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# Pricing new Risks or Insurance Products in a Bayesian Framework

## Case Study: Commercial Space Travel Insurance

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# ABSTRACT

In most pricing exercises, an Actuary makes extensive use of loss data pertaining to the risk under consideration to obtain a sensible risk premium. This is not possible for new risks or insurance products where loss data is sparse or unavailable. Here, initial premiums are set subjectively with little or no actuarial justification and then adjusted over time. We thus propose a Bayesian methodology incorporating expert (underwriter) opinion and loss data from lower and upper benchmark risks to obtain a range of possible premiums for the risk under consideration in a realistic context and on a sound foundation.

The method is illustrated for commercial space travel insurance. Expert opinion is simulated and a sensitivity analysis is performed. A range of possible premiums for the risk of loss of life due to failure of a sub-orbital flight is obtained depending on the expert's opinion. It is found that careful elicitation of expert information is required whenever loss data for a benchmark risk is sparse. Our Bayesian approach also provides the flexibility and scope to incorporate experts' opinions that may differ from our views.



**KEYWORDS:** Pricing, Insurance, Benchmarking, Expert Opinion, Bayesian, Space Travel

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# ACRONYMS

BBC – British Broadcasting Company

CAA – Civil Aviation Authority

MCMC – Markov Chain Monte Carlo

MLE – Maximum Likelihood Estimate

MTWA – Maximum Take-off Weight Authorised

NASA – National Aeronautics and Space Administration

OBPE – Optimal Bayes’ Point Estimate

# 1. INTRODUCTION

## 1.1 Space travel and tourism

Man has ventured into space for decades of years with the first manned missions accomplished in the early 1960's followed by the moon landing in July 1969. Along with great success came devastating losses such as the Challenger disaster in 1986 where the shuttle exploded soon after take-off killing six astronauts and a schoolteacher; and the more recent Columbia shuttle tragedy in 2003.

There is a considerable risk of explosion or loss of onboard life-support involved in space missions. When such an incident occurs, it can potentially lead to enormous losses with regards to both the spacecraft and lives onboard.

Nonetheless, such risks have not deterred mankind from exploring outer space. There have indeed been amazing discoveries and advancements in space travel and exploration, such as evidence for the existence of water on Mars in the distant past, which has been supported by spectacular images and telemetry of the Mars' surface sent back by probes. In 2001, an important milestone was marked in space history when Dennis Tito became the first paying space tourist to go to outer space. According to BBC (2001), he paid $20m for his ticket aboard the Soyuz space shuttle which was scheduled for a supply mission to the International Space Station.

Another such important milestone was the award of the $10 million Ansari X prize in 2004, the largest prize in history, to Mojave Aerospace Ventures for the flight of SpaceShipOne. In order to win the prize, designer Burt Rutan financially backed by Paul Allen (Microsoft co-founder), had to build and launch a reusable spacecraft capable of carrying three people to over 100 kilometres above the earth's surface, twice within two weeks, which they accomplished successfully. These events will, if not already done so, lead to the birth of a whole new industry for commercial space travel.

Other names for commercial space travel are 'space tourism', 'public space travel' and 'personal space flight'. Lindskold (1999) predicted that within the next 25 years, a remarkable sequence of events would take place in space and once an infrastructure between Earth and space is constructed, there would be regular passenger trips to space and back, carrying mainly tourists. He found that there is a great yearning among people to travel to space. In his opinion, the first space tourism trips will most likely be sub-orbital jumps where a spacecraft makes a ballistic trip to an altitude of 100km, flying in a wide arc from the ground to the top of the arc where a few minutes of weightlessness would be experienced and then returning to Earth without reaching orbit. With the progress of time and technology, this would be followed by orbital trips around the Earth, to orbital hotels and perhaps even trips to the Moon and lunar hotels.

In the future, individuals may be able to purchase tickets as frequently as required for travel to space for business purposes, sightseeing or even research, just as for aeroplanes. One may argue that Lindskold's sequence of events is already in motion, for example the award of the Ansari X prize which may allow this vision to become a reality. Virgin Galactic, founded by Sir Richard Branson to become the world's first spaceline, bought out the exclusive rights to Burt Rutan's design and technology of SpaceShipOne. Rutan's team, backed by Virgin Galactic have already started work on SpaceShipTwo and further successors of SpaceShipOne and plan to commence commercial sub-orbital flights by 2009 followed by orbital flights in the near future, with the long-term potential for space voyages to space hotels and the moon.

## 1.2 Insuring space travel

According to Virgin Galactic (2007), the hybrid rocket motor and the feathered-wing re-entry mechanism of SpaceShipOne increase the safety and reliability of the craft in comparison to a space shuttle. However, there is still a considerable risk of failure during flight, the consequences of which may be enormous and therefore passengers would want some kind of insurance cover for the duration of the flight.

The following extract is from the collectSPACE (2006) website in response to a question on insurance for the passengers of SpaceShipTwo:

*"Whitehorn explained that while they anticipate no problem receiving insurance for third party liability (as SpaceShipOne had), the insurance market has told them that it will be several years into passenger operations before they will be able to offer a personal liability package. Individuals may be able to purchase coverage for themselves earlier (at what is sure to be high premiums) but Virgin Galactic will need to prove to the market that they have a safe vehicle before they become comfortable with extending protection."*

These recent developments in space travel and tourism will affect the Insurance industry, in particular entities such as Lloyds, insurance companies and others like Virgin who are involved in insuring space risk one way or another. In the long term, it could lead to the establishment of entirely new markets for insurers and related parties such as a market for commercial space travel insurance, provided that the risk in question is insurable. Hart *et al* (1996: pp1-2) details the basic criteria for a risk to be insurable, for example, the frequency and severity of the expected loss must be assessable, the premium should be affordable, etc.

One may define commercial space travel as the widespread act of travelling into space for reasons such as business, sightseeing or research once a price has been paid to undertake the activity including the means of transportation to and from the destination. Insurance for such a trip away from Earth would be readily available to purchase on the marketplace with the flexibility to tailor the policy according to the individual circumstances and coverage in terms of duration, sum assured, etc. required by the passengers. The market for commercial space travel insurance in the future may well resemble that of the travel insurance one existent today, where passengers are covered for travel to another destination that is not considered home.

However, in the short term, insurers may not be willing to offer such a product or cover against risks involved in space travel, as commented by Whitehorn. This is probably due to the lack of loss data available for potential risks that might be covered under such a policy. This poses a difficulty for the insurers in assessing and quantifying the risk involved in terms of the chance of occurrence of a peril and its associated loss and consequences. It is therefore no surprise that the insurance market has informed Virgin Galactic that it would be a while before they are able to offer a personal liability package. This would provide insurers with enough time to collect loss data and assess the risks involved from the experience of the flight operations up to that time. However, as previously mentioned, passengers would want insurance cover for themselves, which the insurers may be willing to extend provided that Virgin Galactic can prove the safety of the flight and operations to the market. In this case, one may pose the obvious question of whether it is possible to calculate a premium for such a policy or risk and if so, how?

## 1.3 Premium calculation

A premium is the payment made by a policyholder for complete or partial insurance cover against a risk. The issues surrounding the setting of premiums in fact apply to any new risk or insurance product, especially in the London market where loss data is scarce or unavailable. Examples include the risks posed by terrorism, new business risks due to climate change and other catastrophe risks which have all recently featured in the headlines.

When new risks with no loss data available become a concern for businesses or individuals and first appear on the market, underwriters initially use their expert judgement to price these risks. As relevant loss data is obtained over time, the underwriters adjust their premium levels.

However, one cannot be certain in these cases as to whether the initial premiums are a true reflection of the exposure to the risk, since there is little or no actuarial justification for the initial premiums charged, and it is difficult to quantify the credibility and expertise of underwriters who are the experts in pricing the risk under consideration. We thus suggest a Bayesian approach to price such risks in a realistic context and on a sound foundation. The definition of "realistic context" is debatable, however here, it refers to the idea that it is better to provide a reliable estimate based on some related data regardless of the strength of this basis, rather than using any number without justification.

This paper is set out as follows. Section 2 explains the background theory on Bayesian statistics, credibility theory, benchmarking, expert opinion and premium setting used in subsequent sections whilst subsection 2.5 provides a literature review on these areas and how they have been incorporated with each other for application in fields such as engineering, aerospace programs, nuclear energy. Section 3 describes the proposed Bayesian methodology. Section 4 illustrates its application to commercial space travel insurance. In particular, subsections 4.3.3 and 4.3.4 contain the simulations and sensitivity analysis respectively. The data and Mathematica code used in the implementation along with screenshots of the output results are supplied in Appendices A and B. Section 5 concludes.

# 2. BACKGROUND THEORY

## 2.1 Bayesian Statistics

Klugman (1992) and Congdon (2001) provide an in-depth description and understanding of the Bayesian approach in statistical analyses. The basic idea of Bayesian techniques is that unknown parameters are treated as random variables, i.e. a probability distribution gives the likelihood that various possible values of the parameters are the true values. Thus in comparison to the frequentist approach, both the data, $\underline{X}$ and the parameters, $\theta$ are treated as random quantities.

Suppose we have probability distribution $\pi(\theta)$ that contains information about parameter $\theta$. This prior distribution for $\theta$ is the starting point of the Bayesian approach and can be deduced from past experience, known fact or guesswork. Now suppose we have a vector of $n$ observations $\underline{x} = (x_1, x_2, ..., x_n)^T$. The likelihood function is then denoted by $f(\underline{x} \mid \theta)$, which describes the likelihood of various values of $\underline{X}$ being obtained given that $\theta$ is the true parameter value. The prior information is updated with knowledge obtained from conducting experiments or collecting data to give posterior information. The posterior distribution of $\theta$ obtained using the laws of probability and Bayes Theorem is:

$$\pi(\theta \mid \underline{x}) = \frac{f(\underline{x} \mid \theta)\pi(\theta)}{f(\underline{x})}$$

where $f(\underline{x}) = \int f(\underline{x} \mid \theta)\pi(\theta)d\theta$ is the marginal distribution of $\underline{X}$. Replace the integral sign with a summation for discrete distributions. Since $f(\underline{x})$ does not depend on $\theta$ and effectively provides the 'normalising' constant for $\pi(\theta|\underline{x})$, it is sufficient to determine the form of $\pi(\theta|\underline{x})$ and hence one can write;

$$\pi(\theta|\underline{x}) \propto f(\underline{x}|\theta)\pi(\theta)$$

This process of Bayesian updating can be repeated continuously in which case the current posterior information becomes the new prior information.

$\pi(\theta|\underline{x})$ contains all of our current knowledge about the unknown parameter $\theta$ and thus is the foundation for all Bayesian inference and summaries of $\theta$ such as point estimates or interval estimates. We are particularly interested in point estimates, i.e. the mean.

Bayesian decision theory in conjunction with appropriately chosen loss functions can then be used to obtain an Optimal Bayes' Point Estimate (OBPE) of $\theta$. Define a loss function $L(g(\underline{x}),\theta)$ where $g(\underline{x})$ is the OBPE of $\theta$. For given $L$, $g(\underline{x})$ is the function which minimises the expected loss with respect to the posterior distribution, $\pi(\theta|\underline{x})$, given by:

$$E[L] = \int L(g(\underline{x}),\theta)\pi(\theta|\underline{x})d\theta$$

The form of $L$ determines the form of $g(\underline{x})$. We use the quadratic (squared error) loss function given by $L(g(\underline{x}),\theta) = (g(\underline{x})-\theta)^2$, where $g(\underline{x})$ is the mean of the posterior distribution. For absolute error and zero-one loss functions, $g(\underline{x})$ is the posterior median and mode respectively. Young *et al*

(2005: pp1-60) provide a thorough and more detailed introduction to Decision Theory and Bayesian inference.

A proof showing that the posterior mean is the OBPE for $\theta$ under squared error loss follows:

Differentiate $E[L]$ with respect to $g(\underline{x})$ in order to find the minimum and the corresponding value of $g(\underline{x})$.

$$\frac{\partial E}{\partial g} = \int 2\left[g(\underline{x}) - \theta\right] \pi(\theta \mid \underline{x})\, d\theta = 0 \quad \text{when}$$

$$\int g(\underline{x})\, \pi(\theta \mid \underline{x})\, d\theta = \int \theta\, \pi(\theta \mid \underline{x})\, d\theta$$

i.e. $g(\underline{x}) = \int \theta\, \pi(\theta \mid \underline{x})\, d\theta = E[\theta \mid \underline{x}]$ which is the posterior mean.

$\pi(\theta \mid \underline{x})$ represents our updated state of knowledge of $\theta$ having observed the data $\underline{x}$ and thus the mean is a useful summary value.

Schuckers (2002) demonstrates the robustness and flexibility of Bayesian techniques by using such methods to find an interval estimate for the matching performance of a biometric identification device when no errors are detected, which is not possible in classical statistics.

However, one of the most criticised aspects of the Bayesian approach is the subjective choice of the prior distribution as statistical analyses should be as objective as possible. Young *et al* (2005: pp39-42) present an elaborate discussion and history of the work surrounding the choice of prior distributions and Bayesian statistics in general. They conclude that particular section with the main approaches used in the selection of prior distributions, such as, physical reasoning, using flat/uniform priors including improper priors such as Laplace, Jeffreys, subjective priors (de Finetti, Savage) and finally convenient prior distributions e.g. conjugate priors which are often used to simplify calculations. A further subjective choice may also arise when selecting the model to be used.

## 2.2 Credibility Theory and Benchmarking

According to Klugman *et al* (2004: pp515-610), "*Credibility theory is a set of quantitative tools which allows an insurer to perform prospective experience ratings (adjust future premiums based on past experience) on a risk or group of risks.*" Boland (2007: pp159-190) also provides an introduction to Credibility Theory and its application to Actuarial Science.

Credibility Theory provides a mechanism for systematically adjusting premiums in the light of claims experience. It incorporates Bayesian ideas and uses individual sample data from a group of policies for a recent period and collateral data (prior information) from similar policies in earlier periods. Let $\hat{\theta}_S$ and $\hat{\theta}_C$ be estimates of $\theta$ obtained from the sample data and collateral data respectively. One may combine these two sources of information to provide a good estimate of $\theta$ by attaching a credibility factor or weight $Z$ to the sample individual data, where $0 \le Z \le 1$. A credibility estimate of $\theta$ can be formed as a linear combination or weighted average of $\hat{\theta}_S$ and $\hat{\theta}_C$ as shown below:

$$Z\hat{\theta}_S + (1-Z)\hat{\theta}_C$$

The credibility factor $Z$ reflects how much confidence is placed in the sample data from the risk compared with the collateral data. The value of $Z$ is determined by the volume, reliability and future relevance of the sample data, i.e. the more sample data there are, the higher the value of $Z$ should be and conversely, the more relevant the collateral data, the lower the value of $Z$ should be. Full credibility to the sample data is when $Z = 1$ whilst partial credibility has $Z < 1$. However, note that the value of $Z$ is not a function of the actual data values from the risk itself.

In the context of Bayesian statistics, the OBPE derived from the posterior distribution of $\theta$ can be expressed as a linear combination of the sample data and the prior estimate with weightings of $Z$ and $1-Z$ respectively. In particular, for the quadratic loss function, the posterior mean can be expressed as a credibility estimate of $\theta$ as:

$$E\left[\theta \mid \underline{X} = \underline{x}\right] = Z\hat{\theta}_S + \left(1-Z\right)\mu_0$$

where the statistic $\hat{\theta}_S$ depends on the sample data $\underline{x}$ and is usually the mean of the data.
$\mu_0$ is the prior mean for $\theta$.

In our methodology, we shall actually use benchmark data rather than that corresponding to the risk in question. Benchmarking can be defined as the process of making a comparison with some point of reference. In our particular context, we shall refer to benchmarking as the process of making use of loss data from risks similar to the new risk in question.

## 2.3 Expert Opinion

An expert is someone with far-reaching knowledge or ability in a particular field or subject that goes beyond that possessed by the average person. The individual's experience and knowledge would be adequate for others to rely upon his or her opinion. The field of Expert opinion analysis is an ever-growing one due to the increasing desire to assess safety in various programs such as nuclear and aerospace where the proper functioning of new systems is critical.

Cooke (1991) gives a historical review and discussion of the subject whilst detailing various methods for eliciting expert opinion and uncertainty such as the Delphi method. He examines the application of expert opinion in practical areas like policy analysis, the aerospace industry. He demonstrates the problems of assessing the likelihood of rare or unobserved catastrophic events by using NASA shuttles as an example. The author also discusses three models for combining expert opinion; weighted combinations, Bayesian approach and psychological scaling models.

## 2.4 Premium setting

Hart *et al* (1996: pp281-324) explains the process of premium setting in great detail. He explains four methods; 'market rate' pricing, "target pricing", theoretical approach and supply & demand analysis. In particular, the theoretical approach involves assessing the pure risk premium which is simply the expected cost of claims for the risk under consideration and an allowance is then made to cater for expenses like brokerage, overheads and profit margins. However in practice, other issues have to also be taken into consideration when setting premiums such as competitor rates, market demand, insurer's market share and firm's other objectives.

Anderson *et al* (2007) describe common pricing techniques implemented in the London Market business where there is some available data. Risks are priced using either the experience of the historic risk under consideration (Experience rating), information about a risk's exposure (Exposure rating), or in most cases a combination of both. They also suggest a "top down" exposure rating approach that may be adopted in cases with insufficient relevant data.

In contrast, Dickson (2005: pp38-50) employs premium calculation principles like the pure premium principle, expected value principle which are functions of the loss random variable. It is then a subjective issue of selecting a principle which satisfies desirable properties such as non-negative loading, additivity that are relevant to the risk under consideration. Lane (2005) examines the use of the standard deviation premium principle that was theorised by Kreps (1999), in the aviation industry. He explores the risk associated with the aviation industry, discussing various simple pricing rules and methodologies, their features and related pricing issues. In particular, the author highlights the consensus that the premium load factor should allow for an element of riskiness of the insurance in question through measures such as standard deviation. We however, concentrate on calculating the expected loss for the risk in question.

## 2.5 Literature Review

There is extensive academic literature on Bayesian techniques and expert opinion analysis and how one may incorporate the latter in a Bayesian framework. Cooke (1991), for example, discusses various methods for eliciting expert opinions and uncertainty, which are then either combined with one another or incorporated as data values into approaches like Bayesian ones in order to analyse the usefulness of expert judgement in fields such as aerospace programs, military intelligence and nuclear energy. Such approaches have been widely applied in an engineering context in order to assess the reliability or failure probability/rate of a new critical system. In particular, Droguett *et al* (2004) applies this approach to motor-driven pumps where it is shown that incorporation of expert information reduces uncertainty in the population variability distribution of failure rates of a group of similar systems, especially where limited data is available. We see similar results in Singpurwalla's (1988) analysis of the impact of Weibull survival data on the posterior distributions for the parameters, in particular for ball bearings and gas-turbine disks, where combining prior expert information obtained using a PC-Based procedure and the little data available helped reduce uncertainty that was present in the prior parameter distributions.

In contrast, there is limited literature on such an approach in an actuarial context, particularly with regards to pricing. However, related actuarial literature include works of Verrall *et al* (2005) who develop a Bayesian approach that combines a negative binomial stochastic model for the claims triangle with expert opinion in the context of claims reserving in General Insurance. Two situations are considered in particular; when a certain row development factor is changed or a certain number of years are chosen to be used in the estimation. Hsieh (2004) proposes a model that makes use of expert knowledge to obtain parameter values for the prior, and Extreme Value Theory in a Bayesian framework in order to forecast next record catastrophe loss. Catastrophe data is then used to demonstrate that this method provides admissible and theoretically sound forecasts and in particular, one notices that forecasts become more conservative (larger) as the prior becomes less informative.

# 3. PROPOSED METHODOLOGY

A Bayesian approach is suggested in order to resolve the difficulties involved in pricing new risks or insurance products where there is little or no loss data available. The proposed method incorporates expert opinion with benchmark loss data from similar risks in a Bayesian framework in order to strengthen the reliability of estimates and price the new risk under consideration in a realistic context.

In this method however, there is an element of subjectivity in the choice of the benchmark and, in certain circumstances, an appropriate benchmark similar to the risk in question may not even exist. This did not pose a concern in the literature since the authors had access to either limited failure data from the system in question or data from identical systems. It is thus suggested to use two sets of loss data from different benchmarks at the opposite ends of the risk scale, i.e. a lower and upper bound. The choice of the upper and lower benchmark risks is still subjective, however, one can widen the range between these bounds to eliminate any concerns of whether the risk in question lies between these benchmarks or not. Bayesian analyses and inference can then be undertaken for each of the bounds separately in order to obtain reliable estimates and hence a range of possible premiums for the risk in question. Note that these analyses have to be performed separately for the frequency and severity distributions and then combined appropriately to find a sensible range of premiums per unit exposure. However, there are exceptions to every rule and thus one would have to adapt the methodology to the particular risk under consideration. The basic idea is nevertheless demonstrated below.

## 3.1 Description of methodology

We propose three variations of a Bayesian approach that combine expert opinion with benchmark loss data:

(i) Update the analyst's uninformative prior with upper and lower bound benchmark data to obtain 1st stage upper and lower bound posterior distributions for the parameters. Perform a further Bayesian update on each of the 1st stage posterior bounds with the same expert information to obtain 2nd stage upper and lower bound posteriors. If these are of a recognisable form, one can use standard results such as expectation or mode, otherwise, MCMC methods can be implemented to find various moments of the posterior distributions in order to obtain a range of possible premiums. The expert information can either take the form of data observations or densities representing their opinions on the parameters of interest and the corresponding uncertainty. In the former case, the Bayesian update is straightforward as we have the expert opinion in the form of data values. However, when we have densities representing the expert's view and in the particular case where the Bayesian update involves conjugate priors, Cooke (1991: pp115-120, pp178-180) discusses the application of Winkler's (1968) natural conjugate theory which basically involves interpreting an expert's probability density function for the parameter of interest as an equivalent observation, provided that the density is of the required form. For non-conjugate priors, Huseby's (1987) theory on "imaginary observations" may be applicable, however, we shall be considering conjugate priors for our case study and therefore do not tackle this issue any further.

(ii) Elicit expert opinion to obtain a prior distribution for the parameters. Update this with upper bound benchmark data to obtain an upper bound posterior distribution for the parameters. The lower bound posterior is obtained similarly. Analyses are then performed on the posteriors to obtain a sensible range for the premiums.

(iii) Update an uninformative prior with upper bound benchmark data to obtain an upper bound posterior distribution. Perform a similar update for the lower bound. Combine these posterior bounds using weighting factors *w* and $1-w$, where $0 \leq w \leq 1$ (since we assume that the risk in question lies between these bounds). The expert's opinion on how risky the peril under consideration is to the benchmarks and the uncertainty of the opinion is elicited via a beta distribution for the weight *w*. MCMC simulation may then be performed to compute the $95^{th}$ percentiles of the combined distribution in order to obtain a 95% Bayesian Credibility interval for the premium.

The order in which benchmark data and expert opinion are incorporated in a Bayesian framework is crucial and deserves discussion to some length. Ideally for a new risk or insurance product, one would expect the benchmark loss data to be observed before expert opinion is elicited. In saying so, we assume that the risk was not an issue for insurers and institutions until recently and as such did not possess a view or knowledge of it before this time. On the other hand, the benchmark loss data would have been observed for the past few years and therefore it is sensible to update with the data first and then followed by the expert opinion. This is precisely the reason why an uninformative prior was assumed in the first variation. The analyst should have a vague opinion of the parameters of interest before an update with the benchmark data takes place and then followed by the incorporation of expert opinion. A similar update process has been undertaken in the third variation.

The second approach however violates this principle. Nevertheless, it was mentioned since one would usually expect the expert to specify the prior which is then updated with data to give a posterior. Singpurwalla (1988) and Droguett *et al* (2004) follow this approach, however it is perfectly legitimate in their cases as the new systems under their consideration were in existence and known before the failure data from identical systems was observed. In order to remedy this violation in our second approach, a bold assumption can be made in that for subsequent periods going forward, the benchmark loss data for identical risks follows a similar pattern as that observed to date and thus the expert opinion would then be obtained before the data.

For the purposes of this research, we decide not to explore the second and third approaches since a bold assumption has to be made in the former case and in the latter case, MCMC simulation would be required. We shall instead concentrate on the first variation, making particular use of conjugate priors and Winkler's (1968) natural conjugate theory so that the analysis can be mathematically tractable and computationally less-demanding.

Suppose $\underline{u}$ and $\underline{l}$ represent the upper and lower bound benchmark data respectively. Then the 1st stage upper and lower bound posteriors are respectively given by:

$$\pi(\theta \mid \underline{u}) \propto f(\underline{u} \mid \theta)\pi(\theta) \text{ and}$$

$$\pi(\theta \mid \underline{l}) \propto f(\underline{l} \mid \theta)\pi(\theta)$$

where $\pi(\theta)$ is the analyst's uninformative prior for $\theta$, the parameter of interest.

Now let $E$ represent the expert information which may either take the form of data observations or probability density. In the latter case, if the density is of the required form, it can be interpreted as an equivalent observation using Winkler's natural conjugate theory. A Bayesian update can then be performed on each of the 1st stage upper and lower bound posteriors to give:

$\pi(\theta \mid \underline{u}, E) \propto f(E \mid \theta, \underline{u}) \pi(\theta \mid \underline{u})$ and

$\pi(\theta \mid \underline{l}, E) \propto f(E \mid \theta, \underline{l}) \pi(\theta \mid \underline{l})$

which are respectively the 2nd stage upper and lower bound posteriors used to obtain a sensible range of premiums for the risk in question. We now demonstrate the application of this methodology to a particular case study and data set.

# 4. CASE STUDY: COMMERCIAL SPACE TRAVEL INSURANCE

In practice, travel insurance policies provide cover against a wide range of risks such as loss of life or injury, loss of baggage, travel delays, etc. We shall however only examine the risk of loss of life due to failure of a sub-orbital flight i.e. crash or mid-air explosion of crafts such as SpaceShipOne.

Suppose that a defined benefit is payable on the death of a policyholder during a sub-orbital flight due to crash or mid-air explosion. The loss event can be seen as a Bernoulli trial.

**Table 4.1: Discrete loss distribution for death risk during suborbital flight**

| Loss Amount, $L$ | $0$ | $S$ |
|---|---|---|
| $\mathrm{P}(L=l)$ | $1-q$ | $q$ |

where $S$ is the sum assured and $q$ is the failure probability of the SpaceShip flight. The pure risk premium is thus:

$$\mathrm{E}L = Sq$$

In a Bayesian framework, $q$ would be defined via a probability distribution and we would thus need to find the expected value of $q$ in order to calculate the pure risk premium. Therefore $q$ is our parameter of interest.

Since the exposure here is the sum assured, we see that the risks for different policyholders are fairly homogeneous and thus we can either aggregate the risks or examine each individually. We shall in fact consider the pure risk premium per unit exposure, which is known as the premium rate. The chargeable premium for a policy with sum assured $S$ is then found by multiplying this premium rate by $S$.

As discussed earlier, SpaceShip crafts use safer and more reliable mechanisms than space shuttles which employ either solid or liquid fuelled rockets that require more precautions during storage and operation. The initial flight phase of SpaceShips is similar to that of a jet carrier aircraft, after which the rocket motor ignites to take the craft to an altitude over 100 km. We thus assume that the SpaceShip crafts are less risky than shuttles but more risky than aeroplanes, i.e. the failure probability, $q$ for the SpaceShip craft should lie between that of the space shuttle and aeroplane. This allows us to use space shuttle and aeroplane loss data as our upper and lower bound benchmark data respectively in the Bayesian methodology outlined earlier.

## 4.1 Description of the data

The aviation and space shuttle loss data are obtained from CAA (2006) and NASA (2007) respectively, in the form of the number of failures due to either crash or mid-air explosion each year. We use accident data rather than fatalities data and the total number of flights flown each year as the corresponding utilisation or exposure data for the losses rather than the hours or passengers since we are interested in the failure probability of the system (flight). Aviation data is obtained for the years 1995-2004 whilst space shuttle data is obtained for the years 1981-2006.

In particular, we extract the number of reportable accidents by class of aircraft for UK registered or operated public transport large and small aeroplanes and the corresponding utilisation data in terms of total hours and flights flown each year from CAA (2006) as shown in **Tables A-1, A-2, A-3** and **A-4**. The actual accident data is presented graphically in CAA (2006) and so the underlying numerical data was requested from the safety unit at CAA. A ‘reportable accident’ is defined comprehensively in CAA (2006), however, for simplicity we assume that it refers to a crash or mid-air explosion, i.e. a failure. The accident data is broken down by class of aircraft, i.e. business jet, jet and turboprop for large aeroplanes and piston, turboprop for small aeroplanes. We shall however amalgamate the data and consider the total number of reportable accidents in each year for large and small aeroplanes. The aircraft are UK registered or operated but are involved in operations worldwide. We use public transport operations data rather than non-public data as the former involves transportation of passengers and/or cargo whilst the latter involves all other activities e.g. aerial surveys, construction work etc.

CAA (2006) defines large aeroplanes as those with maximum take-off weight authorised (MTWA) of over 5700 kg whilst small aeroplanes have MTWA of under 5700 kg. One may argue that the accident data for small aeroplanes is more relevant than that for large aeroplanes to our case study as the SpaceShip crafts will have MTWA of under 5700 kg and therefore one should only use the small aeroplane accident data for the lower benchmark. However, the failure probability of small aeroplanes seems to be approximately twice that of large aeroplanes, i.e. they are twice as risky as large planes. Since we would like to ensure that the risk in question comfortably lies between the benchmark risks at the opposite ends of the risk scale, we subjectively choose to aggregate the accident data by year for large and small aeroplanes as shown in **Table A-5**. The corresponding utilisation data in terms of flights flown by year is also aggregated and shown in that table.

The number of flights flown by large aeroplanes is about 15 times that of the small aeroplanes implying that the exposure is much greater for large aeroplanes. This gives us another reason to incorporate the large aeroplanes accident data into our lower benchmark since we would like to use as much data as possible to provide a reliable lower bound estimate for the failure probability.

**Table A-6** displays the mission numbers, launch and landing dates for all shuttle missions during the years 1981-2006 extracted from NASA (2007). There were 117 flights in total out of which two were unsuccessful, i.e. the Challenger disaster in 1986 and the Columbia shuttle tragedy in 2003. Brief descriptions of the failures are also included.

## 4.2 Elicitation of expert opinion

We assume that the experts, who in most cases will be underwriters, have some statistical knowledge. The analyst's opinion on the expertise of the experts is ignored and we shall also only incorporate the opinion of one expert. The case of multiple experts could be further research with dependencies between experts modelled using tools like copulas.

We would like to elicit the expert's opinion on $q$. We shall ask the expert for frequencies, such as the number of failures out of say, 100 flights, rather than probabilities as people tend to comprehend frequencies (5 out of 100) better than probabilities (0.05). The expert can either be 100% certain of his or her view of $q$, i.e. he or she states the number of failures out of, say 100 flights; or the expert can specify a probability distribution for $q$ which also incorporates his or her uncertainty of the estimate. The former case does simplify the analysis, however it is not very practical as specifying the failure probability of a new system is not an exact science and the expert may have doubts over his or her estimate of $q$. We therefore choose to elicit a beta probability distribution for $q$ from expert opinion, where $q$ ranges over $(0,1)$.

Experts find it easier to specify distributions in terms of moments such as mean, variance or percentiles rather than distributional parameters. We could ask the expert for his or her estimate of the mean and variance of the beta distribution for $q$ and it would then be a straightforward exercise to find the corresponding values of $\alpha$ and $\beta$ by the Method of Moments principle. However, specifying the variability of $q$, especially in terms of number of failures out of say, 100 flights, may still not be an easy task for an expert such as an underwriter. We therefore decide to elicit the $2.5^{th}$ and $97.5^{th}$ percentiles of the distribution of $q$ from the expert in order to compute the parameters $\alpha$ and $\beta$. One may do so by asking the expert a question along the lines of:

*"Out of 100,000 SpaceShip flights, specify a range for the number of crashes or mid-air explosions that would occur in your opinion, so that 95% of the time, your interval contains the true number of crashes or mid-air explosions from these 100,000 flights, i.e. a 95% confidence interval."*

The failure probability for planes has an order of magnitude of $10^{-5}$ whilst the space shuttle failure probability is of magnitude $10^{-2}$ as seen from **Tables A-5** and **A-6**, so the failure probability for the SpaceShip craft should have an order of magnitude lying between these benchmarks. This is the reason why we ask the expert to specify the number of failures out of 100,000 flights, however, this is a subjective choice and any other value that the reader feels appropriate may be used. The analysis nevertheless remains the same. Note that the elicitation must be carefully undertaken as the probabilities being dealt with are fairly small.

Suppose $(a,b)$ is the 95% confidence interval for $q$ obtained by dividing the lower and upper bound of the elicited range for the number of failures by 100,000, i.e. $a$ and $b$ are some specified values in the interval $(0,1)$.

Numerical methods must be used to compute the parameters $\alpha$ and $\beta$ accurately since the beta density has gamma functions. Spiegelhalter *et al* (1994) provide a normal approximation for the beta distribution that requires much less computing power and time than numerical methods, however, after experimentation in Excel, we found that this approximation worsens as the interval $(a,b)$ widens. We therefore do not consider Spiegelhalter's approximation but in fact use Mathematica's *NMinimize* function to numerically minimise the squared sum of errors expression below with respect to $\alpha$ and $\beta$:

$$f(\alpha,\beta) = \left( \int_0^a \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}dx - 0.025 \right)^2 + \left( \int_0^b \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}dx - 0.975 \right)^2$$

The *NMinimize* function finds the global minimum of $f(\alpha,\beta)$ and the corresponding values of $\alpha$ and $\beta$ for a specified range $(a,b)$. We now have the expert's beta density for $q$ and can thus go on to incorporate this information with benchmark data in a Bayesian framework as described earlier.

## 4.3 Application of the methodology

In practice, expert information would be obtained by conducting interviews or surveys. Due to time and space constraints, this will be simulated by means of generating two random numbers; $b$ from $\mathrm{U}(0,1)$ distribution and $a$ from

$U(0,b)$ distribution. We shall run 5 simulations and examine the results in each case.

3-Dimensional plots of the upper and lower bound premium rate for values of *a* and *b* varying over $(0,1)$ will also be analysed to determine whether or not the range of possible premiums obtained is sensitive to *a* and *b*. If so, careful and correct elicitation of the expert's opinion would be required in any pricing exercise using the above methodology.

Since we have binomial loss data for shuttles and planes, a classical statistician would use the Maximum Likelihood Estimates (MLEs) for the failure probabilities found by dividing the number of failures by the number of flights to obtain possible premium bounds, i.e. considering the MLE over all the years, we find the upper bound premium rate to be $\frac{2}{117} = 0.017094$ and the lower bound premium rate as $\frac{184}{10835000} = 0.000016982$. The failure probability for shuttles is thus approximately thousand times higher than that of planes and in theory, any premium rate between these bounds can be charged for the risk in question.

### 4.3.1 1st stage Bayesian update

We now apply the 1st stage of the Bayesian methodology described earlier, where updating with only benchmark data is performed.

Let $y_s$ and $y_p$ represent the binomial failure data for the upper bound (shuttles) and lower bound (planes) respectively, aggregated over all available years. Assume a uniform prior for *q*.

Upper Bound

$q \sim \mathrm{U}(0,1)$
$y_s \sim \mathrm{Bin}(117, q)$ with 2 failures observed.
Then

$$\pi(q \mid y_s) \propto f(y_s \mid q)\pi(q)$$
$$\propto q^2(1-q)^{117-2}$$
$$\propto q^2(1-q)^{115}$$

i.e. $q \mid y_s \sim \mathrm{Bet}(3,116)$ which is the upper bound posterior distribution.

We can then find the optimal Bayes point estimator (OBPE) w.r.t. squared error loss as:

$$\hat{q}_s = \mathrm{E}[q \mid y_s] = \frac{3}{3+116} = \frac{3}{119} = 0.025210084$$

which is greater than $\frac{2}{117} = 0.017094017,$ the MLE for space shuttles.

We can write $\hat{q}_s$ in the form of a credibility estimate:

$$\hat{q}_s = \frac{3}{119} = \frac{2}{117}Z + \frac{1}{2}(1-Z)$$

where the credibility factor, $Z = \frac{117}{119}$ which shows that great weight is placed on the shuttle data since the collateral mean of the uniform prior is uninformative in comparison to the amount of available data.

We can also check the variance of the posterior and compare it with that of the prior:

$$\mathrm{Var}[q \mid y_s] = \frac{3\text{x}116}{(3+116)^2(3+116+1)} = 2.0479\text{x}10^{-4} \ll \mathrm{Var}[q] = \frac{(1-0)}{12} = 0.08\overline{3}$$

Lower Bound

Similarly,

$q \sim \mathrm{U}(0,1)$

$y_p \sim \mathrm{Bin}(10835000, q)$ with 184 failures observed.

Then

$$\pi(q \mid y_p) \propto f(y_p \mid q)\pi(q)$$
$$\propto q^{184}(1-q)^{10835000-184}$$
$$\propto q^{184}(1-q)^{10834816}$$

i.e. $q \mid y_p \sim \mathrm{Bet}(185, 10834817)$ which is the lower bound posterior distribution.

We can then find the optimal Bayes point estimator (OBPE) w.r.t. squared error loss as:

$$\hat{q}_p = \mathrm{E}[q \mid y_p] = \frac{185}{185+10834817} = \frac{185}{10835002} = 1.7074 \text{ x } 10^{-5}$$

which is greater than $\frac{184}{10835000} = 1.6982 \text{ x } 10^{-5}$, the MLE for planes.

We can write $\hat{q}_p$ in the form of a credibility estimate:

$$\hat{q}_p = \frac{185}{10835002} = \frac{184}{10835000}Z + \frac{1}{2}(1-Z)$$

where the credibility factor, $Z = \frac{10835000}{10835002}$ which shows that even greater weight is placed on the plane data since the collateral mean of the uniform prior is uninformative in comparison to the amount of available data.

We can also check the variance of the posterior and compare it with that of the prior:

$$\mathrm{Var}[q \mid y_p] = \frac{185\text{x}10834817}{(185+10834817)^2(185+10834817+1)}$$
$$= 1.5758\text{x}10^{-12} \ll \mathrm{Var}[q] = \frac{(1-0)}{12} = 0.08\overline{3}$$

Observe that $\hat{q}_s$ and $\hat{q}_p$ are greater than the respective MLEs for space shuttles and planes and the range between $\hat{q}_s$ and $\hat{q}_p$ is greater than that between the MLEs, i.e. the range has increased. This is because we started off with an uninformative prior for $q$ and thus the Bayesian approach widens the range of premium rates chargeable for the risk in question as well as raises the lower bound premium rate, which one may interpret as a prudent or conservative measure. Therefore in theory, with only benchmark data available, one may charge any premium rate between the values $\hat{q}_s$ and $\hat{q}_p$ for the risk in question.

Note also that the variance of the posteriors is less than that of the uniform priors for both the lower and upper bound cases. This should be no surprise as the uniform prior is a flat $\text{Bet}(1,1)$ distribution whereas the updated posteriors are unimodal beta distributions that have smaller variance than the flat prior.

### 4.3.2 2$^{nd}$ stage Bayesian update

Now consider the incorporation of expert information into the analysis. We use the *RandomReal* function in Mathematica to generate $b$ and a variation of *RandomReal* function to generate $a$. The *NMinimize* function is then used to find the values of $\alpha$ and $\beta$ that correspond to the specified range $(a,b)$.

Winkler's natural conjugate theory is used to interpret the expert's $\text{Bet}(\alpha,\beta)$ density for $q$ as an equivalent binomial observation, i.e. "$\alpha$ failures out of $\alpha+\beta$ flights". The 1$^{st}$ stage upper and lower bound posteriors are now treated as priors and each updated with the expert information, $E$ to obtain the 2$^{nd}$ stage posteriors as shown below.

Upper Bound

$q \mid y_s \sim \text{Bet}(3,116)$ which is now our upper bound prior distribution.
Then updating with $E$ gives

$$\pi(q \mid y_s, E) \propto f(E \mid q, y_s)\pi(q \mid y_s)$$
$$\propto q^{\alpha}(1-q)^{\beta} q^{2}(1-q)^{115}$$
$$\propto q^{\alpha+2}(1-q)^{\beta+115}$$

i.e. $q \mid y_s, E \sim \text{Bet}(\alpha+3, \beta+116)$ which is the new upper bound posterior distribution.

We can then find the optimal Bayes point estimator (OBPE) w.r.t. squared error loss as:

$$\hat{q}_{s,e} = \text{E}[q \mid y_s, E] = \frac{\alpha+3}{\alpha+3+\beta+116} = \frac{\alpha+3}{\alpha+\beta+119}$$

We can also write $\hat{q}_{s,e}$ in the form of a credibility estimate:

$$\hat{q}_{s,e} = \frac{\alpha+3}{\alpha+\beta+119} = \frac{\alpha}{\alpha+\beta}Z + \frac{3}{119}(1-Z)$$

where the credibility factor, $Z = \dfrac{\alpha+\beta}{\alpha+\beta+119}$ which shows that as the values of $\alpha$ and $\beta$ get larger, i.e. the beta density is more peaked around a certain value, the greater is the weight attached to the expert information.

Lower Bound

Similarly,

$q \mid y_p \sim \text{Bet}\left(185, 10834817\right)$ which is now our lower bound prior distribution. Then updating with $E$ gives

$$\pi\left(q \mid y_p, E\right) \propto f\left(E \mid q, y_p\right) \pi\left(q \mid y_p\right)$$

$$\propto q^{\alpha}\left(1-q\right)^{\beta} q^{184}\left(1-q\right)^{10834816}$$

$$\propto q^{\alpha+184}\left(1-q\right)^{\beta+10834816}$$

i.e. $q \mid y_p, E \sim \text{Bet}\left(\alpha+185, \beta+10834817\right)$ which is the new lower bound posterior distribution.

We can then find the optimal Bayes point estimator (OBPE) w.r.t. squared error loss as:

$$\hat{q}_{p,e} = \text{E}\left[q \mid y_p, E\right] = \frac{\alpha+185}{\alpha+185+\beta+10834817} = \frac{\alpha+185}{\alpha+\beta+10835002}$$

We can also write $\hat{q}_{p,e}$ in the form of a credibility estimate:

$$\hat{q}_{p,e} = \frac{\alpha+185}{\alpha+\beta+10835002} = \frac{\alpha}{\alpha+\beta} Z + \frac{185}{10835002}\left(1-Z\right)$$

where the credibility factor, $Z = \dfrac{\alpha+\beta}{\alpha+\beta+10835002}$ which shows that as the values of $\alpha$ and $\beta$ get larger, i.e. the beta density is more peaked around a certain value, the greater is the weight attached to the expert information.

Observe that the variance of the posteriors may be less than or greater than the prior variances depending on the values $\alpha$ and $\beta$, i.e. the expert's opinion of $q$. The variance increases if the expert's density assessment of $q$ is different to the estimates from data and vice versa.

Note that the results and comments above assume that $\alpha$ and $\beta$ take values greater than 1, however in theory they can take any value greater than zero.

Using the Bayesian approach described above and the particular data set provided, we see that $\hat{q}_{s,e}$ and $\hat{q}_{p,e}$ give the upper and lower bounds for the range of possible premium rates chargeable for the risk of loss of life due to crash or mid-air explosion of the SpaceShip craft.

### 4.3.3 Simulation

We now perform 5 simulations of the expert information and include the results for the bounds in **Table 4.2** below. We round *a* and *b* down to 5 decimal places as in practice, the expert would usually give whole numbers for the number of failures out of 100,000 flights.

**Table 4.2: Simulation results**

| *a* (5 d.p.) | *b* (5 d.p.) | $\alpha$ | $\beta$ | $\hat{q}_{p,e}$ | $\hat{q}_{s,e}$ |
|---|---|---|---|---|---|
| 0.25733 | 0.41263 | 46.5563 | 93.3724 | 0.0000213709 | 0.1913896206 |
| 0.02656 | 0.60305 | 1.65755 | 5.08245 | 0.0000172273 | 0.0370411530 |
| 0.36215 | 0.74539 | 13.5702 | 10.7353 | 0.0000183267 | 0.1156283385 |
| 0.06157 | 0.12684 | 27.2367 | 270.082 | 0.0000195875 | 0.0726286898 |
| 0.01837 | 0.40470 | 1.81743 | 9.74253 | 0.0000172420 | 0.0368981919 |

Note that the narrower the range $(a,b)$ is, the larger the values of $\alpha$ and $\beta$ are. This is because the expert's probability density for *q* becomes more peaked around a certain value of *q* as the difference between *a* and *b* gets smaller and in this case, it is characteristic for the Beta distribution to have larger parameter values for $\alpha$ and $\beta$.

Also observe that the values for $\hat{q}_{p,e}$ are fairly close to each other for the different simulations in contrast to those for $\hat{q}_{s,e}$ which seem to span a wide range. This is attributable to the fact that we have far more loss data for planes than space shuttles and thus less weight is attached to the expert information in the case of planes in comparison to shuttles. One can see this more clearly by examining the forms of $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ when expressed as credibility estimates with a weighting factor *Z* attached to the expert information as described in section 4.3.2. We thus conclude that our estimates for the lower bound premium rate will be fairly insensitive to the expert information whereas those for the upper bound premium rate will be sensitive, in which case careful elicitation of expert opinion will be required. This should become more apparent when the sensitivity analysis is performed.

A final point to note for the above results is that of the range between $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$. We observe that all the values of $\hat{q}_{p,e}$ are greater than $\hat{q}_p$ and the MLE for planes. Similarly, all the values of $\hat{q}_{s,e}$ are greater than $\hat{q}_s$ and the MLE for shuttles. One would have expected the range between the lower and upper bound premium rates to reduce by incorporating expert information. However, this is not the case above because when performing the simulations, we allowed the expert opinion's on *q* to cover the whole range $(0,1)$, yet in practice, one would assume that the expert views the risk for SpaceShip crafts to lie between that for planes and shuttles, just as the decision maker (we) assumed earlier. If the expert's values for *a* and *b* lie between the lower and upper bound MLEs, say, then the range between $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ indeed decreases. For instance, let *a* = 0.001 and *b* = 0.015, then $\alpha$ = 2.53361 and $\beta$ = 427.43 and hence $\hat{q}_{p,e}$ = 0.0000173074 and $\hat{q}_{s,e}$ = 0.0100800949. The bounds are now closer to each other as expected. The discussion above does not imply that there is an imperfection in our analysis. On the contrary, our method provides the flexibility to incorporate the expert's opinion in the different cases where he or she views the risk in question to lie above, below or in between the benchmarks and this is then reflected in the higher or lower bounds as well as the range of premium rates chargeable for the risk in question.

### 4.3.4 Sensitivity Analysis

The limitations of Mathematica in terms of the maximum array size permitted and processing time requirements for the *NMinimize* function force us to discretise the range into 100 intervals i.e. the expert is asked to specify the lower and upper bounds for the number of failures out of 100 flights. Nevertheless, we should still be able to perform the analysis for the full ranges of *a* and *b*, albeit approximately, and thus deduce whether or not the results are sensitive to expert information.

For all combinations of $a < b$, the corresponding $\alpha$ and $\beta$ values are used to compute $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$. We set $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ to zero for $a \geq b$, i.e. only half of the 3-Dimensional region is significant. 3-D graphs are then plotted for $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ against *a* and *b* to determine the sensitivity of the results to expert information as shown below.

**Figure 4.1: 3-D Scatter plot for $\hat{q}_{p,e}$ (blue) and $\hat{q}_{s,e}$ (purple) – Zoomed In**

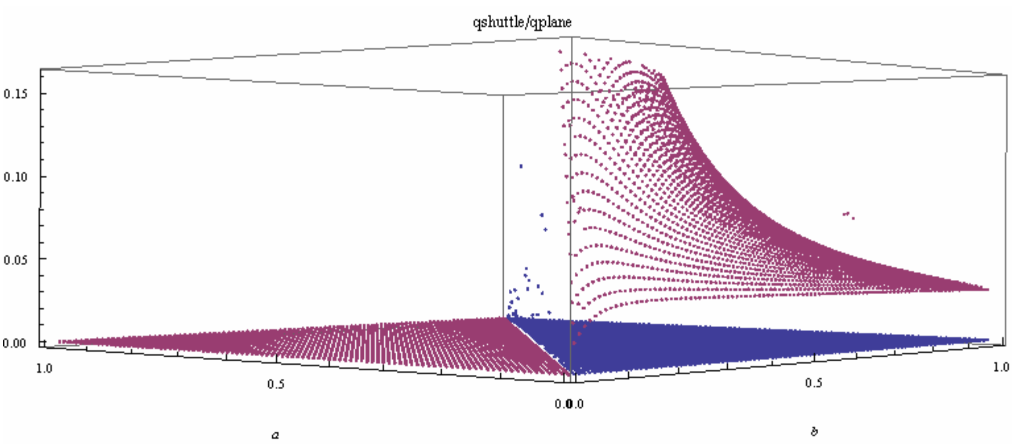


**Figure 4.2: 3-D Scatter plot for $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ – Zoomed Out**

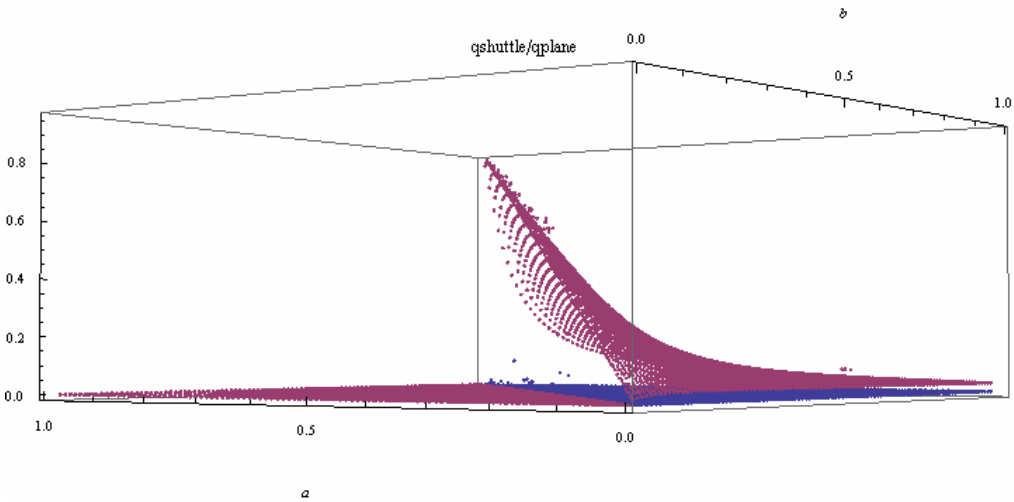



**Figure 4.3: 3-D Scatter plot for $\hat{q}_{p,e}$ – Zoomed In**

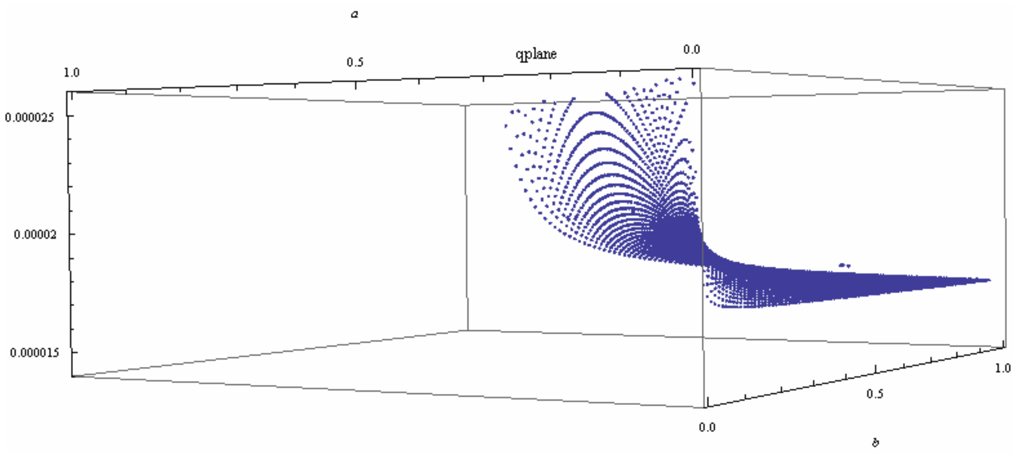


**Figure 4.4: 3-D Scatter plot for $\hat{q}_{p,e}$ – Zoomed Out**

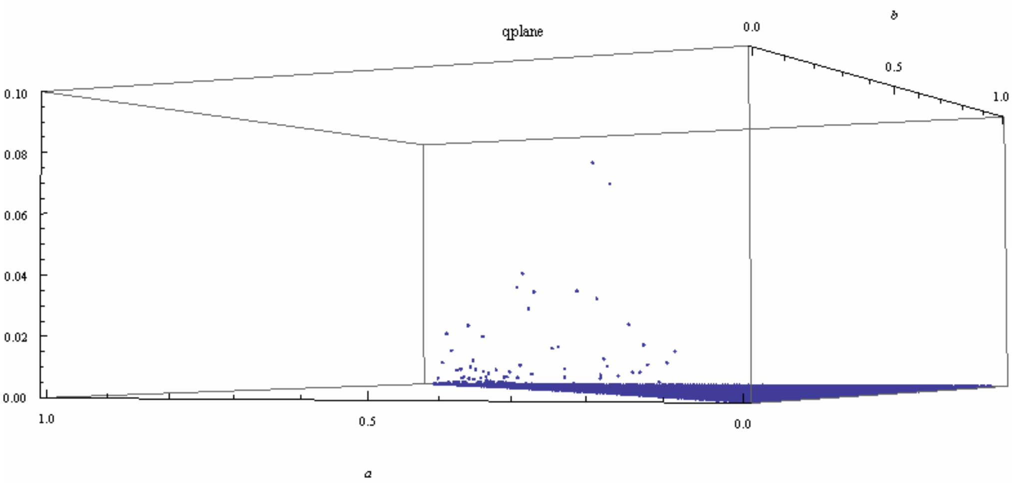



**Figure 4.5: 3-D Scatter plot for $\hat{q}_{s,e}$ – Zoomed In**

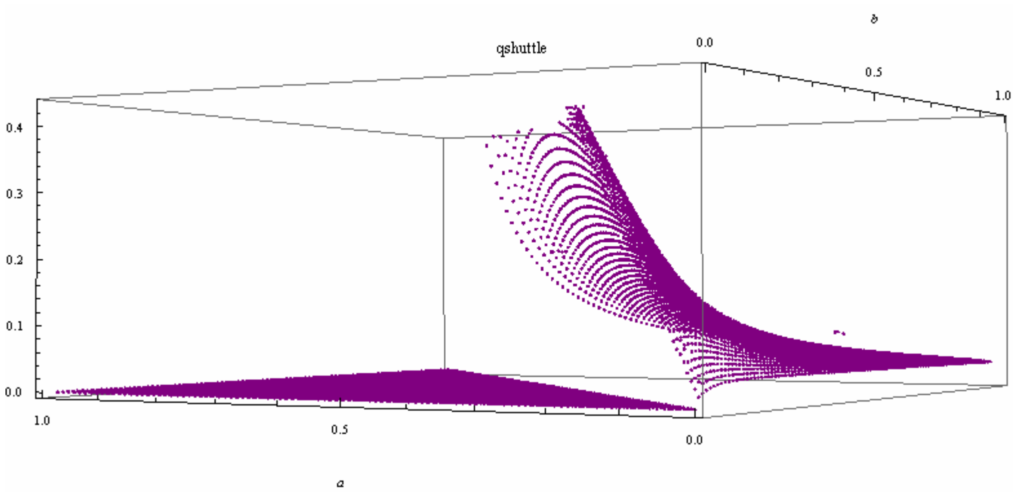
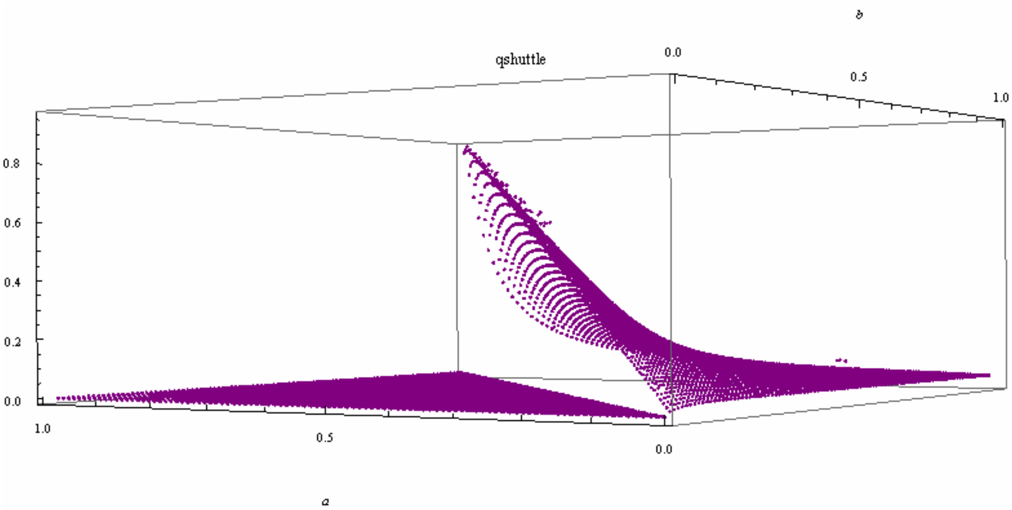


**Figure 4.6: 3-D Scatter plot for $\hat{q}_{s,e}$ – Zoomed Out**




In general, we observe that the values $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ initially increase gradually and then exponentially from the corner where $a = 0.01$ and $b = 0.99$ up to the $45^{\circ}$ line on the *a*-*b* plane where the values of *a* and *b* are almost equal. This is because the smaller the difference between *a* and *b* i.e. the more peaked the expert's density is around a certain value of *q*, the greater the values of $\alpha$ and $\beta$, thus leading to greater values for $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$.

Note that there are some points that seem out of place with the gradual increasing trend which occurs as a result of the numerical optimisation failing to converge to the required precision within the maximum set number of iterations. This is the same reason why we observe certain unusually large values for $\hat{q}_{p,e}$ as seen in **Figure 4.4**, such as the maximum value of about 0.1 occurring when $a = 0.72$ and $b = 0.73$. Spiegelhalter's approximation can be used to obtain a more accurate and lower value for $\hat{q}_{p,e}$ in this case. Similar corrections can be made for the cases where *a* and *b* are very close to each other and then in theory, one should have a lower maximum value for the $\hat{q}_{p,e}$ values. By trial and error, we find the maximum value for $\hat{q}_{p,e}$ to be less than

0.05 and, it may even be lower than this. The same can be applied to the $\hat{q}_{s,e}$ values, however the effect is less pronounced due to limited amount of shuttle data and we thus find the maximum value for $\hat{q}_{s,e}$ to be around 0.95. We decide not to dwell on this issue any further as we are interested in the overall picture of the results rather than the individual points.

Observe from **Figures 4.1** and **4.6** that $\hat{q}_{s,e}$ decreases as *a* and *b* become smaller and closer to each other, with the lowest value at $a = 0.01, b = 0.02$, i.e. the range between the upper ($\hat{q}_{s,e}$) and lower ($\hat{q}_{p,e}$) bounds becomes narrower. This is in agreement with our earlier result where if the expert views the risk in question to lie between that for planes and shuttles, then incorporation of expert information would lead to contraction of the premium range.

If the expert's opinion is otherwise, then we see from **Figures 4.1** and **4.2** that the range between the upper and lower bounds is greater than that between the MLEs and increases as the expert's beta density is peaked around higher values of *q*, i.e. as *a* and *b* become larger. In particular, the maximum value of $\hat{q}_{s,e}$ and hence the largest range between the upper and lower bounds occurs at $a = 0.98, b = 0.99$, where the lower and upper bound premium rate is almost zero and 0.95 respectively. We see from **Figure 4.3** that $\hat{q}_{p,e}$ also increases, however it does so gradually with a much smaller order of magnitude, which is why an almost flat surface for $\hat{q}_{p,e}$ can be visualised from **Figure 4.4**. In contrast, we see from **Figures 4.1, 4.5** and **4.6** that $\hat{q}_{s,e}$ increases by a much larger order of magnitude and hence the visualised surface is more like the slope of a hill. This is because we have far more loss data for planes than space shuttles and thus less weight is attached to the expert information in the case of planes in comparison to shuttles. For this particular case study, we thus conclude that our estimates for the lower bound premium rate are fairly insensitive to the expert information whereas those for the upper bound premium rate are sensitive, in which case careful elicitation of expert opinion would be recommended.

One may argue that the range of premium rates obtained may not be very practical when the expert views the risk in question differently from the decision maker, for instance the extreme case of zero and 0.95. However, he or she is the expert in the field whose opinion is obtained and if the decision maker rates his or her expertise highly, then our Bayesian approach provides the flexibility to incorporate this different view into the analysis without having to worry too much about the benchmarks chosen, and yet still obtaining reliable estimates based on related data and expert information rather than using any number without justification.

One could choose a different set of benchmarks, possibly wider so that the risk, as viewed by the expert lies between them and then perform the above analysis again, however there would be issues surrounding the order of information flow and updating in this case. Also choosing different benchmarks may narrow the range relative to the MLEs for the new benchmark data after incorporation of the expert information, but in absolute terms, it may be almost equal to the range obtained using our original benchmarks and thus may not be necessary to alter the benchmarks after all. The general rule of thumb however is that careful elicitation of expert information should be performed whenever loss data for a benchmark risk is sparse.

# 5. CONCLUSION

We have developed a method to price a new risk or insurance product in a realistic context and strengthen the reliability of estimates in cases where little or no loss data is available for the risk in question. The method incorporates expert opinion in a Bayesian framework with loss data from lower and upper benchmark risks in order to obtain a range of possible risk premiums that may be charged to insure against the risk in question.

We demonstrated the application of this method to a specific case study, i.e. commercial space travel insurance, where in particular, we were interested in finding a sensible range of premiums to provide cover against the risk of loss of life due to crash or mid-air explosion of a sub-orbital flight. As a result of incorporating more loss data for planes than space shuttles, we found the estimates for the lower bound premium rate to be fairly insensitive to the expert information whereas those for the upper bound premium rate were sensitive. Consequently, careful elicitation of expert information needs to be undertaken whenever loss data for a benchmark risk is sparse.

It was also observed that if the expert views the risk in question to lie between the benchmark risks, then incorporation of expert information leads to a narrowing of the premium range, but if the expert's opinion is otherwise, then the opposite is true. We thus see that our Bayesian approach is a very robust and flexible one, allowing us to incorporate the expert's opinion in the different cases where he or she views the risk in question to lie above, below or between the benchmarks, which is then reflected in the range of possible risk premiums chargeable for the risk in question.

The practitioner can then choose a risk premium from within the range and add an allowance for expenses and profit to obtain the chargeable office premium. The method can also be used to determine whether or not premiums to be charged by competitors and the market lie within a sensible range and thus gauge the potential profitability before venturing into the new business area.

Possible extensions to this research include comparing and contrasting the 3 variations of the Bayesian methodology suggested in section 3.1 and their corresponding results, exploring other distributions such as non-conjugate priors where the analysis may not be mathematically tractable and thus computationally demanding by way of requiring MCMC simulation. The actual setting of office premiums to be charged using various premium calculation principles could be another area for further research. Finally, one may also apply an adapted version of the methodology to other new risks or insurance products and analyse the results, if possible comparing and contrasting them with those obtained for our case study.

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# APPENDIX A: DATA

**Table A-1: Number of reportable accidents by class of aircraft, UK Public Transport Large Aeroplanes 1995-2004**

| Year | Business jet | Jet | Turboprop | Total by Year |
|---|---|---|---|---|
| 1995 | | 17 | 5 | 22 |
| 1996 | | 11 | 8 | 19 |
| 1997 | | 9 | 5 | 14 |
| 1998 | | 12 | 8 | 20 |
| 1999 | | 6 | 7 | 13 |
| 2000 | 1 | 7 | 7 | 15 |
| 2001 | | 5 | 3 | 8 |
| 2002 | | 13 | 5 | 18 |
| 2003 | | 14 | 2 | 16 |
| 2004 | 1 | 16 | 0 | 17 |
| **Total** | **2** | **110** | **50** | **162** |

**Source:** CAA (2006) - CAP 763 Aviation Safety Review 2005, Figure 4.1

*Note: The data values corresponding to the figure were obtained from the Safety Unit at CAA.*

**Table A-2: Hours and flights flown by UK Public Transport large airlines and air taxi operators 1995-2004**

| Year | Hours ('000) | Flights ('000) |
|---|---|---|
| 1995 | 1751 | 833 |
| 1996 | 1856 | 871 |
| 1997 | 1988 | 922 |
| 1998 | 2140 | 978 |
| 1999 | 2289 | 1032 |
| 2000 | 2442 | 1088 |
| 2001 | 2505 | 1136 |
| 2002 | 2408 | 1083 |
| 2003 | 2473 | 1078 |
| 2004 | 2637 | 1138 |
| **Total** | **22489** | **10159** |

**Source:** CAA (2006) - CAP 763 Aviation Safety Review 2005, Table 4.3

*Note: This is the corresponding utilisation/exposure data for Table A-1*

**Table A-3: Number of reportable accidents by class of aircraft, UK Public Transport Small Aeroplanes 1995-2004**

| Year | Piston | Turboprop | Total by Year |
|---|---|---|---|
| 1995 | 0 | 1 | 1 |
| 1996 | 4 | 0 | 4 |
| 1997 | 1 | 0 | 1 |
| 1998 | 3 | 0 | 3 |
| 1999 | 2 | 0 | 2 |
| 2000 | 1 | 1 | 2 |
| 2001 | 2 | 1 | 3 |
| 2002 | 1 | 0 | 1 |
| 2003 | 2 | 0 | 2 |
| 2004 | 3 | 0 | 3 |
| **Total** | **19** | **3** | **22** |

**Source:** CAA (2006) - CAP 763 Aviation Safety Review 2005, Figure 4.7

*Note: The data values corresponding to the figure were obtained from the Safety Unit at CAA.*

**Table A-4: Hours and flights flown by UK Public Transport small airlines and air taxi operators 1995-2004**

| Year | Hours ('000) | Flights ('000) |
|---|---|---|
| 1995 | 51 | 78 |
| 1996 | 44 | 71 |
| 1997 | 43 | 69 |
| 1998 | 45 | 73 |
| 1999 | 40 | 67 |
| 2000 | 39 | 63 |
| 2001 | 36 | 61 |
| 2002 | 38 | 66 |
| 2003 | 38 | 60 |
| 2004 | 44 | 68 |
| **Total** | **418** | **676** |

**Source:** CAA (2006) - CAP 763 Aviation Safety Review 2005, Table 4.11

*Note: This is the corresponding utilisation/exposure data for Table A-3*

**Table A-5: Amalgamated accident and utilisation/exposure data (flights flown) for UK Public Transport large and small aeroplanes 1995-2004**

| Year | Total Accidents by Year | Total Flights by Year ('000) | Probability Of Failure |
|---|---|---|---|
| 1995 | 23 | 911 | 0.0000252470 |
| 1996 | 23 | 942 | 0.0000244161 |
| 1997 | 15 | 991 | 0.0000151362 |
| 1998 | 23 | 1051 | 0.0000218839 |
| 1999 | 15 | 1099 | 0.0000136488 |
| 2000 | 17 | 1151 | 0.0000147698 |
| 2001 | 11 | 1197 | 0.0000091896 |
| 2002 | 19 | 1149 | 0.0000165361 |
| 2003 | 18 | 1138 | 0.0000158172 |
| 2004 | 20 | 1206 | 0.0000165837 |
| **Total** | **184** | **10835** | 0.0000169820 |

*Note: Tables A-1, A-2, A-3 and A-4 were used to create this table. The number of flights flown is used as the corresponding exposure data as we are interested in the probability rather than the rate of failure. The probability of failure was calculated as the number of accidents divided by the number of flights flown each year.*

**Table A-6: NASA Space shuttle mission details 1981-2006**

| Year | Mission | Launch | Landing | Count | Failure |
|---|---|---|---|---|---|
| 2006 | STS-116 | December 9, 2006 | December 22, 2006 | 1 | 0 |
| | STS-115 | September 9, 2006 | September 21, 2006 | 1 | 0 |
| | STS-121 | July 4, 2006 | July 17, 2006 | 1 | 0 |
| 2005 | STS-114 | July 26, 2005 | Aug. 9, 2005 | 1 | 0 |
| 2004 | | | | 0 | |
| 2003 | STS-107 | Jan. 16, 2003 | Vehicle and crew lost during entry on Feb. 1, 2003 | 1 | 1 |
| 2002 | STS-113 | Nov. 23, 2002 | Dec. 7, 2002 | 1 | 0 |
| | STS-112 | Oct. 7, 2002 | Oct. 18, 2002 | 1 | 0 |
| | STS-111 | June 5, 2002 | June 19, 2002 | 1 | 0 |
| | STS-110 | April 8, 2002 | April 19, 2002 | 1 | 0 |
| | STS-109 | March 1, 2002 | March 12, 2002 | 1 | 0 |
| 2001 | STS-108 | Dec. 5, 2001 | Dec. 17, 2001 | 1 | 0 |
| | STS-105 | Aug. 10, 2001 | Aug. 22, 2001 | 1 | 0 |
| | STS-104 | July 12, 2001 | July 24, 2001 | 1 | 0 |
| | STS-100 | April 19, 2001 | May 1, 2001 | 1 | 0 |
| | STS-102 | March 8, 2001 | March 21, 2001 | 1 | 0 |
| | STS-98 | Feb. 7, 2001 | Feb. 20, 2001 | 1 | 0 |
| 2000 | STS-97 | Nov. 30, 2000 | Dec. 11, 2000 | 1 | 0 |
| | STS-92 | Oct. 11, 2000 | Oct. 24, 2000 | 1 | 0 |
| | STS-106 | Sept. 8, 2000 | Sept. 20, 2000 | 1 | 0 |
| | STS-101 | May 19, 2000 | May 29, 2000 | 1 | 0 |
| | STS-99 | Feb. 11, 2000 | Feb. 22, 2000 | 1 | 0 |
| 1999 | STS-103 | Dec. 19, 1999 | Dec. 27, 1999 | 1 | 0 |
| | STS-93 | July 23, 1999 | July 27, 1999 | 1 | 0 |
| | STS-96 | May 27, 1999 | June 6, 1999 | 1 | 0 |
| 1998 | STS-88 | Dec. 4, 1998 | Dec. 15, 1998 | 1 | 0 |
| | STS-95 | Oct. 29, 1998 | Nov. 7, 1998 | 1 | 0 |
| | STS-91 | June 2, 1998 | June 12, 1998 | 1 | 0 |
| | STS-90 | April 17, 1998 | May 3, 1998 | 1 | 0 |
| | STS-89 | Jan. 22, 1998 | Jan. 31, 1998 | 1 | 0 |
| 1997 | STS-87 | Nov. 19, 1997 | Dec. 5, 1997 | 1 | 0 |

| Year | Mission | Launch | Landing | Count | Failure |
|---|---|---|---|---|---|
| | STS-86 | Sept. 25, 1997 | Oct. 6, 1997 | 1 | 0 |
| | STS-85 | Aug. 7, 1997 | Aug. 19, 1997 | 1 | 0 |
| | STS-94 | July 1, 1997 | July 17, 1997 | 1 | 0 |
| | STS-84 | May 15, 1997 | May 24, 1997 | 1 | 0 |
| | STS-83 | April 4, 1997 | April 8, 1997 | 1 | 0 |
| | STS-82 | Feb. 11, 1997 | Feb. 21, 1997 | 1 | 0 |
| | STS-81 | Jan. 12, 1997 | Jan. 22, 1997 | 1 | 0 |
| 1996 | STS-80 | Nov. 19, 1996 | Dec. 7, 1996 | 1 | 0 |
| | STS-79 | Sept. 16, 1996 | Sept. 26, 1996 | 1 | 0 |
| | STS-78 | June 20, 1996 | July 7, 1996 | 1 | 0 |
| | STS-77 | May 19, 1996 | May 29, 1996 | 1 | 0 |
| | STS-76 | March 22, 1996 | March 31, 1996 | 1 | 0 |
| | STS-75 | Feb. 22, 1996 | March 9, 1996 | 1 | 0 |
| | STS-72 | Jan. 11, 1996 | Jan. 20, 1996 | 1 | 0 |
| 1995 | STS-74 | Nov. 12, 1995 | Nov. 20, 1995 | 1 | 0 |
| | STS-73 | Oct. 20, 1995 | Nov. 5, 1995 | 1 | 0 |
| | STS-69 | Sept. 7, 1995 | Sept. 18, 1995 | 1 | 0 |
| | STS-70 | July 13, 1995 | July 22, 1995 | 1 | 0 |
| | STS-71 | June 27, 1995 | July 7, 1995 | 1 | 0 |
| | STS-67 | March 2, 1995 | March 18, 1995 | 1 | 0 |
| | STS-63 | Feb. 3, 1995 | Feb. 11, 1995 | 1 | 0 |
| 1994 | STS-66 | Nov. 3, 1994 | Nov. 14, 1994 | 1 | 0 |
| | STS-68 | Sept. 30, 1994 | Oct. 11, 1994 | 1 | 0 |
| | STS-64 | Sept. 9, 1994 | Sept. 20, 1994 | 1 | 0 |
| | STS-65 | July 8, 1994 | July 23, 1994 | 1 | 0 |
| | STS-59 | April 9, 1994 | April 20, 1994 | 1 | 0 |
| | STS-62 | March 4, 1994 | March 18, 1994 | 1 | 0 |
| | STS-60 | Feb. 3, 1994 | Feb. 11, 1994 | 1 | 0 |
| 1993 | STS-61 | Dec. 2, 1993 | Dec. 13, 1993 | 1 | 0 |
| | STS-58 | Oct. 18, 1993 | Nov. 1, 1993 | 1 | 0 |
| | STS-51 | Sept. 12, 1993 | Sept. 22, 1993 | 1 | 0 |
| | STS-57 | June 21, 1993 | July 1, 1993 | 1 | 0 |
| | STS-55 | April 26, 1993 | May 6, 1993 | 1 | 0 |

| Year | Mission | Launch | Landing | Count | Failure |
|---|---|---|---|---|---|
| | STS-56 | April 8, 1993 | April 17, 1993 | 1 | 0 |
| | STS-54 | Jan. 13, 1993 | Jan. 19, 1993 | 1 | 0 |
| 1992 | STS-53 | Dec. 2, 1992 | Dec. 9, 1992 | 1 | 0 |
| | STS-52 | Oct. 22, 1992 | Nov. 1, 1992 | 1 | 0 |
| | STS-47 | Sept. 12, 1992 | Sept. 20, 1992 | 1 | 0 |
| | STS-46 | July 31, 1992 | Aug. 8, 1992 | 1 | 0 |
| | STS-50 | June 25, 1992 | July 9, 1992 | 1 | 0 |
| | STS-49 | May 7, 1992 | May 16, 1992 | 1 | 0 |
| | STS-45 | March 24, 1992 | April 2, 1992 | 1 | 0 |
| | STS-42 | Jan. 22, 1992 | Jan. 30, 1992 | 1 | 0 |
| 1991 | STS-44 | Nov. 24, 1991 | Dec. 1, 1991 | 1 | 0 |
| | STS-48 | Sept. 12, 1991 | Sept. 18, 1991 | 1 | 0 |
| | STS-43 | Aug. 2, 1991 | Aug. 11, 1991 | 1 | 0 |
| | STS-40 | June 5, 1991 | June 14, 1991 | 1 | 0 |
| | STS-39 | April 28, 1991 | May 6, 1991 | 1 | 0 |
| | STS-37 | April 5, 1991 | April 11, 1991 | 1 | 0 |
| 1990 | STS-35 | Dec. 2, 1990 | Dec. 10, 1990 | 1 | 0 |
| | STS-38 | Nov. 15, 1990 | Nov. 20, 1990 | 1 | 0 |
| | STS-41 | Oct. 6, 1990 | Oct. 10, 1990 | 1 | 0 |
| | STS-31 | April 24, 1990 | April 29, 1990 | 1 | 0 |
| | STS-36 | Feb. 28, 1990 | March 4, 1990 | 1 | 0 |
| | STS-32 | Jan. 9, 1990 | Jan. 20, 1990 | 1 | 0 |
| 1989 | STS-33 | Nov. 22, 1989 | Nov. 27, 1989 | 1 | 0 |
| | STS-34 | Oct. 18, 1989 | Oct. 23, 1989 | 1 | 0 |
| | STS-28 | Aug. 8, 1989 | Aug. 13, 1989 | 1 | 0 |
| | STS-30 | May 4, 1989 | May 8, 1989 | 1 | 0 |
| | STS-29 | March 13, 1989 | March 18, 1989 | 1 | 0 |
| 1988 | STS-27 | Dec. 2, 1988 | Dec. 6, 1988 | 1 | 0 |
| | STS-26 | Sept. 29, 1988 | Oct. 3, 1988 | 1 | 0 |
| 1987 | | | | 0 | |
| 1986 | STS-51L | Jan. 28, 1986 | Vehicle and crew lost 73 seconds after lift off | 1 | 1 |
| | STS-61C | Jan. 12, 1986 | Jan. 18, 1986 | 1 | 0 |

| Year | Mission | Launch | Landing | Count | Failure |
|---|---|---|---|---|---|
| 1985 | STS-61B | Nov. 26, 1985 | Dec. 3, 1985 | 1 | 0 |
| | STS-61A | Oct. 30, 1985 | Nov. 6, 1985 | 1 | 0 |
| | STS-51J | Oct. 3, 1985 | Oct. 7, 1985 | 1 | 0 |
| | STS-51I | Aug. 27, 1985 | Sept. 3, 1985 | 1 | 0 |
| | STS-51F | July 29, 1985 | Aug. 6, 1985 | 1 | 0 |
| | STS-51G | June 17, 1985 | June 24, 1985 | 1 | 0 |
| | STS-51B | April 29, 1985 | May 6, 1985 | 1 | 0 |
| | STS-51D | April 12, 1985 | April 19, 1985 | 1 | 0 |
| | STS-51C | Jan. 24, 1985 | Jan. 27, 1985 | 1 | 0 |
| 1984 | STS-51A | Nov. 8, 1984 | Nov. 16, 1984 | 1 | 0 |
| | STS-41G | Oct. 5, 1984 | Oct. 13, 1984 | 1 | 0 |
| | STS-41D | Aug. 30, 1984 | Sept. 5, 1984 | 1 | 0 |
| | STS-41C | April 6, 1984 | April 13, 1984 | 1 | 0 |
| | STS-41B | Feb. 3, 1984 | Feb. 11, 1984 | 1 | 0 |
| 1983 | STS-9 | Nov. 28, 1983 | Dec. 8, 1983 | 1 | 0 |
| | STS-8 | Aug. 30, 1983 | Sept. 5, 1983 | 1 | 0 |
| | STS-7 | June 18, 1983 | June 24, 1983 | 1 | 0 |
| | STS-6 | April 4, 1983 | April 9, 1983 | 1 | 0 |
| 1982 | STS-5 | Nov. 11, 1982 | Nov. 16, 1982 | 1 | 0 |
| | STS-4 | June 27, 1982 | July 4, 1982 | 1 | 0 |
| | STS-3 | March 22, 1982 | March 30, 1982 | 1 | 0 |
| 1981 | STS-2 | Nov. 12, 1981 | Nov. 14, 1981 | 1 | 0 |
| | STS-1 | April 12, 1981 | April 14, 1981 | 1 | 0 |
| | | **Total** | | **117** | **2** |

**Source:** NASA (2007) Space Shuttle Mission Archives [Online]. Available: http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/list_main.html [17 July 2007]

# APPENDIX B: MATHEMATICA CODE, OUTPUT RESULTS AND SCREENSHOTS

The Mathematica code and the corresponding output results for the simulation exercise performed in Section 4.3.3 are included below:

```
RandomReal[]

0.412634

RandomReal[{0, %}]

0.257325


NMinimize[
 {(CDF[BetaDistribution[α, β], 0.25733] - 0.025)^2 +
   (CDF[BetaDistribution[α, β], 0.41263] - 0.975)^2, α > 0 && β > 0},
 {α, β}]

{3.0415 × 10^-12, {α → 46.5563, β → 93.3724}}
```

```
(46.55625346279727 + 185) /
 (46.55625346279727 + 93.3723827803198 + 10 835 002)
```

0.0000213709

```
(46.55625346279727 + 3) / (46.55625346279727 + 93.3723827803198 + 119)
```

0.19139

```
RandomReal[]
```

0.417403

```
RandomReal[{0, %}]
```

0.222135

```
RandomReal[]
```

0.603047

```
RandomReal[{0, %}]
```

0.0265603

```
NMinimize[
 {(CDF[BetaDistribution[α, β], 0.02656] - 0.025)^2 +
   (CDF[BetaDistribution[α, β], 0.60305] - 0.975)^2, α > 0 && β > 0},
 {α, β}]
```

$\{1.2636 \times 10^{-30}, \{\alpha \to 1.65755, \beta \to 5.08245\}\}$

```
(1.6575546192023471 + 185) /
 (1.6575546192023471 + 5.08244665331523 + 10 835 002)
```

0.0000172273

```
(1.6575546192023471 + 3) / (1.6575546192023471 + 5.08244665331523 + 119)
```

0.0370412

```
RandomReal[]
```

0.655641

```
RandomReal[{0, %}]
```

0.0813414

```
RandomReal[]
```

0.692231

```
RandomReal[]
```

0.745391

```
RandomReal[{0, %}]
```

0.362148

```
NMinimize[
 {(CDF[BetaDistribution[α, β], 0.36215] - 0.025)^2 +
   (CDF[BetaDistribution[α, β], 0.74539] - 0.975)^2, α > 0 && β > 0},
 {α, β}]
```

$\{1.72977 \times 10^{-24}, \{\alpha \to 13.5702, \beta \to 10.7353\}\}$

```
(13.57016743926401 + 185) /
 (13.57016743926401 + 10.735251094525953 + 10835002)
```

0.0000183267

```
(13.57016743926401 + 3) / (13.57016743926401 + 10.735251094525953 + 119)
```

0.115628

```
RandomReal[]
```

0.126843

```
RandomReal[{0, %}]
```

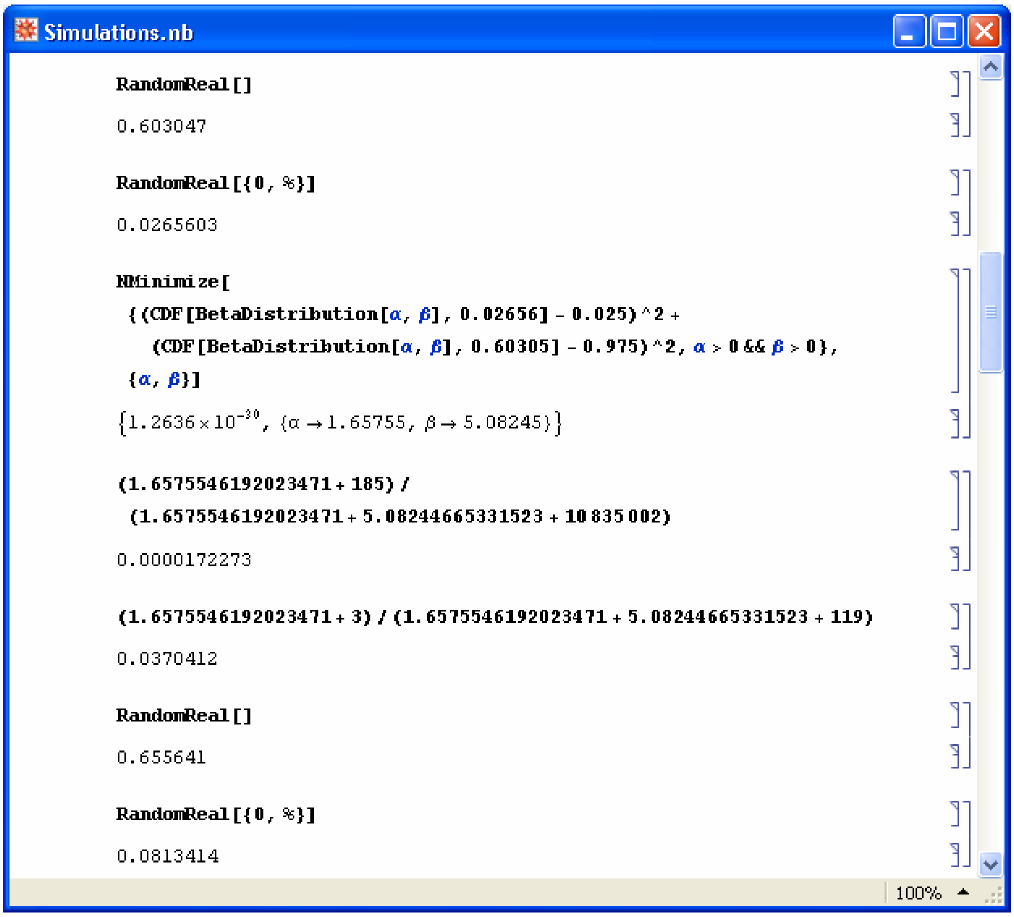
0.0615701

```
NMinimize[
 {(CDF[BetaDistribution[α, β], 0.06157] - 0.025)^2 +
   (CDF[BetaDistribution[α, β], 0.12684] - 0.975)^2, α > 0 && β > 0},
 {α, β}]
```

$\{7.51993 \times 10^{-17}, \{\alpha \to 27.2367, \beta \to 270.082\}\}$

```
(27.23670615325979 + 185) /
 (27.23670615325979 + 270.08233037966835 + 10 835 002)

0.0000195875

(27.23670615325979 + 3) / (27.23670615325979 + 270.08233037966835 + 119)

0.0726287

RandomReal[]

0.404703

RandomReal[{0, %}]

0.0183744

NMinimize[
 {(CDF[BetaDistribution[α, β], 0.01837] - 0.025)^2 +
   (CDF[BetaDistribution[α, β], 0.40470] - 0.975)^2, α > 0 && β > 0},
 {α, β}]

{6.96344×10^-32, {α → 1.81743, β → 9.74253}}

(1.8174262647560675 + 185) /
 (1.8174262647560675 + 9.742528530639985 + 10 835 002)

0.000017242

(1.8174262647560675 + 3) / (1.8174262647560675 + 9.742528530639985 + 119)

0.0368982
```

**Figure B-1: Screenshot of the simulation implementation**

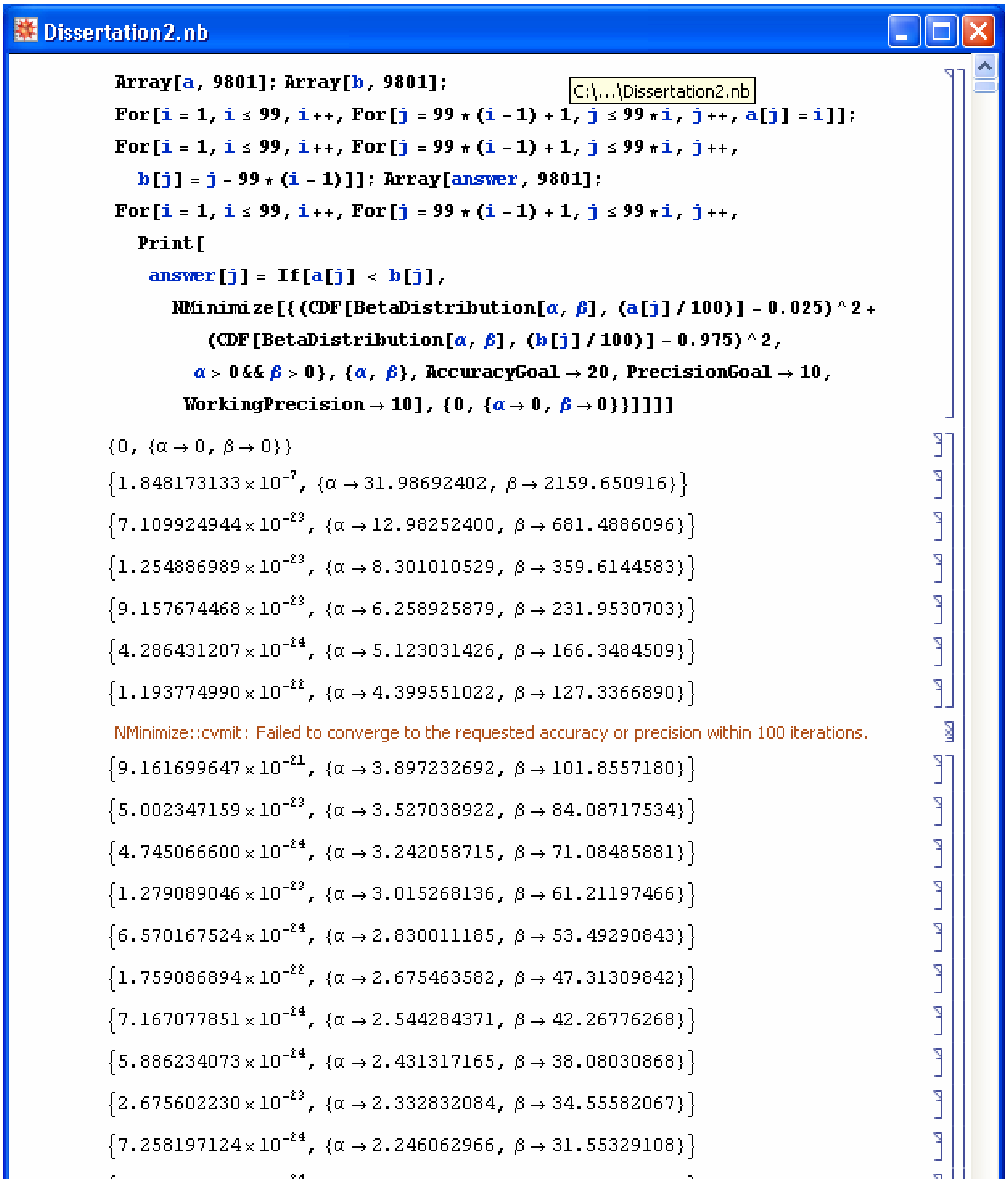


The Mathematica code and a screenshot showing the output results for the sensitivity analysis performed in Section 4.3.4 are included below.

The following piece of code computes the values of $\alpha$ and $\beta$ for different feasible combinations of *a* and *b*.

```
Array[a, 9801]; Array[b, 9801];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++, a[j] = i]];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  b[j] = j - 99 * (i - 1)]]; Array[answer, 9801];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  Print[
   answer[j] = If[a[j] < b[j],
     NMinimize[{(CDF[BetaDistribution[α, β], (a[j] / 100)] - 0.025) ^ 2 +
        (CDF[BetaDistribution[α, β], (b[j] / 100)] - 0.975) ^ 2,
       α > 0 && β > 0}, {α, β}, AccuracyGoal → 20, PrecisionGoal → 10,
      WorkingPrecision → 10], {0, {α → 0, β → 0}}]]]]
```

**Figure B-2: Screenshot of the sensitivity analysis implementation**



The next piece of code extracts $\alpha$ and $\beta$ from the output and then calculates the values of $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$.

```
Array[c, 9801]; Array[d, 9801];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  c[j] = Part[answer[j], 2, 1, 2]]];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  d[j] = Part[answer[j], 2, 2, 2]]]; Array[qshuttle, 9801];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  qshuttle[j] = (c[j] + 3) / (c[j] + d[j] + 119)]]; Array[qplane, 9801];
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  qplane[j] = (c[j] + 185) / (c[j] + d[j] + 10 835 000)]];
```

```
For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  qplane[j] = If[a[j] < b[j], qplane[j], 0]]]

For[i = 1, i ≤ 99, i++, For[j = 99 * (i - 1) + 1, j ≤ 99 * i, j++,
  qshuttle[j] = If[a[j] < b[j], qshuttle[j], 0]]]
```

The following lines of code plot the 3-Dimensional figures shown in Section 4.3.4 in their respective order:

```
ListPointPlot3D[{Table[{b[i] / 100, a[i] / 100, qplane[i]}, {i, 1, 9801}],
  Table[{b[i] / 100, a[i] / 100, qshuttle[i]}, {i, 1, 9801}]},
 AxesLabel → {b, a}, PlotLabel → "qshuttle/qplane"]
```

```
ListPointPlot3D[{Table[{b[i] / 100, a[i] / 100, qplane[i]}, {i, 1, 9801}],
  Table[{b[i] / 100, a[i] / 100, qshuttle[i]}, {i, 1, 9801}]},
 AxesLabel → {b, a}, PlotLabel → "qshuttle/qplane", PlotRange → All]
```

```
ListPointPlot3D[Table[{b[i] / 100, a[i] / 100, qplane[i]}, {i, 1, 9801}],
 AxesLabel → {b, a}, PlotLabel → qplane, PlotRange → {0.000014, 0.000026}]
```

```
ListPointPlot3D[Table[{b[i] / 100, a[i] / 100, qplane[i]}, {i, 1, 9801}],
 AxesLabel → {b, a}, PlotLabel → qplane, PlotRange → {0.0000000001, 0.1}]
```

```
ListPointPlot3D[Table[{b[i] / 100, a[i] / 100, qshuttle[i]}, {i, 1, 9801}],
 AxesLabel → {b, a}, PlotLabel → qshuttle, PlotStyle → {Purple}]
```

```
ListPointPlot3D[Table[{b[i] / 100, a[i] / 100, qshuttle[i]}, {i, 1, 9801}],
 AxesLabel → {b, a}, PlotLabel → qshuttle, PlotStyle → {Purple},
 PlotRange → All]
```

This last piece of code exports all the values for $a$, $b$, $\alpha$ , $\beta$ , $\hat{q}_{p,e}$ and $\hat{q}_{s,e}$ into separate *".dat"* files in case the reader wishes to examine the results and possibly use them in further research.

```
str = OpenWrite["C:/a.dat"]

OutputStream[C:/a.dat, 33]

For[i = 1, i ≤ 9801, i++, Write[str, N[a[i] / 100]]]; Close[str]

C:/a.dat

str = OpenWrite["C:/b.dat"]

OutputStream[C:/b.dat, 34]

For[i = 1, i ≤ 9801, i++, Write[str, N[b[i] / 100]]]; Close[str]

C:/b.dat
```

```
str = OpenWrite["C:/alpha.dat"]

OutputStream[C:/alpha.dat, 35]

For[i = 1, i ≤ 9801, i++,
 If[c[i] == 0, Write[str, c[i]], Write[str, N[c[i]]]]];
Close[str]

C:/alpha.dat

str = OpenWrite["C:/beta.dat"]

OutputStream[C:/beta.dat, 36]

For[i = 1, i ≤ 9801, i++,
 If[d[i] == 0, Write[str, d[i]], Write[str, N[d[i]]]]];
Close[str]

C:/beta.dat

str = OpenWrite["C:/qse.dat"]

OutputStream[C:/qse.dat, 37]

For[i = 1, i ≤ 9801, i++,
 If[qshuttle[i] == 0, Write[str, qshuttle[i]],
  Write[str, N[qshuttle[i]]]]];
Close[str]

C:/qse.dat

str = OpenWrite["C:/qpe.dat"]

OutputStream[C:/qpe.dat, 38]

For[i = 1, i ≤ 9801, i++,
 If[qplane[i] == 0, Write[str, qplane[i]], Write[str, N[qplane[i]]]]];
Close[str]

C:/qpe.dat
```

# APPENDIX C: SPIEGELHALTER'S APPROXIMATION FOR COMPUTING THE PARAMETERS OF THE BETA DISTRIBUTION

Suppose that $(a,b)$ is the elicited 95% confidence interval for the probability of failure, $q$. Approximate the beta distribution by a normal distribution so that we can assume that the interval $(a,b)$ approximately represents (mean $\pm$ $W$ standard deviations) of a beta $(\alpha,\beta)$ distribution. Since we are considering 95% confidence intervals, $W$ would take the value 1.96.

The mean and variance of a Beta distribution are given by $\frac{\alpha}{\alpha+\beta}$ and $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ respectively. Then

$$\frac{\alpha}{\alpha+\beta}+1.96\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}=b$$

and

$$\frac{\alpha}{\alpha+\beta}-1.96\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}=a$$

Adding the above two equations:

$$\frac{\alpha}{\alpha+\beta}=\frac{a+b}{2}$$

where we define $g=\frac{a+b}{2}$ as the midpoint of the interval.

Note that the mean of the beta distribution is equal to the midpoint of the interval $(a,b)$ due to the Normal approximation. The approximation will thus get less reliable as the difference between *a* and *b* values becomes larger.

Subtracting the two equations:

$$2W\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}=b-a$$

where $W=1.96$ in the case of a 95% confidence interval.

$$2W\sqrt{\frac{g(1-g)}{(\alpha+\beta+1)}}=b-a$$

$$4W^2\frac{g(1-g)}{(\alpha+\beta+1)}=(b-a)^2$$

$$\therefore \alpha+\beta+1=g(1-g)\left(\frac{W}{h}\right)^2$$

where $h=\frac{(b-a)}{2}$ is the half range of the interval $(a,b)$.

Then the 2 simultaneous equations give us the required values of $\alpha$ and $\beta$ as:

$$\alpha=\frac{W^2g^2(1-g)}{h^2}-g \quad \text{and} \quad \beta=\frac{W^2g(1-g)^2}{h^2}-(1-g)$$