(\mathbb{E}_1\left[B_1^{(0)}-\Pi_1^{(0)}\right]-V_1^{(0)}\right).$$

On the other hand, from time 1 on, the remaining premium plan $\underline{\pi}_1^{(0)}$ is replaced by $\underline{\pi}_1^{(1)}$:

$$\underline{\pi}_1^{(1)} = \left(\pi_1^{(1)},\pi_2^{(1)},...\right), \tag{5.3.5}$$

and the remaining benefit package $\underline{b}_1^{(0)}$ is replaced by $\underline{b}_1^{(1)}$:

$$\underline{b}_1^{(1)} = \left(b_1^{(1)}, b_2^{(1)}, ...\right). \tag{5.3.6}$$

We denote the expected actuarial present value at time 1 of the updated remaining premiums and benefits as:

$$\mathbb{E}_1\left[\Pi_1^{(1)}\right] = \sum_{j=0}^{\infty} \pi_{j+1}^{(1)} v^j \ {}_jp_{x+1}^{(1)}, \tag{5.3.7}$$

and

$$\mathbb{E}_1\left[B_1^{(1)}\right] = \sum_{j=0}^{\infty} b_{j+1}^{(1)} v^j \ {}_jp_{x+1}^{(1)}, \tag{5.3.8}$$

respectively. The updated premium plan and benefit package are determined such that the actuarial equivalence is restored at time 1, i.e. when the following equation is satisfied:

$$\mathbb{E}_1\left[\Pi_1^{(1)} - B_1^{(1)}\right] = \mathbb{E}_1\left[\Pi_1^{(0)} - B_1^{(0)}\right] + \alpha_1\left(\mathbb{E}_1\left[B_1^{(0)} - \Pi_1^{(0)}\right] - V_1^{(0)}\right).$$

Taking into account the contributions of both the insurer and the policyholders, one finds that:

$$V_1^{(1)} = \mathbb{E}_1\left[B_1^{(1)} - \Pi_1^{(1)}\right],$$

which means that the actuarial equivalence has been restored at time 1.

A similar reasoning can be applied at time 2 where the available reserve based on the updating up to time 1 is given by:

$$V_2^{(1)} = \frac{V_1^{(1)} + \pi_1^{(1)} - b_1^{(1)}}{vI(1,2)},$$

with $I(1,2)$ being the survival index of the portfolio from time 1 to time 2, and assumed to be strictly positive. Using the new information available at time 2, the estimates of future survival probabilities at that time are updated to ${}_jp_{x+2}^{(2)}$, for $j = 1, 2, ...$, such that the value of the required reserve is as follows:

$$\mathbb{E}_2\left[B_2^{(1)} - \Pi_2^{(1)}\right] = \sum_{j=0}^{\infty} b_{j+2}^{(1)} v^j \ {}_jp_{x+2}^{(2)} - \sum_{j=0}^{\infty} \pi_{j+2}^{(1)} v^j \ {}_jp_{x+2}^{(2)}.$$

The amount $\mathbb{E}_2\left[B_2^{(1)} - \Pi_2^{(1)}\right] - V_2^{(1)}$ is needed to restore the actuarial equivalence. For $\alpha_2 \in [0,1]$, the share of the insurer is given by $1-\alpha_2$, such that:

$$V_2^{(2)} = V_2^{(1)} + (1-\alpha_2)\left(\mathbb{E}_2\left[B_2^{(1)} - \Pi_2^{(1)}\right] - V_2^{(1)}\right).$$

The contribution of the policyholder is $\alpha_2\left(\mathbb{E}_2\left[B_2^{(1)} - \Pi_2^{(1)}\right] - V_2^{(1)}\right)$. The updated premium plan $\underline{\pi}_2^{(2)}$and benefit package $\underline{b}_2^{(2)}$ are determined from the following updated actuarial equivalence:

$$\mathbb{E}_2\left[\Pi_2^{(2)} - B_2^{(2)}\right] = \mathbb{E}_2\left[\Pi_2^{(1)} - B_2^{(1)}\right] + \alpha_2\left(\mathbb{E}_2\left[B_2^{(1)} - \Pi_2^{(1)}\right] - V_2^{(1)}\right).$$

Insurance regulation requires that having arrived at time k, the actuarial equivalence has to be restored. In case of a classical life insurance contract, where benefits and premiums are fixed at policy issue, the insurer is fully responsible for restoring the actuarial equivalence, i.e. $\alpha_k = 0$. In our present setting, the cost of restoring the actuarial equivalence is covered by both the insurer and policyholders. The available provision as well as all future benefits and premiums are updated according to a pre-specified risk-sharing scheme which is characterized by the coefficients $\alpha_1, \alpha_2, ...$ This flexible approach to manage longevity risk for newly underwritten contracts can be seen as a series of successive (yearly) applications of the fundamental static equivalence principle. We say here that such a contract is managed by a *dynamic equivalence principle*, which is defined hereafter for any time k.

Definition 5.1 (Dynamic equivalence principle)**.** *At any time k, the $k-1$ values of the remaining contract features*

$$\mathcal{C}_k^{(k-1)} = \left\{\underline{b}_k^{(k-1)}, \underline{\pi}_k^{(k-1)}\right\},$$

are replaced by

$$\mathcal{C}_k^{(k)} = \left\{\underline{b}_k^{(k)}, \underline{\pi}_k^{(k)}\right\},$$

taking into account the information stream over time, such that the actuarial equivalence is restored at that time, i.e.

$$\mathbb{E}_k\left[\Pi_k^{(k)} - B_k^{(k)}\right] = \mathbb{E}_k\left[\Pi_k^{(k-1)} - B_k^{(k-1)}\right] + \alpha_k\left(\mathbb{E}_k\left[B_k^{(k-1)} - \Pi_k^{(k-1)}\right] - V_k^{(k-1)}\right), \tag{5.3.9}$$

where $\alpha_k \in [0,1]$ is the share of the loss born by the policyholder at time k. The retrospective reserve at time k before updating, i.e. $V_k^{(k-1)}$, is given by:

$$V_k^{(k-1)} = \frac{V_{k-1}^{(k-1)} + \pi_{k-1}^{(k-1)} - b_{k-1}^{(k-1)}}{vI(k-1,k)},$$

where $V_{k-1}^{(k-1)}$ is the retrospective reserve at time $k-1$ after having been increased by the insurer's participation in the deviation risk:

$$V_{k-1}^{(k-1)} = V_{k-1}^{(k-2)} + (1-\alpha_{k-1})\left(\mathbb{E}_{k-1}\left[B_{k-1}^{(k-2)} - \Pi_{k-1}^{(k-2)}\right] - V_{k-1}^{(k-2)}\right),$$

and where $I(k-1,k)$ represents the (strictly positive) survival index from time $k-1$ to time k in the portfolio which is composed of policyholders aged $x+k-1$ at time $k-1$. Taking into account the contribution of both the insurer and the policyholders, one finds that:

$$V_k^{(k)} = \mathbb{E}_k\left[B_k^{(k)} - \Pi_k^{(k)}\right],$$

which means that the actuarial equivalence has been restored at time k.

5.4 Pure endowment with single premium

5.4.1 The updating mechanism

In this section, we focus on a portfolio of t-year pure endowments sold to l_x policyholders aged x at time 0. The contract pays a benefit of $b_t^{(0)}$ at time t upon survival of the policyholder at that time. The single pure premium for this contract is denoted by $\pi_0^{(0)}$. We assume that the policy is flexible, in the sense that it allows the contract elements to be updated over time. Based on the estimate ${}_tp_x^{(0)}$ at time 0 of the t-year survival probability, we use the actuarial equivalence principle (5.3.2) to determine the value of the pure premium $\pi_0^{(0)}$ at contract inception:

$$\pi_0^{(0)} = b_t^{(0)} v^t \; {}_tp_x^{(0)}. \tag{5.4.1}$$

We include a premium loading in this section and we denote it by φ, which is assumed to be positive. This results in the loaded single premium $P_0^{(0)}$:

$$P_0^{(0)} = \pi_0^{(0)} + \varphi.$$

This setting includes a wide range of possible pricing principles; see Kaas et al. (2008) for some examples. The loading φ can also be determined by applying shocks on the estimated survival probabilities, in the spirit of the Solvency II regulation. Because the loaded premium is the sum of the actuarial value of the benefit $b_t^{(0)}$ and a loading, we can interpret $P_0^{(0)}$ as the pure premium for a contract paying the benefit $b_t^{(0)} + \frac{\varphi}{v^t \ {}_tp_x^{(0)}}$ at maturity. Thus, the contract can be described by the benefit package $\underline{b}_0^{(0)} = (0, ..., 0, b_t^{(0)} + \frac{\varphi}{v^t \ {}_tp_x^{(0)}}, 0, ...)$ and the pure premium plan $\underline{P}_0^{(0)} = \left(P_0^{(0)}, 0, 0, ...\right)$. Under this setting, the insurer estimates at time 0 both the pure premium and the loading. As the future unfolds and new information becomes available, the estimate of the premium may require an adjustment and the estimate of the loading may turn out to be insufficient to cope with the deviation risk for the remaining years. In the sequel, the time-0 estimates of the pure premium as well as the loading will both be updated over time. Note that another simpler setting which is not considered here and requires only minor modifications in the subsequent results is to update the pure premium only, without taking into account the loading in the dynamic equivalence principle.

At time 1, the value of the retrospective reserve per policy still in force is:

$$V_1^{(0)} = \frac{P_0^{(0)}}{vI(0,1)}. \tag{5.4.2}$$

The new information available at time 1 leads to a new estimate ${}_{t-1}p_{x+1}^{(1)}$ of the $(t-1)$-year survival probability of the policyholder now aged $x+1$. From time 1 on, the benefit package $\underline{b}_1^{(0)} = (0, ..., 0, b_t^{(0)} + \frac{\varphi}{v^t \ {}_tp_x^{(0)}}, 0, ...)$ and the premium plan $\underline{P}_1^{(0)} = (0, 0, 0, ...)$ at that time are replaced by $\underline{b}_1^{(1)} = (0, ..., 0, b_t^{(1)} + \frac{\varphi}{v^t \ {}_tp_x^{(0)}}, 0, ...)$ and $\underline{P}_1^{(1)} = \left(P_1^{(1)}, 0, 0, ...\right)$, respectively, where we assume that only a single extra-premium $P_1^{(1)}$ is paid at time 1. This means that the benefit is updated from $b_t^{(0)}$ to $b_t^{(1)}$, and/or an additional amount $P_1^{(1)}$ is paid by the policyholders.

Let us now apply the dynamic equivalence principle (5.3.9) to obtain the values of the updated benefit and the additional premium. The present values of future premiums using the information available at time 1 before and after the updating are given by:

$$\mathbb{E}_1\left[\Pi_1^{(0)}\right] = 0, \tag{5.4.3}$$

and

$$\mathbb{E}_1\left[\Pi_1^{(1)}\right] = P_1^{(1)}, \tag{5.4.4}$$

respectively. Moreover, the present values of future benefits using the information available at time 1 before and after the updating are given by:

$$\mathbb{E}_1\left[B_1^{(0)}\right] = \left(b_t^{(0)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right) v^{t-1}\ {}_{t-1}p_{x+1}^{(1)}, \tag{5.4.5}$$

and

$$\mathbb{E}_1\left[B_1^{(1)}\right] = \left(b_t^{(1)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right) v^{t-1}\ {}_{t-1}p_{x+1}^{(1)}, \tag{5.4.6}$$

respectively. Plugging Expressions (5.4.3)–(5.4.6) as well as (5.4.2) in Equation (5.3.9) and multiplying both sides by $\left(-v^{t-1}\ {}_{t-1}p_{x+1}^{(1)}\right)^{-1}$ leads to:

$$b_t^{(1)} - \frac{P_1^{(1)}}{v^{t-1}\ {}_{t-1}p_{x+1}^{(1)}} = b_t^{(0)} - \alpha_1\left(\left(b_t^{(0)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right) - \frac{P_0^{(0)}}{v^t I(0,1)\ {}_{t-1}p_{x+1}^{(1)}}\right). \tag{5.4.7}$$

Furthermore, by noting that $P_0^{(0)} = \left(b_t^{(0)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right) v^t\ {}_tp_x^{(0)}$, we find that $b_t^{(1)}$ and $P_1^{(1)}$ satisfy the following equation:

$$b_t^{(1)} - \frac{P_1^{(1)}}{v^{t-1}\ {}_{t-1}p_{x+1}^{(1)}} = b_t^{(0)} - \alpha_1\left(b_t^{(0)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right)\left(1 - \frac{{}_tp_x^{(0)}}{I(0,1)\ {}_{t-1}p_{x+1}^{(1)}}\right). \tag{5.4.8}$$

Using (5.4.8), we find that the retrospective reserve at time 2 is given by:

$$V_2^{(1)} = \left(b_t^{(1)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right) v^{t-2}\frac{{}_{t-1}p_{x+1}^{(1)}}{I(1,2)}.$$

If we apply the dynamic equivalence principle (5.3.9), we find that the time-2 updated value of the benefit $b_t^{(2)}$ and the additional premium $P_2^{(2)}$ satisfy the equation:

$$b_t^{(2)} - \frac{P_2^{(2)}}{v^{t-2}\ {}_{t-2}p_{x+2}^{(2)}} = b_t^{(1)} - \alpha_2\left(b_t^{(1)} + \frac{\varphi}{v^t\ {}_tp_x^{(0)}}\right)\left(1 - \frac{{}_{t-1}p_{x+1}^{(1)}}{I(1,2)\ {}_{t-2}p_{x+2}^{(2)}}\right).$$

In general, the retrospective reserve $V_k^{(k-1)}$ at time k, for $k = 2, ..., t$, is given by:

$$V_k^{(k-1)} = \left(b_t^{(k-1)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) v^{t-k} \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)},$$

such that $b_t^{(k)}$ and $P_k^{(k)}$ satisfy:

$$b_t^{(k)} - \frac{P_k^{(k)}}{v^{t-k} \ {}_{t-k}p_{x+k}^{(k)}} = b_t^{(k-1)} - \alpha_k \left(b_t^{(k-1)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) \left(1 - \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k) \ {}_{t-k}p_{x+k}^{(k)}} \right). \tag{5.4.9}$$

The values of the couple $(b_t^{(k)}, P_k^{(k)})$ depend on the realization of the survival index, the new estimate of the mortality table and the participation of the insurer to cover the systematic deviation through the choice of the parameters $\alpha_1, ..., \alpha_k$. Thus, there is no unique couple which can be determined from (5.4.9). Hereafter, we consider two particular cases; updating the premium plan only, and updating the benefit package only.

Case 1 (Updating the premium plan only)**.** *In case the benefit is not subject to revision, we find that the reserve at time k is as follows:*

$$V_k^{(k-1)} = \frac{\left(b_t^{(0)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) v^{t-k+1} \ {}_{t-k+1}p_{x+k-1}^{(k-1)}}{vI(k-1,k)} = \frac{P_0^{(0)}}{v^k \ {}_t p_x^{(0)}} \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)}.$$

*Let us now derive a general expression for the additional premium. Starting with the time-*1 *additional amount $P_1^{(1)}$, we find from (5.4.8):*

$$\begin{aligned} P_1^{(1)} &= \alpha_1 \left(\left(b_t^{(0)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) v^{t-1} \ {}_{t-1}p_{x+1}^{(1)} - \frac{P_0^{(0)}}{vI(0,1)} \right) \\ &= \alpha_1 \frac{P_0^{(0)}}{v \ {}_t p_x^{(0)}} \left({}_{t-1}p_{x+1}^{(1)} - \frac{{}_t p_x^{(0)}}{I(0,1)} \right). \end{aligned}$$

The premium $P_1^{(1)}$ corresponds to the value at time 1 *of the loaded single premium using the time-*0 *information. This is then corrected from the deviation $\left({}_{t-1}p_{x+1}^{(1)} - \frac{{}_t p_x^{(0)}}{I(0,1)} \right)$ and scaled by the contribution α_1 of the policyholders. Thus, the contribution of the policyholders takes into account changes of both the past realizations and the new estimates. At time* 2*, the* *additional amount* $P_2^{(2)}$ *is given by:*

$$\begin{aligned} P_2^{(2)} &= \alpha_2 \left(\left(b_t^{(0)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) v^{t-2} \ {}_{t-2}p_{x+2}^{(2)} - V_2^{(1)} \right) \\ &= \alpha_2 \frac{P_0^{(0)}}{v^2 \ {}_t p_x^{(0)}} \left({}_{t-2}p_{x+2}^{(2)} - \frac{{}_{t-1}p_{x+1}^{(1)}}{I(1,2)} \right), \end{aligned}$$

*and has a similar interpretation as the time-*1 *additional amount.*

In general, we find that the additional amounts $P_k^{(k)}$ *required from the policyholders at the successive times* $k = 2, 3, ..., t$ *are given by:*

$$P_k^{(k)} = \alpha_k \frac{P_0^{(0)}}{v^k \ {}_t p_x^{(0)}} \left({}_{t-k}p_{x+k}^{(k)} - \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)} \right). \tag{5.4.10}$$

The numerical value of $P_k^{(k)}$ *can be negative, implying that the insurer pays back the policyholder for being too conservative. Also, by considering that* k *takes values* $1, 2, ..., t$, *we implicitly assume that policyholders could pay an additional positive amount* $P_t^{(t)}$ *at contract expiration. This assumption may not be realistic in practice. However, in such a case, this can be compensated by a benefit reduction.*

Let us notice that in case we use the multiplicative form $P_0^{(0)} = \pi_0^{(0)} (1 + \varphi)$ *instead of the additive form* $P_0^{(0)} = \pi_0^{(0)} + \varphi$, *the updating formula* (5.4.10) *implies that*

$$P_k^{(k)} = \pi_k^{(k)} (1 + \varphi),$$

where $\pi_k^{(k)}$ *is the time-*k *pure additional amount which satisfies* (5.4.10), *such that:*

$$\pi_k^{(k)} = \alpha_k \frac{\pi_0^{(0)}}{v^k \ {}_t p_x^{(0)}} \left({}_{t-k}p_{x+k}^{(k)} - \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)} \right).$$

This means that the loading φ *is constant over time and is applied to the future pure additional amounts, even when they are negative. However, we can still write the future required amounts in the additive form*

$$P_k^{(k)} = \pi_k^{(k)} + \varphi^{(k)},$$

where $\varphi^{(k)} = \frac{\varphi}{\pi_k^{(k)}}$ *is a time-varying loading.*

Case 2 (Updating the benefit package only)**.** *Let us now assume that policyholders do not pay additional premiums. Instead, the benefit at time t can be revised throughout the contract to account for deviations. In this case, it follows directly from* (5.4.9) *that the time-k updated value of the benefit has the following expression:*

$$b_t^{(k)} = b_t^{(k-1)} - \alpha_k \left(b_t^{(k-1)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) \left(1 - \frac{p_{x+k-1}^{(k-1)}}{I(k-1,k)} \frac{{}_{t-k}p_{x+k}^{(k-1)}}{{}_{t-k}p_{x+k}^{(k)}} \right). \tag{5.4.11}$$

Note that we also find the following expression for the reserve at time k:

$$V_k^{(k-1)} = \left(b_t^{(k-1)} + \frac{\varphi}{v^t \ {}_t p_x^{(0)}} \right) v^{t-k} \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)}.$$

Cases 1 and 2 are two particular risk-sharing schemes which have the same goal of restoring the actuarial equivalence. In the remainder of this section, we focus on Case 1 and set $b_t^{(t)} = b_t^{(0)} = b_t$. Moreover, we simplify the setting by assuming that $\alpha_k = \alpha$ for all $k = 1, 2, ..., t$, meaning that the insurer transfers back to the policyholders the same share of the shortfall every year. We answer two specific questions. The first question is raised by the insurer who wants to determine under what condition a contract managed by the dynamic equivalence principle provides more safety than a classical one, where safety is measured by the probability of loss. Second, from the point of view of policyholders, we search for conditions under which buying a contract priced under the dynamic equivalence principle will be cheaper than its classical counterpart. Here, a classical pure endowment contract is a contract without updating (i.e. $\alpha = 0$) whose single premium is denoted by P_Ψ:

$$P_\Psi = \pi_0^{(0)} + \Psi, \tag{5.4.12}$$

with $\Psi > 0$. Note that despite not being formally imposed here, it is reasonable to expect that $\varphi \leq \Psi$. Moreover, the contracts should have the same loading when the insurer covers all the shortfall, i.e. $\varphi = \Psi$ for $\alpha = 0$, whereas there should be no loading when the policyholders are covering the shortfall, i.e. $\varphi = 0$ for $\alpha = 1$.

5.4.2 Impact on the insurer's solvency

The analysis in this subsection is carried out from the point of view of the insurer who has the choice between selling a pure endowment under the classical setting or under the risk-sharing scheme described in Case 1. The time-0 shortfall risk per-policy for a portfolio of pure endowments priced in the classical static setting is denoted by R_Ψ, whereas its counterpart priced in the dynamic setting is denoted by $R_{\alpha,\varphi}$. For the classical contract, we have:

$$R_\Psi = I(0,t)b_t v^t - P_\Psi, \tag{5.4.13}$$

which can also be written as:

$$R_\Psi = \left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right)\pi_0^{(0)} - \Psi. \tag{5.4.14}$$

On the other hand, we find in the dynamic case:

$$R_{\alpha,\varphi} = I(0,t)b_t v^t - P_0^{(0)} - \sum_{k=1}^{t} v^k I(0,k) P_k^{(k)},$$

From (5.4.10), we see that $R_{\alpha,\varphi}$ can also be written as:

$$R_{\alpha,\varphi} = I(0,t)b_t^{(0)}v^t - P_0^{(0)} - \alpha\frac{P_0^{(0)}}{{}_tp_x^{(0)}}\sum_{k=1}^{t} I(0,k)\left({}_{t-k}p_{x+k}^{(k)} - \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)}\right),$$

or, in its simplified form:

$$R_{\alpha,\varphi} = \left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right)\left(\pi_0^{(0)} - \alpha P_0^{(0)}\right) - \varphi, \tag{5.4.15}$$

where we use

$$\sum_{k=1}^{t} I(0,k)\left({}_{t-k}p_{x+k}^{(k)} - \frac{{}_{t-k+1}p_{x+k-1}^{(k-1)}}{I(k-1,k)}\right) = I(0,t) - {}_tp_x^{(0)}.$$

The insurer will have a loss in case the shortfall is positive, i.e. in case the payments to the policyholders are higher than expected. In the following theorem, we provide a condition on Ψ, φ and α such that the probability of a loss for a contract priced under the dynamic equivalence principle is lower compared to its classical counterpart.

Theorem 5.1. *A pure endowment contract with single premium and loading φ priced under the dynamic equivalence principle has a lower loss probability than its counterpart with loading Ψ and priced using a classical premium principle, if the yearly share $\alpha \in [0,1]$ satisfies the following condition:*

$$\frac{\pi_0^{(0)}}{\pi_0^{(0)} + \varphi}\left(1 - \frac{\varphi}{\Psi}\right) \leq \alpha. \tag{5.4.16}$$

Proof. The loss probability of the insurer under the dynamic equivalence principle is given by:

$$\mathbb{P}\left[R_{\alpha,\varphi} > 0\right] = \mathbb{P}\left[\left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right)\left(\pi_0^{(0)} - \alpha P_0^{(0)}\right) \geq \varphi\right], \tag{5.4.17}$$

whereas its counterpart under the classical setting is given by:

$$\mathbb{P}\left[R_\Psi > 0\right] = \mathbb{P}\left[\left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right)\pi_0^{(0)} \geq \Psi\right]. \tag{5.4.18}$$

First, suppose that $\frac{\pi_0^{(0)}}{\pi_0^{(0)}+\varphi}\left(1 - \frac{\varphi}{\Psi}\right) \leq \alpha$ holds with $\alpha < \frac{\pi_0^{(0)}}{\pi_0^{(0)}+\varphi}$. The latter inequality implies that

$$\pi_0^{(0)} - \alpha P_0^{(0)} > 0,$$

and hence, the former inequality can be written as follows:

$$\frac{\varphi}{\pi_0^{(0)} - \alpha P_0^{(0)}} \geq \frac{\Psi}{\pi_0^{(0)}}. \tag{5.4.19}$$

Therefore, we find from (5.4.19) that:

$$\mathbb{P}\left[\left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right) > \frac{\varphi}{\pi_0^{(0)} - \alpha P_0^{(0)}}\right] \leq \mathbb{P}\left[\left(\frac{I(0,t)}{{}_tp_x^{(0)}} - 1\right) > \frac{\Psi}{\pi_0^{(0)}}\right],$$

or, equivalently, $\mathbb{P}\left[R_{\alpha,\varphi} > 0\right] \leq \mathbb{P}\left[R_\Psi > 0\right]$.

Suppose now that $\frac{\pi_0^{(0)}}{\pi_0^{(0)}+\varphi} \leq \alpha$, which is equivalent to $\pi_0^{(0)} - \alpha P_0^{(0)} \leq 0$. Since the upper bound of the support of $I(0,1)$ is 1, we find for $\alpha \in [0,1]$ that:

$$R_{\alpha,\varphi} \leq -\left(1-\alpha\right)P_0^{(0)} \leq 0,$$

and therefore, we have that $\mathbb{P}\left[R_{\alpha,\varphi} > 0\right] = 0 \leq \mathbb{P}\left[R_\Psi > 0\right]$.

□

Theorem 5.1 shows that in case the insurer wants to reduce the loss probability, the proportion α of the risk that is borne by the policyholders has to be set according to (5.4.16). This observation raises the question of whether policyholders would be interested in buying such contracts. We investigate this question in the following subsection.

5.4.3 Policyholders' perspective

The goal of this subsection is to derive an additional constraint on the loadings and on the risk-sharing scheme. In particular, we take into account the constraint that contracts priced under the dynamic equivalence principle should have lower premiums compared to their classical counterparts.

Let us first derive the actuarial value of the premiums per-policy for each contract. Obviously, for the classical contract we simply have P_Ψ. For the dynamic contract, policyholders pay the single premium $P_0^{(0)}$ at time 0 and potentially some additional amounts $P_k^{(k)}$. The time-0 random present value of all payments per-policy is given by

$$\Pi = P_0^{(0)} + \sum_{k=1}^{t} I(0,k) P_k^{(k)} v^k = P_0^{(0)} + \alpha \frac{P_0^{(0)}}{{}_tp_x^{(0)}} \left(I(0,t) - {}_tp_x^{(0)} \right). \quad (5.4.20)$$

We find immediately that $\mathbb{E}_0[\Pi] = P_0^{(0)}$, which implies that policyholders are not expected to pay additional premiums.

We now state the theorem showing that under a condition on Ψ, φ and α, it remains favorable to buy a contract under the dynamic setting, although policyholders may have to pay additional amounts in the future. The reasoning here is based on the fact that if the dynamic contract leads to lower premiums (including the future potential payments), then it would be preferred by profit-seeking policyholders regardless of their risk preference.

Theorem 5.2. *A pure endowment contract with single premium and loading φ priced under the dynamic equivalence principle is more favorably priced than its counterpart with loading Ψ and priced using a classical premium principle if the yearly share $\alpha \in [0,1]$ satisfies the following condition:*

$$\alpha \leq \frac{{}_tp_x^{(0)}}{1 - {}_tp_x^{(0)}} \frac{\Psi - \varphi}{\pi_0^{(0)} + \varphi}. \quad (5.4.21)$$

Proof. For a fixed benefit b_t, if $\Pi \leq P_\Psi$, then the dynamic contract is more favorably priced than the classical one. Since P_Ψ is a constant, this inequality is always fulfilled if and only if the upper bound of the support of Π, that is

$$P_0^{(0)} + \alpha \frac{P_0^{(0)}}{{}_tp_x^{(0)}} (1 - {}_tp_x^{(0)}),$$

is smaller than P_Ψ, which completes the proof. □

Remark 5.1. *The result in Theorem 5.2 provides a general upper bound on the share α from the viewpoint of the policyholders. However, it is worth noting that this upper bound is rather conservative, since it corresponds to the worst case scenario for the number of survivors. Thus, it is possible to obtain alternative upper bounds for α which are greater than the upper bound in (5.4.21) under some specific settings. For instance, this can be achieved in an expected utility framework, which requires additional assumptions on the utility function of policyholders and on the distribution of the survival index $I(0,t)$.*

5.4.4 Viable risk-sharing scheme for pure endowments

It appears from Theorems 5.1 and 5.2 that an appropriate updating mechanism has to comply with two conditions. The first one aims at improving the solvency situation of the insurer whereas the second one follows from the fact that the contract has to remain appealing to policyholders. It is however important that both conditions do not conflict. In the following, we introduce a definition for a viable risk-sharing scheme.

Definition 5.2 (Viable risk-sharing scheme for pure endowments)**.** *Consider a pure endowment contract priced under a dynamic equivalence principle (characterized by a loading φ and a yearly share α) and a classical pure endowment (with loading Ψ). Moreover, assume that*

$$b_t v^t \leq P_\Psi \tag{5.4.22}$$

holds in case:

$$\Psi - \varphi \leq b_t v^t \left(1 - {}_tp_x^{(0)}\right). \tag{5.4.23}$$

Then, compared to the classical contract, the proposed updating mechanism improves the solvency situation of the insurer and is more favorably priced for policyholders if the following condition holds:

$$\frac{\pi_0^{(0)}}{\Psi} \frac{\Psi - \varphi}{\pi_0^{(0)} + \varphi} \leq \alpha \leq \min\left\{ \frac{{}_tp_x^{(0)}}{1 - {}_tp_x^{(0)}} \frac{\Psi - \varphi}{\pi_0^{(0)} + \varphi}, 1 \right\}. \tag{5.4.24}$$

The above definition provides a condition on the contract such that both the insurer and the policyholders are better off with the updating mechanism. We can extract from (5.4.24) some limiting cases. On the one hand, setting $\varphi = \Psi$ leads to $\alpha = 0$, which means that if the dynamic contract is as expensive as the classical one, then there must be no additional premiums.

On the other hand, setting $\varphi = 0$ leads to $\alpha = 1$, which means that if the dynamic contract does not include any loading, then the policyholder should bear all the deviation risk. Moreover, imposing $\alpha \in [0, 1]$ implies that the initial price of the dynamic contract has to be cheaper than that of the classical one, i.e. $\varphi \leq \Psi$. In this sense, the viable risk-sharing scheme is consistent with intuition.

Inequality (5.4.22) ensures the existence of a range of α on which both parties agree in case (5.4.23) is fulfilled. Inequality (5.4.23) compares the difference between worst-estimate premium (i.e. when the t-year survival probability is set to 1) and the best-estimate premium (i.e. when the t-year survival probability is given by ${}_tp_x^{(0)}$) with the difference between the loadings Ψ and φ. Thus, these inequalities mean that if the difference between the prices of the classical and dynamic contracts is lower than the difference between the worst- and best-estimate premiums, then the loaded premium for the classical contract has to be at least equal to the worst-estimate premium.

5.4.5 Analysis of the viable risk-sharing scheme

We study the pairs (α, φ) leading to a viable risk-sharing scheme by considering three specific cases. The analysis is performed by determining the possible values of that pair for three different values of the ratio:

$$\gamma = \frac{\Psi}{\pi_0^{(0)}}.$$

For illustrative purpose only, assume that the time-0 estimate of the t-year survival probability ${}_tp_x^{(0)}$ is given by 0.981. This number has been estimated from a Lee-Carter model, using data for both Belgian males and females which covers the period $1974 - 2015$ and ages $35 - 99$. It corresponds to the 30-year survival probability of a life aged 35. The estimation procedure follows the methodology described in Pitacco et al. (2009).

Figure 5.4.1 displays an example where γ is such that inequality (5.4.22) is not satisfied. It is then straightforward to show that for $\varphi \geq 0$, inequality (5.4.23) will always be satisfied. In this case, the lower bound from (5.4.24) is greater than the upper bound. As a consequence, we cannot find any α on which the insurer and policyholders would both agree. The reason is that the classical contract is too cheap. Thus, against a too cheap contract, policyholders would chose the dynamic one only if α is sufficiently low, and in particular, lower than the minimum required by the insurer to enhance

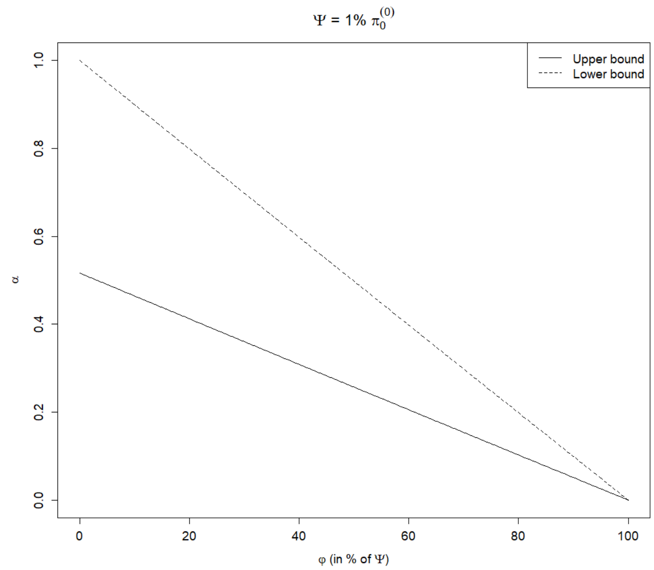


Figure 5.4.1: Bounds of α from inequality (5.4.24) as a function of φ, where γ is such that inequality (5.4.23) is satisfied, while (5.4.22) is not.

its solvency situation. This translates into a conflict between the constraints of the two parties.

In the left panel of Figure 5.4.2, the ratio γ is such that (5.4.22) becomes an equality. Again, we have that for $\varphi \geq 0$, inequality (5.4.23) is always fulfilled. Moreover, since (5.4.22) is an equality, we have that:

$$\gamma = \gamma^\star = \frac{1 - {}_tp_x^{(0)}}{{}_tp_x^{(0)}},$$

and in this example we have $\gamma^\star \approx 1.94\%$. Additionally, combining the fact that (5.4.23) is fulfilled with $\gamma = \gamma^\star$, we find that this case implies an equality between the upper and lower bounds from (5.4.24). This means that for each value of φ, there exists a unique value of α on which the two parties agree. In particular, any pair (α, φ) on that line of the graph will have the same loss probability for the insurer, and the same price for policyholders, taking into account their potential future payments. We can also conclude that a loading $\Psi = \gamma^\star \pi_0^{(0)}$ is the optimal loading for a classical contract in the sense of (5.4.24). The right panel of Figure 5.4.2 displays the ratio $\gamma^\star$ as a function of the initial estimate ${}_tp_x^{(0)}$. Clearly, higher values of this estimate

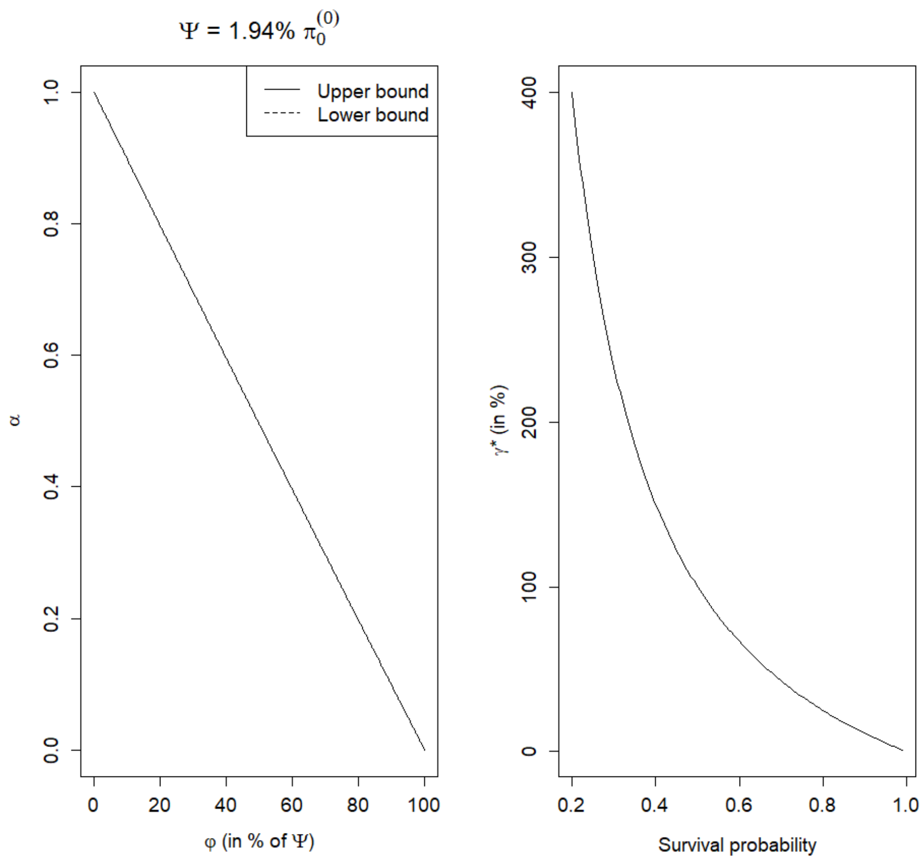


Figure 5.4.2: Left: Bounds of α from inequality (5.4.24) as a function of φ where γ is such that (5.4.22) becomes an equality. Right: Ratio $\gamma^\star$ as a function of the initial estimate of the t-year survival probability.

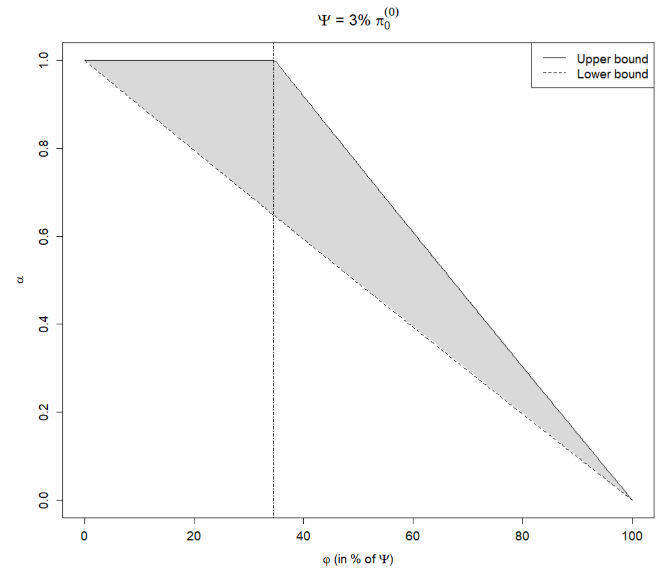


Figure 5.4.3: Bounds of α from inequality (5.4.24) as a function of φ, where γ is such that inequality (5.4.22) is satisfied. The shaded area provides the possible (α, φ) leading to a viable risk-sharing scheme.

imply lower values of $\gamma^\star$. This would typically be the case for (relatively) short-term contracts or for young policyholders. Therefore, for high value of ${}_tp_x^{(0)}$, the threshold level of the loading Ψ (relatively to the pure premium) such that a viable risk-sharing scheme can be determined will be very low.

In Figure 5.4.3, inequality (5.4.23) does not hold before the point $\varphi \approx 35.4\%\Psi$, but it does hold after that point. Moreover, the ratio γ is chosen such that (5.4.22) is satisfied in both cases, although this condition is not necessary for the existence of α before $\varphi \approx 35.5\%\Psi$. This implies that for each value of φ, we can find a range of α on which both parties agree. Thus, the possible pairs (α, φ) leading to a viable risk-sharing scheme constitute a surface, which depends on the value of γ. In particular, we observe that if the loading of the classical contract is too high, then even for high values of φ (i.e. before $\varphi \approx 35.4\%\Psi$), policyholders would still prefer to bear a significant part of the deviation risk. Note that the point $\varphi \approx 35.4\%\Psi$ is such that (5.4.23) becomes an equality, and thus, the ratio 35.4% from this example is determined from:

$$1 - \frac{\gamma^\star}{\gamma}.$$

5.5 Term annuity with single premium

5.5.1 The updating mechanism

Consider on a portfolio of t-year term annuity contracts sold to l_x policyholders aged x at time 0. The contract pays a yearly benefit of $b^{(0)}$ at times $1, 2, ..., t$, as long as the policyholder is alive. The single pure premium for this contract is denoted by $\pi_0^{(0)}$. Similarly to the procedure of Section 5.4, we derive here updating rules for a term annuity with single premium, as well as conditions on the loadings and the yearly shares of the loss which is transferred back to policyholders.

From the actuarial equivalence principle (5.3.2), we find that the pure premium $\pi_0^{(0)}$ at contract inception is given by:

$$\pi_0^{(0)} = b^{(0)} a^{(0)}_{\overline{x:t|}}, \tag{5.5.1}$$

with

$$a^{(0)}_{\overline{x:t|}} = \sum_{j=1}^{t} v^j \ {}_jp_x^{(0)}.$$

The loaded single premium $P_0^{(0)}$ is the sum of the pure single premium and a positive loading φ, i.e.:

$$P_0^{(0)} = \pi_0^{(0)} + \varphi.$$

Notice again that $P_0^{(0)}$ can be interpreted as the pure premium for a contract paying a yearly benefit $b^{(0)} + \frac{\varphi}{a^{(0)}_{\overline{x:t|}}}$.

At time 1, the value of the retrospective reserve per-policy still in force is:

$$V_1^{(0)} = \frac{P_0^{(0)}}{vI(0,1)}. \tag{5.5.2}$$

The new information available at time 1 leads to new estimates ${}_jp_{x+1}^{(1)}$, $j = 1, ..., t-1$, of the j-year survival probabilities of the policyholder now aged $x+1$. From time 1 on, the remaining yearly benefits $b^{(0)}$ paid at times $1, 2, ..., t$ are replaced by $b^{(1)}$, and/or an additional amount $P_1^{(1)}$ is paid by the policyholder at time 1. Thus, the present values of future premiums using the information available at time 1 before and after the updating are given by:

$$\mathbb{E}_1\left[\Pi_1^{(0)}\right] = 0 \qquad \text{and} \qquad \mathbb{E}_1\left[\Pi_1^{(1)}\right] = P_1^{(1)}, \tag{5.5.3}$$

respectively, whereas the present values of future benefits using the information available at time 1 before and after the updating are given by:

$$\mathbb{E}_1\left[B_1^{(0)}\right]=\left(b^{(0)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}, \tag{5.5.4}$$

and

$$\mathbb{E}_1\left[B_1^{(1)}\right]=\left(b^{(1)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}, \tag{5.5.5}$$

respectively. Plugging Expressions (5.5.3)–(5.5.5) as well as (5.5.2) in Equation (5.3.9) and multiplying both sides by $\left(-\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}\right)^{-1}$ leads to:

$$b^{(1)}-\frac{P_1^{(1)}}{\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}}=b^{(0)}-\alpha_1\left(\left(b^{(0)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)-\frac{P_0^{(0)}}{vI(0,1)\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}}\right). \tag{5.5.6}$$

Furthermore, by noting that $P_0^{(0)}=\left(b^{(0)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)a^{(0)}_{x:t|}$, we find that $b^{(1)}$ and $P_1^{(1)}$ satisfy the following equation:

$$b^{(1)}-\frac{P_1^{(1)}}{\sum_{j=0}^{t-1}v^j\ {}_jp^{(1)}_{x+1}}=b^{(0)}-\alpha_1\left(b^{(0)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)\left(1-\frac{a^{(0)}_{x:t|}}{I(0,1)\sum_{j=0}^{t-1}v^{j+1}\ {}_jp^{(1)}_{x+1}}\right). \tag{5.5.7}$$

Using (5.5.7) and rearranging leads to the following expression of the available reserve at time 2:

$$V_2^{(1)}=\left(b^{(1)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)\frac{\sum_{j=0}^{t-1}v^{j+1}\ {}_jp^{(1)}_{x+1}}{v^2I(1,2)}.$$

Applying again the dynamic equivalence principle (5.3.9) at time 2 leads to the following equation for the updated value of the benefits $b^{(2)}$ and the additional premium $P_2^{(2)}$:

$$b^{(2)}-\frac{P_2^{(2)}}{\sum_{j=0}^{t-2}v^j\ {}_jp^{(2)}_{x+2}}=b^{(1)}-\alpha_2\left(b^{(1)}+\frac{\varphi}{a^{(0)}_{x:t|}}\right)\left(1-\frac{\sum_{j=1}^{t-1}v^{j+1}\ {}_jp^{(1)}_{x+1}}{I(1,2)\sum_{j=0}^{t-2}v^{j+2}\ {}_jp^{(2)}_{x+2}}\right).$$

At time k, for $k = 2, ..., t$, the retrospective reserve $V_k^{(k-1)}$ is given by:

$$V_k^{(k-1)} = \left(b^{(k-1)} + \frac{\varphi}{a_{\overline{x:t|}}^{(0)}} \right) \frac{\sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_jp_{x+k-1}^{(k-1)}}{v^k I(k-1,k)}, \tag{5.5.8}$$

such that $b^{(k)}$ and $P_k^{(k)}$ satisfy:

$$b^{(k)} - \frac{P_k^{(k)}}{\sum_{j=0}^{t-k} v^j \ {}_jp_{x+k}^{(k)}} = b^{(k-1)} - \alpha_k \left(b^{(k-1)} + \frac{\varphi}{a_{\overline{x:t|}}^{(0)}} \right) \left(1 - \frac{\sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_jp_{x+k-1}^{(k-1)}}{I(k-1,k) \sum_{j=0}^{t-k} v^{j+k} \ {}_jp_{x+k}^{(k)}} \right). \tag{5.5.9}$$

We focus now on the two particular cases, where the contract allows for the adjustment of either the premium plan only, or the benefit package only.

Case 3 (Updating the premium plan only)**.** *In case the benefit is not subject to revision, we find from* (5.5.8) *that the reserve at time k is as follows:*

$$V_k^{(k-1)} = \frac{P_0^{(0)}}{v^k a_{\overline{x:t|}}^{(0)}} \frac{\sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_jp_{x+k-1}^{(k-1)}}{I(k-1,k)}.$$

Moreover, we find from (5.5.9) *that the additional amounts $P_k^{(k)}$ required from the policyholders at the successive times $k = 1, 3, ..., t$ are given by:*

$$P_k^{(k)} = \alpha_k \frac{P_0^{(0)}}{v^k a_{\overline{x:t|}}^{(0)}} \left(\sum_{j=0}^{t-k} v^{j+k} \ {}_jp_{x+k}^{(k)} - \frac{1}{I(k-1,k)} \sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_jp_{x+k-1}^{(k-1)} \right). \tag{5.5.10}$$

Notice the similarity between this expression and that of the additional amount for pure endowments given in (5.4.10)*. Indeed, it is straightforward to show that if all yearly benefits of the term annuity are equal to* 0*, except that of year t, then the expression for the additional amount in case of a term annuity reduces to the expression obtained for the pure endowment.*

Case 4 (Updating the benefit package only)**.** *Let us now assume that policyholders do not pay additional premiums. Instead, the benefit at time t can be revised throughout the contract to account for deviations. In this case, it follows directly from* (5.5.9) *that the time-k updated value of the benefit has the following expression:*

$$b^{(k)} = b^{(k-1)} - \alpha_k \left(b^{(k-1)} + \frac{\varphi}{a_{\overline{x:t|}}^{(0)}} \right) \left(1 - \frac{\sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_jp_{x+k-1}^{(k-1)}}{I(k-1,k) \sum_{j=0}^{t-k} v^{j+k} \ {}_jp_{x+k}^{(k)}} \right), \tag{5.5.11}$$

which is again similar to that of the pure endowment contract. Note that the expression of the available reserve (5.5.8) at time k still holds in this case, as it is expressed in terms of the updated benefit.

Let us now set $b^{(k)} = b$, for $k = 0, 1, ..., t$ and focus on Case 3. Moreover, we assume that $\alpha_k = \alpha$ for all $k = 1, 2, ..., t$ and determine bounds for α such that it is advantageous for both parties to engage in an annuity contract managed by the dynamic equivalence principle. Analogously to the pure endowment contract, we define a classical term annuity as a contract without updating (i.e. $\alpha = 0$) whose single premium is denoted by P_Ψ:

$$P_\Psi = \pi_0^{(0)} + \Psi, \tag{5.5.12}$$

with $\Psi > 0$.

5.5.2 Impact on the insurer's solvency

We determine in this subsection conditions on the risk-sharing scheme such that it is safer for the insurer to sell a term annuity managed with a dynamic equivalence principle instead of its counterpart under the classical setting. The time-0 shortfall risk per-policy for a portfolio of term annuities priced in the classical static setting is denoted by R_Ψ, whereas its counterpart priced in the dynamic setting is denoted by $R_{\alpha,\varphi}$. For the classical contract, we have:

$$R_\Psi = b \sum_{k=1}^{t} v^k I(0,k) - P_\Psi, \tag{5.5.13}$$

which can also be written as:

$$R_\Psi = \left(\frac{\sum_{k=1}^{t} v^k I(0,k)}{a_{\overline{x:t|}}^{(0)}} - 1 \right) \pi_0^{(0)} - \Psi. \tag{5.5.14}$$

On the other hand, we find in the dynamic case:

$$R_{\alpha,\varphi} = \sum_{k=1}^{t} v^k I(0,k) - P_0^{(0)} - \sum_{k=1}^{t} v^k I(0,k) P_k^{(k)}. \tag{5.5.15}$$

First, note that the sum

$$\sum_{k=1}^{t} \left(I(0,k) \sum_{j=0}^{t-k} v^{j+k} \ {}_j p_{x+k}^{(k)} - I(0,k-1) \sum_{j=1}^{t-k+1} v^{j+k-1} \ {}_j p_{x+k-1}^{(k-1)} \right)$$

reduces to $\sum_{k=1}^{t} v^k I(0,k) - \sum_{k=1}^{t} v^k \ {}_kp_x^{(0)}$. Therefore, plugging Expression (5.5.10) of $P_k^{(k)}$ in (5.5.15) leads to following expression of $R_{\alpha,\varphi}$:

$$R_{\alpha,\varphi} = b\sum_{k=1}^{t} v^k I(0,k) - P_0^{(0)} - \alpha \frac{P_0^{(0)}}{a_{x:t|}^{(0)}} \left(\sum_{k=1}^{t} v^k I(0,k) - \sum_{k=1}^{t} v^k \ {}_kp_x^{(0)} \right),$$

or, in its simplified form:

$$R_{\alpha,\varphi} = \left(\frac{\sum_{k=1}^{t} v^k I(0,k)}{a_{x:t|}^{(0)}} - 1 \right) \left(\pi_0^{(0)} - \alpha P_0^{(0)} \right) - \varphi. \tag{5.5.16}$$

The following theorem shows that the condition on Ψ, φ and α such that the probability of a loss for a pure endowment contract priced under the dynamic equivalence principle is lower compared to its classical counterpart, remains the same for the term annuity contract.

Theorem 5.3. *A term annuity contract with single premium and loading φ priced under the dynamic equivalence principle has a lower loss probability than its counterpart with loading Ψ and priced using a classical premium principle, if the yearly share $\alpha \in [0,1]$ satisfies (5.4.16).*

Proof. The proof is similar to that of Theorem 5.1. □

5.5.3 Policyholders' perspective

In this subsection, we take the perspective of policyholders and derive conditions on Ψ, φ and α such that contracts priced under the dynamic equivalence principle have lower premiums compared to their classical counterparts.

The actuarial value at time 0 of the premiums per-policy for the classical contract is simply P_Ψ, whereas for the dynamic contract, it is given by:

$$\Pi = P_0^{(0)} + \sum_{k=1}^{t} I(0,k) P_k^{(k)} v^k = P_0^{(0)} + \alpha \frac{P_0^{(0)}}{a_{x:t|}^{(0)}} \left(\sum_{k=1}^{t} v^k I(0,k) - a_{x:t|}^{(0)} \right). \tag{5.5.17}$$

Again, we find for the term annuity managed with the dynamic equivalence principle that $\mathbb{E}_0[\Pi] = P_0^{(0)}$, i.e. policyholders are not expected to pay additional premiums.

We now state the theorem showing that under a condition on Ψ, φ and α, it remains favorable to buy a term annuity under the dynamic setting, although policyholders may have to pay additional amounts in the future.

Theorem 5.4. *A term annuity contract with single premium and loading φ priced under the dynamic equivalence principle is more favorably priced than its counterpart with loading Ψ and priced using a classical premium principle if the yearly share $\alpha \in [0, 1]$ satisfies the following condition:*

$$\alpha \leq \frac{a^{(0)}_{x:t|}}{\sum_{k=1}^{t} v^k - a^{(0)}_{x:t|}} \frac{\Psi - \varphi}{\pi^{(0)}_0 + \varphi}. \tag{5.5.18}$$

Proof. For a fixed benefit b_t, if $\Pi \leq P_\Psi$, then the dynamic contract is more favorably priced than the classical one. Since P_Ψ is a constant, this inequality is always fulfilled if and only if the upper bound of the support of Π, that is

$$P^{(0)}_0 + \alpha \frac{P^{(0)}_0}{a^{(0)}_{x:t|}} \left(\sum_{k=1}^{t} v^k - a^{(0)}_{x:t|} \right),$$

is smaller than P_Ψ, which completes the proof. □

5.5.4 Viable risk-sharing scheme for term annuities

Taking into account the constraints of the insurer and policyholders derived in Theorems 5.3 and 5.4, we can now define a viable risk-sharing scheme for term annuity contracts.

Definition 5.3 (Viable risk-sharing scheme for term annuities)**.** *Consider a term annuity contract priced under a dynamic equivalence principle (characterized by a loading φ and a yearly share α) and a classical pure endowment (with loading Ψ). Moreover, assume that*

$$b \sum_{k=1}^{t} v^k \leq P_\Psi \tag{5.5.19}$$

holds in case:

$$\Psi - \varphi \leq b \left(\sum_{k=1}^{t} v^k - a^{(0)}_{x:t|} \right). \tag{5.5.20}$$

Then, compared to the classical contract, the proposed updating mechanism improves the solvency situation of the insurer and is more favorably priced for policyholders if the following condition holds:

$$\frac{\pi_0^{(0)}}{\Psi}\frac{\Psi-\varphi}{\pi_0^{(0)}+\varphi} \leq \alpha \leq \min\left\{\frac{a_{x:t|}^{(0)}}{\sum_{k=1}^{t} v^k - a_{x:t|}^{(0)}}\frac{\Psi-\varphi}{\pi_0^{(0)}+\varphi}, 1\right\}. \tag{5.5.21}$$

The conditions derived in the above definition for term annuities are very similar to those of the pure endowments, and the discussion on their limiting cases, as well as on the interpretation of Inequalities (5.5.19) and (5.5.20) remain valid here. In particular, Inequalities (5.5.19) and (5.5.20) mean that if the difference between the prices of the classical and dynamic contracts is lower than the difference between the worst- and best-estimate premiums, then the loaded premium for the classical contract has to be at least equal to the worst-estimate premium.

5.6 Concluding remarks

In this chapter, we have addressed systematic longevity risk in long-term insurance business in a setting where both the assumption of independence and the assumption of known survival probabilities are violated. Increasing the size of the portfolio remains efficient for reducing the diversifiable part of the risk, but the deviation risk cannot be eliminated in this way. It appears that transferring the risk, or at least part of it, to policyholders is an efficient solution. However, in order for a risk-sharing scheme to be viable, it should meet both the insurer's and the policyholders' constraints.

Any updating mechanism may suffer from a transparency drawback when the initial estimates are compared to the portfolio survival index. A solution coping with this issue consists in comparing the initial predictions with their corresponding realizations in a reference group (e.g. the general population of the country) instead of the realizations in the portfolio. This would enhance the transparency of the updating scheme, but in turn leaves the insurance company with an extra basis risk, on top of the random variations in the number of survivors. This approach can be a topic for future research.

Part III

On the interplay between longevity and mortality

CHAPTER 6

Pricing insurance contracts with offsetting relationship

6.1 Introduction

Exploiting the offsetting relationship between the values of annuity and life insurance portfolios provides insurers with two main advantages. The first one is risk reduction, as insurers active on these two businesses can benefit from a *natural hedging* effect to cope with unforeseeable systematic longevity and mortality evolutions. The existing literature has focused mostly on this aspect, and in particular on determining the optimal product mix under different settings which reduces the portfolio's risk exposure according to some meaningful risk measure. For instance, Gründl et al. (2006) use the probability of default as a constraint to determine the optimal weights and include the fact that the maximum number of policyholders in each business line may be impacted by the probability of default of the insurance company. Tsai et al. (2010) base their optimization problem on the conditional value-at-risk and set an arbitrary Sharp ratio for the loaded premium. Wang et al. (2010) derive a duration-based optimal mix which immunizes the combined portfolio against parallel shifts of the force of mortality. We also refer to Wang et al. (2013), Cox et al. (2013), Gatzert and Wesker (2014), Li and Haberman (2015), Luciano et al. (2017) and Wong et al. (2017).

The second benefit of the offsetting relationship is competitiveness, and has barely been addressed in the literature. Intuitively speaking, a company selling both annuities and life insurances would experience a loss on the annuity portfolio (resp. life insurance portfolio) in case people live longer (resp. shorter) than expected, but this loss would be compensated for on the life insurance portfolio (resp. annuity portfolio). Therefore, since the insurer is less exposed to systematic longevity and mortality deviations, the risk loadings of both business lines may be reduced, which results in more favorably priced contracts. Empirical research conducted by Cox and Lin (2007) supports this intuition but there is little analytical research in this stream.

Bayraktar and Young (2007) show that the prices of pure endowments and term insurance contracts are lower when their offsetting relation is exploited, but only under some specific assumptions. However, on the one hand, they do not provide insights on each business line, and on the other hand, they do not analyze the challenges of insurers relying on joint pricing. Moreover, the premium loading in Bayraktar and Young (2007) is exogenous and independent of the portfolio composition. As most papers have already shown, the risk of insurers relying on natural hedging depends on the relative contribution of each business line to the overall risk. Thus, analyzing the premium loadings and the benefits of joint pricing has to take into account the impact of the different factors at play, in particular that of portfolio composition. Another issue which has not yet been addressed is how switching from a stand-alone pricing to a joint pricing can impact the demand in each business line.

This chapter attempts to fill this gap by shedding light on some challenges in pricing offsetting businesses together. We present the work in progress of Hanbali and Villegas (2019), in which we consider a setting where the insurer charges on top of the pure premium a loading determined from a target risk reduction at portfolio level. Throughout the analysis, we assume that the insurer's risk is equal under the two competing settings. In the numerical analysis, we work under two mortality models, namely the standard Lee-Carter model and a two-population gravity extension where future paths of mortality in the two business lines are not perfectly dependent. As a first step, we analyze the conditional required loaded premium given the business composition and benefit ratio. We find that joint pricing does not necessarily lead to lower premiums, especially on the annuity side, which is the business line that has the lowest per-policy contribution to the overall portfolio risk under our assumptions. Moreover, the choice of competitiveness requires a careful monitoring of the portfolio. Since the insurer has less control over the business composition than over the product structure, we investigate how the insurer can gain more flexibility over business composition at the expense of the flexibility over product design. In a second step, we consider an insurer active on both businesses with a stand-alone pricing strategy and analyze how switching to a joint pricing strategy impacts the demand in each business line and the total collected premiums. Using a model inspired from Gründl et al. (2006), we find that the switch can result in a decrease of the demand of term annuities, and insurers may experience a reduction in the total collected premiums. More importantly, we argue that the decision of the switch has to be carefully analyzed taking into account several factors, and the reaction of policyholders to a change of tariff as well as the contribution of each business line to the overall risk are prime ones.

The remainder of the chapter is organized as follows. Section 6.2 contains the expressions for the stand-alone and conditional joint pricing. Moreover, we present in the same section the model used to analyze whether it is favorable to switch from the stand-alone to the joint pricing. Section 6.3 is devoted to the mortality models, namely, the standard Lee-Carter model and a gravity-Lee-Carter model. The numerical analyses are conducted in Section 6.4, where we start with the analysis of the conditional joint pricing and then discuss the consequences of the switch. We conclude in Section 6.5 with a discussion on possible future directions. The Appendix 6.6 contains additional figures supporting the robustness of the analysis.

6.2 Pricing models

6.2.1 Actuarial quantities

We consider an insurer with a portfolio of term annuity and term insurance contracts. In order to clear out the paper from cumbersome notations, we assume that there is a single age group in each business line. The random present value at contract inception of a term annuity sold at time 0 to a policyholder aged x at that time is denoted by V_A. Similarly, for a term insurance sold at time 0 to a policyholder aged y which is denoted by V_I. For T_A and T_I the respective maturities of the two contracts, we write:

$$V_A = B_A \sum_{k=1}^{T_A} I_A(0,k)v(0,k), \tag{6.2.1}$$

and

$$V_I = B_I \sum_{k=1}^{T_I} I_I(0, k-1)\,(1 - I_I(k-1, k))\,v(0, k), \tag{6.2.2}$$

where $v(0, k)$ is the discount factor on the time interval $(0, k)$, B_A is the yearly annuity benefit and B_I is the insurance benefit which is paid at the end of the year of death. We assume throughout the paper that there is no market risk and we take a constant discounting factor $v(0, k) = v^k$ for all k's. This assumption is used essentially to ease the presentation and can be relaxed for a broader setting. The random variables $I_A(j, k)$ (or equivalently $I_I(j, k)$), for $j \leq k = 0, 1, ...$, are the survival indices of the corresponding business line from time j to time k, such that $I_A(j, k)$ (or $I_I(j, k)$) is equal to 1 if all policyholders in that business line at time 0 who survived until time j are still alive at time k, and it is equal to 0 if they all die.

Each of the contracts introduced above is sold against a single premium. The premiums are determined from the actuarial equivalence principle and include an additive loading which allows the insurer to compensate for the uncertainty on the survival-or-not of the policyholders. The loaded premiums of the contracts in (6.2.1) and (6.2.2) are given by:

$$P_A = \pi_A + \Psi_A, \tag{6.2.3}$$

and

$$P_I = \pi_I + \Psi_I, \tag{6.2.4}$$

respectively, where π refers to the pure premium derived from the actuarial equivalence, i.e. the expected present value of the contract, and the Ψ's are the risk premiums, or premium loadings, which are here determined from a risk measure that quantifies the risk associated to the contracts payoff. More details about premium principles can be found in Kaas et al. (2008), among others.

6.2.2 Stand-alone pricing

In case the insurer does not rely on natural hedging, the loadings are set according to the specific risk of each contract. Suppose that the insurer determines the loaded premiums such that the loss, which is measured by some appropriate risk measure, for each contract separately is reduced by a certain factor compared to the case where there is no loading. Thus, the loaded premiums in case of stand-alone pricing are denoted by P_A^{sa} for the term annuity and P_I^{sa} for the term insurance, and follow from:

$$\varphi\left[V_A - P_A^{sa}\right] = (1-\zeta)\,\varphi\left[V_A - \pi_A\right],$$

and

$$\varphi\left[V_I - P_I^{sa}\right] = (1-\zeta)\,\varphi\left[V_I - \pi_I\right].$$

The factor ζ is the target risk reduction and φ is a risk measure assumed to be positive homogeneous and translation invariant. Additionally, φ may be subadditive, which will help to account explicitly for the offsetting relationship between the two business lines. Thus, the stand-alone loaded premiums P_A^{sa} and P_I^{sa} are given by:

$$P_A^{sa} = \pi_A + \zeta\left(\varphi\left[V_A\right] - \pi_A\right), \tag{6.2.5}$$

and

$$P_I^{sa} = \pi_I + \zeta\left(\varphi\left[V_I\right] - \pi_I\right). \tag{6.2.6}$$

Note that the contribution to the overall risk of one business line does not impact the premium of the product in the other business line.

6.2.3 Conditional joint pricing

We consider now the case where the insurer exploits the offsetting relationship between the two business lines. We introduce the notation N_A and N_I for the number of term annuities and number of term insurances, respectively, underwritten at contract initiation. The proportion of term insurance contracts is given by $n = \frac{N_I}{N_A+N_I}$. An important remark is that determining the loaded premiums when the contracts are priced jointly requires a knowledge about the numbers N_A and N_I. However, these quantities are unknown when the loaded premium is set, and will eventually be impacted by the tariff. Therefore, we shall interpret the loaded premiums in this section as the conditional required premiums associated to a given realization of N_A and N_I.

Let P_A^{nh} and P_I^{nh} be the required loaded premiums for an insurer exploiting the offsetting relationship between term insurances and term annuities. We write:

$$P_A^{nh} = \pi_A + \Psi_{ptf}(n), \tag{6.2.7}$$

and

$$P_I^{nh} = \pi_I + \Psi_{ptf}(n), \tag{6.2.8}$$

where $\Psi_{ptf}(n)$ is the required risk premium conditionally on n, which is the same for both business lines. Note that in the notation $\Psi_{ptf}(n)$, we emphasize on the dependence on the proportion n. In more general terms, the portfolio risk premium depends on the contribution of each business line to the overall risk. This contribution does not depend on the number of underwritten policies only, but also on the characteristics of the contracts, e.g. the benefit level and the maturity. However, capturing the contribution of each business line is not straightforward. Instead, we investigate the effect of each component separately.

The loaded premiums P_A^{nh} and P_I^{nh} can be determined analogously to the stand-alone case. Nevertheless, in order to isolate the impact of the offsetting relationship on the pricing only, we determine P_A^{nh} and P_I^{nh} such that the overall risk in the joint pricing case and in the stand-alone case are equal:

$$\varphi\left[N_A\left(V_A - P_A^{nh}\right) + N_I\left(V_I - P_I^{nh}\right)\right] = N_A\varphi\left[V_A - P_A^{sa}\right] + N_I\varphi\left[V_I - P_I^{sa}\right].$$

Therefore, the required conditional risk premium is given by:

$$\begin{aligned}\Psi_{ptf}(n) = \varphi\left[(1-n)\left(V_A - \pi_A\right) + n\left(V_I - \pi_I\right)\right] \\ - (1-\zeta)\left((1-n)\varphi\left[V_A - \pi_A\right] + n\varphi\left[V_I - \pi_I\right]\right),\end{aligned} \tag{6.2.9}$$

which can be plugged in (6.2.7) and (6.2.8) to obtain the corresponding loaded premiums.

It is clear from Expression (6.2.9) that the conditional required risk premium $\Psi_{ptf}(n)$ is a function of the proportion n. As mentioned above, other factors are actually at play. In particular, the portfolio risk premium (and hence the loaded premiums) is also a function of the ratio $b = \frac{B_I}{B_A}$ and is likely to be impacted by the dependence structure between V_A and V_I, via the term $\varphi\left[(1-n)V_A + nV_I\right]$. We explore the impact of all these factors in the numerical section.

6.2.4 Switching from the stand-alone to the joint pricing

Let us now present the setting used to investigate whether it is favorable for an insurer who is active on both business lines to switch from the stand-alone pricing to the joint pricing. Consider an insurer who prices the contracts separately, with premiums P_A^{sa} and demand N_A^{sa} on the term annuity business line, and with premiums P_I^{sa} and demand N_I^{sa} on the term insurance business line. We write $w = \frac{N_I^{sa}}{N_I^{sa}+N_A^{sa}}$. After the tariff has been adjusted to account for the offsetting effect, the premiums P_A^{sa} and P_I^{sa} are adjusted to P_A^{nh} and P_I^{nh}, respectively. Due to this adjustment, the number of policyholders is expected to change from N_A^{sa} to N_A^{nh} on the term annuity side, and from N_I^{sa} to N_I^{nh} on the term insurance side. In particular, we assume that:

$$N_A^{nh} = N_A^{sa}\left(1 - q_A \frac{P_A^{nh} - P_A^{sa}}{P_A^{sa}}\right), \tag{6.2.10}$$

and

$$N_I^{nh} = N_I^{sa}\left(1 - q_I \frac{P_I^{nh} - P_I^{sa}}{P_I^{sa}}\right). \tag{6.2.11}$$

This model implies that if pricing both contracts leads to lower premiums for one of the business lines (e.g. $\frac{P_A^{nh} - P_A^{sa}}{P_A^{sa}} < 0$ for annuity contracts), the insurer can expect the demand in that business line to increase (e.g. $N_A^{nh} > N_A^{sa}$). The impact of the change of tariff on the demand is scaled by a reaction factor q which is assumed to be strictly positive. The reaction factor implies that a 1% increase (resp. decrease) of the tariff for a product leads to a q% decrease (resp. increase) of the number of policyholders in the corresponding business line. This model is inspired by that of Gründl et al. (2006) which constrains the maximum demand in their optimization problem from the hedging perspective by a similar linear relationship involving the ruin probability instead of the change in the price.

We assume again that the loaded premiums in the stand-alone case are determined from the risk reduction constraints with target risk reduction factor ζ. Thus, P_A^{sa} and P_I^{sa} are still given by Expressions (6.2.5) and (6.2.6), respectively. For the joint pricing case, we assume that the insurer's risk before and after the switch remains unchanged. This means that $\Psi_{ptf}(w)$ is such that:

$$\varphi\left[N_A^{nh}\left(V_A - P_A^{nh}\right) + N_I^{nh}\left(V_I - P_I^{nh}\right)\right] = N_A^{sa}\left(\varphi\left[V_A\right] - P_A^{sa}\right) + N_I^{sa}\left(\varphi\left[V_I\right] - P_I^{sa}\right),$$

and rearranging leads to:

$$\begin{aligned}\Psi_{Ptf}(w) = \tfrac{1}{N_A^{nh} + N_I^{nh}}\big(&\varphi\left[N_A^{nh}\left(V_A - \pi_A\right) + N_I^{nh}\left(V_I - \pi_I\right)\right] \\ &- (1-\zeta)\left(N_A^{sa}\varphi\left[V_A - \pi_A\right] + N_A^{sa}\varphi\left[V_I - \pi_I\right]\right)\big).\end{aligned} \tag{6.2.12}$$

In contrast with the setting described in Subsection 6.2.3, the risk premium $\Psi_{ptf}(w)$ which determines the loaded premiums P_A^{nh} and P_I^{nh} is the solution of a non linear equation. This is because N_A^{nh} and N_I^{nh} are functions of $\Psi_{ptf}(w)$. Note also the difference between $\Psi_{ptf}(n)$ from Subsection 6.2.3 and $\Psi_{ptf}(w)$ in Expression (6.2.12) above. The former is the conditional required portfolio risk premium given the portfolio composition n after the products have been launched. The latter is the unconditional required portfolio risk premium associated to the demand proportion w before the switch, where the number of policyholders is modeled explicitly. In both cases, the risk under the competing settings is equal.

6.3 Mortality models

This section describes the mortality models which will support the numerical analyses in Section 6.4. The first one is the so-call Lee-Carter model (henceforth LC model) proposed in Lee and Carter (1992). The second one consists in two correlated Lee-Carter models in the fashion of Carter and Lee (1992), and we add a gravity term to ensure that projections for the two sub-populations do not diverge, as suggested in Dowd et al. (2011). This second model allows to account for the fact that the dependence between the forces of mortality in the two subpopulations is not necessarily perfect. We refer to this adapted version by gravity-Lee-Carter (henceforth gLC).

Under the LC model, the force of mortality $\mu(x, t)$ at age x and time t, which is given by

$$\mu(x, t) = \exp\left(\alpha(x) + \beta(x)\kappa(t)\right), \tag{6.3.1}$$

implies that future paths of mortality for all ages are driven by the same common trend $\kappa(t)$. This is however not necessarily the case in practice due to adverse selection and age basis risk; see e.g. Gatzert and Wesker (2014) and references therein. Moreover, as shown in e.g. Zhu and Bauer (2014) and Li and Haberman (2015), the model used for the dependence between policyholders buying annuities and those buying life insurances can have a substantial impact on the performance of natural hedging. Thus, we relax the extreme positive dependence assumption by considering the gravity based gLC model, in which the forces of mortality for policyholders buying annuities and policyholders buying life insurances are not perfectly dependent. At time t, let $\mu_A(x, t)$ be the force of mortality for policyholders buying term annuity contracts and aged $x = x_1, ..., x_A$. Moreover, let $\mu_I(y, t)$ be the force of mortality for policyholders buying term insurance contracts and aged $y = y_1, ..., y_I$. We write:

$$\mu_A(x, t) = \exp\left(\alpha_A(x) + \beta_A(x)\kappa_A(t)\right), \text{for } x = x_1, ..., x_A, \tag{6.3.2}$$

and

$$\mu_I(y, t) = \exp\left(\alpha_I(y) + \beta_I(y)\kappa_I(t)\right), \text{for } y = y_1, ..., y_I. \tag{6.3.3}$$

Under the gLC model, each subpopulation has its specific time trend, namely, $\kappa_A(t)$ for the portfolio of term annuities and $\kappa_I(t)$ for the portfolio of life insurances. In addition, the dependence between the processes $\kappa_A(t)$ and $\kappa_I(t)$ is modeled using correlated mean-reversion processes, i.e.

$$\kappa_A(t) = m_A + \kappa_A(t-1) + \frac{\gamma_A}{2}\left(\kappa_A(t-1) - \kappa_I(t-1)\right) + \xi_A(t),$$

and

$$\kappa_I(t) = m_I + \kappa_I(t-1) + \frac{\gamma_I}{2}\left(\kappa_I(t-1) - \kappa_A(t-1)\right) + \xi_I(t),$$

where the ξ's are i.i.d. normally distributed over time, with $Cov\left[\xi_A(t), \xi_I(t)\right] = \sigma_A\sigma_I\rho$. Note that for the dynamics of the processes κ_A and κ_I, an alternative which is not investigated here would include jumps in order to account for mortality shocks.

We use data of the US population for the term insurance business line, and that of the English and Welsh population for the annuity business line, obtained from the Human Mortality Database (`www.mortality.org`), over the period 1933-2014. The parameters of $\mu(x,t)$ are estimated by combining data for ages 30-65 of the US population and data for ages 66-90 of the English and Welsh population. For the gLC model, the forces of mortality $\mu_A(x,t)$ and $\mu_I(y,t)$ are estimated using ages 60-90 and 30-65, respectively, from the relevant country. The parameters α and β as well as the patterns of the processes κ from (6.3.1), (6.3.2) and (6.3.3) are obtained using maximum-likelihood estimation with Poisson distribution of the number of deaths, and are subject to the standard identifiability constraints; see e.g. Pitacco et al. (2009) for more details on estimating the Lee-Carter model.

Figure 6.3.1 displays the estimates for these parameters for μ, μ_A and μ_I. The estimates of α, α_A and α_I are very close to each other. However, there are some differences between the estimates of $\beta(x)$ with those of $\beta_A(x)$, for $x = 60, ..., 90$, and between the estimates of $\beta(y)$ with those of $\beta_I(y)$, for $y = 30, ..., 65$. This is also the case for the time trends κ, κ_A and κ_I.

Finally, the estimates of the drift and volatility parameters of the LC model are -0.823 and 1.316, respectively. The parameters of the gravity model are reported in Table 6.1.

6.4 Numerical analyses

The results reported in this section are obtained using Monte-Carlo simulations from the distribution of the time trends determined in Section 6.3.

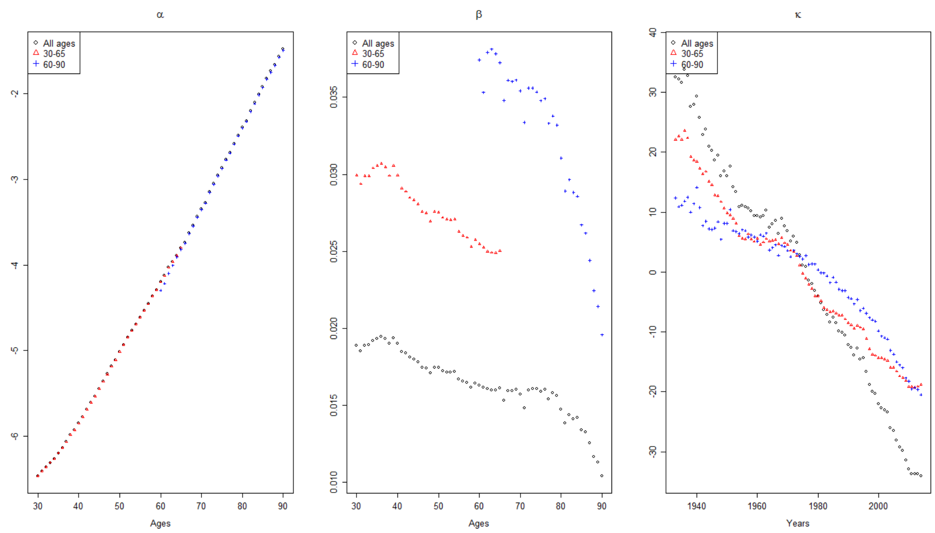


Figure 6.3.1: Estimated parameters of μ, μ_A and μ_I.

Term annuity portfolio		**Term insurance portfolio**	
m_A	-0.403	m_I	-0.504
	(0.131)		(0.078)
σ_A	1.179	σ_I	0.704
	(0.093)		(0.055)
γ_A	-0.161	γ_I	-0.017
	(0.005)		(0.003)
ρ		0.280	
		(0.107)	

Table 6.1: Estimated parameters of the gravity model with standard errors in brackets.

We also apply the jump-off correction described in e.g. Pitacco et al. (2009) and references therein, in order to avoid a jump between the last observed mortality rates and the first simulated ones.

We consider 30-year term annuities sold to policyholders aged 60 and 35-year term insurances sold to policyholders aged 30. The risk measure φ is the conditional value-at-risk at a level ε, which is defined as follows:

$$\mathrm{CVaR}\left[X;\varepsilon\right] = \mathbb{E}\left[X | X \geq F_X^{-1}\left(\varepsilon\right)\right],$$

where F_X^{-1} is the inverse cumulative distribution function of X; see e.g.

Denuit et al. (2005) and Kaas et al. (2008) for further details and properties of CVaR. We set ε to 0.95. The interest rate is assumed to be equal to 1%.

Under this setting, we find that the risk of the individual contracts per unit of benefit is given by $\varphi[V_A]-\pi_A \approx 0.695\times B_A$ and $\varphi[V_I]-\pi_I \approx 0.015\times B_I$ under the LC model, and by $\varphi[V_A]-\pi_A \approx 0.782\times B_A$ and $\varphi[V_I]-\pi_I \approx 0.013\times B_I$ under the gLC model. Thus, we have that $\varphi[V_I]-\pi_I > \varphi[V_A]-\pi_A$ for $b=\frac{B_I}{B_A} > 46$ under the LC model and for $b=\frac{B_I}{B_A} > 60$ under the gLC model. This means that for benefit ratios b greater than 46 under the LC model or greater than 60 under the gLC model, the term insurance business line has a greater per-policy contribution in the overall risk. In what follows, we consider values of b which are sufficiently high (e.g. $b = 100$), such that the term insurance contract has a higher per-policy risk. For low values of b, the conclusions must be adjusted accordingly.

6.4.1 Analysis of the conditional joint pricing

We investigate the impact of the portfolio composition on the loaded premium per unit of benefit by varying n. We consider three values of the risk reduction target ζ, namely, 0%, 50% and 100%. The results are displayed in Figure 6.4.2, where we set $b = 100$. This figure shows clearly that the required loaded premiums under natural hedging and their differences with their stand-alone counterparts depend on n, and also on the dependence between mortality rates of the two subpopulations. Recall that the risk premiums depend also on the benefit ratio b, whose effect will be studied later. Note that we performed the analysis using other values of the interest rate and other functional forms for the risk measure φ. We report in the appendix of this chapter results obtained from the value-at-risk at the confidence level 0.99, and also those obtained with an interest rate of 5%.

Figure 6.4.2 reveals several challenges for insurers who price the contracts jointly. Along the line of previous research which tackles the issue of natural hedging from the portfolio optimization perspective, we observe here that there is also a proportion of underwritten businesses which minimizes the required value of the loaded premium. This implies the existence of a *competitiveness region* for the insurer. In particular, in order to be more competitive compared to insurers who do not rely on joint pricing, the actual premium has to be set between the stand-alone loaded premium (i.e. the horizontal line) and the minimum required loaded premium from the joint pricing. This competitiveness region is clearly influenced by the mortality model, but it is also affected by the value of the target risk reduction factor

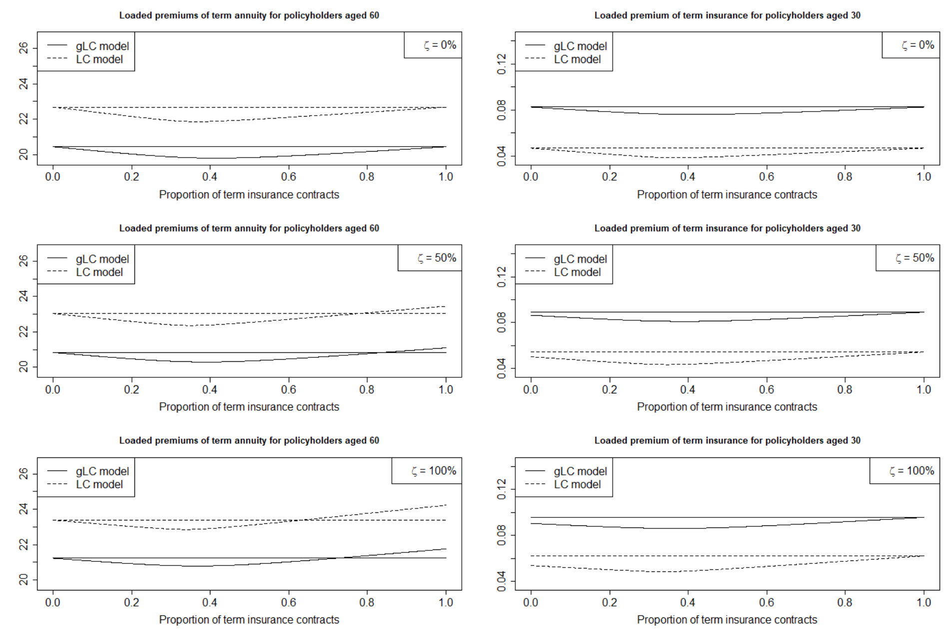


Figure 6.4.2: Loaded premium per unit of benefit in function of n with joint pricing and stand-alone pricing (flat lines) for term annuities (right column) and term insurances (left column), where $b = 100$.

ζ.

Figure 6.4.2 also highlights the fact that pricing both contracts jointly does not necessarily lead to lower premiums for both business lines. For the term insurance contracts which are displayed on the right column of the figure, we observe that the loaded premiums are always lower when the offsetting relationship is exploited. This implies that joint pricing may provide the insurer with a competitive advantage on the business line which has the highest per-policy contribution to the overall risk. However, for term annuity contracts which are displayed on the left column, this is not always the case. In particular, for $\zeta > 0$ (i.e. when the stand-alone contracts are not priced at the pure premium), there is a *critical threshold* in the proportion n beyond which the conditional required loaded premiums for the annuity portfolio are higher than the loaded premiums set in a stand-alone way. One implication of the existence of such critical thresholds is that if the insurer is expecting the number of underwritten term insurances to be too high compared to that of term annuities (at least under the present assumptions), then the joint pricing is not necessarily desirable.

We illustrate the importance of this finding by focusing on the gLC model with $\zeta = 50\%$. Suppose that the tariff of the insurer on the term annuity business line is 20.625 per unit of benefit. In case the tariff for the term insurance business line is consistent with that of the term annuity business line, then we have that the tariff per unit of death benefit is 0.084. Compared to insurers who do not rely on joint pricing, the joint pricer has a competitive advantage of 1% on the term annuity side and of 5.3% on the term insurance business line. Thus, since the competitive advantage is higher on the term insurance portfolio, we could expect higher proportion of policyholders on this business line, and eventually, this proportion may turn out to be beyond the critical threshold.

Another challenge in the joint pricing is revealed from the convex shape of the loaded premiums in Figure 6.4.2: having set the loaded premium upfront in a competitive way (i.e. as low as possible on both business lines), the insurer has to limit the proportion n between the two businesses in order for that premium to be sufficient. This means that insurers relying on joint pricing to be more competitive have the burden of portfolio monitoring.

To illustrate this challenge, consider again the previous example under the gLC model with $\zeta = 50\%$ and tariff equal to 20.625 per unit of yearly survival benefits, and to 0.084 per unit of death benefit. From Figure 6.4.2, these loaded premiums correspond to the proportions $n = 11.11\%$ and $n = 70.77\%$. This insurer has a competitive advantage on both business lines over insurers who determine the loaded premium individually for each contract, while having the same overall risk. Nevertheless, Figure 6.4.2 shows that this insurer has to maintain the proportion n in the interval $[11.11\%, 70.77\%]$ in order for the pre-specified loaded premium to be sufficient for both businesses. In case the proportion n remains in this interval, the pre-specified loaded premiums would be higher than the required ones and the insurer can also benefit from the offsetting effect. It is also worth to mention that setting the tariff equal to the lower bound of the competitiveness region (as this may arise from an analogy with previous studies on optimal natural hedging strategies) is in fact the most risky choice for the insurer, since there is only a single proportion n for which the conditional loaded premium matches the actual one.

The question which arises now is how the above results can be used to derive a reasonable strategy such that joint pricing becomes desirable in terms of competitiveness and also in terms of flexibility in portfolio monitoring. Since the insurer has (to some extent) more control over the design of the contract than over the allocation of the businesses, one solution that partially enhances the joint pricing consists in setting a constraint on the ratio b between the benefits of the two business lines such that the interval for the ratio n is wide enough to allow for some flexibility. The reason behind is that the ratio b controls the per-policy contribution of each business line, and hence, this strategy can be viewed as a risk adjustment. An illustration is provided in Figure 6.4.3 which is obtained using the gLC model. For both values of ζ, we observe that low ratios b push the critical threshold of the proportion n closer to 1. This effect can be exploited to meet the characteristics of the insurer's portfolio. For instance, suppose that the insurer sets the tariff of the term annuity equal to 20.625 for $\zeta = 50\%$. This implies that whereas the insurer has to maintain the ratio n in the interval $[11.11\%, 70.77\%]$ for $b = 100$, this interval is given by $[6.20\%, 50.05\%]$ for $b = 200$ and by $[18.31\%, 88.99\%]$ for $b = 50$. Note also that the interval is wider for low values of b.

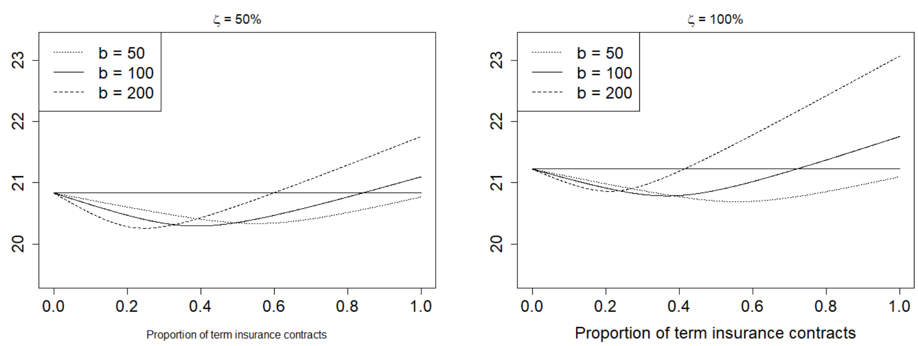


Figure 6.4.3: Loaded premium per unit of benefit using the gLC model in function of n for 3 different values of the ratio b.

6.4.2 Analysis of the switch

We pursue the analysis under the gLC model. We study the cases with $b = 100$ and $b = 200$. We consider two values of the target risk reduction factor ζ, namely, a low target reduction of 25% and a high target reduction of 100%. For the reaction factors q_A and q_I, we consider four different cases. For the first case, policyholders in both business lines have the same reaction factor of 0.5, i.e. $q_A = q_I = 0.5$. For the second case, we assume that policyholders buying term insurances have a reaction factor $q_I = 2$ whereas policyholders buying term annuities have a reaction factor $q_A = 0.5$. For the third case, we consider the opposite situation with $q_A = 2$ and $q_I = 0.5$, and for the fourth case, we assume equally high reaction factors with $q_A = q_I = 2$.

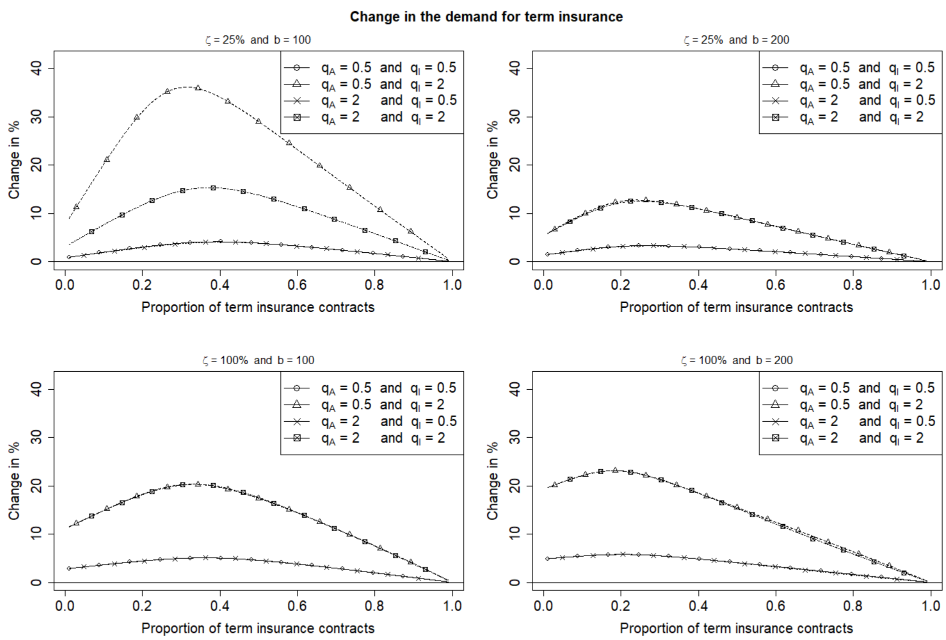


Figure 6.4.4: Percentage change in the demand of term insurance policies.

Concerning term insurance contracts, we observe in Figure 6.4.4 that the demand in this business line increases in all cases after the switch. The magnitude of the increase is lower for high target risk reduction ζ and for high reaction factor of the corresponding business line q_I. Naturally, the demand increases more when the reaction factor of term insurance buyers is high. We also observe that for low benefit ratio and low target risk reduction (i.e. for $b = 100$ and $\zeta = 25\%$), the demand of term insurances is impacted by the reaction factor of term annuity buyers.

The results for term annuity contracts are displayed in Figure 6.4.5. Unlike on the term insurance business line, we observe here that the insurer might experience a decrease of the demand for term annuities in a case of a switch. In particular, the consequence of the critical ratio is that the demand of term annuities can decrease for insurers with high proportion of term insurance contracts in their portfolio. The level of the increase or decrease in the demand is also determined by ζ and the ratio b. Moreover, the reaction factor of the corresponding business line q_A plays a role. Here, the reaction of term

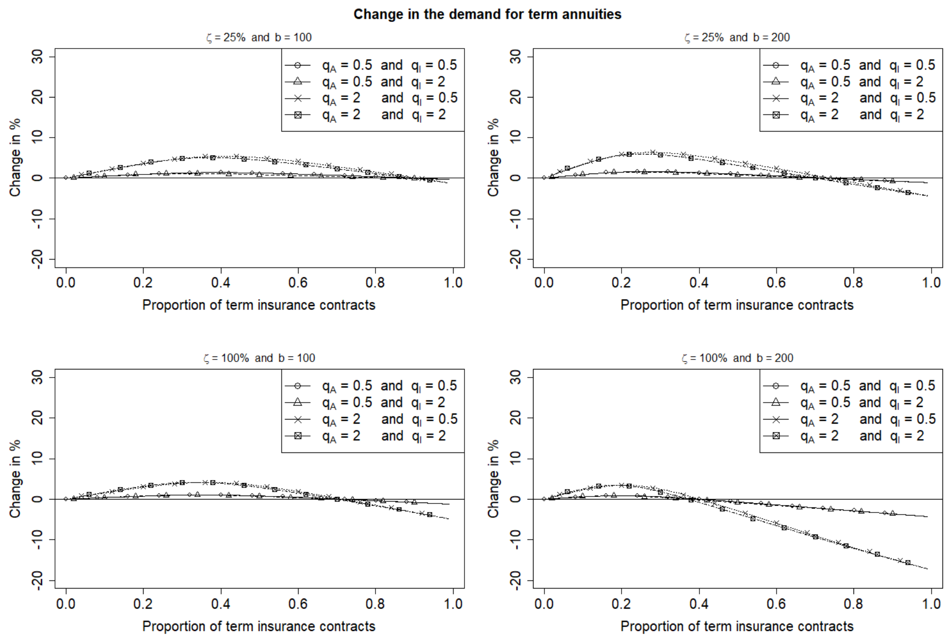


Figure 6.4.5: Percentage change in the demand of term annuity policies.

insurance buyers barely impacts the change in the demand for annuities.

Despite the decrease in the demand of annuity contracts, the insurer may be more concerned about changes in the total collected premiums. Figure 6.4.6 displays the change in the total collected premiums before and after the switch. Recall that the insurer's risk remains unchanged. We observe on Figure 6.4.6 that when the reaction factor is the same on both business lines (i.e. $q_A = q_I$), the total collected premiums will either always increase or always decrease, depending on the values of q_A and q_I. For instance, in case of low reaction factors on both business lines, then the tariffs are lower but policyholders do not react enough to offset the loss due to the tariff decrease. As a consequence, the total collected premiums decreases. Similarly, for equally high reaction factors, the tariffs are lower and policyholders react enough to offset the loss due to the tariff decrease. As a consequence, the total collected premiums increases.

For $q_A \neq q_I$, the effect of the switch is different. We observe that when the reaction factor of policyholders from the risky business line is higher (i.e. for $q_A = 0.5$ and $q_I = 2$), then the switch will often lead to an increase of the total collected premiums, and the decrease corresponding to low proportions

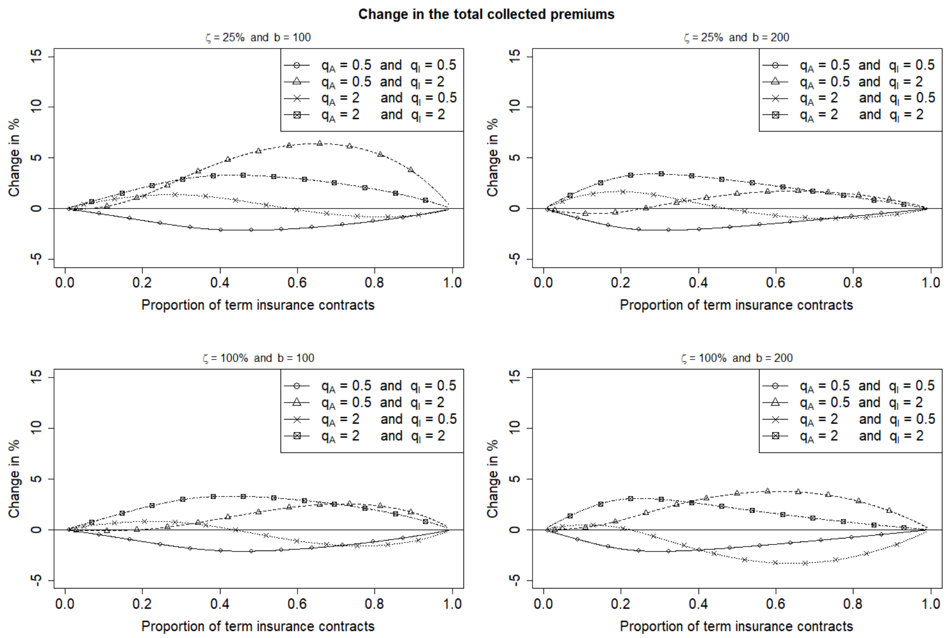


Figure 6.4.6: Percentage change in the total collected premiums.

w is marginally low. In the opposite situation where $q_A = 2$ and $q_I = 0.5$, the total collected premiums can increase only for high proportion of term annuities, and the decrease in case of high proportion of term insurance is substantial.

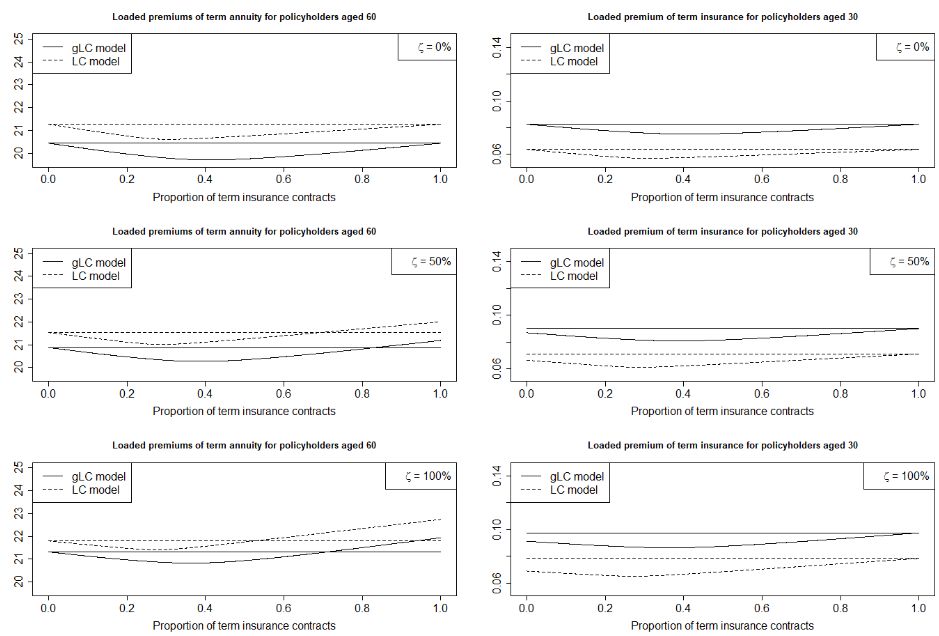
6.5 Concluding remarks

In this chapter, we investigate some challenges related to the joint pricing of term annuities and term insurances. We find that relying on the offsetting relationship between these contracts in the pricing is not always desirable. Moreover, we also find that switching from the stand-alone pricing to the joint pricing is not recommended in some cases.

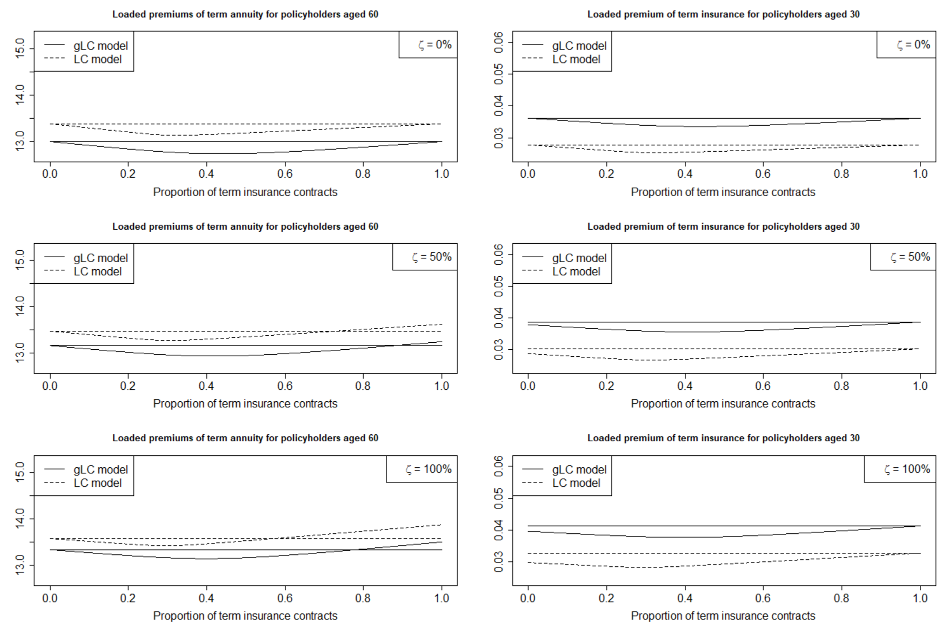
A general conclusion of this chapter is that the benefit of exploiting the offsetting relationship between two business lines depends on several factors, such as the composition of the portfolio, the benefit ratio, the target risk reduction of the insurer and the dependence between the mortality patterns in the two subpopulations. Therefore, the decision of joint pricing has to be carefully analyzed in practice to account for the specific features of the contracts at hand and using different mortality models.

This work allows to answer some ambiguities related to joint pricing and paves the path for potential future topics of research. Some directions of research would consist in following the literature on optimal product mix from the hedging perspective by including other risk management tools and incorporating the uncertainty on the assets and interest rate risk (e.g. Gatzert and Wesker (2012), Gatzert and Wesker (2014) and Luciano et al. (2017)). Moreover, the analysis conducted in this chapter can be combined with the updating mechanism described in Chapter 5. More specifically, an insightful study would consist in investigating the share of systematic longevity and mortality risks when the dynamic equivalence principle is applied for offsetting businesses jointly.

6.6 Appendix – Sensitivity tests



Loaded premium per unit of benefit in function of n with joint pricing and stand-alone pricing (flat lines) for term annuities (right column) and term insurances (left column), where $b = 100$, using the value-at-risk at the confidence level 0.99 and interest rate of 1%.



Loaded premium per unit of benefit in function of n with joint pricing and stand-alone pricing (flat lines) for term annuities (right column) and term insurances (left column), where $b = 100$, using the conditional value-at-risk at the confidence level 0.95 and interest rate of 5%.

Part IV

Conclusions

Conclusions

The classical approach to manage insurance risk consists in selling a sufficiently large number of policies. This strategy allows the insurer to reduce the variations of the claim payments per-policy around the corresponding theoretical mean. Nevertheless, the insurer remains exposed to a systematic risk which cannot be completely eliminated by increasing the portfolio size. The presence of this risk is essentially due to the fact that the two components of the claim payment, i.e. the claim amount and its probability of occurrence, may be time-varying and subject to uncertainty. Managing this risk is all the more important in the context of long-term insurance business, where the premium estimate at policy issue has to account for the unpredictable changes in the underlying risk factors.

In this thesis, we have addressed systematic risk in long-term insurance business which stems from the uncertainty on the claim amount (medical inflation risk) and on the probability of the benefit payment (longevity and mortality risks).

Part I was devoted to the management of medical inflation risk in Belgian private health insurance contracts. The Belgian legislator introduced in 2007 a mechanism allowing insurers to transfer medical inflation risk back to policyholders via premium adjustments. Our contribution is threefold. First, we focus on the medical index, which captures the evolution of claim amounts over time. We highlight some deficiencies in its current construction. We also propose an alternative method which accounts for the heterogeneity in the structure of products sold in the Belgian market, while satisfying to some extent the transparency requirement of consumers' representatives. Second, we focus on the adjustment mechanism, whose goal is to include in the contract elements the information on medical inflation over time. We derive actuarially fair mechanisms to assess the adjustment rule prescribed by the Belgian regulator in the Royal Decree of 18 March 2016. Our analysis shows that the Belgian rule often leads to conservative premium adjustments from the viewpoint of the insurer. We also find that this rule may be insufficient under some scenarios of medical inflation and interest rate. Third, we discuss some remaining challenges in the Belgian system with a focus on the transferability of the reserve and on age discrimination. We also provide recommendations to enhance the Belgian system and ensure more protection for policyholders, taking into account the solvency constraint of insurers.

In Part II, we address the management of longevity risk in long-term insurance business. We further analyze the solution consisting in transferring the systematic risk back to policyholders. We extend the existing literature by considering a situation where only part of the risk is transferred, and hence, the risk-sharing mechanism is not limited to the no-transfer/full-transfer binarism. We propose a dynamic equivalence principle, and derive updating rules for the contract elements. We compare contracts managed with the proposed risk-sharing scheme and their classical counterparts. Our setting allows us to derive conditions for a viable risk-sharing scheme, such that both the insurer and policyholders are better off by sharing the risk over time.

Finally, Part III considers the interplay between longevity and mortality risks, and we compare the case where the offsetting relationship between these two risks is exploited in the pricing (i.e. joint pricing), with the case where it is not exploited (i.e. stand-alone pricing). We highlight several challenges for insurer's relying on joint pricing. In particular, we show that for the less risky business line, the required loaded premium in case of joint pricing can be higher than its counterpart in case of stand-alone pricing. We also find that the choice of competitiveness comes with the burden of portfolio monitoring. Moreover, for the same level of risk, our analysis suggests that switching from the stand-alone to the joint pricing may lead to a decrease of the total collected premiums.

The literature on the management of long-term insurance risks is broad, and this thesis focuses on some parts only. Future research in this direction are thus numerous. As an example, a new life market is emerging which would allow institutions bearing long-term risks to transfer them to financial markets in the form of longevity- or mortality-linked securities (Blake et al., 2013). These new financial instruments can be rather appealing to investors, since they provide diversification opportunities due to the fact that longevity and mortality risks are likely to be independent to other traditional financial risks. Of course, they also raise a new set of challenges. One of them is the problem of pricing, since the market price of long-term risks is not straightforward to determine for such emerging markets. As a consequence, the market risk premiums of these securities (sometimes called bonds) are set using expert judgment, and may have a strong impact on the success of the launch. Another challenge is basis risk which comes from the fact the mortality of the hedging instrument can be different from that of the portfolio. Some answers have been proposed to manage this risk (see e.g. (Coughlan et al., 2011)), but there are still other directions to explore.

List of Figures

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