INTRODUCTION

As medical and particularly genetic advances are made with respect to critical illnesses, there are concerns that genetic discrimination will become inherent in the insurance industry, rendering some people unable to purchase insurance or increasing the premium they are charged massively, for products such as life, income protection and critical illness insurance. The other side of the argument is that without full disclosure of all the information an individual has about his or her true risk, the insurance company may be exposed to adverse selection. The two main concerns for the insurance industry with respect to adverse selection are that high-risk individuals will purchase higher levels of insurance than low and medium-risk individuals, and secondly if the insurance company does not know where the risk lies, premiums will increase for everyone in the market to cover the increased risk. This may lead to low-risk individuals withdrawing from the market because premiums they are charged are expensive relative to the risk they actually present.

Genetically transmitted diseases that are of interest to the insurer are diseases which develop late in the life of an otherwise healthy adult. Currently these types of diseases for which predictive genetic tests exist, are very rare and to date most studies conclude that asymmetric knowledge of the risk of these diseases is not a significant threat to the insurance industry.

As the attention of geneticists turns towards more common multifactorial diseases, it is reasonable to assume that future genetic discoveries may have a much greater effect on the insurance industry. This dissertation aims to look at the risk of adverse selection by examining the level of market withdrawal that may occur when predictive genetic test results of common multifactorial diseases are not disclosed to the insurer. The disease focused on is heart attack and the market examined is the critical illness insurance market. An equilibrium model is proposed to examine market withdrawal. In the model the critical illness insurance market is split into different risk strata depending on a person's genetic and environmental risk. The Premium charged is calculated under the assumption that genetic test results are not disclosed to the insurer. Using utility theory the maximum premium an individual in a particular risk stratum is willing to pay is calculated and then the level of market withdrawal is quantified. Should a proportion of the low-risk strata decline to purchase insurance, the amount of risk in the market, and hence premiums, will increase, which may lead

to further market withdrawal. The market equilibrium is defined as the point where no further market withdrawal or increases in the premium charged occurs. The model examines the amount of withdrawal that has occurred once equilibrium has been reached, if this equilibrium does exist.

In section 1 the reader is introduced to the basics of human genetics and its impact on the insurance industry. Utility theory and the calculation of the maximum premium people are willing to pay for insurance is discussed in section 2. A basic critical illness pricing model and the estimation of its parameters are described in section 3.1, and in section 3.2 we expand this model by introducing gene-environment risk strata. Section 4.1 provides a complete description of the equilibrium model and how market withdrawal is quantified, and discusses the results calculated. The conclusions are discussed in section 5.

1.1 BASIC HUMAN GENETICS

Deoxyribonucleic acid, DNA, is the material located in the cells of a person's body, which contains the information or "genetic code" required for the body to carry out functions. This information may be thought of as the instructions to construct other components of cells such as proteins. The part of DNA, which contains this genetic code, is the gene. Genes are located on a large molecule of DNA called a chromosome. Humans have 46 chromosomes per cell, 22 pairs of "autosomes" and two sex chromosomes, XX in the case of a female and XY in the case of a male. Each of these 46 chromosomes should be replicated perfectly in all other cells of the body, as new cells are produced by a process of division, known as mitosis.

Offspring inherit 23 chromosomes from each parent totalling the required 46. This means, for the process of reproduction, the sperm and egg cells must each dispense with half of their chromosomes and become "gametes". This occurs through a process called meiosis. Initially sections of differing lengths of a chromosome are interchanged with that of its partnering chromosome. Division of the cell then occurs such that four daughter cells are formed, each containing 23 chromosomes, which are all a different mix of their former pair. In this way a parent passes on different aspects of genetic information to its offspring.

One of the functions of the gene is to carry a code, which in turn produces a protein chain. Different genes generate different proteins, which carry out different functions in the body. It is unsurprising, then, that occasionally a cell will make an error in expressing these codes. These errors, or mutations, may be harmless or advantageous to the organism, or they may cause disease. Only genetic variations of the egg or sperm cells may be passed on to the offspring and hence most genetic mutations are not passed on.

Genes may be present in different forms known as alleles. An example of the simplest case is eye colour. The gene has two alleles, denoted *A* and *a*. These alleles can result in genotypes, *AA*, *Aa*, *aA* or *aa*. Let us assume that *A* represents brown eyes, and *a* represents blue eyes. Then an individual with genotype *AA*, *Aa* or *aA* would have brown eyes, and one with *aa* would have blue eyes. We would say that brown eyes are dominantly inherited and blue eyes recessively inherited, as *A* overrides *a* and genotype *Aa* results in brown eyes. In this example, the individual's

eye colour is said to be their phenotype. Mutations in genes create new alleles, causing altered proteins, which may be fatal, harmful or beneficial.

Just as harmless differences in alleles like eye colour can be dominantly or recessively inherited, so to can variations which cause harmful diseases. A dominantly inherited disease such as Huntington's disease is one such that an individual only needs to inherit one mutated gene in order to develop the condition in the future. An offspring of a gene carrier of a rare dominantly inherited single-gene disorder has a fifty percent chance of inheriting the gene, independent of any other offspring. Dominantly inherited disorders are attacking diseases, in that the dominant allele produces a harmful protein. Recessively inherited diseases occur when two copies of the mutated gene are necessary to cause the disease. In the case of recessively inherited disorders, such as cystic fibrosis, non-functioning proteins are produced by the allele, which purports to a deficiency rather then an attacking disease.

In reality, there exist much more common diseases, which are caused by a combination of environmental and genetic factors that have not yet been discovered. For an example, consider smoking as the environmental factor. People who smoke are more likely to suffer from certain critical illnesses. However, illness will not occur unless the individual possesses the genetic potential for it to develop. It is probable that many diseases are influenced by more then one gene, and by numerous environmental factors. These multifactorial diseases are of growing interest within genetic research, and are the focus of this dissertation. Obviously genetic test results for these more common diseases, when discovered, will have implications on a much broader scale than those for the rare single-gene disorders for which predictive genetic tests are now available. From an insurer's point of view, as the number of people who possess genetic information related to the health risk they pose, unavailable to the insurer, rises, the more susceptible to adverse selection (see section 1.3.4) the insurer may become.

1.2 PREDICTIVE GENETIC TESTS

A predictive genetic test is one carried out on a healthy individual, perhaps prompted by a family history of a certain disease, in order to determine whether this individual is a carrier of a mutation in a specific gene which has a known association with a particular, and generally serious, disease. The following are points which

should be noted in reference to predictive genetic testing according to Evans et al, 2001:

a) Predictive tests have the potential for accurately assessing risk, and for targeting screening and preventative measures appropriately.
b) In general, these predictive genetic tests do not imply that an individual, who has a positive result, will certainly develop the disease in question. There is always an element of uncertainty attached to these tests although in some genetic disorders the identified risk may be higher (for example, Huntington's disease). The predictive genetic tests are unable to identify the severity of the condition or predict the time of onset.
c) Medically the value of the test depends on the nature of the disease being tested for and the effectiveness and cost of the treatment available.

The above points are not just of medical relevance, but also of some importance to the insurer. Accurately assessing risk is fundamental to the insurer for pricing all insurance products. Presently, underwriting evaluates the risk posed by an individual's health for life insurance, critical illness (CI) insurance and similar insurance products. Questions about family history, occupation, age and sex are used routinely, to ensure a fair premium is charged for the cover provided. Clearly it is of interest to the insurer how likely it is that an individual with a positive test result or negative test result will develop the disorder.

One would assume that the medical value of the test, such as importance of early detection, availability of effective treatment etc. will have some impact on the number of people who take the test, and the number of people who take the test will have some effect on the risk of adverse selection (see section 1.3.4) to an insurance company.

1.3 GENETICS AND INSURANCE CONCERNS

The fair premium charged to a population of people grouped into a similar risk category is calculated by estimating the risk that this population poses. For example, a very broad risk category might be females aged forty. As was stated previously, it is through underwriting that an insurer estimates some of the risks associated with offering cover to a person. For policies related to an individual's health or future life

expectancy such as life insurance policies, income protection insurance or CI insurance, medical underwriting is carried out. Results of predictive genetic tests may offer further information relating to an individuals medical risk. Opinion varies as to whether this predictive genetic information should be made available to the insurer. Does this extra information pose a threat to the general insured population, or does it benefit them? Is it possible that if applicants have additional information that they need not disclose to the insurance company relating to the risk that they individually pose, this information could be used to the financial disadvantage of the insurer? In the case where individuals possess additional information to that of the insurance company, we say that there exists asymmetric knowledge.

1.3.1 Ethical issues

There are fears that genetic discrimination may occur if genetic test results must be given to the insurer. Genetic information may be misinterpreted leading to inaccurate decision-making. There are also worries that a small group of people may be deemed uninsurable, these people being the very group who most require it. (Ashcroft (2007)). To the contrary, Holm (2007) argued that genetic information is no more or less at risk of misinterpretation or discrimination then other types of medical and personal information, which is submitted freely, thus far, to the insurer.

Concerns also exist that people will elect not to undertake potentially beneficial genetic tests because of their trepidation about genetic discrimination and becoming subject to significantly increased premiums. Similarly concerns exist that people may be reluctant to take part in clinical trials. Currently several organisations are attempting to address some of these issues.

1.3.2 Current legislation and regulation

The Human Genetics Commission (HGC) is a UK Government advisory body, who together with the Association of British Insurers (ABI), have agreed on and published a “Concordat and Moratorium on Genetics and Insurance”. This document aims to regulate the use of genetic test results in the underwriting process. Currently applicants are only required to disclose genetic test results when applying for life insurance with sum assured of £500,000 or larger, CI insurance with sum assured of £300,000 or larger, or income protection insurance with sum assured of £30,000 per annum or larger. The moratorium ends in November 2011. The Genetics and

Insurance Committee (GAIC), is a scientific Advisory Committee, run by the Department of Health, to whom the ABI must present evidence if they believe insurers are at risk due to a specific genetic test result. In some instances, such as Huntington's disease, family history implies a specific genetic mutation. Currently negative test results may be revealed to the insurer to over-ride such a family history. Inevitably this would reveal to the insurer, by default, who the mutation carriers are, which defeats the purpose of the moratorium in these cases, but is deemed acceptable by the HGC and GAIC.

Together the HGC and GAIC monitor the progress of genetic testing and emerging problems and work together to attempt to protect all stakeholders as advancements are made. Macdonald (2003) summarised how the UK government, through the HGC and GAIC, has taken an evidence-based approach to their policy-making. In response the insurance industry are asked to do likewise and take an evidence-based approach to underwriting.

Patient groups such as breakthrough cancer and the Alzheimer's society inform people of the outcomes of the moratorium, and it's ending date. While the moratorium tries to satisfy all interested parties, its short-term nature means that people may still be deterred from taking these potentially beneficial genetic tests, as they fear what the end of the moratorium in 2011 might bring. GAIC, the ABI and HGC refer to this as the "test now, buy later" problem. However it is important to note that there may not exist a satisfactory long-term solution. Each genetic disorder should be considered separately as relevant advances are made. A long-term legislative document may unintentionally disadvantage an insurer/applicant in the future. It is the nature of genetic and medical fields that advances are ongoing, and hence it is difficult to legislate now for what is essentially unknown territory.

1.3.3 Epidemiology

In order to calculate the transition intensities mentioned in section 1.2, often known as onset rates in medical terms, actuaries rely heavily upon the published results of epidemiological studies. This involves the study of the patterns, frequencies and influencing factors of diseases within the population. However, such studies are generally undertaken from a medical perspective, and hence are often incomplete from an actuarial point of view.

Single-gene disorders, by the evident Mendelian inheritance patterns of their phenotypes, are more obvious as genetically inherited diseases. Multifactorial disorders can be infinitely more complex. As genetic research advances with speed, it is quite probable that epidemiology, long-term by its very nature, will struggle to keep up. The relevant onset intensities of diseases, levels of penetrance and frequency of mutations may not be produced for some time after both genetic and environmental discoveries have been made.

1.3.4 Adverse selection

The most central issue for an insurance company posed by predictive genetic test results is adverse selection. Should an individual find that they are a mutation carrier of a gene which indicates they are more likely to suffer from a particular disease, are they more likely to take out life, income protection, critical illness or long term care insurance? A mutation carrier might be inclined to purchase insurance for a much larger amount then they would have considered were they not a mutation carrier. Adverse selection suggests that an insurance company could potentially be exposed to significantly more risk then they would have allowed for in calculating the cost of the insurance. Currently, most studies conclude that adverse selection does not seem to be a major threat to the insurance industry in large markets, for example Gutiérrez and Macdonald (2003) and Macdonald et al (2003b), but as more common multifactorial diseases become the focus of genetic research, and as predictive genetic testing becomes more widespread, adverse selection may increase significantly.

1.4 ASYMMETRIC KNOWLEDGE

While knowing the overall risk that the whole population poses, insurance companies are only able to place individuals into risk strata as accurately as the received underwriting information allows. Within each stratum every person pays the same premium, dictated by the averaged risk the group represents. The less information available to the insurance company, the wider the categories of risk will be. Large variations of risk within a stratum will mean that the individuals whose risk is among the highest in the stratum will receive cover at a reduced premium. The lower risk individuals will pay a higher premium than that which reflects their true risk. This might be of little or no consequence if individuals themselves do not know

what risk they represent and hence whether their insurance policy is relatively expensive or relatively cheap. However, if asymmetric knowledge exists within the market then it is possible that people who are low-risk may consider the cost of insuring too high and choose not to insure. If the cover the higher risk proportion of the market receives is inexpensive and if people consider themselves at high-risk of developing a critical illness, they may be inclined to purchase larger amounts of insurance then they generally would have were they unaware of this risk. Of course the number of people in each true risk stratum purchasing insurance and the amount which they purchase, will alter the risk that the insurance company is exposed to. The insurer will have no choice but to respond by altering the premiums charged appropriately to cover the risk. This will inevitably have further effect on the number of people purchasing insurance and so a cycle of changing premiums and insurance buying behaviour may ensue.

2.1 THE THEORY OF UTILITY

Utility theory is one of the aspects of economics that is concerned with human behaviour. In this respect, it is not possible to have controlled and repeatable experiments. Instead the assumptions of rational and utility-maximising behaviour are relied upon to create models in order to infer the decisions individuals may make. A person's utility is a measure of their well-being or satisfaction arising from the wealth that they have, and what it (or a fraction of it) may be exchanged for. Macdonald and Tapadar (2010), used an individual's utility function, *U(w)*, to measure their preference between the risk of loss in wealth or certain wealth less the premium paid for this security. The same approach and notation will be used in this dissertation.

A utility function, *U(w)*, must represent a person's attitude to different levels of wealth denoted, *W*, and risk. These attitudes are deemed to be:

a) Non-satiation, whereby more wealth is always preferable to less wealth. Hence the rate of change of the function will always be positive, $U'(w) > 0$.
b) Risk-aversion, such that an increase in wealth is valued less highly then not having a decrease in wealth. This property will result in a concave function, $U''(w) < 0$.

It is assumed that everyone in the insurance buying population has the same utility function, initial wealth *W*, and future wealth *X*, which is random. Consider an insurance product that charges a premium, *P*, to cover a loss of *L*, which occurs with probability *q*. The premium charged is the actuarially fair premium such that $P = qL$. People will choose to purchase insurance only if the utility of their current wealth less the premium they are charged is greater then the expected utility:

$$U(W - P) > E[U(X)] = qU(W - L) + (1 - q)U(W). \qquad (1)$$

However, because of risk aversion, a person will be willing to pay a larger premium to insure against a loss in wealth. The maximum premium a person would be willing to pay, *P**, can be found by working backwards from their utility function. By rearranging equation *(1)* we find:

$$P^* = W - U^{-1}\left[qU(W-L)) + (1-q)U(W)\right]. \qquad (2)$$

Clearly the maximum premium people are willing to pay as in equation *(2)* depends on wealth, loss and probability of a loss, and on the utility function assumed (section 2.3) and degree of risk aversion (section 2.2).

2.2 MEASUREMENT OF RISK AVERSION

To quantify the trade off between risk and the cost of avoiding it, we need a measure of aversion to risk. A more risk averse person is willing to pay more to avoid the risk of a loss. All utility functions have two coefficients which specify this trade off:

a) The Arrow-Pratt measure of absolute risk-aversion, $A_U(w)$, known as the absolute risk aversion coefficient:

$$A_U(w) = -\frac{U'(w)}{U''(w)}. \qquad (3)$$

Utility functions with the same absolute risk aversion coefficient give rise to the same preferences and decisions.

b) The (Arrow-Pratt) relative risk-aversion coefficient, $R_U(w)$, weights the absolute risk aversion coefficient by the level of wealth, w, the individual has:

$$R_U(w) = wA_U(w) = -\frac{wU'(w)}{U''(w)}. \qquad (4)$$

The risk aversion coefficients are selected by what are deemed to be the most realistic values. Working backwards from the chosen risk aversion coefficients, the parameters of the utility functions can be calculated. Many published papers estimate risk aversion coefficients using various survey and other information, for example Booij and Van Praag (2009), however here the calculation of the coefficients follows Macdonald and Tapadar (2010), so that the results may be compared.

2.2.1 Numerical estimation of risk-aversion coefficients

Eisenhauer and Ventura (2003) used a Bank of Italy survey of wealth and income conducted in 1995 to calculate absolute and relative risk aversion coefficients for different sections of the population. When they combined all sections they calculated that an individual with an average annual income of 46.7777 million lira had absolute risk aversion coefficient equal to 0.1837 and relative risk coefficient of 8.59. These figures require adjustment to make them relevant to the current UK population.

The average annual income of 46.7777 million lira must be converted to the equivalent amount in sterling. The average exchange rate over the year of 1995 was £1 = 2570.6 lira. Hence the corresponding income in sterling is £18,197.22, which is updated for inflation using the retail price index (RPI). The RPI figures used by Macdonald and Tapadar are those for July 1995, 149.1 and July 2006, 198.5. The equivalent average annual income earned in the UK in 2006 adjusted for inflation is equal to £24,226. The actual average income in the UK in 2006 was £25,810. This enables the calculation of an error coefficient of sorts, *(24,266/25,810)* by which the risk aversion coefficients can be adjusted. In the UK, the Inland Revenue's latest figures for 2003 state that 83% of the population have personal wealth less than £100,000. The absolute and relative risk aversion coefficients are updated as follows:

a) Absolute risk aversion: $\frac{0.1837}{2570.6} \times \frac{198.5}{149.1} \times \frac{24226}{25810} = 8.93 \times 10^{-5}.$

b) Relative risk aversion: $8.93 \times 10^{-5} \times 100{,}000 \approx 9.$

While these figures provide a good starting point, the utility for some other values of relative risk aversion will also be considered in the model.

2.3 UTILITY FUNCTIONS

Again similar to Macdonald and Tapadar (2010), utility is measured here using two different types of utility function, and various levels of risk aversion. The first of these functions is the iso-elastic utility function calculated as follows:

$$U_{I(\lambda)}(w) = \begin{cases} (w^{\lambda} - 1)/\lambda & \lambda < 1, \lambda \neq 0 \\ \log(w) & \lambda = 0. \end{cases} \qquad (5)$$

It is clear from equation *(3)* and equation *(5)* above that as λ decreases, an individual's risk aversion will increase. To obtain a relative risk aversion coefficient of 9, using equation *(4)* and equation *(5)*, with wealth equal to £100,000, λ must equal -8. λ = 0.5 and 0 will also be considered, which yield relative risk aversion coefficients of 0.5 and 1 respectively.

The second function used is the negative exponential utility function:

$$U_{N(A)}(w) = -\exp(-Aw) \qquad A > 0. \qquad (6)$$

where *A* is the absolute risk aversion coefficient. The value of A that will be used in the model is $A = 9 \times 10^{-5}$, which is equal to the absolute risk aversion coefficient of the iso-elastic utility function when λ = -8.

3.1 THE CRITICAL ILLNESS PRICING MODEL

A six state Markov model, as shown in Figure 1 is used to calculate the single premium paid at the outset for a CI insurance policy. Everyone begins in the healthy state, and by the definition of a critical illness, there is no return into the healthy state once an outward transition has occurred. A Markov approach employs two assumptions that enable the calculation of the occupancy probabilities for each state using the transition intensities into each state. The assumptions are as follows:

a) The transition intensity from one state, for example state 0; healthy, into another state, say state 1 heart attack, denoted μ_{x+t}^{01}, depends only on the current age and not on any other aspect of the life's history.

b) The probability of making a transition into a particular state in an infinitesimal time interval denoted *dt* is:

$$_{dt}p_{x+t}^{01} = \mu_{x+t}^{01}.dt + o(dt). \qquad (7)$$

($_{dt}p_{x+t}^{01}$ = The probability that a person in state 0, healthy, at aged x+t is in state 1, heart attack, at age x+t+dt)

The above assumptions allow the derivation of a system of differential equations for the occupancy probabilities of every state, $_{t}p_{x}^{ij}$, as follows:

$$\frac{d}{dt}{}_{t}p_{x}^{ij} = \sum_{k\neq j} {}_{t}p_{x}^{ik}\mu_{x+t}^{kj} - {}_{t}p_{x}^{ij}\sum_{k\neq j}\mu_{x+t}^{jk}. \qquad (8)$$

These equations are known as the Kolmogorov differential equations and can be solved numerically to find the age related occupancy probabilities. A more rigorous explanation of Markov models for insurance pricing purposes can be found in Habermann & Pitacco (1998).

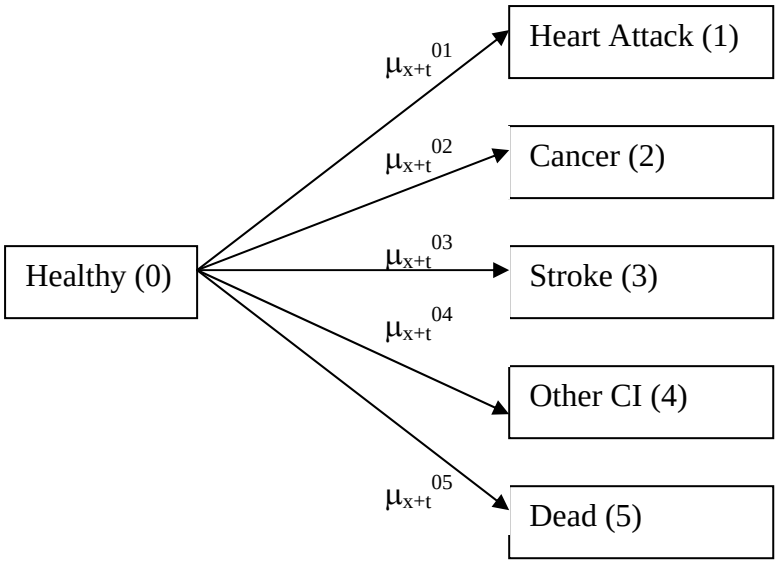

Figure 1
CI insurance Markov model for pricing

3.1.1 Estimation of transition intensities

The transition intensities used in the pricing model are estimated in the same way as Gutiérrez and Macdonald (2003) estimated them. They used various medical studies and population data to estimate the onset rates of various critical illnesses. The major causes of CI insurance claims are heart attacks, strokes and cancer. The related transition intensities are estimated as follows:

a) Heart attacks in males:

$$\mu_x^{Heart} = \exp(-13.2238 + 0.152568x) \qquad (x < 44) \qquad (9)$$

$$\mu_x^{Heart} = (-0.01245109 + 0.000315605x) \qquad (x > 49) \qquad (10)$$

Using linear interpolation between ages 44 and 49.

b) Heart attacks in Females:

$$\mu_x^{Heart} = \frac{0.598694}{\Gamma(15.6412)} \times 0.15317^{15.6412} \exp(-0.15317x)x^{14.6412}. \qquad (11)$$

c) Cancer in males:

$$\mu_x^{Cancer} = \exp(-11.25 + 0.105x) \qquad (x < 51) \qquad (12)$$

$$\mu_x^{Cancer} = (0.2591585 - 0.01247354x + 0.000191691x^2 - 8.92933 \times 10^{-7} x^3) \qquad (x \geq 60) \qquad (13)$$

Using linear interpolation between ages 51 and 60.

d) Cancer in Females:

$$\mu_x^{Cancer} = \exp(-10.78 + 0.123x - 0.00033x^2) \qquad (x < 53) \qquad (14)$$

$$\mu_x^{Cancer} = -0.01545632 + 0.0003805097x \qquad (x \geq 53) \qquad (15)$$

e) Strokes in males:

$$\mu_x^{Stroke} = \exp(-16.9524 + 0.294973x - 0.001904x^2 + 0.00000159449x^3). \qquad (16)$$

f) Strokes in Females:

$$\mu_x^{Stroke} = \exp(-11.1477 + 0.081076x). \qquad (17)$$

In general, to distinguish between life insurance claims and CI insurance claims, an individual must survive for 28 days post diagnosis to be considered as a CI insurance claim. This is largely irrelevant where cancer is concerned because usually an individual diagnosed with cancer will survive at least 28 days. However this is not the case for heart attacks or strokes and so the transition intensities into these states must be amended accordingly by the 28-day survival probability of an individual who has suffered one of these CI's. The 28-day survival probability for a male who has suffered a heart attack, ($_{28days}p_x^{Heart}$), is dependant on age and is shown in Table 1. The 28-day survival probability of a female who has suffered a heart attack is 0.79 at ages 20-80. Finally, the 28-day survival probability for any individual aged *x* who has

suffered a stroke is *(0.9 – 0.002x)/0.9*. These probabilities are taken from Gutiérrez and Macdonald (2003).

Table 1:
28-Day Survival Probabilities for Males post Heart Attack

Age	${}_{28days}p_x^{Heart}$	Age	${}_{28days}p_x^{Heart}$	Age	${}_{28days}p_x^{Heart}$
20-39	0.85	53-56	0.81	62-64	0.77
40-42	0.84	57	0.80	65-74	0.76
43-46	0.83	58-59	0.79	75-79	0.75
47-52	0.82	60-61	0.78	80+	0.74

Utilising the fact that claims arising from other critical illnesses amounts to 15% of those arising from heart attacks, strokes or cancers, the transition intensity into the "other CI" state is calculated as:

$$\mu_x^{OtherCI} = 0.15 \times (\mu_x^{Cancer} + \mu_x^{Heart} \times_{28days} p_x^{Heart} + \mu_x^{Stroke} \times_{28days} p_x^{Stroke}). \qquad (18)$$

The final transition intensity is that into the dead state, which does not result in a claim. The rate of mortality is taken from the English Life Tables number 15, denoted μ_x^{ELT15}. This is adjusted to remove any deaths from causes leading to a CI insurance claim. The ratios of the number of deaths from causes that could lead to a CI claim to the total number of deaths are as follows:

a) Males:

$$\theta_x = 0.0185408 + 0.0655723x - 0.00667105x^2 + 0.000223974x^3 - 0.00000228356x^4)$$

$$(x < 30) \qquad (19)$$

$$\theta_x = -2.80056 + 0.149759x - 0.00203616x^2 + 0.00000881081x^3)$$

$$(x > 44) \qquad (20)$$

Using linear interpolation between ages 30 and 44.

b) Females:

$$\theta_x = -0.0261291 + 0.104641x - 0.0118145x^2 + 0.000467135x^3 - 0.00000579010x^4$$

$$(x < 30) \qquad (21)$$

$$\theta_x = -1.34514 + 0.0897216x - 0.00119978x^2 + 0.00000486785x^3$$

$$(x > 35) \qquad (22)$$

Again, using linear interpolation between the ages of 30 and 35.

Deaths from heart attacks and strokes that occur within 28 days of the event must also be added back making the total rate if mortality:

$$\mu_x^{Dead} = (1-\theta_x)\mu_x^{ELT15} + (1-{}_{28days}\,p_x^{Heart})\mu_x^{Heart} + (1-{}_{28days}\,p_x^{Stroke})\mu_x^{Stroke}. \qquad (23)$$

3.1.2 Calculation of the single, actuarially fair premium

Now that the CI model in Figure 1 is parameterised, a Euler scheme is employed to solve the Kolmogorov equations. A Euler scheme is a numerical method of solving ordinary differential equations with a given initial value. The initial values here are occupancy probabilities and are, ${}_0 p_x^{Healthy} = 1$, with all other occupancy probabilities equal to zero. More information on the Euler scheme can be found in Ascher and Petzold (1998). To simplify the calculation of the actuarially fair premium, the force of interest is chosen to equal zero. This means that the premium of the CI insurance policy with sum assured equal to one is equal to the probability that an individual has transferred into a critical illness state within the term of the policy. That is:

$$P = {}_t\,p_x^{Heart} + {}_t\,p_x^{Cancer} + {}_t\,p_x^{Stroke} + {}_t\,p_x^{OtherCI}. \qquad (24)$$

where t is the policy term and x is the age at policy inception.

The above premium is calculated under the assumption that every individual is believed to be at the same risk of developing a critical illness by both the insurer and the insured. With the introduction of knowledge about genetic and environmental factors which influence the onset rate of a disease, a person's probability of making a CI claim changes depending on an their exposure, and hence their actuarially fair premium will also vary.

3.2 RISK STRATA

In a greatly simplifying approach, we assume that one of two genotypes is more vulnerable to heart attacks. The adverse genotype is denoted, *G*, while the beneficial genotype is denoted *g*. Similarly it is supposed that an individual will be exposed to one of two possible environmental factors. Exposure to the adverse environmental factor is denoted *E*, and exposure to the beneficial factor is denoted *e*. Thus we have four risk strata in addition to sex. The lowest risk strata is *ge*, the highest is *GE*. Because heart attacks are relatively common we assume that the risk factors associated with heart attacks are also relatively common. We will suppose that genotype *G* and environmental exposure *E* each occur with probability 0.1. This implies that the beneficial genotype *g* and the beneficial exposure *e* each occur with probability 0.9. Table 2 shows the distribution of the population in each stratum before asymmetric knowledge and utility theory has played a part. These probabilities should change as age, *x*, increases, because people in the higher risk stratum should die before those in the lower risk stratum. Therefore the frequency of adverse exposures in the healthy population should decrease as age increases but this has negligible impact on the model and is therefore ignored.

Table 2:
Starting population proportions for each stratum.

	e	E
g	0.81	0.09
G	0.09	0.01

3.2.1 Proportional transition intensities

The epidemiology of the various factors, which affect the rate of heart attacks, is as yet unknown. This means that the transition intensities for individuals in each of

the risk strata, *ge*, *gE*, *Ge* and *GE*, from the healthy state into the heart attack state are unknown. However, we know the transition intensity from the healthy state into the heart attack state for the whole population, $\mu^{01}(x)$, as calculated in equations *(9)*, *(10)* and *(11)*. The transition intensities from the healthy state into the heart attack state for a particular risk stratum, *s*, is denoted $\mu_s^{01}(x)$, and is calculated by assuming a proportional hazards model:

$$\mu_s^{01}(x) = k \times \rho_s \times \mu^{01}(x). \qquad (25)$$

where ρ_s is a measure of the level of penetrance that a particular gene-environment interaction results in. We assume ρ_s is independent of age and sex. The starting values of ρ_s used in the equilibrium model (see section 4.1) are taken from Macdonald, Pritchard and Tapadar (2006), and are given in table 3. These values are selected to represent modest penetrance. Calculations are carried out for varying values of ρ_s reflecting the fact that the epidemiology is currently unknown. This allows us to examine the impact of the epidemiological discoveries and the range of the potential affects these discoveries may have on the insurer.

k is a constant specific to age, the term of the policy and sex. It is calculated such that the transition intensities for each of the strata are consistent, overall, with the population transition intensity. Macdonald, Pritchard and Tapadar (2006) derived an equation *(27)* for the calculation of *k*. This is used here to calculate *k* for the various policy terms and ages for which the model is run. Let w_s denote the proportion of the population in stratum *s* and healthy, then:

$$\mu^{01}(x+t) = \frac{\sum_s w_s \times \exp\left(-\int_0^t \mu_s^{01}(x+y)dy\right) \times \mu_s^{01}(x+t)}{\sum_s w_s(x) \times \exp\left(-\int_0^t \mu_s^{01}(x+y)dy\right)} \qquad (26)$$

By substituting in equation *(25)*, the formula becomes:

$$\mu^{01}(x+t) = \frac{\sum_s w_s \times \exp\left(-\int_0^t \mu^{01}(x+y)dy\right)^{k\rho_s} \times k \times \rho_s \times \mu^{01}(x+t)}{\sum_s w_s \times \exp\left(-\int_0^t \mu^{01}(x+y)dy\right)^{k\rho_s}} \qquad (27)$$

Macdonald, Pritchard and Tapadar calculated $k = 1.317274$ for males with $x = 60$ and $t = 5$ for the values of ρ_s given in table 3 and the population frequencies given in table 2, and $k = 1.316406$ for females with the same variables. The corresponding values calculated for males and females using equation *(27)* in the equilibrium model (see section 4.1) are $k = 1.316882$ and $k = 1.316258$ respectively. The small differences in value we attribute to differences in method of calculation. Now using equation *(25)* and different values of ρ_s for varying levels of penetrance, different transition intensities for each stratum may be calculated. From these transition intensities the occupancy probabilities and premiums can now be calculated. This is done in the same manner as described in section 3.1.2, the only difference being the initial occupancy probabilities used in the Euler scheme to solve the Kolmogorov equations. The initial probability that an individual in a particular stratum will begin in the healthy state becomes the starting population frequency of that stratum. For example using the population frequencies given in table 2, for stratum *ge* the initial occupancy probabilities are ${}_0p_x^{healthy,ge} = 0.81$ and all other initial occupancy probabilities are equal to zero. The premium charged assuming genetic information is not disclosed to the insurer is calculated as follows:

$$\frac{\sum_s w_s P_s}{\sum_s w_s} \qquad (28)$$

where P_s is the actuarially fair premium which should be charged to individuals in stratum s and w_s is the population frequency of stratum s.

Table 3:
Values of ρ_s for each stratum representing modest penetrance.

	e	E
g	0.7	0.9
G	1.1	1.3

4.1 THE EQUILIBRIUM MODEL

The equilibrium model measures the level of market withdrawal which occurs within the CI insurance industry, under the assumption that genetic test results are not disclosed to insurance companies so premiums for low risk-individuals are expensive relative to the risk they represent. When withdrawal occurs, the level of risk in the market is altered. The model employs an iterative procedure to recalculate premiums and withdrawal until such time as an equilibrium is reached where any further increases in premium causes no further market withdrawal. At the point of equilibrium the total amount of withdrawal is calculated as a percentage of the initial market.

To begin building the equilibrium model, we need to estimate values of the sum assured. Unlike car insurance for example, it is difficult to quantify the loss an individual will make in the event of a CI insurance claim. Therefore, within the equilibrium model it is assumed that the sum assured ranges from £10,000 to £90,000 in steps of £10,000, recalling that initial wealth is equal to £100,000. Here we introduce a new assumption that, within each stratum the distribution of policy purchasing is uniform over all possible values of the sum assured. So the number of people in stratum *ge* who purchase a CI policy with sum assured equal to £10,000, is the same as the number of people in stratum *ge* who purchase a CI policy with sum assured £20,000, and £30,000 and so on, up to a maximum of £90,000.

The starting point of the model is selecting the age and terms of the policies under which the level of market withdrawal will be examined. The values of ρ_s given in table 3 and the population frequencies in each stratum shown in table 2 are the starting point used in the model. The value of k for the policy in question, required to calculate the transition intensity of heart attacks in each strata from equation (25), can then be calculated using equation *(27)*. The model allows us to calculate the actuarially fair premium which should be charged to each individual within a stratum for each value of sum assured (see section 3.2.1). The actuarially fair premium for each sum assured, charged by the insurer to individuals in all strata is then calculated

under the assumption of asymmetric knowledge (see section 3.2.1). Using equation *(2)* and the utility functions described in section 2.3, the maximum premium an individual in a particular risk stratum is willing to pay is for each sum assured is calculated. This is compared to the actual premium charged under asymmetric knowledge to determine whether an individual will choose to continue to purchase insurance or withdraw from the market. Using the assumption of uniform distribution of the number of policies held for each sum assured within a stratum, the extent of market withdrawal can be quantified. Market withdrawal within a stratum will be denoted, ψ_s. The first group of people to withdraw from the market will be those who would intend to purchase policies with sum assured equal to £10,000. This would represent one ninth of all the people in the stratum. If the level of market withdrawal, ψ_s is say, equal to £50,000, this implies that withdrawal has occurred at every level of sum assured up to and including policies with sum assured equal to £50,000.

When withdrawal occurs, population frequencies in the lower risk categories of the remaining insured population will fall meaning a larger proportion of the population then reside in the higher risk strata. When insurance companies recognise the increased risk, through an increased number of claims, the premium will be adjusted accordingly, which may lead to further market withdrawal. This continues until such time as a market equilibrium is reached (policy purchasing behaviour and premiums stabilise) if such a market equilibrium exists. The number of iterations the model has calculated is shown in the tables as a superscript on the notation used to represent population proportions and withdrawal. For example the population proportion in a stratum *s*, after the *ith* calculation is denoted, ω_s^i, and the level of market withdrawal in stratum *s* after the *ith* calculation is denoted, ψ_s^i.

Table 4:
Market equilibrium for males where age at policy inception, x = 35 and 45, and policy term, t = 15 years. k=1.336894. Individuals in the high-risk stratum purchase insurance at the standard rate. Loadings are not included in the calculation of the premium.

Utility Function		ge	gE	Ge	GE
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
$U_{\log}(w)$	ω_s^1	0.81	0.09	0.09	0.01
	ψ_s^1	0	0	0	0
$U_{I(-8)}(w)$	ω_s^1	0.81	0.09	0.09	0.01
	ψ_s^1	0	0	0	0
$U_{N(9e-5)}(w)$	ω_s^1	0.81	0.09	0.09	0.01
	ψ_s^1	0	0	0	0

Table 4 shows the degree of market withdrawal for each utility function for males aged 35 and policy term 15 years. No loadings have been added to the charged premium and modest penetrance is assumed (ρ_s from table 3). For the majority of utility functions everybody within the lower risk strata, *ge* and *gE*, will continue to purchase insurance regardless of the expensive premium. Only the iso-elastic utility function with $\lambda = 0.5$, gives rise to any market withdrawal. Individuals in stratum *ge*

purchasing insurance with sum assured of £10,000 will discontinue insurance purchase, but the resulting increase in premium is so small that it has no further effect and equilibrium is quickly reached. Males aged 45 with a policy term of 15 years yield the same results.

The model was also run for males aged 55 for a policy term of 15 years. No market withdrawal is witnessed at these ages after the calculation of the initial premium, therefore figures are not shown. For females aged 35, 45 and 55 with a policy term of 15 years and a modest level of penetrance (ρ_s from table 3), no market withdrawal is witnessed after the calculation of the initial premium, and again figures are not shown.

To try to account for the differences between male and female results and differing results at differing ages, the actuarially fair premiums and transition intensities are examined more closely. Table 5 shows the premium charged per £1 sum assured under asymmetric knowledge and the percentage of this premium which is equal to the actuarially fair premium within each stratum for a policy term of 15 years. It is immediately obvious, that while the actuarially fair premium within the lowest risk stratum *ge* as a percentage of the premium charged, increases with age for males, the opposite is true for females. For males within the stratum *GE*, the benefit of asymmetric knowledge is reduced as age increases and again we see the opposite is true for females. By comparing the transition intensities into the states where a claim will be made for males and females of different ages, shown in table 6, these differences can be explained. The transition intensity for a female aged 35 into the heart attack state is a very small proportion of the combined transition intensities into a claiming state. This means that this particular intensity makes a relatively small contribution to the number of CI claims made (2.16% of all CI claims at age 35) and hence contributes less to the premium charged. Therefore the extra cost in premiums for individuals in the low risk strata, due to asymmetric knowledge related to the transition intensity of a heart attack is extremely small, insufficient to result in any market withdrawal. While the claims contribution of heart attacks increases to 17.3% for females at age 65, this is still too small to lead to any market withdrawal. Table 5 shows that as females get older, individuals in stratum *ge* are increasingly disadvantaged by asymmetric knowledge which is in agreement with the increasing percentage of claims attributed to heart attacks. A contribution of 29.2% of all CI

claims arise from heart attacks for males aged 35; this increases to 37.5% at age 45, 33.1% at age 50 and reduces to 25.1% at age 65. This is opposite to the trend present in females; however heart attacks are sufficiently common in males within the age bracket of 40 - 50, to have a significant enough effect on the premium charged such that market withdrawal occurs.

Table 5:
The premium charged by the insurance company, P, per £1 sum assured under asymmetric knowledge, and the percentage of this premium which should actually be charged of individuals in each stratum for a term of 15 years.

Males:			% of premium charged that a stratum should be charged			
x	k	P	ge	gE	Ge	GE
35	1.336889	0.0477	97.16	106.65	116.09	125.50
45	1.368820	0.1355	97.45	105.99	114.42	122.73
55	1.391743	0.2635	98.16	104.36	110.43	116.35
65	1.399415	0.3880	98.69	103.12	107.39	111.59
Females:						
x	k	P	ge	gE	Ge	GE
35	1.319859	0.0495	99.49	101.20	102.91	104.63
45	1.331049	0.1097	99.10	102.10	105.09	108.07
55	1.351279	0.1935	98.81	102.79	106.74	110.65
65	1.371468	0.2863	98.75	102.95	107.08	111.14

Table 6:
Transition intensities, times the 28 day survival probability where relevant, into all states where a CI claim will be made for males and females aged x.

x	Sex	$\mu_x^{Heart-population}$	μ_x^{Cancer}	μ_x^{Stroke}	$\mu_x^{OtherCI}$
35	M	0.000319	0.000513	0.000119	0.00014
35	F	0.0000329	0.001029	0.000227	0.00019
45	M	0.001485	0.001466	0.000488	0.00052
45	F	0.000282	0.002703	0.000498	0.00051
55	M	0.003975	0.005103	0.001371	0.00157
55	F	0.001151	0.005472	0.001093	0.00110

65	M	0.006128	0.012408	0.002643	0.00318
65	F	0.002872	0.009277	0.002396	0.00204

By changing the length of the term of the policy for different age groups and running the model, the age bracket at highest risk and therefore with the biggest effect on the premium is further highlighted. For males with a policy term of 5 years, withdrawal up to £10,000, (ψ_s^1 = £10,000), occurred for the following ages at policy inception: *x = 40, 45, 50* and *55*. For males with a policy term of 10 years, withdrawal up to £10,000, (ψ_s^1 = £10,000), occurred for ages: *x = 35, 40, 45* and *50*, at policy inception. No additional market withdrawal is witnessed after the recalculation of premiums in the above mentioned cases. These figures indicate that heart attacks contribute most towards CI claims in males within the age bracket of approximately 45 – 50, which is in agreement with Figure 2. Figure 2 shows the transition intensity of a heart attack at age *x*, as a percentage of the total transition intensity into a claiming state at this age. Using this information, further calculations run by the model are only considered for ages 35, 45 and 55 and a policy term of 15 years for males as this represents a reasonable spread of risk, and contains the highest risk age groups.

When the policy term was changed to 5 and 10 years in the model for females, unsurprisingly, no market withdrawal was observed as the contribution of heart attacks towards the total number of CI claims for females is relatively small at all ages in comparison to males (Figure 2). Further calculations are performed for the ages of 35, 45 and 55 and for a policy term of 15 years, similar to that for males, to avoid confusion.

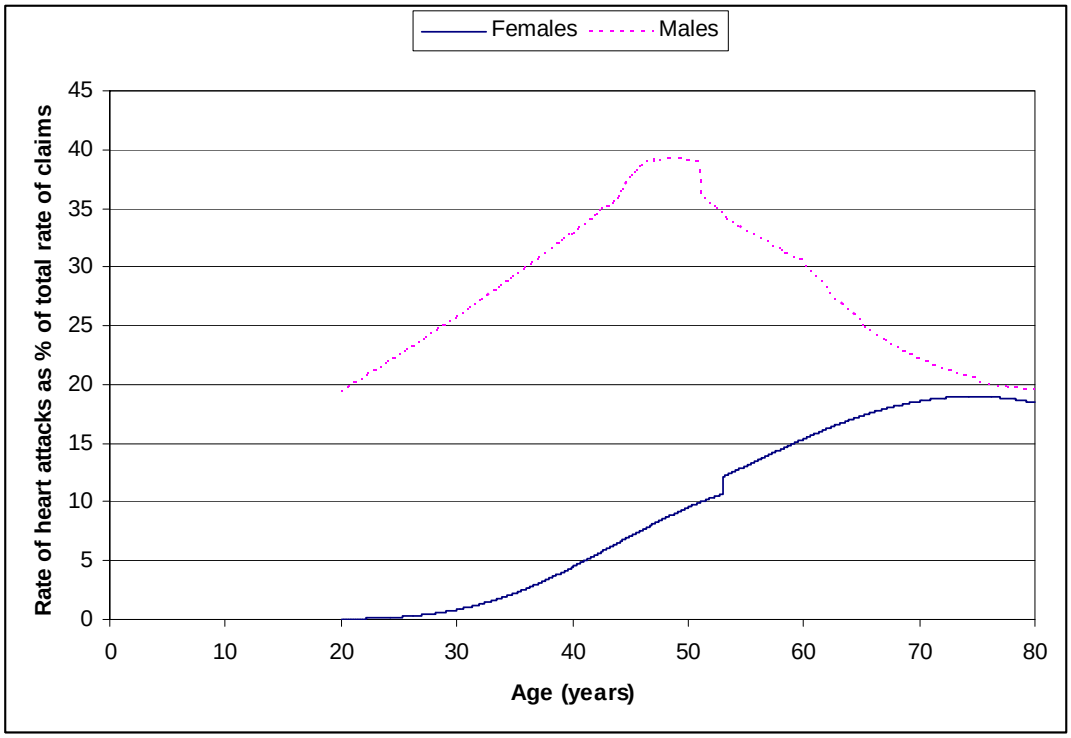

Figure 2:
Transition intensity into heart attack state at varying age, as a percentage of the total transition intensity into a state where a CI claim will be made.

4.1.1 Market withdrawal when loadings are added to premiums

The measurement of market withdrawal in the above section is calculated based on a premium which is just sufficient to cover the expected costs of claims. In reality premiums charged must also cover any expenses incurred by selling the policy, and be large enough such that a profit is expected to be made for the insurance company. These additional costs on top of the expected cost of claims are called loadings. We extend on previous published work by introducing the inclusion of loadings in the calculation of the premium. This does not alter the maximum premium an individual is willing to pay, P^*, and so we would expect to see greater market withdrawal when loadings are added to the premium charged. Tables 7, 8 and 9 show the amount of market withdrawal and the level at which market equilibrium is reached for males aged 35, 45 and 55 with a policy term of 15 years and loadings of 5% and 10% added to the premium charged. In the case where a utility function has given rise to zero market withdrawal, it has not been shown in the tables.

Table 7:
Market equilibrium for males with age at policy inception, x = 35, and policy term, t = 15 years. k=1.336879. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£20,000	0	0	0
	ω_s^2	0.76829	0.10975	0.10975	0.01219
	ψ_s^2	£30,000	0	0	0
	ω_s^3	0.73973	0.12329	0.12329	0.01370
	ψ_s^3	£30,000	0	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	£10,000	0	0
	ω_s^2	0.71429	0.12698	0.14286	0.01587
	ψ_s^2	£40,000	£10,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	0	0	0
	ω_s^2	0.76829	0.10976	0.10976	0.01219
	ψ_s^2	£20,000	0	0	0

Table 8:
Market equilibrium for males with age at policy inception, x = 45, and policy term, t = 15 years. k=1.368777. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£30,000	0	0	0
	ω_s^2	0.73973	0.12329	0.12329	0.01370
	ψ_s^2	£30,000	0	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	£10,000	0	0
	ω_s^2	0.71429	0.12698	0.14286	0.01587
	ψ_s^2	£40,000	£20,000	0	0
	ω_s^3	0.72581	0.11290	0.14516	0.01613
	ψ_s^3	£40,000	£20,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	0	0	0
	ω_s^2	0.76829	0.10976	0.10976	0.01219
	ψ_s^2	£20,000	0	0	0

Table 9:
Market equilibrium for males with age at policy inception, x = 55, and policy term, t = 15 years. k=1.391743. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£30,000	0	0	0
	ω_s^2	0.73973	0.12329	0.12329	0.01370
	ψ_s^2	£30,000	0	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	£20,000	0	0
	ω_s^2	0.72581	0.11290	0.14516	0.01613
	ψ_s^2	£50,000	£20,000	0	0
	ω_s^3	0.67925	0.13208	0.16981	0.01887
	ψ_s^3	£50,000	£30,000	0	0
	ω_s^4	0.69231	0.11538	0.17308	0.01923
	ψ_s^4	£50,000	£30,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01234
	ψ_s^2	£20,000	£10,000	0	0

The level of market withdrawal for females where the premium charged includes loadings of 5% and 10%, assuming moderate levels penetrance, are shown in tables 10, 11 and 12 for ages 35, 45 and 55 respectively. The policy term is 15 years and the population frequencies are as noted in the tables.

Table 10:
Market equilibrium for females with age at policy inception, x = 35, and policy term, t = 15 years. k=1.3198544. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01235
	ψ_s^2	£20,000	£10,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£30,000	£30,000	0	0
	ω_s^2	0.77143	0.08571	0.12857	0.01429
	ψ_s^2	£30,000	£30,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	£10,000	0	0
	ω_s^2	0.80000	0.08889	0.10000	0.01111
	ψ_s^2	£10,000	£10,000	0	0

Table 11:
Market equilibrium for females with age at policy inception, x = 45, and policy term, t = 15 years. k=1.331049. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01235
	ψ_s^2	£20,000	£10,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£30,000	£20,000	0	0
	ω_s^2	0.76056	0.09859	0.12676	0.01408
	ψ_s^2	£30,000	£30,000	0	0
	ω_s^3	0.77143	0.08571	0.12857	0.01429
	ψ_s^3	£30,000	£30,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01235
	ψ_s^2	£20,000	£10,000	0	0

Table 12:
Market equilibrium for females with age at policy inception, x = 55, and policy term, t = 15 years. k=1.391743. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 5% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01235
	ψ_s^2	£20,000	£10,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	£20,000	0	0
	ω_s^2	0.72581	0.11290	0.14516	0.01613
	ψ_s^2	£40,000	£30,000	0	0
	ω_s^3	0.73770	0.09836	0.14754	0.01639
	ψ_s^3	£40,000	£30,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01234
	ψ_s^2	£20,000	£10,000	0	0

Under these penetrance levels and population frequencies and the worst case scenario of 10% loadings, the maximum level of market withdrawal for males is 48% of the entire initial market. This withdrawal occurs for a relative risk aversion coefficient, $R(w) = 0.5$, at age 55 and out of the two lowest risk strata *ge* and *gE*. For females, the biggest proportion of market withdrawal under these same levels of penetrance, population frequencies, and relative risk aversion also occurs at age 55 and total withdrawal once equilibrium has been reached represents 39% of the initial market. These are clearly massive proportions of the market and although equilibrium is reached in every case, these levels of withdrawal would obviously represent a huge volume of this section of the insurance industry. However as mentioned above, the premiums calculated based on loadings of 10% are the worst-case scenario. Market withdrawal for premiums calculated based on 5% loadings yield the much-reduced level of market withdrawal of 27% for both males and females. However, this is still a significant proportion of the market.

Future tables show the level of market withdrawal for premiums calculated based on 0% loadings which yield the lowest level of withdrawal and 10% loadings which is deemed to be the worst-case scenario. The actual premium charged in reality is most likely somewhere between these figures.

4.1.2 Varying levels of penetrance

As the illness under consideration is a multifactorial disorder for which the genetics and epidemiology has not yet been discovered and examined, the level of penetrance of any genetic mutations associated with heart attacks is as yet unknown. It is important therefore to examine not just different levels of penetrance for the results that they actually yield themselves, but also to see the magnitude of the effect differing levels of penetrance have on the premiums and market withdrawal. In doing this we can look at the range of impact that future discoveries may have on the insurance industry.

As the level of penetrance increases for the high-risk strata, they decrease for the low-risk strata. The value of k as calculated from equation *(27)* increases when these changes are made to the level of penetrance. So for the lower risk strata, ρ_s decreases and k increases. Hence the transition intensity into the heart attack state for these intensities, as calculated by equation *(25)*, can either increase or decrease. As a

low-risk individual's risk increases, the premium they are willing to pay, P^* also increases and visa-versa. Thus it is not trivial that the level of market withdrawal increases for these increased levels of penetrance. This said, we do expect an overall increase in market withdrawal for increased levels of penetrance.

Table 13 shows the market withdrawal for males aged 45, policy term 15 years and population frequencies (ω_s) and modest to high levels of penetrance (ρ_s) as shown. For a premium calculated based on 0% loadings, only small levels of market withdrawal are witnessed (9% of the initial market for $U_{I(0.5)}$). When 10% loadings are included in the calculation of the premium, we notice for the first time that an equilibrium does not exist. Market withdrawal alternates between 46% and 47% of the initial market. This is easily explained if the withdrawal is subdivided by stratum. 45% of the withdrawal is from stratum *ge*, the lowest risk stratum. 1% of the withdrawal is from stratum *gE*, but when the premium is recalculated this changes to 2%. When this small percentage is removed from *gE*, the percentage of the total insured population remaining in the other strata is increased relatively. This means that as more people are excluded from stratum *gE* the percentage of the market who are now in stratum *ge* is increased and the risk over the entire population, and hence the premium, decreases again.

Table 14 gives the level of market withdrawal for males aged 55 at this modest to high level of penetrance. In this age group we see that market withdrawal at its worst case ($U_{I(0.5)}$) is actually reduced from 47% to 46% but the log utility function, U_{log}, showed an increase in withdrawal from 19% of the market at modest penetrance to 28% of the market at modest to high penetrance.

Modest to high penetrance levels within females aged 35 and with policy term of 15 years, cause no further market withdrawal then those with modest penetrance, although in some cases it required more iterations for equilibrium to be reached. The results for females aged 45 are the same as those yielded from females aged 55 and are both are shown in table 15. In the worst case scenario of $U_{I(0.5)}$ and premium calculated assuming loadings of 10% at modest to high penetrance, there is an increase from 30% withdrawal from the market at modest penetrance to 38% at modest to high penetrance.

Table 13:
Market equilibrium for males with age at policy inception, x = 45, and policy term, t = 15 years. k=1.532870. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 0% are included in Calculation of premium:					
	ρ_s	0.6	0.85	1.15	1.4
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.6	0.85	1.15	1.4
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	0	0	0
	ω_s^2	0.70313	0.14063	0.14063	0.01563
	ψ_s^2	£50,000	£10,000	0	0
	ω_s^3	0.66667	0.14815	0.16667	0.01852
	ψ_s^3	£50,000	£20,000	0	0
	ω_s^4	0.67925	0.13208	0.16981	0.01887
	ψ_s^4	£50,000	£10,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	0	0	0
	ω_s^2	0.76829	0.10976	0.10976	0.01295
	ψ_s^2	£20,000	0	0	0

Table 14:
Market equilibrium for males with age at policy inception, x = 55, and policy term, t = 15 years. k=1.560278. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 0% are included in Calculation of premium:					
	ρ_s	0.6	0.85	1.15	1.4
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.79121	0.09890	0.09890	0.01099
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.6	0.85	1.15	1.4
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£50,000	£10,000	0	0
	ω_s^2	0.66667	0.14815	0.16667	0.01852
	ψ_s^2	£50,000	£20,000	0	0
	ω_s^3	0.67925	0.16981	0.13208	0.01887
	ψ_s^3	£50,000	£20,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09877	0.11111	0.01235
	ψ_s^2	£30,000	£10,000	0	0
	ω_s^3	0.75000	0.11111	0.12500	0.01389
	ψ_s^3	£30,000	£10,000	0	0

Table 15:
Market equilibrium for females with age at policy inception, x = 45, 55, and policy term, t = 15 years. k(55)=1.512256 and k(45)=1.488433. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 0% are included in Calculation of premium: No market withdrawal is witnessed.					
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.6	0.85	1.15	1.4
	ω_s^1	0.81	0.09	0.09	0.01
$U_{I(0.5)}(w)$	ψ_s^1	£40,000	£20,000	0	0
	ω_s^2	0.72581	0.11290	0.14516	0.01613
	ψ_s^2	£40,000	£20,000	0	0
	ω_s^1	0.81	0.09	0.09	0.01
$U_{\log}(w)$	ψ_s^1	£20,000	£10,000	0	0
	ω_s^2	0.77778	0.09878	0.11111	0.01235
	ψ_s^2	£20,000	£10,000	0	0

The model was also run for both males and females aged 35, 45 and 55 with population frequencies as before, where the adverse gene and environmental factors are assumed to have a high level of penetrance. The values of ρ_s used to represent this high level of penetrance are given in table 16. Table 17 and 18 summarise, for males and females respectively, the total percent of the initial market that has withdrawn for different levels of penetrance. Only utility functions where market withdrawal has occurred have been included. These tables show the importance of the actual unknown level of penetrance in assessing the impact asymmetric knowledge may have on insurance companies. Also highlighted is the importance of the inclusion of loadings in the calculation of the premium charged.

Table 16:
Values of ρ_s for each stratum representing high penetrance.

	e	E
g	0.5	0.75
G	1.25	1.5

Table 17:
% of initial market that has withdrawn when equilibrium has been reached for males for the given penetrance, utility function and age. All policy terms are 15 years. Individuals in the high-risk stratum purchase insurance at the standard rate. Starting population frequencies are as follows: $\omega_s^1 = 0.81$, $\omega_s^1 = 0.09$, $\omega_s^1 = 0.09$, $\omega_s^1 = 0.01$.

Males: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	9%	35	modest	37%
45	modest	0%	45	modest	38%
55	modest	0%	55	modest	48%
35	modest-high	9%	35	modest-high	46%
45	modest-high	9%	45	modest-high	46%-47%
55	modest-high	9%	55	modest-high	47%
35	high	18%	35	high	56%
45	high	18%	45	high	57%
55	high	18%	55	high	57%
Males: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	18%
45	modest	0%	45	modest	18%
55	modest	0%	55	modest	19%
35	modest-high	0%	35	modest-high	18%
45	modest-high	0%	45	modest-high	18%
55	modest-high	0%	55	modest-high	28%
35	high	9%	35	high	27%
45	high	9%	45	high	27%
55	high	9%	55	high	28%

Table 18:
% of initial market that has withdrawn when equilibrium has been reached for females for the given penetrance, utility function and age. All policy terms are 15 years. Individual in the high-risk stratum purchase insurance at the standard rate. Starting population frequencies are as follows: $\omega_s^1 = 0.81$, $\omega_s^1 = 0.09$, $\omega_s^1 = 0.09$, $\omega_s^1 = 0.01$.

Females: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	30%
45	modest	0%	45	modest	30%
55	modest	0%	55	modest	39%
35	modest-high	0%	35	modest-high	30%
45	modest-high	0%	45	modest-high	38%
55	modest-high	0%	55	modest-high	38%
35	high	0%	35	high	30%
45	high	0%	45	high	39%
55	high	9%	55	high	48%
Females: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	10%
45	modest	0%	45	modest	19%
55	modest	0%	55	modest	19%
35	modest-high	0%	35	modest-high	10%
45	modest-high	0%	45	modest-high	19%
55	modest-high	0%	55	modest-high	19%
35	high	0%	35	high	19%
45	high	0%	45	high	19%
55	high	0%	55	high	19%

4.1.3 Varying population frequencies

Though it is probably more realistic to assume that an individual is an adverse gene-carrier, or exposed to the adverse environmental factor with probability 0.1, this is also an assumption that will only be confirmed when the genetic discoveries and epidemiology have been evaluated. Again, to investigate the differing potential outcome from the point of view of the insurance company, the model was also run under the assumption that an individual is an adverse gene-carrier or is exposed to the adverse environmental factor with probability 0.5. This redistributes the population frequencies into those shown in table 19. Tables 20 and 21 show the level of withdrawal for policy-holders aged 55 with policy term 15 years for these new population frequencies. Tables 20 and 21 are included for comparison with tables 9 and 12 respectively for the premium calculated based on 10% loadings. This

comparison illustrates an increased level of market withdrawal under these altered population frequencies. This increase in the level of withdrawal occurs with every utility function at every age. The total market withdrawal as a percentage of the initial insurance purchasing market for all the three levels of penetrance are shown in tables 22 for males and 23 for females. Again by comparing these results with those in tables 17 and 18 for the initial population frequencies given in table 2, it is clear that where withdrawal was at its lowest (0% loadings), withdrawal increases significantly when the population frequencies are spread evenly over all the risk strata. Where withdrawal was at its highest for the initial population frequencies, the percentage of the initial market who withdraw due to expensive premiums actually decreases even though as discussed above the level of withdrawal increases. This is because, while the level of withdrawal actually increases within the strata *ge* and *gE* in all cases, this now represents a smaller percentage of the population, as *ge* is now only 25% of the initial insured population. This suggests that increased numbers of people with the adverse genotype and adverse environmental exposure can be either a greater or lesser threat to the insurance industry depending on the levels of loadings charged by an insurance company.

Table 19:
Population proportions for each stratum.

	e	E
g	0.25	0.25
G	0.25	0.25

Table 20:
Market equilibrium for males with age at policy inception, x = 55, and policy term, t = 15 years. k=1.059349. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 0% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.25	0.25	0.25	0.25
$U_{I(0.5)}(w)$	ψ_s^1	£30,000	£10,000	0	0
	ω_s^2	0.18750	0.25000	0.28125	0.28125
	ψ_s^2	£30,000	£10,000	0	0
	ω_s^1	0.25	0.25	0.25	0.25
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.22857	0.25714	0.25714	0.25741
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.25	0.25	0.25	0.25
$U_{I(0.5)}(w)$	ψ_s^1	£60,000	£40,000	0	0
	ω_s^2	0.11538	0.19231	0.34615	0.34615
	ψ_s^2	£60,000	£50,000	0	0
	ω_s^3	0.12000	0.16000	0.36000	0.36000
	ψ_s^3	£60,000	£50,000	0	0
	ω_s^1	0.25	0.25	0.25	0.25
$U_{\log}(w)$	ψ_s^1	£30,000	£20,000	0	0
	ω_s^2	0.19355	0.22581	0.29032	0.29032
	ψ_s^2	£30,000	£20,000	0	0

Table 21:

Market equilibrium for females with age at policy inception, x = 55, and policy term, t = 15 years. k=1.027629. Individuals in the high-risk stratum purchase insurance at the standard rate.

Utility Function		ge	gE	Ge	GE
Loadings of 0% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.25	0.25	0.25	0.25
$U_{I(0.5)}(w)$	ψ_s^1	£20,000	0	0	0
	ω_s^2	0.20588	0.26471	0.26471	0.26471
	ψ_s^2	£20,000	0	0	0
	ω_s^1	0.25	0.25	0.25	0.25
$U_{\log}(w)$	ψ_s^1	£10,000	0	0	0
	ω_s^2	0.22857	0.25714	0.25714	0.25741
	ψ_s^2	£10,000	0	0	0
Loadings of 10% are included in Calculation of premium:					
	ρ_s	0.7	0.9	1.1	1.3
	ω_s^1	0.25	0.25	0.25	0.25
$U_{I(0.5)}(w)$	ψ_s^1	£50,000	£40,000	0	0
	ω_s^2	0.14815	0.18519	0.33333	0.33333
	ψ_s^2	£50,000	£40,000	0	0
	ω_s^1	0.25	0.25	0.25	0.25
$U_{\log}(w)$	ψ_s^1	£30,000	£20,000	0	0
	ω_s^2	0.19355	0.22581	0.29032	0.29032
	ψ_s^2	£30,000	£20,000	0	0

Table 22:
% of initial market that has withdrawn when equilibrium has been reached for males for the given penetrance, utility function and age. All policy terms are 15 years. Individual in the high-risk stratum purchase insurance at the standard rate. Starting population frequencies are as follows: $\omega_{ge}^{1}=0.25$

$\omega_{gE}^{1}=0.25\ \omega_{Ge}^{1}=0.25\ \omega_{GE}^{1}=0.25$

Males: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	13.89%	35	modest	30.56%
45	modest	13.89%	45	modest	30.56%
55	modest	11.11%	55	modest	30.56%
35	modest-high	22.22%	35	modest-high	36.11%
45	modest-high	22.22%	45	modest-high	36.11%
55	modest-high	16.67%	55	modest-high	36.11%
35	high	33.33%	35	high	41.67%
45	high	27.78%	45	high	41.67%
55	high	22.22%	55	high	41.67%
Males: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	5.56%	35	modest	13.89%
45	modest	5.56%	45	modest	13.89%
55	modest	2.78%	55	modest	13.89%
35	modest-high	11.11%	35	modest-high	19.44%
45	modest-high	8.33%	45	modest-high	19.44%
55	modest-high	8.33%	55	modest-high	19.44%
35	high	13.89%	35	high	22.22%
45	high	13.89%	45	high	25%
55	high	11.11%	55	high	19.44%

Table 23:
% of initial market that has withdrawn when equilibrium has been reached for females for the given penetrance, utility function and age. All policy terms are 15 years. Individual in the high-risk stratum purchase insurance at the standard rate
Starting population frequencies are as follows: $\omega_{ge}^{1}=0.25$

$\omega_{gE}^{1}=0.25\ \omega_{Ge}^{1}=0.25\ \omega_{GE}^{1}=0.25$

Females: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	19.44%
45	modest	2.78%	45	modest	22.22%
55	modest	5.56%	55	modest	25%
35	modest-high	2.78%	35	modest-high	19.44%
45	modest-high	2.78%	45	modest-high	25%
55	modest-high	8.33%	55	modest-high	30.56%
35	high	2.78%	35	high	22.22%
45	high	8.33%	45	high	25%
55	high	13.89%	55	high	30.56%
Females: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	8.33%
45	modest	0%	45	modest	11.11%
55	modest	2.78%	55	modest	13.89%
35	modest-high	0%	35	modest-high	11.11%
45	modest-high	2.78%	45	modest-high	11.11%
55	modest-high	2.78%	55	modest-high	13.89%
35	high	0%	35	high	11.11%
45	high	2.78%	45	high	13.89%
55	high	2.78%	55	high	16.67%

4.1.4 Increased rate of purchase within high-risk stratum

The assumption that individuals who know that they are at high risk of developing a critical illness may purchase larger amounts of insurance than they would have done if they considered themselves to be at low risk is not an unrealistic one. However, with common multifactorial disorders, the increase in risk is not as extreme as some of the rare single gene diseases. Also, it is likely that a high-risk individual would need to have the same level of knowledge, relating to their risk, as an epidemiologist to make the decision to purchase higher levels of insurance. In the model, market withdrawal is measured under the assumption that individuals in stratum *GE* purchase insurance at *(a)* twice the standard rate and *(b)* four times the standard rate. For the reasons described above these are probably quite extreme rates

of purchase and would be unlikely to occur in reality, but are included so that the examination of withdrawal is as complete as possible.

There is an upper limit to the sum assured that people are able to purchase policies with. It is unrealistic to propose that people will purchase insurance for amounts greater than their initial wealth, for this reason we assume that only the individuals who would have initially purchased insurance with sum assured less than or equal to £40,000 increase their rate of purchase. This distorts the assumption described in section 4.1 that policy purchasing is uniform over all values of sum assured within stratum *GE*. The distribution of policy purchasing over all values of sum assured within stratum *GE* under the assumption of *(a)* and *(b)* above, is shown in table 24. We represent the increase in the total amount of sums assured in stratum *GE* by increasing the starting population frequency in this stratum equivalently. Obviously the population frequencies in all of the strata must be adjusted accordingly. The resulting starting population frequencies are shown in the tables of results below:

a) Table 25 shows the percentage of the market who withdraw, for males when individuals in the high risk stratum *GE* purchase insurance at twice the standard rate.
b) Table 26 shows the percentage of the market who withdraw, for males when individuals in the high risk stratum *GE* purchase insurance at four times the standard rate.
c) Table 27 shows the percentage of the market who withdraw, for females when individuals in the high risk stratum *GE* purchase insurance at twice the standard rate.
d) Table 28 shows the percentage of the market who withdraw, for females when individuals in the high risk stratum *GE* purchase insurance at four times the standard rate.

Again, these results are compared to those in table 17 for males and 18 for females, which give the percentage withdrawal for the population frequencies given in table 2 under the assumption that individuals in the high-risk stratum purchase insurance at the standard rate. The level of market withdrawal increases sporadically with no clear pattern for both males and females, when individuals in stratum *GE* purchase insurance at both twice the standard rate, and four times the standard rate. Upon closer

inspection of these results it became apparent that an increase in withdrawal only occurs where the premium charged is just below the maximum premium individuals would be willing to pay when all insurance is purchased at the standard rate. In other words, an increase in withdrawal only occurs where it is already very close to occurring under the standard rate of insurance purchase in stratum *GE*.

Table 24:
The distribution of insurance purchasing over all values of sum assured denoted SA, within stratum GE to represent increased rate of purchase.

(a) Insurance purchase at twice the standard rate.		*(b) Insurance purchase at four times the standard rate.*	
SA	fraction of strata who Purchase	SA	fraction of strata who Purchase
£10,000	0	£10,000	0
£20,000	1/9	£20,000	0
£30,000	0	£30,000	0
£40,000	1/9	£40,000	1/9
£50,000	1/9	£50,000	1/9
£60,000	2/9	£60,000	1/9
£70,000	1/9	£70,000	1/9
£80,000	2/9	£80,000	2/9
£90,000	1/9	£90,000	3/9

Notes: (a) Where four times the sum assured exceeds £90,000, we assume that insurance is purchased for sum assured equal to £90,000. (b) The total sum assured in stratum GE for insurance purchased at twice the standard rate is 22.22% higher then the total sum assured as calculated under the original assumption of uniform distribution of purchase over all values of sum assured. (c) The total sum assured in stratum GE for insurance purchased at four times the standard rate is 44.44% higher then the total sum, assured calculated under the original assumption of uniform distribution of purchase over all values of sum assured.

Table 25:
% of initial market that has withdrawn when equilibrium has been reached for males for the given penetrance, utility function and age. All policy terms are 15 years. Individuals in the high-risk stratum purchase insurance at twice the standard rate. Starting population frequencies are as follows: $\omega_{ge}^{1}=0.80818$, $\omega_{gE}^{1}=0.08979$, $\omega_{Ge}^{1}=0.08979$, $\omega_{GE}^{1}=0.01222$.

Males: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	8.98%	35	modest	36.91%
45	modest	8.98%	45	modest	37.89%
55	modest	8.98%	55	modest	47.86%
35	modest-high	8.98%	35	modest-high	45.88%
45	modest-high	8.98%	45	modest-high	46.87%
55	modest-high	8.98%	55	modest-high	46.79%
35	high	17.96%	35	high	55.85%
45	high	17.96%	45	high	56.84%
55	high	17.96%	55	high	56.84%
Males: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	17.96%
45	modest	0%	45	modest	17.96%
55	modest	0%	55	modest	18.95%
35	modest-high	0%	35	modest-high	17.96%
45	modest-high	0%	45	modest-high	17.96%
55	modest-high	0%	55	modest-high	27.93%
35	high	8.98%	35	high	26.94%
45	high	8.98%	45	high	26.94%
55	high	8.98%	55	high	27.93%

Table 26:
% of initial market that has withdrawn when equilibrium has been reached for males for the given penetrance, utility function and age. All policy terms are 15 years. Individuals in the high-risk stratum purchase insurance at four times the standard rate. Starting population frequencies are as follows: ω_{ge}^{1} *=0.80636,* ω_{gE}^{1} *=0.08959,* ω_{Ge}^{1} *=0.08959,* ω_{GE}^{1} *=0.01444.*

Females: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	8.96%	35	modest	36.83%
45	modest	8.96%	45	modest	37.83%
55	modest	8.96%	55	modest	47.48%
35	modest-high	8.96%	35	modest-high	45.79%
45	modest-high	8.96%	45	modest-high	46.79%
55	modest-high	8.96%	55	modest-high	46.79%
35	high	17.92%	35	high	55.74%
45	high	17.92%	45	high	56.74%
55	high	17.92%	55	high	56.74%
Females: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	17.92%
45	modest	0%	45	modest	17.92%
55	modest	0%	55	modest	18.91%
35	modest-high	8.96%	35	modest-high	17.92%
45	modest-high	8.96%	45	modest-high	17.92%
55	modest-high	0%	55	modest-high	27.87%
35	high	8.96%	35	high	26.88%
45	high	8.96%	45	high	26.88%
55	high	8.96%	55	high	27.93%

Table 27:
% of initial market that has withdrawn when equilibrium has been reached for females for the given penetrance, utility function and age. All policy terms are 15 years. Individuals in the high-risk stratum purchase insurance at twice the standard rate. Starting population frequencies are as follows: $\omega_{ge}^{1}=0.80818$, $\omega_{gE}^{1}=0.08979$, $\omega_{Ge}^{1}=0.08979$, $\omega_{GE}^{1}=0.01222$.

Females: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	29.91%
45	modest	0%	45	modest	38.89%
55	modest	0%	55	modest	38.89%
35	modest-high	0%	35	modest-high	29.91%
45	modest-high	0%	45	modest-high	37.89%
55	modest-high	0%	55	modest-high	37.89%
35	high	0%	35	high	29.91%
45	high	0%	45	high	38.89%
55	high	8.98%	55	high	47.86%
Females: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	9.97%
45	modest	0%	45	modest	18.95%
55	modest	0%	55	modest	18.95%
35	modest-high	0%	35	modest-high	9.97%
45	modest-high	0%	45	modest-high	18.95%
55	modest-high	0%	55	modest-high	18.95%
35	high	0%	35	high	18.95%
45	high	0%	45	high	18.95%
55	high	0%	55	high	18.95%

Table 28:
% of initial market that has withdrawn when equilibrium has been reached for females for the given penetrance, utility function and age. All policy terms are 15 years. Individuals in the high-risk stratum purchase insurance at four times the standard rate. Starting population frequencies are as follows: $\omega_{ge}^{1}=0.80636$, $\omega_{gE}^{1}=0.08959$, $\omega_{Ge}^{1}=0.08959$, $\omega_{GE}^{1}=0.01444$.

Females: Utility function = $U_{I(0.5)}(w)$					
Loadings of 0% added to premium.			Loadings of 10% added to premium.		
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	29.86%
45	modest	0%	45	modest	38.82%
55	modest	0%	55	modest	38.82%
35	modest-high	0%	35	modest-high	29.86%
45	modest-high	0%	45	modest-high	37.83%
55	modest-high	0%	55	modest-high	37.83%
35	high	0%	35	high	29.86%
45	high	0%	45	high	38.82%
55	high	8.98%	55	high	47.78%
Females: Utility function = $U_{\log}(w)$					
x	penetrance	%withdrawal	x	penetrance	%withdrawal
35	modest	0%	35	modest	9.95%
45	modest	0%	45	modest	18.91%
55	modest	0%	55	modest	18.91%
35	modest-high	0%	35	modest-high	9.95%
45	modest-high	0%	45	modest-high	18.91%
55	modest-high	0%	55	modest-high	18.91%
35	high	0%	35	high	18.91%
45	high	0%	45	high	18.91%
55	high	0%	55	high	18.91%

5.1 MODEL LIMITATIONS

While considering the implications of the results found by the model, it is important to be conscious of the limitations within the model. This allows us to be more aware of the range of potential effects differing assumptions may have on the actual results. The limitations are as follows:

a) The model relies heavily on utility theory to predict the behaviour of people within the market. Utility theory requires inferring people's attitude to risk and wealth. However there are often inconsistencies in people's preferences (Luce and Raiffa, 1951, pages 25-29), and different people will most probably have different attitudes to risk and wealth unlike the assumption made in the model that everyone in the population has the same attitude to risk. The model was run for three different levels of risk aversion to try to overcome this somewhat.

b) The assumption that everyone in the market has the same level of wealth is clearly incorrect. We have used figures from 2003 stating that 83% of the population have wealth less then £100,000 to choose wealth equal to £100,000 for all individuals for the purposes of calculating utility in the model. This should most likely be increased to accurately reflect today's population, which would increase the relative risk aversion coefficient and hence decrease market withdrawal. This assumption also implies that everyone in the market has the same coefficient of relative risk aversion.

c) In most cases, adverse environmental exposures such as smoking will be known to the insurer. From the insurer's point of view, this will split the insurance buying population into at least two different groups who will pay different premiums representative of their environmental risk, irrespective of their genetic risk. This would in reality decrease the level of market withdrawal.

d) Adverse environmental factors are likely to contribute to the risk of other critical illnesses not just heart attacks as assumed in the model. For example smoking increases the risk of cancer as well as heart attacks. This is ignored in the model.

e) As mentioned in section 3.2, population frequencies should be defined for a particular age. As individuals in stratum *GE* are more likely to suffer a critical illness then those in *ge*, the percentage of the population in *GE* should

decrease as the population ages. The population frequencies of stratum *ge* and possibly *gE* should increase relatively and the population frequency of *Ge* is most likely to decrease with increasing age. These population frequencies are assumed to be constant at all ages once defined in the model.

f) It is assumed that everyone in the population knows their individual risk and behaves accordingly.

g) The model bases a person's decision to withdraw from the market purely on their risk of having a heart attack. These individuals may be at higher risk for some other critical illness, and therefore a higher premium may not dissuade them from purchasing insurance, regardless of the fact that the premium is high relative to their risk of a heart attack.

h) The level of market withdrawal is quantified based on the assumption that the number of people in each stratum who purchase insurance with sum assured equal to £10,000 is the same as the number of people who purchase insurance for any other sum assured in that stratum, (£20,000, £30,000,….., £90,000). It is difficult to know the accuracy of this assumption.

i) The insurance company will only recognise the increased risk as the number of claims rise, meaning there may be substantial delays before premiums are increased sufficiently. Within this time further factors may have taken place to effect the true level of risk within the population.

5.2 CONCLUSIONS

Market equilibrium was reached in almost every calculation run in the model. Only the iso-elastic utility functions with relative risk aversion coefficients equal to 0.5 and 1 give rise to any market withdrawal. As was discussed in section two, the most realistic value of the relative risk aversion coefficient based on the survey carried out by the Bank of Italy in 1995, is 9. If this coefficient does accurately reflect the current population's attitude towards risk aversion, then the results of the model show that there will be no market withdrawal at any level from the lower risk strata under asymmetric knowledge and that market withdrawal is not a threat to the insurance industry. This includes instances where high-risk individuals purchase insurance up to four times that of the standard rate of purchase and where penetrance

of the adverse genotype and environmental factor is high. These results are not shown as no market withdrawal was witnessed.

Should the population have relative risk aversion coefficient equal to 0.5 or 1, market withdrawal may pose a significant threat to the insurance industry. The most crucial factor effecting withdrawal for all levels of penetrance and population frequencies is the percentage loadings added to the premium by insurance companies to cover expenses and profit. This being said even with no loadings added to the premiums, should the geneticists and epidemiologists discover that the adverse genotype and adverse environmental factor have a high level penetrance, market withdrawal between 17.45% and 22.22% of the initial insurance buying market will occur. This is a significant proportion of the market. The maximum withdrawal occurring for high levels of penetrance and loadings of 10% added to the premium is 57% which is clearly a massive reduction in volume of the CI proportion of the insurance market.

Should geneticists and epidemiologists find that the adverse genotypes and adverse environmental exposure is very common within the population (occurs with probability 0.5) market withdrawal will certainly occur. In both males and females the amount of withdrawal, for the even distribution of population frequencies over all risk strata, increased where it was initially low (0% loadings) and decreased where it was initially high (10% loadings). Therefore the number of people who are exposed to adverse genotype and adverse environmental exposure must be considered in conjunction with the loadings added to the premium to assess the effect on market withdrawal. The level of withdrawal depends more heavily on the loadings added to the premium and on the level of penetrance of the adverse exposure factors.

Typically for males for each age group, population frequency and rate of purchase looked at in the model, there is a maximum increase in withdrawal of approximately 10% between modest levels of penetrance and high levels of penetrance for adverse gene and environmental factors. However in a few instances this was as high as 20%. This suggests that the level of penetrance that is discovered by geneticists and epidemiologists will be important in determining levels of market withdrawal and adverse selection from the insurer’s point of view. This adds weight to the argument made in section 1.3.2, stating the difficulty in legislating now for or against the use of genetic information while the genes and levels of penetrance are undiscovered. In 50% of cases, females show a maximum increase in withdrawal of 10% between

modest and high levels of penetrance for each age group, population frequency and rate of purchase. The other 50% of cases display very little increase in withdrawal between modest and high levels of penetrance. This suggests that levels of penetrance are more important amongst males for quantifying market withdrawal, possibly because heart attacks make up a bigger proportion of CI claims for males than females.

Insurance purchase at twice the standard rate or four times the standard rate by individuals in stratum *GE* has little or no effect on market withdrawal in males or females, and is therefore of no concern to the insurer.

Market withdrawal resulting from asymmetric knowledge relevant to heart attacks, is very much dependant on the loadings charged by the insurance company. If coefficient of relative risk aversion is equal to 0.5 or 1 it is almost certain that there will be some market withdrawal but the effect of this on the insurance industry is very difficult to identify without the relevant epidemiology. This being said if loadings on the premium are between 5% and 10%, the amount of market withdrawal may be very damaging to the CI proportion of the market regardless of the other factors. If the coefficient of relative risk aversion is accurately calculated as 9, then there will be no market withdrawal as a result of asymmetric knowledge relevant to heart attacks. These results are specific to heart attacks, however should further genetic and environmental discoveries be made relevant to cancer, market withdrawal is likely to be greater then the results given here, as cancer is a more common critical illness. The opposite is true for strokes.