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Surrender analysis in a segregated fund

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Abstract

This paper is focused on the analysis of the surrender option embedded in a participating endowment insurance. Two main sections can be identified. In the first one we investigate factors which could affect the decision of the policyholder to surrender his contract. The study has been developed using some data from a segregated fund of a Bancassurance. Research is conducted by means of the discrete-time logistic regression model, which is supported by a non-parametric data analysis through the actuarial method. The second section concerns the pricing of the surrender option embedded in a single-premium participating endowment insurance. To this purpose we adopt the Least Squares Monte Carlo method, which is used in a purely financial field to price American options. The aim is to determine a value of the surrender option under the assumption that the policyholders are perfectly rational and, in addition, a more realistic value is calculated relaxing the before mentioned assumption.

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1. Introduction

Over time, insurance undertakings have developed mechanisms for the purpose of attributing more flexibility to insurance policies (see [Pit(2002)]). These mechanisms are options that enable the policyholder to make choices related to his credit against the insurer, that may have an impact on both the maturity and benefits of the contract (that could deeply change the contract). For many years insurance undertakings ignored options embedded in their policies, given the lack of interest that the policyholders had in these options. This situation became really serious in the last two decades of the 20th century, when, as a result of the financial markets turmoils, the policyholders have begun to exercise embedded options more frequently and in a more opportunistic way. In fact, insurers had to pay benefits which value exceed the premiums earned. Hence the need of greater attention to insurance product design, in order to determine an explicit value for all benefits, or at least be aware of what is offered to policyholders. Furthermore, for a proper contract valuation, the insurer has to take into account the “dynamic policyholder behavior”, where “dynamic” highlights the policyholders' ability to react to external factors, usually economic factors, and thus to exercise the embedded options in order to maximize their profits (see [KM(2008)]).

The main options embedded in life insurance contracts are: annuity and lump-sum (conversion) option, surrender option, paid-up and resumption option, deferment and extension option and over-depositing option.

In particular, Italian participating life insurance policies (the so-called assicurazioni rivalutabili) offer profit sharing of the particular segregated fund in which policyholder premiums are invested and include interest rate guarantees.

The delicate matter is that, very often, these contracts enable the policyholder to surrender the policy and to receive the surrender value containing the profit sharing accumulated up to that time. If lapse rate increases when market value of assets portfolio decreases, then the company could suffer losses due to unexpected liquidity needs (for payment of surrender values). Many empirical studies show that an increase of interest rates could incite policyholders to give up their contract to enter in more rewarding investment opportunities (see [EK(2011)], [Kim(2005)], for example). This situation could generate a dangerous vicious circle: a remarkable increase of surrender could force insurer to endure losses due to assets selling off; the losses affect rate of return of segregated fund which could be lower than the guaranteed ones; in this case insurer must integrate with own funds his obligations toward insured persons, the latter could lose their trust in the undertaking’s solvency and that could trigger new surrender decisions.

This paper, which summarizes the dissertation based upon previous considerations, is structured as follow. In Section 2 we briefly present some general issues related to options embedded in life insurance policies. Section 3 is dedicated to surrender factors research. In Section 4 we describe the methodology used to price the surrender option. Finally, Section 5 provides some concluding remarks.

2. Surrender factors research

2.1. Factors

The factors that produce early termination of a contract can be classified in two main sets. The first concerns macroeconomic variables meanwhile the second concerns the features of contract, insured and policyholder (see [Kie(2012)]). These latter information are managed by undertaking in a highly confidential way and indeed the existing empirical literature is based mostly on the first set. One of the objectives in such studies is to verify that surrenders’ trend is consistent with two hypotheses (based on empirical studies):

1. *Interest rate hypothesis which assumes that policyholders will be more interested to surrender their contracts when the interest rates rise, in order to enter in more rewarding investment opportunities;*

2. *Emergency fund hypothesis which conjectures that personal financial emergency obliges policyholders to lapse their contracts in order to access the surrender value.*

The interest rate hypothesis is tested using market interest rates, stock indices, and policy’s credited rate, while the emergency fund hypothesis is tested using unemployment rate, gross domestic product and other economic growth indices.

As far as the second set of factors is concerned, the analysis is developed taking into account the features of the contract (such as product type, premium type, policy year,…) and the features of the policyholder and the insured (such as age, gender,…) (see [CEG(2009)]). Furthermore, it can be considered insurance sales channels and ancillary guarantees.

Beyond these two main sets the company characteristics can be included in the research.

The analyzed data are related to a segregated fund of a Bancassurance. The data set comprises information about: policyholder, insured, product type and contract parameters. In addition to these information closely related to contracts, we have included in the analysis return of segregated fund, yield to maturity of BTP (Italian Government Bond) with maturity 5 years, real GDP growth rate, unemployment rate and inflation rate (obviously with refer to italian market).

2.2. Model

The research is conducted using the discrete-time logistic regression model (in survival analysis environment), therefore the subject of analysis is the policy year up to possible surrender event. The model is defined by:

$$(2.1)\quad ln\left(\frac{q_{i,t}}{1-q_{i,t}}\right) = \alpha_t + \boldsymbol{\beta}'\mathbf{X}_{i,t} = \alpha_t + \beta_1 x_{i,t,1} + \cdots + \beta_k x_{i,t,k}\,,$$

where $q_{i,t}$ (discrete-time hazard function) denotes the probability that an individual i surrenders his contract at time t given that he did not surrender it before and $\mathbf{X}_{i,t}$ denotes the design matrix.
The first member in (2.1) is the logit or log-odds of $q_{i,t}$ while the second member is a linear combination of explanatory variables, plus a term α_t .
The model is an extension of the model characterized only by time-independent explanatory variables. The latter model assumes that there is proportionality between the odds and, for this reason, it is called proportional odds model. Anyway, the inclusion of time-dependent variables into the model is a common practice and this implies that, selected any two individuals, the odds ratio is not necessarily constant over time, since the values of time-dependent variables may change in different times for different individuals.
Preliminarily, we have done a non-parametric data analysis through the actuarial method (see [LYL(1996)]), which results have highlighted that it could be better to include in (2.1) some interactions between explanatory variables and time, in order to take into account a possible trend of such variables toward logit (for more details please see the dissertation [Lon(2012)]).

The estimated coefficients confirm both the interest rate hypothesis and the emergency fund hypothesis. Moreover, all the features considered about contract, policyholder and insured have a statistically significant impact on surrender decisions.

3. Surrender option pricing

Market consistent valuation of insurance contracts (see [OP(2005)]), and therefore also of the embedded options, is based on principles and methodologies used in financial markets. In the context of financial derivative instruments, a surrender option is an American put option written on the residual value of the policy with a strike price given by the surrender value and with the peculiarity that its existence is linked to insured survival (knock-out American put option, see [BBM(2009)], [BBM(2010]).

The classical financial perfect-market framework is used to evaluate the surrender option, i.e. we assume: markets are perfectly competitive, frictionless and free of arbitrage opportunities, and all agents are price-takers, rational, non-satiated and share the same information. Therefore, in this framework, we can determine a market consistent value of the surrender option. Nevertheless, this approach overestimates the "actual" value of the option (it can be interpreted as the worst-case value from the company point of view), since the empirical evidence shows that the actual surrender decisions are far away from being perfectly rational. This means that a real evaluation of surrender option has to necessarily consider the irrationality, or better the rationality level, of policyholder behaviour.

3.1. Insurance contract embedding surrender option

We price the surrender option embedded in a single-premium participating endowment insurance with maturity T years. The contract provides a survival benefit b_T^v at time T, if the insured is alive at maturity T, or a death benefit b_τ^d at time $\tau \in (0,T]$ in case τ coincides of the individual's time of death. In addition to these benefits, the policyholder has the right to surrender the contract at any time $\theta \in [t', T)$ receiving b_θ^r as surrender value (note that $(0, t')$ denotes deferment period in which the policyholder cannot surrender the contract). At t=0, against the single-premium payment, the insurer invests the initial capital C_0 (equal to the pure premium earned) in a specific segregated fund, that is a well-defined assets portfolio, which rate of return determines the revaluation of insurance benefits.

Let I_ℓ be the rate of return on the segregated fund during year $\ell \in \{1, \ldots, T\}$. Then the adjusted capital is defined as:

$$(3.1) \quad C_\ell = C_0 \prod_{k=1}^{\ell} (1 + \rho_k),$$

where ρ_k denotes the annual rate of revaluation, that is given by

$$(3.2) \quad \rho_k = \max\left\{ \frac{\min\{\beta I_k, I_k - \eta\} - i'}{1 + i'}, \rho_{\min} \right\},$$

with: i' =technical interest rate, $\beta \in (0,1]$ = participating level, η = annual minimum rate retained by the company and $\rho_{\min}$ = annual minimum guaranteed rate.

For simplicity, let be $b_t^y = C_t$, where $y \in \{v, d, r\}$. Then, for fixed value $t \geq 0$ and a surrender time $\theta \geq 0$ (surrender strategy), the possible benefits linked to the contract, paid up to time t, may be summarized by:

$$(3.3) \quad B_t^A(\theta) = b_T^v \mathbb{I}_{T \leq \min\{t,\theta\}, T < \tau} + b_\tau^d \mathbb{I}_{\tau \leq \min\{t,\theta,T\}} + b_\theta^r \mathbb{I}_{\theta < \min\{\tau,T\}, t' < \theta \leq t},$$

where $\mathbb{I}_E$ denotes indicator variable of the event E and with the convention that $\theta \geq T$ or $\theta \geq \tau$ or $\theta \leq t'$ if the contract is never surrendered. In relation to surrender option, the contract is called American contract, whereas a contract without the surrender option is called European contract and its benefits may be summarized by

$$(3.4) \quad B_t^E = b_T^v \mathbb{I}_{T \leq t, T < \tau} + b_\tau^d \mathbb{I}_{\tau \leq \min\{t,T\}}.$$

3.2. Probabilistic framework

We take as given $(\Omega, \mathcal{F}, \mathbb{Q})$, a probability space where Ω is the set of all possible outcomes of random variables (biometric and financial) that characterize the valuation, $\mathcal{F}$ is a σ-algebra (on Ω) and $\mathbb{Q}$ is a risk-neutral probability measure, meaning that under $\mathbb{Q}$ the market value of any asset is given by the expected present value, conditionally on information available, of its future cash flows, using the risk-free rate for discounting and the risk-neutral probability for the expected value calculation. The existence of a risk-neutral probability measure is guaranteed by the absence of arbitrage, but its uniqueness is not verified (since we operate in an incomplete market framework). Therefore, the measure $\mathbb{Q}$ represents a specific measure assumed to be chosen by both the policyholder and the insurer.

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ be the filtered probability space that is obtained by defining on $(\Omega, \mathcal{F}, \mathbb{Q})$ the filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$. Finally, fixed t, let X_t be the set of random variables that characterize the valuation problem, defined on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$. Thus, the stochastic process $(X_t)_{t\geq 0}$ generates $\mathbb{F}$.

3.3. Biometric model

For each $t \geq 0$, the available information $\mathcal{F}_t$ allows to know whether the insured is dead before time t (as well as the time of death $\tau \leq t$) or he is still alive. Then, by definition, the time of death of the insured τ is a stopping-time. We assume that the random variable τ has a Weibull distribution with shape parameter $\gamma > 0$ and scale parameter $\lambda > 0$. The mortality intensity is given by

$$h_x(t) = \frac{\gamma}{\lambda} \left(\frac{x+t}{\lambda} \right)^{\gamma - 1}$$

where x is the age of the insured at time $t=0$ (hence, $x+\tau$ denotes the age-at-death random variable).

3.4. Financial model

We assume that the rate of return on the segregated fund in $[k\text{-}1,k]$ is given by

$$(3.5) \quad I_k = \frac{F_k - F_{k-1}}{F_{k-1}},$$

where F_k denotes the market value of the segregated fund at time k (of one unit). Moreover, we assume that the segregated fund is composed only by stocks and bonds, thus F_k can be written as a convex combination of a stock index A_k and a bond index O_k:

$$(3.6) \quad F_k = \alpha^A A_k + (1 - \alpha^A) O_k, \qquad \alpha \in [0,1].$$

In order to reflect (approximately) the management strategy of the segregated fund it is necessary to adequately define the market indices A_k and O_k. Therefore, for the segregated fund dynamics we adopt a bivariate diffusion model, in which the interest rate $(r_t)_{t\geq 0}$ follows the Cox-Ingersoll-Ross mean-reverting square root process and the stock index $(A_t)_{t\geq 0}$ evolves according to Black and Scholes model. The bond index is obtained from a buy-and-sell strategy, with a fixed trading horizon, of a zero coupon bond with fixed duration (see [AC(2003)], [DeFM(2002)]).

3.5. Contract valuation

The valuation is based on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$, where the filtration $\mathbb{F}$ is generated by $(\mathrm{X_t})_{t\geq 0} = (\mathrm{r_t}, \mathrm{F_t}, \mathrm{C_t})_{t\geq 0}$. Moreover, let $c_t = \exp(\int_0^t r_s \mathrm{d}s)$ be the bank account value at time $t \geq 0$.The value (price) at time $t \geq 0$ of the European contract is given by the conditional expected value, in the risk-neutral environment, of the contract future cash flows discounted up to time t. In particular, at time t=0 we have:

$$(3.7) \quad V_0^E = E^{\mathbb{Q}}\left[\int_0^\infty c_u^{-1} \mathrm{d}B_u^E\right],$$

where B_u^E is given by (3.4), which involves $\mathrm{d}B_u^E = 0$ for each $u > \min\{\tau, T\}$.
In order to determine the time 0 value of the American contract, as is common practice in purely financial context for the American option pricing, it is necessary to determine the optimal surrender strategy. Note that it is natural to consider any surrender time θ to be a stopping time. Let $\mathrm{T}_{\mathbb{F}}$ be the set of possible stopping times; then the American contract value at time $t=0$ is given by

$$(3.8) \quad V_0^A(\theta^*) = \sup_{\theta \in \mathrm{T}_{\mathbb{F}}} V_0^A(\theta) = \sup_{\theta \in \mathrm{T}_{\mathbb{F}}} \left\{E^{\mathbb{Q}}\left[\int_0^\infty c_u^{-1} \mathrm{d}B_u^A(\theta)\right]\right\},$$

where $B_u^A(\theta)$ is given by (3.3), which involves $\mathrm{d}B_u^A(\theta)$=0 for any u >min $\{\tau, T, \theta\}$. The solution of the optimal stopping problem θ^* maximizes the expected value at contract issue date.

3.6. Least Squares Monte Carlo method

The Least Squares Monte Carlo method has the objective to (approximately) solve the problem (3.8), using Monte Carlo simulation and Least Squares regression (in a Markovian setting).
The idea underlying this method is to estimate the continuation value of the contract in order to identify the optimal surrender strategy by comparing at each time this value with the surrender one. Practically speaking (as clearly shown in the algorithm 3.8), it is necessary to consider a discretization of the time dimension of the problem (3.8) denoted by $\mathbf{T} = \{\mathrm{t_0}, \dots, \mathrm{t_n}\}$ where $\mathrm{t_0} = 0, \mathrm{t_n} = T$ and if $\mathrm{t}' > 0$ then $\mathrm{t_1} = \mathrm{t}'$ (thus, we evaluate a Bermudan-style contract) and a set of functions $\{f_h(\cdot)\}_{h=1}^H$, all independent from each other, taken from a suitable basis, which are used to approximate the continuation value (for more details about the LSMC approach to price the surrender option see the dissertation [Lon(2012)] and/or the reference papers [BBM(2009)], [LS(2001)]).

3.7. Policyholder behaviour model

It has been assumed so far that the policyholder follows a fully rational and optimal exercise strategy when deciding whether or not to exercise the surrender option. As said before, this assumption is too strong with respect to the empirical evidence. The idea underlying the valuation proposed by [DeG(2010)], in case of not full rationality of the policyholder behaviour, is to introduce a surrender probability function depending on rational choices and macroeconomic scenario at each possible surrender time.
The probability that the policyholder decides to surrender the contract in the interval $[t, t+\mathrm{d}t]$ is given by

$$(3.9) \quad q_t^r = 1 - \exp\left\{-\int_t^{t+\mathrm{d}t} h(u)\mathrm{d}u\right\},$$

where $\mathrm{h}(\cdot)$ is the hazard function.
Moreover, let $(\xi_t)_{t\geq 0}$ be a stochastic process defined on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ with $\xi_t \in \{0,1\}$ for each $t \geq 0$. Fixed t, if the surrender value is greater than the continuation value of the contract, i.e. at time t it would be rational to surrender, then $\xi_t = 1$, otherwise $\xi_t = 0$. Actually, the policyholder surrenders the contract even if this decision is not rational. Therefore, relaxing the full rationality assumption, let be

$$(3.10) \quad h(t \mid \xi_t = 0) = \eta^I,$$

where $\eta^I \geq 0$ can be interpreted as a parameter for irrational surrender choice (this choice is due to factors which are not closely related with contract features). For sake of simplicity, let η^I be constant. Thus, the irrational surrender probability is

$$(3.11) \quad q_t^{rI} = 1 - \exp\{-\eta^I \mathrm{d}t\},$$

In case $\xi_t = 1$, let be

$$(3.12) \quad h(t \mid \xi_t = 1) = \eta^R + \eta^I,$$

and the rational surrender probability is given by

$$(3.13) \quad q_t^{rR} = 1 - \exp\{-(\eta^R + \eta^I)\mathrm{d}t\},$$

where $\eta^R \geq 0$ measures the “ability” of the policyholder to recognize that the optimal strategy is to surrender the contract.
The empirical evidence shows that the ability of the policyholder to recognize the optimal surrender strategy increases in stressed market conditions and, vice-versa, decreases in market stability conditions. Therefore, two simple ways to define η^R are

$$\eta^R = A^2 r_t^2 + B, \qquad A, B \in \mathbb{R},$$
$$\eta^R = a r_t^2 + b r_t + c, \quad a, b, c \in \mathbb{R}: h(\cdot) \geq 0.$$

Figure 1a shows the time of rational exercise of option and the spot rate in a simulated scenario. Whilst, Figure 1b shows the surrender probabilities q_t^r (with $\eta^I = 0.1, A = 20$ and $B = 0.2$) and the spot rate in a simulated scenario.

3.8. Algorithm

We indicate each simulated values in m-th scenario with a superscript m.

1. Simulation:
Simulate M times of death τ and M paths of the process $(\mathrm{X_t})_{\mathrm{t}\in\mathbf{T}}$.

2. Initialization:
For $m=1,\ldots,M$:

if $\tau^m > T$ then $\theta^{*,m} = T$ and $P^m_{\theta^*,m} = b_T^{v,m}$,
otherwise $\theta^{*,m} = \tau^m$ and $P^m_{\theta^*,m} = b_{\tau^m}^{d,m}$.

3. Backward induction:
For j=n-1,…,1:

a. Let $I_j = \{m \in \{1,\ldots,M\}: \tau^m > t_j\}$, i.e. I_j is the set of scenarios in which the policy-holder is still alive at time t_j. Moreover, for any $m \in I_j$ let $C_j^m = c_{t_j}^m (c^m_{\theta^*,m})^{-1} P^m_{\theta^*,m}$.

b. Determine the least squares estimation of coefficients $\boldsymbol{\beta}_j$ by regression of $\{C_j^m, m \in I_j\}$ against $\left\{f_h(X_{t_j}^m), m \in I_j\right\}_{h=1}^H$.

c. For any $m \in I_j$, let $\tilde{C}_j^m = \widehat{\boldsymbol{\beta}}_j \boldsymbol{f}(X_{t_j}^m)$ be the continuation value estimated at time t_j in m-th scenario.

d. (Rational decision) For any $m \in I_j$, if $b_{t_j}^{r,m} > \tilde{C}_j^m$, then $\xi_{t_j}^m = 1$. Otherwise $\xi_{t_j}^m = 0$.

e. If $\xi_{t_j}^m = 0$, then simulate the contract surrender with probability q_t^{rI}. Otherwise, this simulation is performed using the probability q_t^{rR}.

f. In case of surrender $\theta^{*,m} = t_j$ and $P^m_{\theta^*,m} = b_{t_i}^{r,m}$.

4. Valuation
In the previous step we have defined for each m=1,…,M the optimal surrender strategy. Therefore, the time 0 value of the American contract is

$$\overline{V_0^A} = \frac{1}{M}\sum_{m=1}^{M} \left(c^m_{\theta^*,m}\right)^{-1} P^m_{\theta^*,m}.$$

In order to compute the present value of European contract in $t = 0$, V_0^E, we can use the algorithm above without executing the Backward induction step. Moreover, along with $\overline{V_0^A}$ and V_0^E, it is possible to obtain the American contract value under the full rationality assumption $V_0^A(\theta^*)$ and this is simply achieved by setting $q_t^{rI} = 0$ and $q_t^{rR} = 1$. Thus, we have estimated V_0^A, $V_0^A(\theta^*)$, V_0^E and we can compute the surrender option value under both the not full rationality and the full rationality assumption as $V_0^A - V_0^E$ (that can take also negative values) and $V_0^A(\theta^*) - V_0^E$, respectively.

Figure 1a

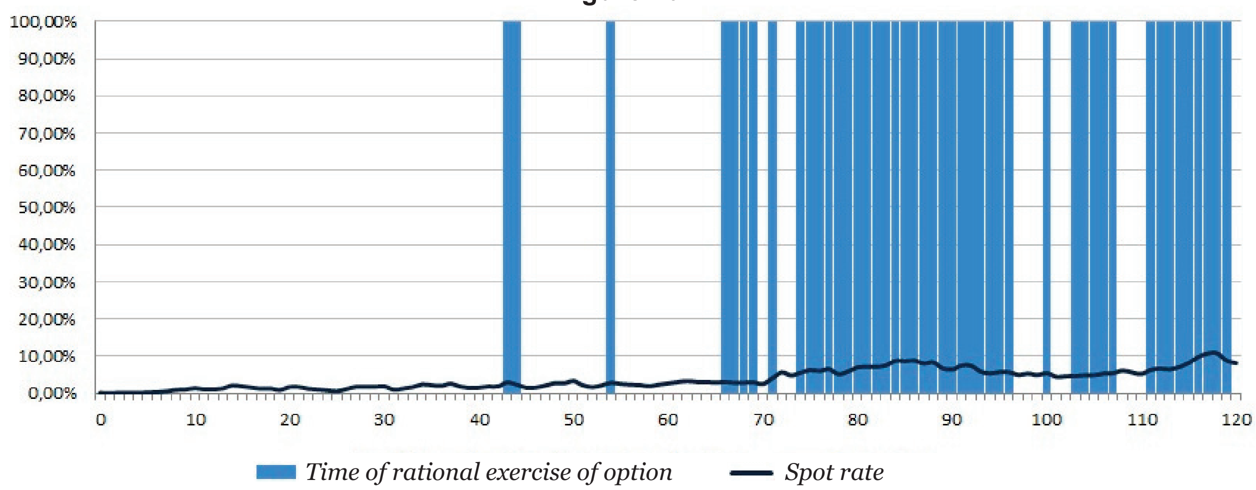
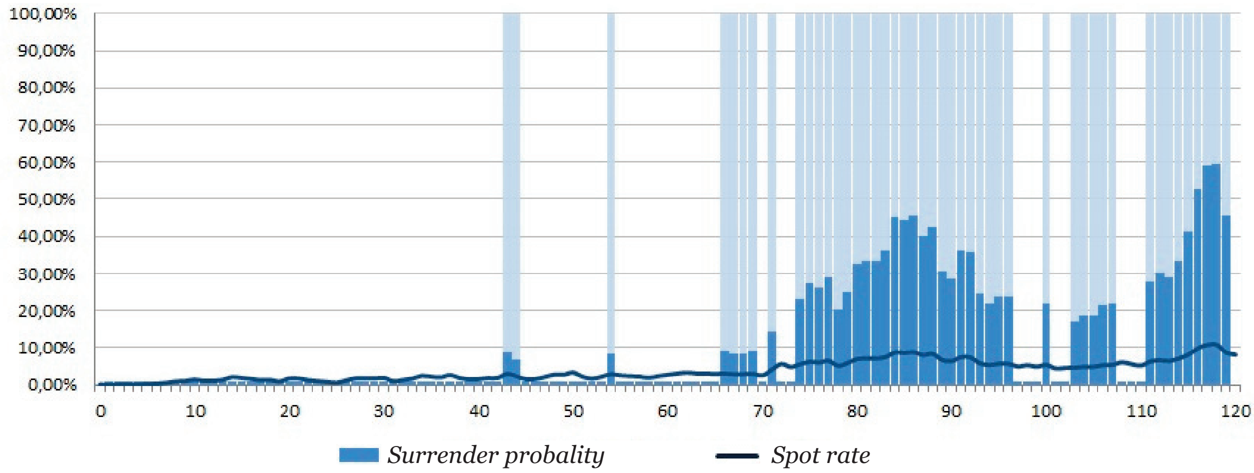


Figure 1b



3.9. Numerical Example: a sensitivity analysis

In this section we present some results obtained by implementing the above described algorithm. The sensitivity analysis regards the participating rule (3.2) and the fund composition (3.6).

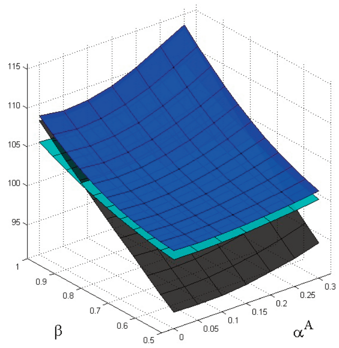


Figure 2 shows American contract values, in the full rationality (blue) and not full rationality (light blue) assumption, and European contract values (grey) on varying of α^A and β, with $\rho_{min} = 0$, $t' = 0$. Valuation date: 31 march 2012.

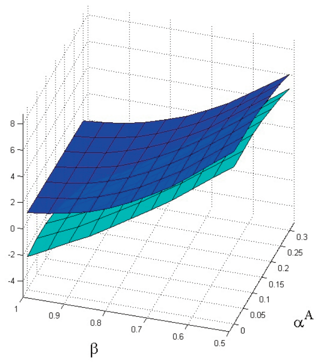


Figure 3 shows surrender option values, in the full rationality (blue) and not full rationality (light blue) assumption on varying of α^A and β, with $\rho_{min} = 0$, $t' = 0$. Valuation date: 31 march 2012.

In particular, we note that a negative value of the surrender option under the not full rationality assumption can be ascribed to the inability of the policyholder to take advantage from the surrender option embedded in the contract.

4. Conclusions

In this work we have proposed a surrender analysis of a segregated fund. The analysis is developed on both surrender factors research and surrender option pricing. As far as the first one is concerned, we have chosen the model, starting from the available data, with the aim to include in the study both macroeconomic variables and features of policyholder, insured and contract, without *a priori* disregarding (as far as possible) variables which could have an influence on surrender decisions.
In the second part we have described the evaluation of the surrender option embedded in a single-premium participating endowment insurance. The proposed approach allows to compute a value of the surrender option under the assumption that the policyholders are perfectly rational and, in addition, a more realistic value relaxing this assumption. The obtained results are in line with expectations and consistent with those reported in reference papers.
The main objectives for further research are two. The first one is to improve the calculation of revaluation rate in order to consider the existing accounting rules (and not only the market value of assets). The second one is related to calibration of the policyholder behaviour model with the aim to reflect his rationality level.

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